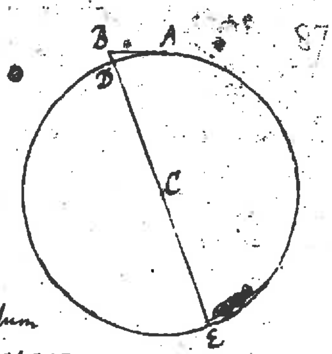


1 Corporis A in circulo AD versus D gyrantis, conatus a centro tantus est quantus in tempore AD (quod pono minutissimum esse) deferret a circumferentia ad distantiam DB: siquidem eam distantiam in eo tempore acquireret si modo conatu non impedito libere moveretur in tangente AB.

Jam cum hic conatus corpora, sic modo in directum ad modum gravitatis continuo urgeret, impelleret per spatia quæ forent ut quadrata temporum: ut noscatur per quantum spatium in tempore unius revolutionis ADEA impellerent, quæro lineam quæ sit ad BD ut est quadratum periferiæ ADEA [illegible] AD². Scilicet est BE . BA :: BA . BD (per 3 elem). Vel cum inter BE [illegible] DE ut et inter BA ac DA differentia supponitur infinite parva, substituo pro se invicem et emergit DE . DA :: DA . DB. ~~[illegible]~~ faciendo denique DA² (sive DE × DB). ADEA² :: DB . $\frac{ADEA^2}{DE}$, obtineo lineam quæsitam (nempe ~~[illegible]~~ tertiam proportionalem in ratione periferiæ ad diametrum) per quam conatus recedendi a centro in directum constanter applicatus propelleret corpus in tempore unius revolutionis.

Verbi gratia cum ista tertia proportionalis æquat 19,7392 semidiametros si conatus recedendi a centro virtute gravitatis tantus esset quantus est conatus in æquatore recedendi a centro propter motum terræ diurnum: in die periodico propelleret grave per $19\frac{3}{4}$ semidiametros terrestres sive per 69087 milliaria, et in hora per 120 mill. Et in minuto primo per $\frac{1}{30}$ mill sive per $\frac{100}{3}$ passus, id est $\frac{500}{3}$ pedes. Et in minuto secundo per $\frac{5}{108}$ ped, sive per $\frac{5}{9}$ digit. Et vero tanta est vis gravitatis ut gravia deorsum pellat 16 pedes circiter in 1", hoc est 350 vicibus longius in eodem tempore quam conatus e centro circiter, adeoque vis gravitatis est toties major, ut ne terra convertendo faciat corpora recedere et in aera prosilire.

2 Coroll. Hinc in diversis circulis conatus a centris sunt ut diametri applicatæ ad quadrata temporum revolutionis, sive ut diametri ductæ in numerum revolutionum factarum in eodem quovis tempore. Sic cum Luna revolvit in 27 dies 7 hors & 43' sive in 27,3216 dies (cujus quadratum est $746\frac{1}{2}$) ac distet 59 vel 60 semidiametris terrestribus a terra. Duco distantiam D 60 in quadratum revolutionis lunaris 1; ac distantiam superficiei terrestris a centro in quadratum revolutionum $746\frac{1}{2}$ [illegible], et sic habeo proportionem 60 ad [illegible]

PLATE V. AN EARLY NEWTON MANUSCRIPT ON GRAVITATION

(FIRST PAGE)

ad $746\frac{1}{2}$, quæ est inter conatum Lunæ et superficiei terrestris recedendi a centro terræ. Itaq conatus superficiei terrestris sub æquatore est $12\frac{1}{2}$ vicibus circiter major quam conatus Lunæ recedendi a centro terræ. Adeoq vis gravitatis est 4000 vicibus major conatu lunæ recedendi a centro terræ, et amplius. Et si conatus ejus a terra efficit ut eandem faciem terram semper respiciat; Hujus lunaris et terrestris systematis conatus recedendi a sole debet esse minor quàm conatus Lunæ recedendi a Terra, aliter luna respiceret solem, potius quam terram.

Sed ut de hac re justiorem æstimationem faciam sit 100000 distantia systematis Lunaris a sole, & y distantia lunæ a terra. Et cum luna conficit 13 revol: 4 sig. 12 gr. 42' in anno stellari, sive 13,369 revolutiones (cujus quadratum est 178,73): duco distantiam solis 100000 in quadratum ejus revolutionis 1, et distantiam Lunæ y in quadratum ejus revolutionum 178,73 et fit 100000 ad 178,73 y, ita conatus terræ a sole ad conatum Lunæ a terra. Unde constat quod distantia lunæ a Terra debet esse major quam $\frac{100000}{178,73}$ sive $559\frac{1}{2}$ respectu distantiæ solis 100000. Et inde Solis maxima parallaxis in orbita lunari non erit minor 19' et solis horizontalis parallaxis in terra non minor 19" puta cum ☉ et ☽ distant 90gr ab Apogæis. Pone vero parallaxin esse 24" et erit distantia lunæ a terra $706\frac{3}{4}$, et conatus ejus recedendi a terra ad conatum terræ recedendi a sole ut 5 ad 4 circiter et sic vis gravitatis erit 5000 vicibus major conatu terræ recedendi a Sole. Sit Magni orbis ½ diam 100000, terræ ½ diam x. Ergo $365\frac{1}{4} \times 365\frac{1}{4} \times x$ (sive 133408x) ita conatus hominis a terra, ad conatum ejus a sole.

Denique in Planetis primarijs cum cubi distantiarum a sole reciproce sunt ut quadrati numeri periodorum in dato tempore: conatus a sole recedendi reciproce erunt ut quadrata distantiarum a sole. Verbi gratia, est in ☿, ♀, ♁, ♂, ♃, ♄ ut $\frac{4}{27}$, $\frac{10}{19}$, 1, $2\frac{5}{16}$, $27\frac{1}{8}$, $90\frac{5}{6}$. sive ut 1, $3\frac{5}{9}$, $6\frac{3}{4}$, $15\frac{2}{3}$, $184\frac{1}{2}$, $614\frac{1}{2}$ reciproce. Vel directe ut 614; 173; 91; 39; $3\frac{1}{3}$; 1.

3 Pendulum gyrans et undulans si sint æque profunda in eodem tempore redeunt.

4 Si pendulum gyrans et undulans sint æque profunda, arcus vibrantis a perpendiculo descriptus a loco initiali ut chorda arcus quem gyrans describit in eodem tempore.

PLATE VI. AN EARLY NEWTON MANUSCRIPT ON GRAVITATION
(SECOND PAGE)

Demonstration.

[2.][2] If $ef = fg = gh = he = 2fa = 2fb = 2gc = 2ed$. And the globe b move from a to b then $2fa : ab :: ab : fa ::$ force or pression of b upon fg at its reflecting : force of b's motion. therefore $4ab = ab + bc + cd + da : fa ::$ force of the reflection in one round (viz: in b, c, d, and a) : force of

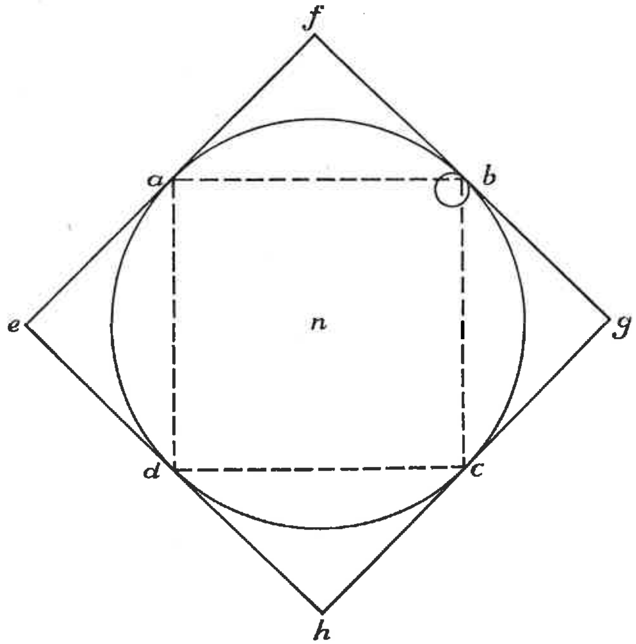


Figure 1.

b's motion. by the same proceeding if the Globe b were reflected by each side of a circumscribed polygon of 6, 8, 12, 100, 1000 sides etc. the force of all the reflections is to the force of the bodys motion as the sume of those sides to the radius of the circle about which they are circumscribed. And so if [the] body were reflected by the sides of an equilaterall circumscribed polygon of an infinite number of sides (i.e. by the circle it selfe) the force of all the reflections are to the force of the bodys motion as all those sides (*id est* the perimiter) to the radius.

[3.] If the body b moved in an Ellipsis[3] that its force in each point (if its motion in that point bee given) [will?] bee found by a tangent circle of Equall crookednesse with that point of the Ellipsis.

[4.] If a body undulate in the circle bd all its undulations of any altitude are performed in the same time with the same radius. Galileus.[4]

[5.] As radius ab to radius ac :: so are the squares of there times in which they undulate.[5]

[6.] If c circulate in the circle $cgef$ [Fig. 2], to whose diameter ce, $ad = ab$ being perpendicular then will the body b undulate in the same time that c circulates.[6]

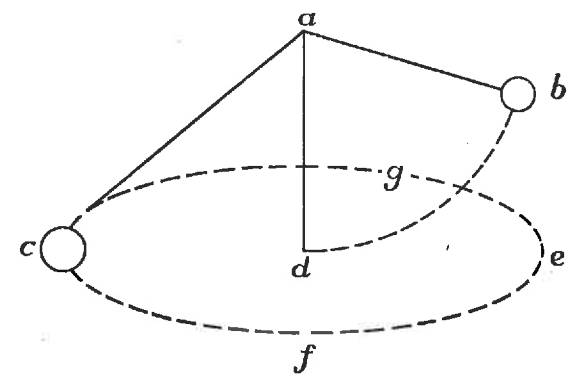


Figure 2.

[7.] And those bodies circulate in the same time whose lines drawne from the center a to the center d are equall.[7]

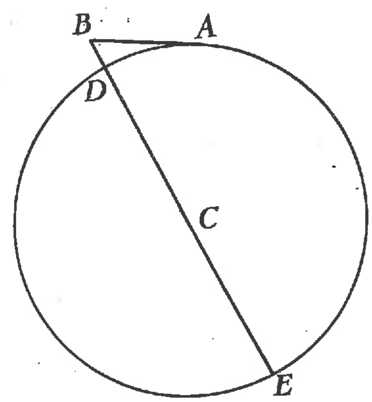
[8.] And $ad : dc ::$ force of gravity to the force of c from its center d.[8]

[9.] Coroll : hence may the force of gravity of the motion of things falling were they not hindered by the aire may very exactly [be] found[9] (viz. [?] $cd : ad ::$ force from d : force from a.

1. For an interpretation of this and the following subsection see above, Part I, Chapter 1.2, pp. 7–11. An equivalent result is derived by an entirely different method in MS. IVa. It seems probable that Newton used this result to derive the peculiar '$\frac{1}{2}R$' formula employed in the calculations of MS. III. See § 2 of the 'Commentary and Interpretation' to that manuscript.

2. This demonstration must have followed Newton's first estimate of the *force of the body's endeavour from the centre in half a revolution* given in Ax.-Prop. 22. Particularly interesting in this connexion is the cancellation of the figure 4+ in Ax.-Prop. 24 and its replacement by 6+ corresponding to the 2π of the present section. For a similar 'polygonal' treatment of circular motion see the demonstration of the law of centrifugal force at the end of the *Scholium* to Prop. 4, Theor. 4, Book, I, *Principia*. Ball ([1], p. 13) suggested that this latter demonstration was the one employed by Newton to calculate 'the force with which a ball revolving within a sphere presses the surface of the sphere' prior to his

Newton's "On Circular Motion"



The endeavour from the centre (*conatus a centro*) "is of such a magnitude that in the time AD (which I set very small) it would carry it away from the circumference to a distance DB."

"Now since the endeavour, provided it were to act in a straight line in the manner of gravity, would impel bodies through distances which are as the squares of the times.

But BD/BA = BA/BE (by Euclid III, 36)

"But since the difference between BE and DE, and also between BA and DA is supposed to be small *infinita*, I substitute one for the other in each case" so that BD/DA=DA/DE

But then in the time in which the body goes through the full circle the endeavour would carry the body a distance equal to BD×(circumference^2/AD^2) = circumference^2/DE.

***Corol.* "Hence the endeavours from the centres of divers circles are as the diameters divided by the squares of the periodic times."**