FOUNDATIONS AND
SOIL MECHANICS

ESTIMATING SETTLEMENTS

OF

STRUCTURES SUPPORTED ON COHESIVE SOILS

BY Charles C. Ladd
Professor of Civil Engineering
Massachusetts Institute of Technology

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I. INTRODUCTION

1.1 TYPES OF SETTLEMENT

When a load of finite dimensions is rapidly applied to a saturated deposit of clay, the resulting settlement can be conveniently divided into three types:

(1) Initial settlement (also called immediate or undrained or shear) denoted by ρ_i.

During application of the load, excess pore pressures will be set up in the clay, but relatively little drainage of water will occur if the clay has a low permeability and a large thickness relative to the rate of loading. The clay will therefore deform at constant volume due to the imposed shear stresses. Under the center line of the load, the vertical compression will be accompanied by a lateral expansion.

(2) Consolidation settlement, denoted by ρ_c

With time, the excess pore pressures dissipate as drainage occurs and the clay undergoes further settlement due to volume changes as stress is transferred from pore pressure u to effective stress $\bar{\sigma}$. The rate of volume change and corresponding settlement is governed by how fast the water can drain out of the clay under the induced hydraulic gradients. This gradual process is called consolidation, or more specifically primary consolidation, and the mathematical model used to describe the process is called consolidation theory.

During consolidation the vertical compression will also be

accompanied by a lateral deformation (either compression or expansion).

(3) Secondary compression, denoted by ρ_s

Even after essentially complete dissipation of the excess pore pressures, so that the effective stresses are practically constant, there will generally be further volume changes and increased settlement. This settlement under seemingly constant effective stress is called secondary compression, or sometimes secondary consolidation, and it is often characterized by a linear relationship between settlement and log time.

The relative importance of the three types of settlement depends on several factors, such as:

(1) The types of clay (sensitivity, amount of organic matter, plasticity, etc.)

(2) The stress history of the clay (normally loaded versus precompressed)

(3) The rate of loading

(4) The magnitude of the load increase

(5) The width of the loaded area in relation to the thickness of the clay deposit.

For a loaded area large in relation to the clay thickness the settlements are close to one-dimensional (no lateral strain) and the initial settlement is practically zero. Secondary compression is often of practical concern with highly organic soils and in cases where primary consolidation occurs rapidly, such as with sand drain installations. Three-dimensional loading of soft clays can cause large initial settlements due to local yielding and contained plastic flow. The initial

settlement of footings on stiff clays is much smaller, but may still be a significant portion of the final total settlement.

1.2 GEOSTATIC STRESSES, K_o, AND STRESS HISTORY

Geostatic stresses are those stresses acting on a horizontal soil deposit due to the weight of the overlying soil. There are no shear stresses on vertical and horizontal planes and the soil underwent only one-dimensional (vertical) compression during deposition. The ratio of the horizontal to vertical consolidation stresses for one-dimensional compression is called the coefficient of lateral stress at rest,* K_o, defined by:

$$K_o = \bar{\sigma}_{hc}/\bar{\sigma}_{vc}$$

If the vertical consolidation stress $\bar{\sigma}_{vc}$ acting on the clay is the largest that has ever existed, the clay is called normally consolidated (N.C.) and loading of a normally consolidated deposit is referred to as virgin compression. If the existing consolidation stress is less than the maximum value that has ever acted, the clay is called overconsolidated (O.C.), preconsolidated, or precompressed. The degree of overconsolidation is expressed by the:

$$\text{Overconsolidation Ratio} = \text{O.C.R.} = \bar{\sigma}_{vm}/\bar{\sigma}_{vc}$$

where $\bar{\sigma}_{vm}$ = maximum past vertical consolidation stress, or more commonly, the maximum past pressure.

The stress history of a clay deposit refers to the existing stresses and the degree of overconsolidation.

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* Also referred to as the coefficient of earth pressure at rest.

For normally consolidated clays, the value of K_o has been empirically correlated to the friction angle of the clay $\overline{\phi}$ by the relationship (Brooker and Ireland, 1965):

$$K_o = 0.95 - \sin \overline{\phi}$$

Since most clays have a friction angle between about 20^o and 30^o, K_o for N.C. clays is typically equal to 0.55 ± 0.10.

K_o increases during one-dimensional unloading and becomes greater than unity for OCR values exceeding about 3.5 ± 1; or in other words, the vertical stress becomes less than the horizontal stress. The variation in K_o with overconsolidation ratio is shown in Fig. 1-1 for a variety of soil types.

For an elastic isotropic material, K_o is a function of Poisson's ratio $\overline{\nu}$,

$$K_o = \frac{\overline{\nu}}{(1 - \overline{\nu})}$$

1.3 ONE-DIMENSIONAL LOADING AND TERZAGHI CONSOLIDATION THEORY

One-dimensional loading of saturated clay causes an initial excess pore pressure Δu_i equal to the increment of applied vertical stress $\Delta \sigma_v$. The stress paths (Lambe, 1964) that occur during undrained loading and during the subsequent consolidation are illustrated in Fig. 1-2 for a normally consolidated and an overconsolidated clay. During loading, the total stress path, TSP, follows oa, while the effective stresses remain at point o. During consolidation, the effective stress path, ESP, proceeds up the K_o line (ob), while the TSP follows ab. At any given time, the value of q must be the same for both the TSP

and the ESP, and thus the excess pore pressure is

$$\Delta u = \text{TSP} - \text{ESP}$$

$$= (p - \bar{p}) \text{ at the appropriate value of } q.$$

Note also that the total horizontal stress decreases during consolidation, but that the total vertical stress remains constant.

The amount of consolidation settlement ρ_c at any time is given by

$$\rho_c = \bar{U}_v \rho_{cf}$$

where

ρ_{cf} = final consolidation settlement

$\bar{U}_v$ = average degree of consolidation for vertical drainage.

The Terzaghi theory of consolidation (see, for example, Lambe and Whitman, 1969) relates $\bar{U}_v$ to a dimensionless time factor T_v by

$$T_v = c_v t/(H_d)^2$$

where

c_v = coefficient of consolidation for vertical drainage (length squared per unit time)

t = time

H_d = height of drainage path.

Figure 1-3 shows the variation with time factor in $\bar{U}_v$ and $\Delta u/\Delta\sigma_v$ at $z/H_d = 0.5$ and 1.0 for one-dimensional consolidation.

1.4 THREE-DIMENSIONAL LOADING

Stress paths illustrating undrained loading and subsequent

consolidation under the center line of a circular load are drawn in Fig. 1-4. The initial excess pore pressure Δu_i is a function of the pore pressure parameters A and B (Skempton, 1954):

$$\Delta u_i = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

$$= \Delta\sigma_h + A(\Delta\sigma_v - \Delta\sigma_h)$$

for B = 1.00 (100% saturation), $\Delta\sigma_3 = \Delta\sigma_h$ and $\Delta\sigma_1 = \Delta\sigma_v$. The ratio $\Delta u_i/\Delta\sigma_v$ will only equal unity for the case where A = 1.00, which may occur if a soft clay is at or near undrained shear failure. Figure 1-4 shows that a relatively small increase in shear stress will bring a normally consolidated clay near the failure envelope. Thus, a soft clay deposit will frequently have a substantial zone that has failed, even though the overall factor of safety is significantly larger than unity.

During consolidation the total stresses in Fig. 1-4 change from point a to point c. The total vertical stress σ_{vf} remains constant, while the horizontal stress decreases. For an elastic medium the change in σ_h is caused by the change in Poisson's ratio from $\nu = 0.5$ to $\nu = \bar{\nu}$ (roughly equal to one-third). The effective stresses change from point b to c, but the exact location of the ESP is indeterminate. In any case, since the path does not follow the K_o line, the strains will not be truly one-dimensional. That is, lateral deformations will accompany the vertical compression.

1.5 ROLE OF THE OEDOMETER TEST AND RECOMMENDED TEST PROCEDURES

The oedometer test is a consolidation test in which the strains and drainage can occur only in the vertical direction since the soil is contained in a rigid solid ring with top and bottom (usually) porous

stones. The load is generally doubled (Load Increment Ratio equal to unity) with readings of vertical deformation versus time obtained during each increment. Information that can be obtained from the test include:

(1) The maximum past pressure $\bar{\sigma}_{vm}$ as determined from empirical procedures;

(2) The compressibility of the soil for one-dimensional loading and unloading (possibly with cycles of unloading and reloading), as defined by the compression curve (vertical strain ϵ_v or void ratio e plotted versus log consolidation stress $\bar{\sigma}_{vc}$).

(3) The coefficient of consolidation c_v from curve fitting procedures, based on the Terzaghi theory of consolidation, applied to the deformation versus time data;

(4) The rate of secondary compression as defined by the slope of the deformation versus log time plot after primary consolidation is completed.

Procedures employed during standard oedometer tests should generally follow the recommendations of Lambe (1951) and ASTM D2435-65T.* Items of particular importance include the following:

(1) Use a diameter to height ratio of at least 2.5, with 3 to 4 preferable, in order to help minimize side friction;

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* However, ASTM D2435 incorrectly defines "initial" and "primary" deformations.

(2) The ring must be non-corrosive and smooth to the touch. A coating of silicone grease on the ring walls is recommended;

(3) Minimum sample dimensions should generally be D = 2 in. and H = 0.5 in.;

(4) Porous stones, such as Norton's P2120, should have a clearance of about 0.01 in. and be replaced or cleaned periodically.

(5) The applied loads must be constant and accurately known (say, within ±1%) and vertical deformations measured to 0.0001 in.;

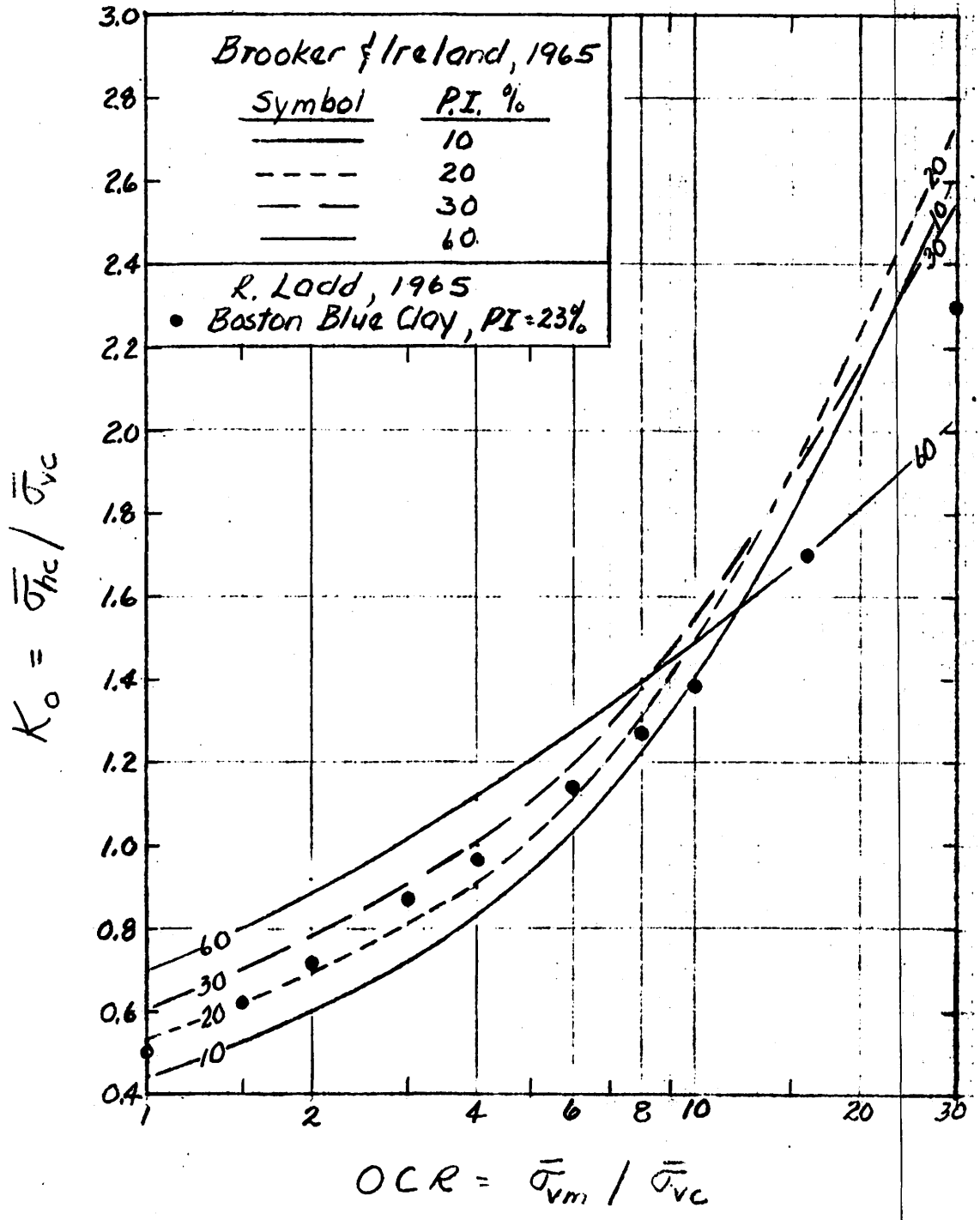
(6) Temperature fluctuations of more than a few degrees (F) will adversely affect secondary compression curves;

(7) Obtain the initial total weight and the final total and dry weights of the entire specimen in order to correctly determine the initial water content, void ratio, and degree of saturation. One should obtain several independent measurements of sample thickness (Z_3 measurements, Lambe, 1951) during the test, and not rely solely on an assumed initial thickness;

(8) An initial "seating" load of about 50 - 100 psf is generally applied to the sample, before adding water, while obtaining an initial dial reading. Then add water and watch the dial reading. If the sample wants

to swell, increase the load in order to prevent swelling. Simply not adding water will not work since the sample may suck water from the porous stones. Or if dry stones are used, the sample may dry out.

Additional comments concerning test procedures and presentation of the data will be made in subsequent sections.



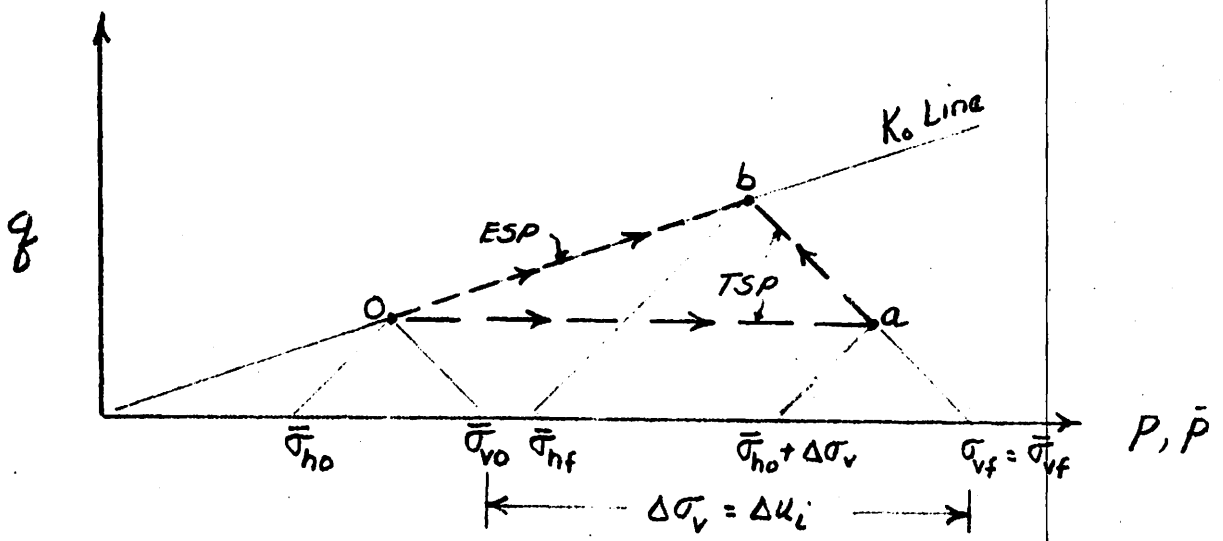
Note: Brooker & Ireland data redrawn from their Figure 11

K_o VERSUS OCR FOR SOILS OF VARYING PLASTICITY

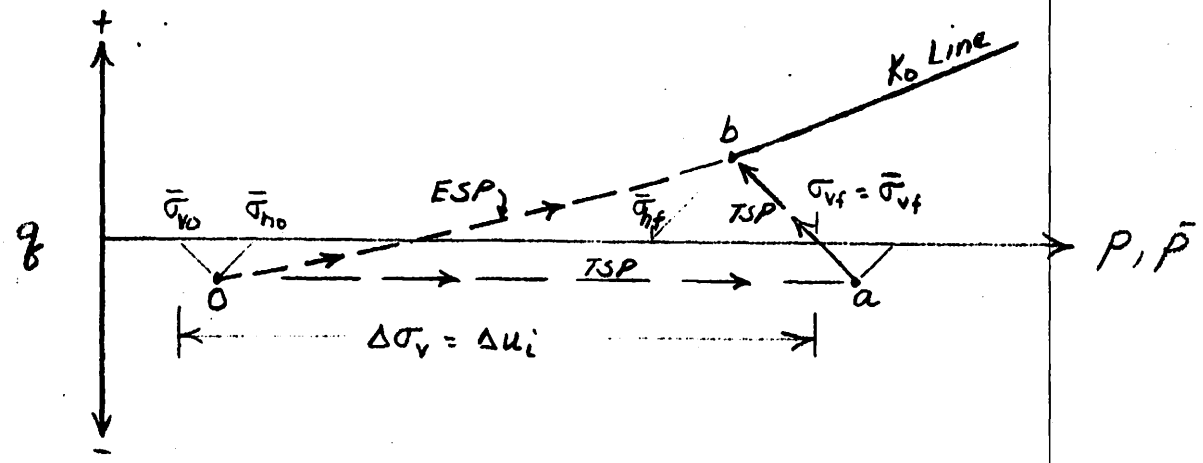
Figure 1-1

$$q = \frac{\sigma_v - \sigma_h}{2} \qquad p = \frac{\sigma_v + \sigma_h}{2} \qquad \bar{p} = \frac{\bar{\sigma}_v + \bar{\sigma}_h}{2}$$

(a) Normally Consolidated Clay, $K_o = 0.5$



(b) Over Consolidated Clay, $K_o = 2.0$



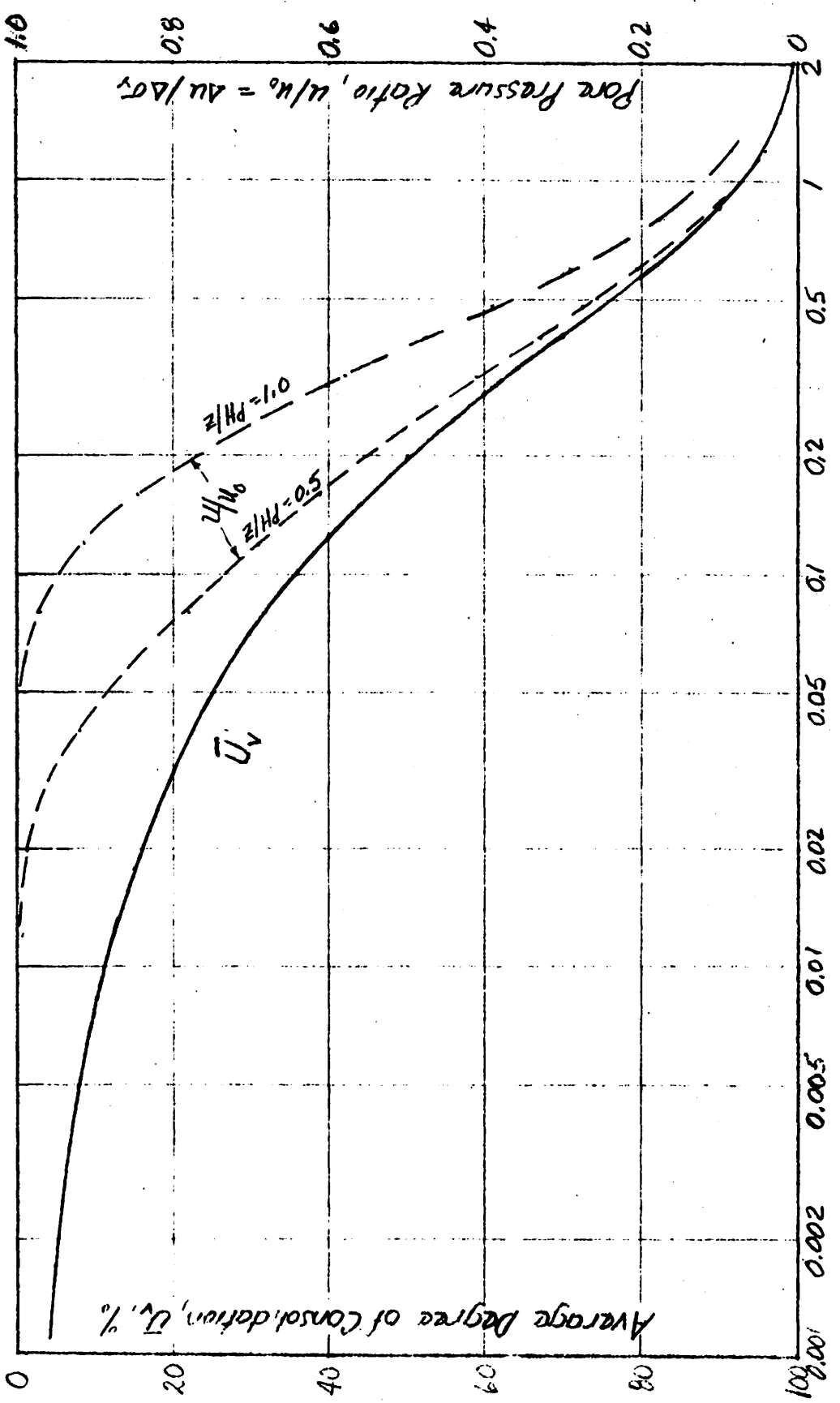
—— —— TSP q vs p

– – – – ESP q vs $\bar{p}$

NOTE: Drawn for $u_o = 0$

STRESS PATHS DURING ONE-DIMENSIONAL CONSOLIDATION

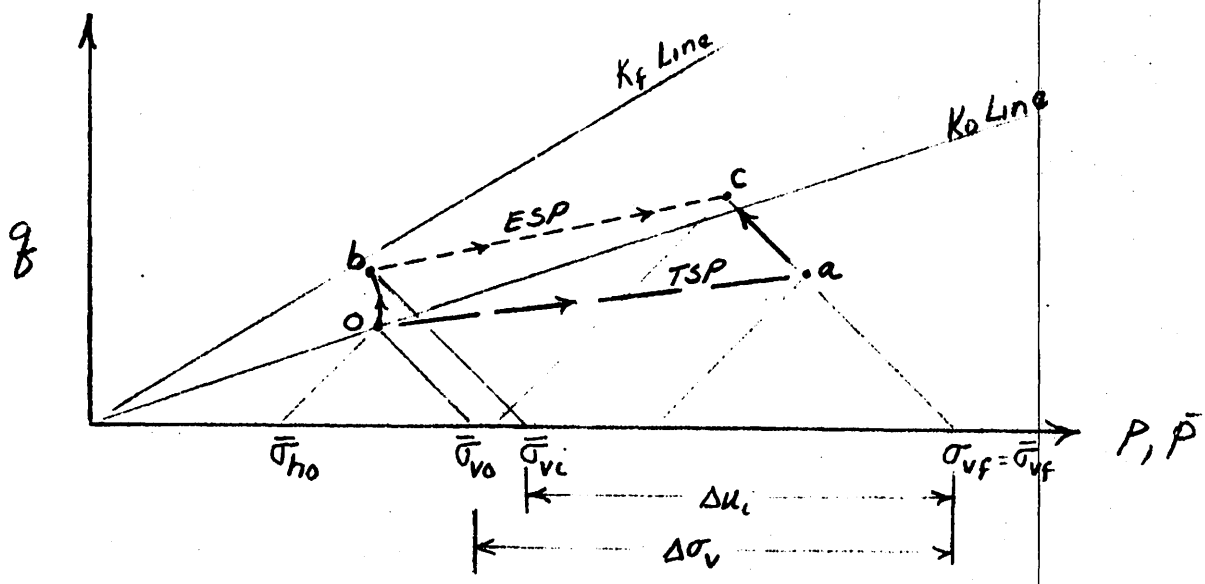
Figure 1-2



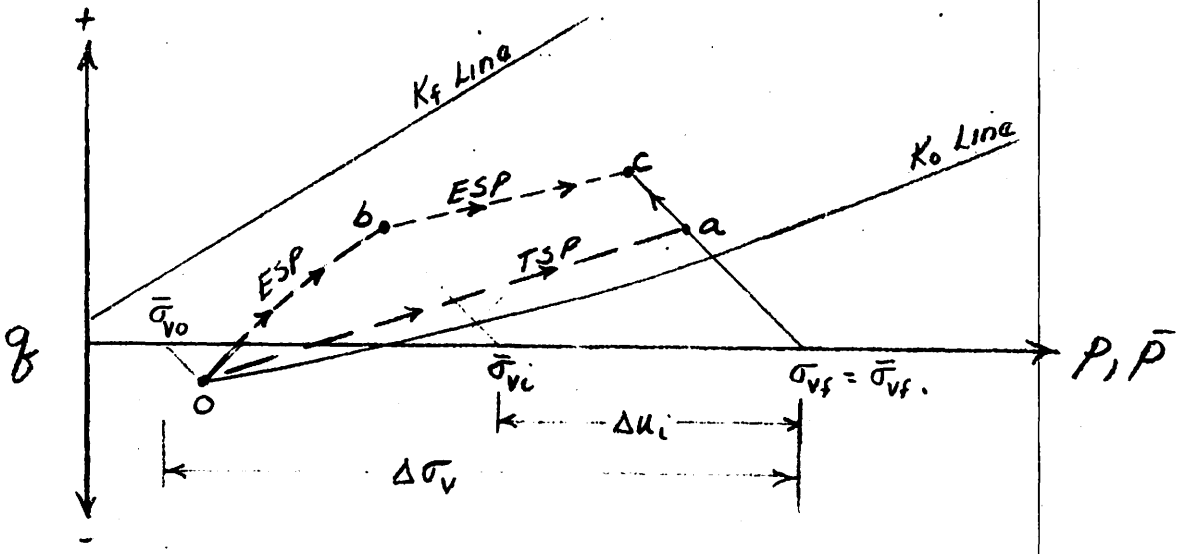
CONSOLIDATION VS TIME FACTOR

Figure 1-3

(a) Normally Consolidated Clay, $K_o = 0.5$



(b) Overconsolidated Clay, $K_o = 2.0$



Undrained Loading

TSP $o \rightarrow a$
ESP $o \rightarrow b$

Consolidation

TSP $a \rightarrow c$
ESP $b \rightarrow c$

(For $u_o = 0$)

STRESS PATHS DURING THREE-DIMENSIONAL LOADING

Figure 1-4

II. ONE-DIMENSIONAL COMPRESSION
(Amount of Consolidation Settlement)

2.1 INTRODUCTION

The results of one-dimensional consolidation (oedometer) tests are generally presented via a graph of void ratio e or vertical strain ϵ_v versus consolidation stress $\bar{\sigma}_{vc}$ plotted to a log scale. This type of plot is used because it exhibits certain characteristic shapes and behavior that have proved useful. Figure 2-1 presents the results of an oedometer test on a plastic deltaic clay and it defines certain parameters that are widely used to characterize compressibility.

When void ratio is used, these parameters are defined in terms of minus Δe per Δlog stress* as follows:

C_c = virgin compression index = slope of compression curve in virgin region

C_r = recompression index = average slope of unloading-reloading cycle

C_s = swelling index = slope of swelling (rebound) curve.

The corresponding parameters in terms of strain per Δlog stress are:

$CR = C_c/(1+e_o)$ = virgin compression ratio

$RR = C_r/(1+e_o)$ = recompression ratio

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* Δ log stress = $\log (\bar{\sigma}_{vc})_2 - \log (\bar{\sigma}_{vc})_1 = \log (\bar{\sigma}_{vc})_2/(\bar{\sigma}_{vc})_1$. For convenience, Δe can be evaluated over one log cycle of stress so that Δ'log stress = log 10 = 1.00.

$SR = C_s/(1+e_o)$ = swelling ratio

The term $(1+e_o)$ enters because:

Strain = $\Delta e/(1+e_o)$

= change in height of sample divided by initial height of sample at start of test.

It should be noted that these parameters are not necessarily unique values for a given test. For example, the swelling index obviously increases with decreasing stress in Fig. 2-1.

Although the void ratio versus log stress plot is more commonly used than the strain versus log stress plot, the latter has several advantages:

(1) Strains are easier to compute than void ratios, which require a knowledge of specific gravity and weight of soil solids;

(2) Two samples may exhibit quite different e-log stress plots, because of differences in initial void ratio, but identical ϵ_v-log stress curves;

(3) Settlements are directly proportional to strains, whereas use of Δe data also requires a knowledge of $(1+e_o)$. Thus the latter introduces two variables, Δe and $(1+e_o)$;

(4) It is easier to standardize strain plots than void ratio plots.

Equations for computation of changes in void ratio or strain from a compression curve based on the foregoing parameters are presented in Fig. 2-2. The actual compression curve is replaced by two straight

lines, one representing recompression up to the maximum past pressure and the second representing virgin compression.

The final consolidation settlement of a clay stratum composed on n layers is given by

$$\rho_{cf} = \sum_{i=1}^{n} H \epsilon_v$$

where

H = initial height of a given layer

ϵ_v = final consolidation strain in that layer.

For the general case consisting of both recompression and virgin compression and replacing ϵ_v by the expression in Fig. 2-2,

Eq. (2-1) $$\rho_{cf} = \Sigma H \left(RR \log \frac{\bar{\sigma}_{vm}}{\bar{\sigma}_{vo}} + CR \log \frac{\bar{\sigma}_{vf}}{\bar{\sigma}_{vm}}\right)$$

where

$\bar{\sigma}_{vo}$ = initial vertical stress

$\bar{\sigma}_{vf}$ = final vertical stress.

If the settlement consists only of recompression, the second term drops out and $\bar{\sigma}_{vm}$ is replaced by $\bar{\sigma}_{vf}$. If the settlement consists only of virgin compression, the first term drops out and $\bar{\sigma}_{vm}$ is replaced by $\bar{\sigma}_{vo}$.

For the case of unloading, the amount of swelling or heave is given by

Eq. (2-2) $$\rho_{cf} = \Sigma H \left(SR \log \frac{\bar{\sigma}_{vf}}{\bar{\sigma}_{vo}}\right)$$

$$= -\Sigma H \left(SR \log \frac{\bar{\sigma}_{vo}}{\bar{\sigma}_{vf}}\right)$$

The method recommended for computing consolidation settlements is based on the use of the parameters RR, CR, etc., rather than the direct use of compression curves, so that one can account for the effects of sample disturbance, secondary compression, and other factors to be discussed.

2.2 MECHANISM OF VOLUME CHANGE AND CHARACTERISTIC BEHAVIOR

The mechanisms occurring during a volume decrease due to loading of a saturated soil can be divided into four categories:

(1) Particle deformations of an elastic nature such as bending of plately shaped clay particles;

(2) Changes in the "closest spacing" of particles at approximately constant orientation:

  (a) Changes in double-layer thickness of widely spaced particles

  (b) Changes in "contact" spacing such as extrusion of "adsorbed" water.

(3) Changes in particle orientation:

  (a) Widely spaced particles assume a more parallel orientation

  (b) Sliding of particles at contacts

(4) Particle deformations of an irreversible nature such as crushing of particles (important in cohesionless soils at high stresses, but thought to be unimportant with ordinary clays).

During unloading the mechanisms controlling swelling are believed to be Items (1) and (2) above. That is, swelling occurs because of

particle unbending, double-layer repulsion, and water adsorption at "contact."

Lambe (1958) and Ladd (1961) present a more detailed discussion of the physico-chemical mechanisms influencing volume changes. Olson and Mesri (1970) discuss the role of physico-chemical versus mechanical effects and show their relative importance as a function of soil type and environment.

Characteristic behavior for one-dimensional compression to be expected with typical clays of moderate sensitivity and plasticity is shown in Fig. 2-3.

(1) The virgin compression ratio is generally constant once the consolidation stress exceeds about 0.5 - 1 TSF (5-10 TSM)*

(2) The shape of an unload-reload cycle is often independent of the maximum past pressure, i.e., $RR_1 = RR_3$;

(3) RR decreases with smaller unload-reload cycles, i.e., RR_2 is less than RR_1;

(4) The slope of the swelling curve increases with increasing rebound, i.e., SR_1 is less than SR_2 in Fig. 2-3. However, SR is independent of the maximum past pressure for a given change in OCR;

(5) The value of SR for one cycle of rebound from $\bar{\sigma}_{vm}$ is often approximately equal to RR for an unload-reload cycle with Δ OCR = 5±;

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* TSM = metric ton per square meter = 0.1 kg/cm^2.

(6) The value of SR for one cycle of rebound from $\bar{\sigma}_{vm}$ is generally equal to 1/5 to 1/10 of the virgin compression ratio CR;

The above behavior is admittedly an oversimplication, but is justified from a practical viewpoint in many cases involving typical clays. The assumption regarding the independence of SR (or C_s) on maximum past pressure is questionable for a large change in pressure. The Navy Manual DM-7 (their Fig. 3-5) and Schmertmann (1955) present methods for estimating the change in C_s with the void ratio at the start of rebound.

There are certain soil types that exhibit behavior markedly different from that shown in Fig. 2-3. Two examples are presented in Fig. 2-4. Very sensitive low plasticity clays and highly plastic clays with a high liquidity index will have a very high compressibility just beyond the maximum past pressure. (the Load Increment Ratio near the $\bar{\sigma}_{vm}$ may have to be reduced to prevent squeezing the clay out of the ring). Moreover, rebound from stresses below the maximum past pressure may be much flatter than from higher stresses. Figure 2-4(b) shows compression data on a highly plastic heavily overconsolidated clay with a very low liquidity index. Such clays exhibit a very high swelling ratio compared to their compression ratio.

2.3 MAXIMUM PAST PRESSURE

2.3.1 Determination

The most widely used method of estimating the in situ maximum past pressure $\bar{\sigma}_{vm}$ is the empirical approach developed by Casagrande (1936), which is illustrated in Fig. 2-1.* In order to obtain meaningful

* The procedure can, of course, be applied to a plot of strain versus log stress.

results, one needs a compression curve with a fairly well defined minimum radius of curvature. This requires a good quality sample to start with. Moreover, the load increment ratio may have to be reduced in the vicinity of $\bar{\sigma}_{vm}$ in order to better define the shape of the curve.

The compression curve that is used to determine the maximum past pressure should be based on the curve corresponding to the end of primary consolidation. This important point will be further discussed in Section IV, where it will be shown that compression curves that include one or more cycles of secondary compression may yield significantly lower values of $\bar{\sigma}_{vm}$.

Other procedures have been suggested for determination of maximum past pressure. The most important one is that of Schmertmann (1955), which is presented in Fig. 2-6 and which will be discussed in the next subsection. Burmister (in ASTM, 1951 and also described by Leonards, 1962, p. 153) proposed a procedure based on characteristic shapes of unload-reload cycles. More recently, Janbu (1969) has suggested that a plot of constrained modulus $D = 1/m_v$ versus $\bar{\sigma}_{vc}$ (natural scale) be used to determine maximum past pressure, since D often undergoes a marked drop in the vicinity of $\bar{\sigma}_{vm}$. This behavior is illustrated in Fig. 2-5, which plots D versus $\bar{\sigma}_{vc}$ for the data previously presented in Fig. 2-1. Jonas (1970), based on experience at TAMS with their controlled gradient consolidation test (Lowe, et al, 1969), suggests that $\bar{\sigma}_{vm}$ equals the midpoint between points a and b in Fig. 2.5. In the writer's opinion, this procedure should not be used in place of the Casagrande and/or Schmertmann methods until considerably more experience is obtained with it.

2.3.2 Meaning of Maximum Past Pressure and Possible Mechanisms

The concept of maximum past pressure is most easily visualized

in terms of the maximum stress that has acted on the clay. At stresses less than this maximum value, the compressibility is less than that in the virgin region and the compression curve exhibits a sharp change in slope as the stresses approach and then exceed $\bar{\sigma}_{vm}$. This same type of behavior (increased compressibility and characteristic shape) can result from mechanisms other than a prior stress increase due to greater overburden.

Table 2-1 lists mechanisms by which a clay might acquire a maximum past pressure greater than the existing vertical effective stress. Changes in total stress are the most obvious cause, followed by changes in the pore pressure conditions. Desiccation is commonly recognized if associated with a clay crust (Parry, 1970, suggests that the location of the minimum shear strength versus depth will occur at the same elevation as the lowest past water table elevation). However, desiccation may also have been an important factor during deposition of many alluvial, backswamp* and tidal "mudflat" deposits.

Clays may also exhibit an increased "maximum past pressure" without having had an increased effective stress. Table 2-1 refers to these mechanisms under the heading of changes in "soil structure" (Lambe, 1958), which denotes soil fabric (orientation and distribution of particles) and interparticle forces (types and relative magnitude). The generation of a maximum past pressure due to secondary compression is now well documented (Leonards and Altschaeffl, 1964; Bjerrum, 1967). Less information is known about the effects of

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* Krinitzsky and Smith (1969) give an excellent description of the depositional environment of backswamp deposits along the lower Mississippi River.

environmental changes and chemical alternations.

Special names, such as quasi-preconsolidation pressure (Leonards and Altschaeffl, 1964) and critical pressure (Bjerrum, 1967), have been used to denote maximum past pressures caused by changes in soil structure. It is likely that the behavior of a clay that is overconsolidated because of an increased past effective stress (Items A and B in Table 2-1) will be different from the same clay overconsolidated because of change in soil structure (Item C in Table 2-1). Likewise, desiccation may have different effects from overburden removal. However, we do not know at present what these differences should be. Also, we often do not know the exact cause of the maximum past pressure. Thus, from a practical viewpoint, one must generally treat all maximum past pressures the same, no matter what their cause.

However, when interpreting the results of oedometer tests and when selecting values of $\bar{\sigma}_{vm}$ to use in design, one should always try to determine the mechanisms involved. This will help decide how the maximum past pressure should vary with depth and whether or not erratic values should be ascribed to testing procedures or natural occurrences.*

2.4 SAMPLE DISTURBANCE AND LABORATORY VERSUS IN SITU COMPRESSION CURVES

Figure 2-6 illustrates the effects of sample disturbance by showing compression curves from oedometer tests on "good" and

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* Soderman and Kim (1970) present an interesting case illustrating the importance of ground water history in selecting the most probable maximum past pressure.

"bad" samples of an overconsolidated clay. Disturbance:

(1) Decreases the void ratio (or increases the strain) at any given value of consolidation stress;

(2) Makes it difficult to define the point of minimum radius, thus obscuring $\bar{\sigma}_{vm}$;

(3) Lowers the estimated value of $\bar{\sigma}_{vm}$ from the Casagrande construction if the disturbance gets excessive,* especially with sensitive clays and clays that have a quasi-preconsolidation pressure;

(4) Increases the compressibility in the recompression region. Terzaghi and Peck (1967), p. 76, state that if the in situ stress increase is small compared to $\bar{\sigma}_{vm}$, the compressibility of even good samples may be 2 to 5 times larger than the in situ compressibility;

(5) Decreases the compressibility in the virgin compression region.

The degree of sample disturbance has been related to the change in effective stress from the theoretical value for "perfect sampling," $\bar{\sigma}_{ps}$, to the actual value of the sample, $\bar{\sigma}_{s}$, (Ladd and

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* Noorany and Poormand (1970) show that samples with a modest amount of disturbance can exhibit the correct $\bar{\sigma}_{vm}$, via the Casagrande method, if the clay is not very sensitive. Leonards and Altschaeffl (1964) show that high quality sampling can indicate the correct quasi-preconsolidation pressure.

Lambe, 1963). The value of $\bar{\sigma}_{ps}$ is given by the equation

$$\bar{\sigma}_{ps} = \bar{\sigma}_{vo} [K_o + A_u(1-K_o)]$$

For normally consolidated clays, $A_u = 0.1 \pm 0.2$, so that $\bar{\sigma}_{ps} = (0.65 \pm 0.2)\ \bar{\sigma}_{vo}$; for heavily overconsolidated clays, $A_u = 0.4 \pm 0.2$ and $\bar{\sigma}_{ps}$ may be larger than $\bar{\sigma}_{vo}$.

A limited amount of data from tube samples of normally consolidated clays suggest that $\bar{\sigma}_s/\bar{\sigma}_{ps}$ typically equals 20±20% indicating a rather substantial amount of disturbance. Tube samples of overconsolidated clays and block samples would be expected to yield higher values of $\bar{\sigma}_s/\bar{\sigma}_{ps}$ and hence less disturbance.

The causes of sample disturbance are, unfortunately, only poorly understood. However, a few comments are worthwhile:

(1) Heavy weight drilling mud is required to prevent squeezing in of clay with deep holes in soft clay. Dry auger holes will seldom yield satisfactory results with soft clays;
(2) Thin wall samples are a necessity;
(3) Larger diameter samples are generally superior;
(4) Block samples have repeatedly been shown to be much superior to the better quality tube samples;
(5) Most of the disturbance probably occurs before the tube is opened and extruded, except possibly for quick clays (Kenney, 1968).*

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* For quick clays, remolding along the edges of the oedometer specimen will lead to lateral expansion due to the increased compressibility of the remolded clay.

Based on observed effects of sample disturbance, Schmertmann (1955) developed a method for reconstructing the in situ compression curve from the results of oedometer tests on reasonably high quality samples. The procedure, shown in Fig. 2-6, consists of the following steps:

(1) Run the oedometer test with a unload-reload cycle after reaching the virgin compression line. The unload-reload cycle should have the same Δ OCR as in situ.

(2) Draw in the recompression index C_r and draw a parallel line through the in situ condition (e_o, $\bar{\sigma}_{vo}$).

(3) Assume a value of $\bar{\sigma}_{vm}$ (start with the Casagrande method);

(4) Draw a line through the intersection of C_r and $\bar{\sigma}_{vm}$ (Point 1) and the intersection of the laboratory C_c and $0.42\ e_o$ (Point 2);

(5) Sketch in the reconstructed in situ curve;

(6) Plot the value of Δe (difference between assumed in situ curve and the laboratory curve) versus $\log \bar{\sigma}_{vc}$, as shown at the bottom of Fig. 2-6;

(7) The assumed value of $\bar{\sigma}_{vm}$ that gives a plot of Δe that is most symmetrical about $\bar{\sigma}_{vm}$ is the "correct" value. (The assumed value of $\bar{\sigma}_{vm}$ in Fig. 2-6 is somewhat too low.)

(8) The in situ virgin compression index is equal to

the line connecting the correct $\bar{\sigma}_{vm}$ (Point 1) and $0.42\ e_o$ (Point 2).

The value of $0.42\ e_o$ is based on experimental evidence that the virgin compression index from samples with varying degrees of disturbance will intersect at a stress at which the void ratio is approximately equal to 42% of the initial void ratio. The actual percentage of e_o might easily vary by ±5% and could be checked by tests on samples of the clay in question.

The procedure outlined in Fig. 2-6 can just as easily be applied to a plot of <u>strain</u> versus log stress, except that the point of intersection is now given by

$$\text{Strain at intersection} = \frac{e_o - 0.42\, e_o}{(1+e_o)} = \frac{0.58\, e_o}{(1+e_o)}$$

Table 2-2 presents data comparing the "corrected" in situ compression curve with the measured laboratory curve for several clays. The Schmertmann correction typically increases CR (or C_c) by 15 ± 5% for moderately good samples of medium to soft clay.

2.5 EFFECT OF TEST PROCEDURES

2.5.1 <u>Side Friction</u>

Side friction will lower the actual effective $\bar{\sigma}_{vc}$ acting on the sample during loading, while the reverse occurs during unloading. For loading in the virgin region with smooth oedometers having D/H = 3 to 4, the average $\bar{\sigma}_{vc}$ acting in the clay probably equals at least 95% of the applied stress (Taylor, 1942; Leonards and Girault, 1961). This effect is of little practical concern. During rebound, the effects of friction will be more pronounced because

K_o becomes much larger. Thus the slope of the swelling curve may be reduced at very large values of OCR. However, one seldom is interested in such data.

2.5.2 Incremental Versus Constant Rate of Strain Loading

Determination of the in situ maximum past pressure from a compression curve requires a well defined curve in the vicinity of $\bar{\sigma}_{vm}$. For most soft clays, standard oedometer tests with a doubling of the load (Load Increment Ratio of unity) are likely to prove unsatisfactory unless the loading is reduced near $\bar{\sigma}_{vm}$. A reduced Load Increment Ratio requires more time, unless the load duration is also reduced. A smaller Load Increment Ratio is also likely to make it difficult to determine the end of primary consolidation and to obtain a value of c_v.

Tests in which the load is constantly changing and with measured pore pressures have recently been developed (Lowe et al, 1969; Smith and Wahls, 1969; Wissa et al, 1971). Such tests have several advantages:

(1) A continuous compression curve corresponding to the end of primary consolidation is easily obtained;

(2) The time required to perform a complete test, even with unload-reload cycles, is greatly reduced (few days versus several weeks);

(3) One can obtain continuous values of c_v;

(4) No loss of soil due to squeezing near the maximum past pressure with soft clays.

Disadvantages are:

(1) The equipment is more complicated* and expensive, and the test is more difficult to set up.

* A very versatile general purpose oedometer developed at MIT is shown in Fig. 2-7. It can perform incremental and constant rate of loading or strain tests.

(2) One does not obtain data concerning the rate of secondary compression, unless the test is converted to a constant load;

(3) Continuous readings generally require a data acquisition system.

A simpler version of the above tests consists of simply loading an oedometer test at a constant rate of strain that is slow enough to preclude a significant development of excess pore pressure, such as performed by Crawford (1964). However, it should not be so slow as to include a significant amount of secondary compression.

The effects of rate of loading on compression curves are illustrated in Fig. 2-8. The curves could have been obtained from constant rate of strain tests at varying slow rates (no excess pore pressures for lines below No. 1) or from an incremental test where the strains correspond to different times after loading. The figure illustrates that the values of $\bar{\sigma}_{vm}$ determined from the curves decreases as the time allowed for compression increases. The effect on $\bar{\sigma}_{vm}$ can be very large for some clays, such as the Leda clay, where Crawford (1964) obtained the following data from incremental tests:

Time After Loading	Casagrande $\bar{\sigma}_{vm}$ (kg/cm^2)
End of primary (t_p = 40 - 55 min)	2.65
One day	1.85
One week	1.33

The effect will obviously be largest with those clays that exhibit a large increasing rate of secondary compression as the stresses

exceed the maximum past pressure. For typical soft CL and CH clays, compression curves based on the end of primary consolidation will yield values of $\bar{\sigma}_{vm}$ that are 10% to 20% larger than "one day" curves. As a general rule, any compression curve for a soft clay that contains at least one cycle of secondary compression may significantly underestimate $\bar{\sigma}_{vm}$.

2.5.3 Environmental Factors

There are some data (Casagrande, 1936; Gray, 1936; Ladd and Luscher, 1965) to show that an increased temperature throughout an oedometer test will yield a lower value of $\bar{\sigma}_{vm}$. Two tests on Boston Blue clay showed a 15 $\pm$ 5% decrease in $\bar{\sigma}_{vm}$ for a 10°C increase in temperature. Since laboratory temperatures might easily be 10°C higher than ground temperature, the error in $\bar{\sigma}_{vm}$ can be significant. However, there are insufficient data concerning temperature to draw general conclusions.

The compressibility of some clays can be affected by changes in pore fluid salt concentration. This may be especially true with illitic marine clays of medium to low plasticity. If clay with a sea water salt concentration (35 g/ℓ) is surrounded by fresh water, a gradual reduction in the pore fluid salt will cause an increased compressibility and a lower $\bar{\sigma}_{vm}$ (see, for example, Leonards and Altschaeffl, 1964). Bjerrum (1967) discusses some of the other physico-chemical effects that might occur due to changes in the nature of the pore fluid.

2.6 EMPIRICAL CORRELATIONS (Also see Johnson, 1970)

2.6.1 Virgin Compression

Several empirical correlations have been suggested for relating C_c or CR to simple index properties. Some of the more widely used

relationships are given below:

Terzaghi and Peck (1967):

$$C_c = 0.009\ (w_L - 10\%)$$

Nishida as quoted in MPMR (1958):

$$C_c = 0.54\ (e_o - 0.35)$$

MPMR (1958) for organic soils, including peats:

$$C_c = 0.010 \text{ to } 0.015\ (w_N,\ \%)$$

Kapp, et al (1966) show for marsh deposits in the New York area:

$$C_c = 0.6\ (e_o - 1) \text{ for } e_o \text{ less than } 6$$
$$C_c = 0.85\ (e_o - 2) \text{ for } e_o = 6 - 14$$

MPMR (1958) show the relationship among C_c, e_o and w_L. This relationship is converted to w_N versus CR for a liquidity index of 1.0 and 0.5 in Fig. 2-9. The Nishida relationship is also plotted in the figure.

Relationships such as those presented in Fig. 2-9 are useful in obtaining a preliminary estimate of compression ratio and in checking data versus what others have found for typical clays. However, one should expect marked deviations with unusual soils. For example, highly sensitive illitic CL clays may have a CR much larger than indicated by Fig. 2-9.

Empirical relationships are very useful in summarizing data that have been gathered for a given soil deposit or certain soil type, such as shown in Fig. 2-10 for varved clays from the Connecticut Valley and glacial Lake Hackensack, New Jersey.

2.6.2 Recompression and Swelling

The writer has found for a wide variety of clays that the

recompression ratio from an unload-reload cycle (say, Δ OCR = 5 - 10) can be estimated from

RR = SR for one cycle of rebound from $\bar{\sigma}_{vm}$

= (0.1 to 0.2) CR

Navdocks DM-7 (1961) present a relationship among C_s, e at rebound, and w_L.

2.7 COMMENTS CONCERNING RECOMMENDED PRACTICE

The greatest uncertainty in an estimate of the amount of final consolidation settlement generally lies in a proper evaluation of the stress history of the soil deposit. This is particularly true with "soft" clays that are slightly overconsolidated. The estimated amount of settlement is often very sensitive to the value of $\bar{\sigma}_{vm}$ that is selected, especially for buildings where the net increase in stress is likely to be rather modest. It is also very difficult to obtain a reliable estimate of $\bar{\sigma}_{vm}$ with soft clays. In almost all cases, the error in $\bar{\sigma}_{vm}$ will be on the low side, mainly because of the effects of sample disturbance. Geologic considerations can sometimes be very useful in ascertaining whether the selected value of $\bar{\sigma}_{vm}$ should be based on an average of measured values or on the maximum values.

As was illustrated in Fig. 2-8, increasing amounts of secondary compression will lower the compression curve and the estimated value of maximum past pressure. It is recommended that the compression curve corresponding to the end of primary consolidation be used to determine the maximum past pressure. This can be accomplished in three ways:

(1) By running a controlled gradient or constant rate of strain (CRSC) consolidation test;

(2) By performing an incremental test in which new loads are applied at the end of primary consolidation, i.e. before a significant amount of secondary compression occurs;

(3) Correction of standard incremental test data to obtain the end of primary compression curve, as demonstrated in Fig. 2-11. In essence, the dial readings d_{100}, rather than d_f, are used to compute strain or void ratio.

Compution of settlements are generally best made with equations such as 2-1 and 2-2, which require a knowledge of the compressibility parameters CR, RR and SR. In selecting these parameters, one should first plot measured (or "corrected" values based on Schmertmann) values from individual tests versus depth. This is illustrated in Fig. 2-12. Then one can see whether or not there is a consistent variation with depth. In the illustrative example, test data denoted by the squares obviously suffer from excessive sample disturbance. Very often values of CR from both overconsolidated and normally consolidated samples will be approximately the same. After looking at all of the data, one can then select values that appear to best represent a given clay deposit. This is generally preferable to selecting values for a given layer based only on the tests performed within that layer. However, one should recognize:

(1) That the compressibility parameters may indeed vary with depth if the soil varies;

(2) That the compressibility parameters may vary with stress level, which will change with depth.

TABLE 2-1

MECHANISMS CAUSING A MAXIMUM PAST PRESSURE
(CRITICAL PRESSURE)

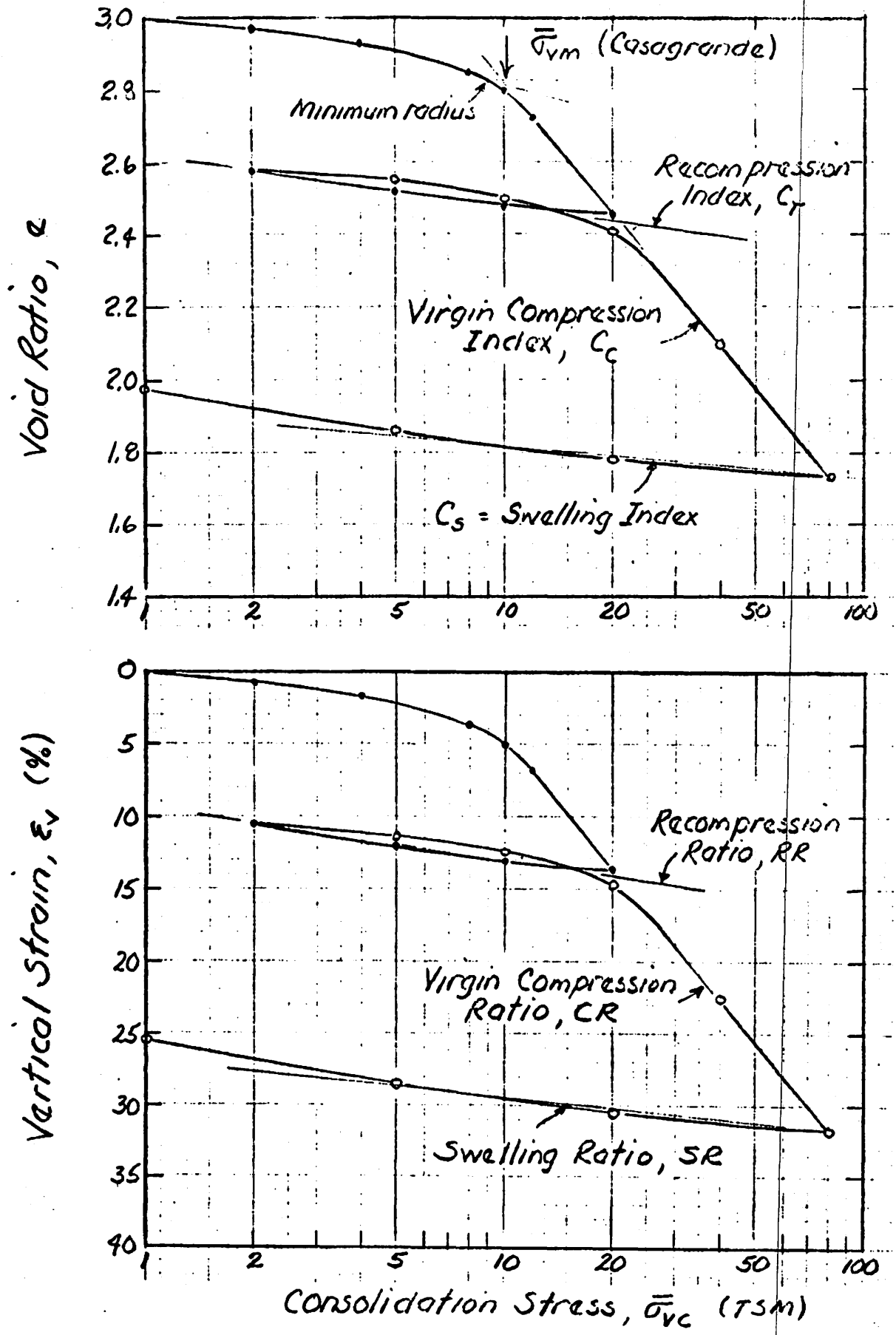
Item	Remarks/References
A. Change in Total Stress Due to:	
1. Removal of overburden	
2. Past structures	
3. Glaciation	
B. Change in Pore Water Pressure Due to:	
1. Change in water table elevation	See Kenney (1964) for sea-level changes
2. Artesian pressures	Common in glaciated areas
3. Deep pumping	Common in many cities
4. Desiccation due to drying	May have occurred during deposition
5. Desiccation due to plant life	May have occurred during deposition
C. Changes in Soil Structure Due to:	
1. Secondary compression (aging)	Importance recently recognized*
2. Changes in environment, such as pH, temperature, salt concentration, etc.	Lambe (1958) Bjerrum (1967) Leonards and Altschaeffl (1964)
3. Chemical alternations due to: "weathering," precipitation of cementing agents, ion exchange, etc.	Bjerrum (1967), Cox (1968)

* The magnitude of $\bar{\sigma}_{vm}/\bar{\sigma}_{vo}$ due to secondary compression for mature natural deposits of highly plastic clays may reach values as high as 1.6 ± 0.3. See Table 4-1.

CORRECTED VS MEASURED VIRGIN COMPRESSION RATIO
BASED ON SCHMERTMANN (1955) METHOD

Name	Description	Depth	Corrected CR	Corrected CR / Measured CR	Reference
Boston Blue Clay	Medium marine CL Clay of moderate sensitivity (overconsolidated)	45 ft. to 75 ft.	0.25 ±.05	1.16±.04	M.I.T.
Kawasaki Clay, Japan	Medium highly plastic slightly organic MH clay with shells & diatoms (normally consolidated)	20 m 35 m	0.27±.02 0.40±.05	1.12±.08 1.21±.03	M.I.T.
Lagunillas Clay, Venezuela	Soft highly plastic CH clay (normally consolidated)	6 m	0.34 ±.03	1.16±.08	M.I.T.
St. Clair Clay Till	Medium to stiff CL clay with appreciable sand and some gravel (overconsolidated)	6 ft. 38 ft.	0.12 0.16	1.12 Block 1.18 Tube 1.34 Tube 1.7 Remolded	Soderman & Kim (1970)

Table 2-2



COMPRESSION CURVES AND COMPRESSIBILITY PARAME

Figure 2-1

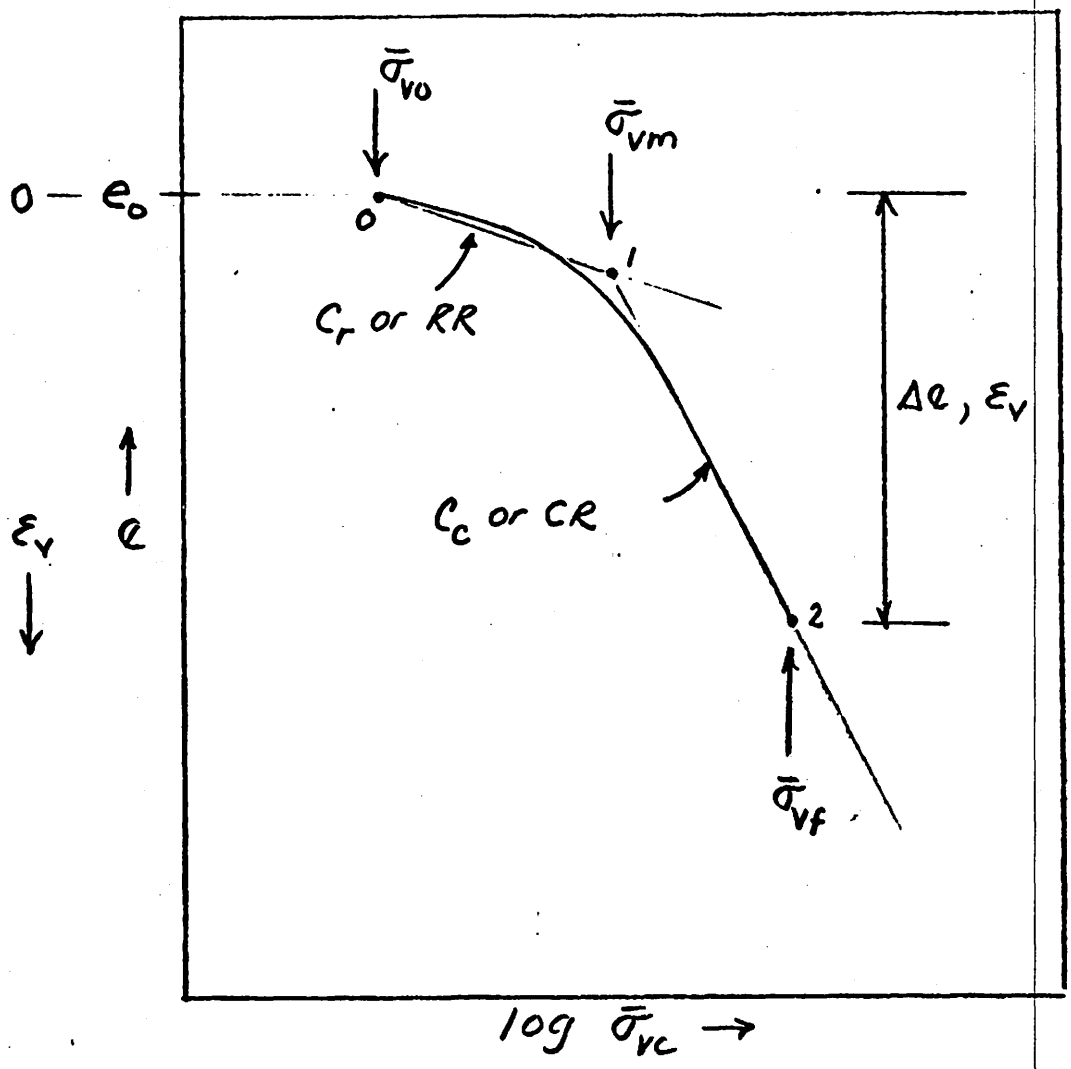


Figure 2-2

At at given depth (e_o, $\bar{\sigma}_{vo}$)

$$\varepsilon_v = \frac{\Delta e}{(1+e_o)} = \frac{a_v \Delta \bar{\sigma}_{vc}}{(1+e_o)} = m_v \Delta \bar{\sigma}_{vc}$$

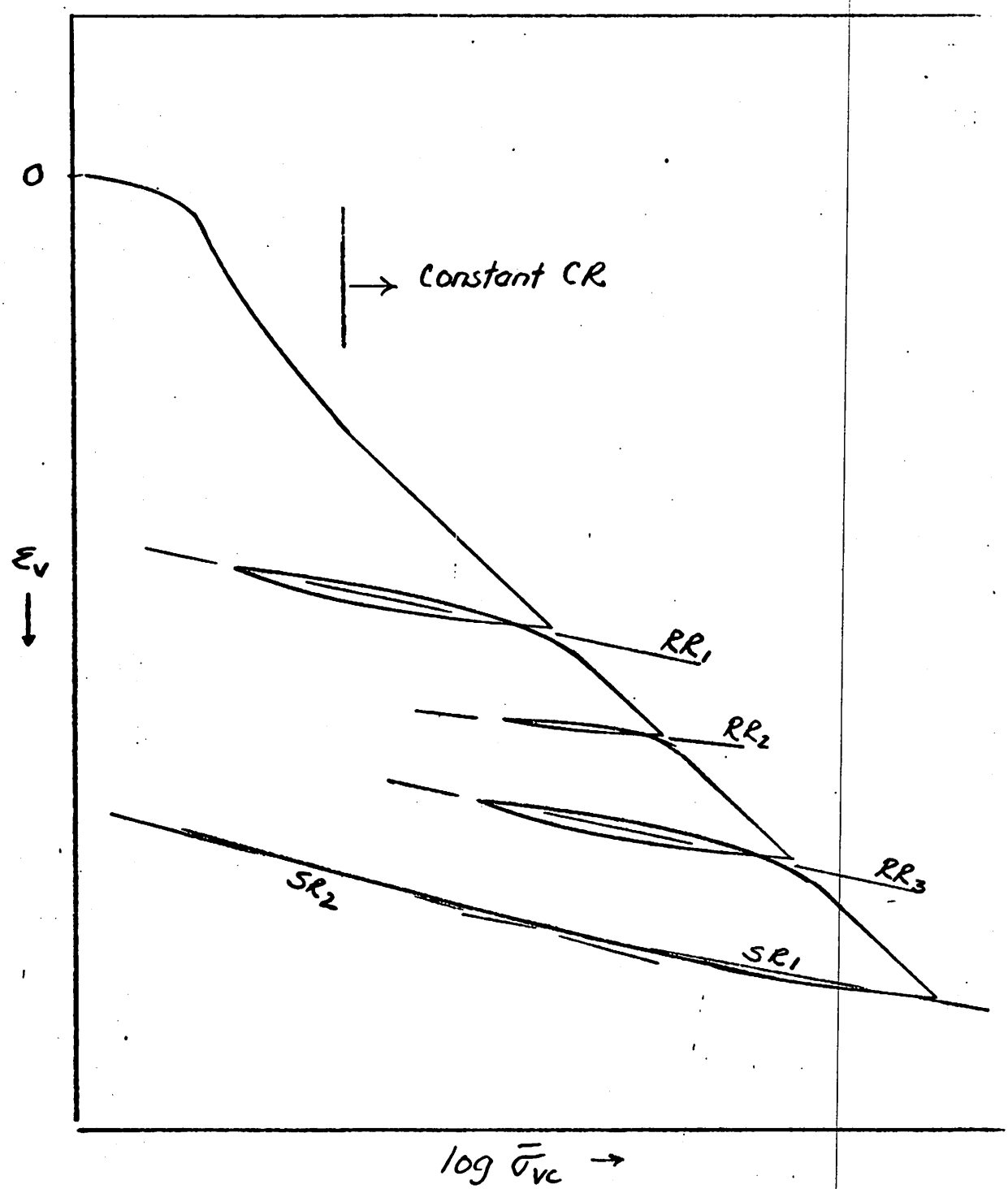
a_v = coef. of compressibility

m_v = coef. of volume change

$$\Delta e = C_r \log \frac{\bar{\sigma}_{vm}}{\bar{\sigma}_{vo}} + C_c \log \frac{\bar{\sigma}_{vf}}{\bar{\sigma}_{vm}}$$

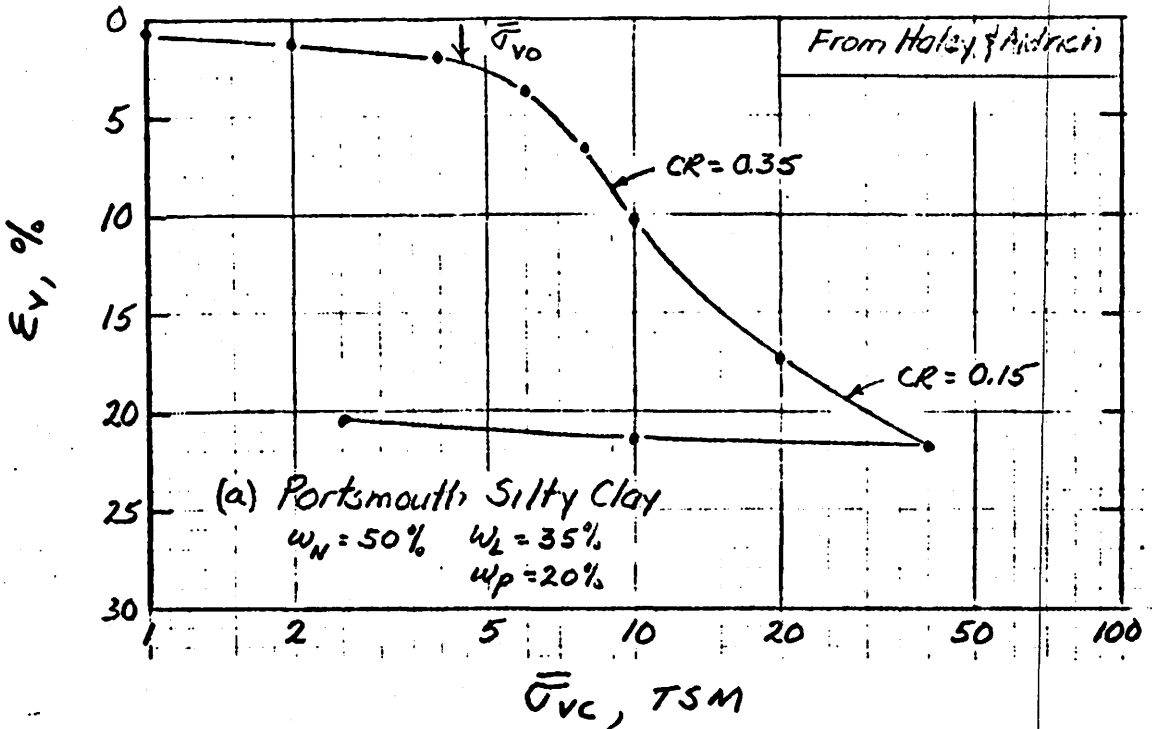
$$\varepsilon_v = RR \log \frac{\bar{\sigma}_{vm}}{\bar{\sigma}_{vo}} + CR \log \frac{\bar{\sigma}_{vf}}{\bar{\sigma}_{vm}}$$

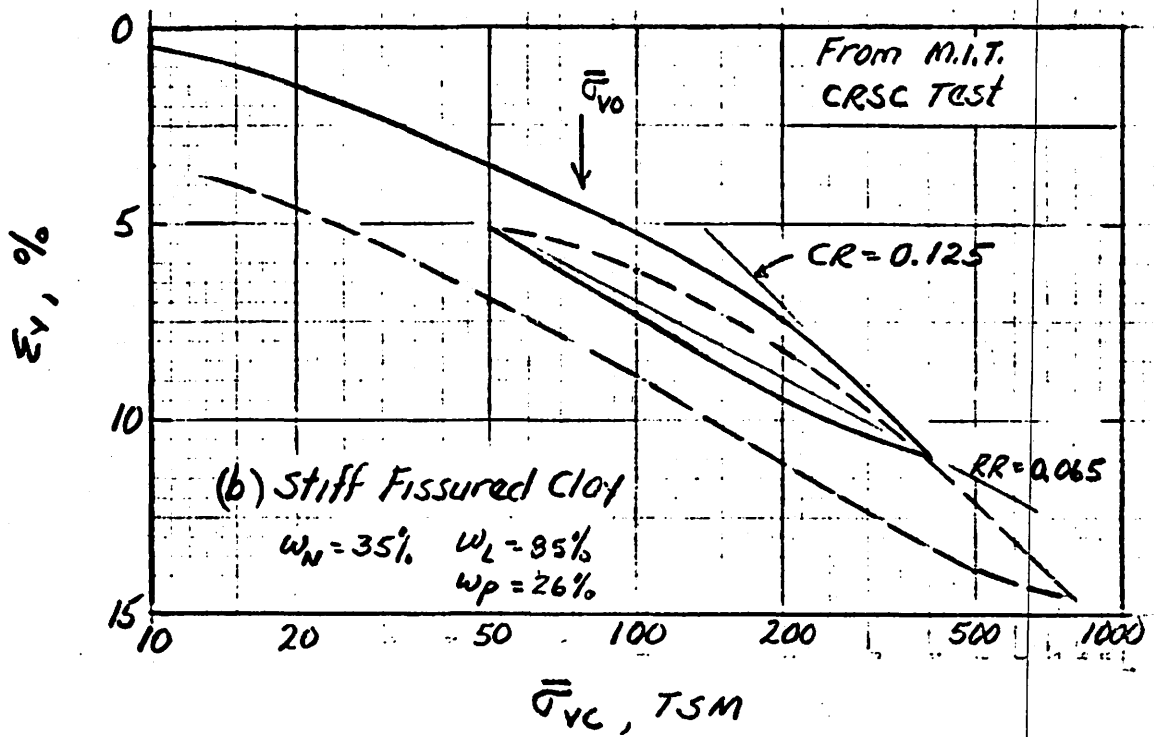
EQUATIONS



CHARACTERISTIC BEHAVIOR OF TYPICAL CLAY OF MODERATE SENSITIVITY AND PLASTICITY

Figure 2-3





COMPRESSION CURVES FOR:

(a) VERY SENSITIVE MARINE CLAY

(b) STIFF FISSURED PLASTIC CLAY

Figure 2-4

CCL 1/26/71

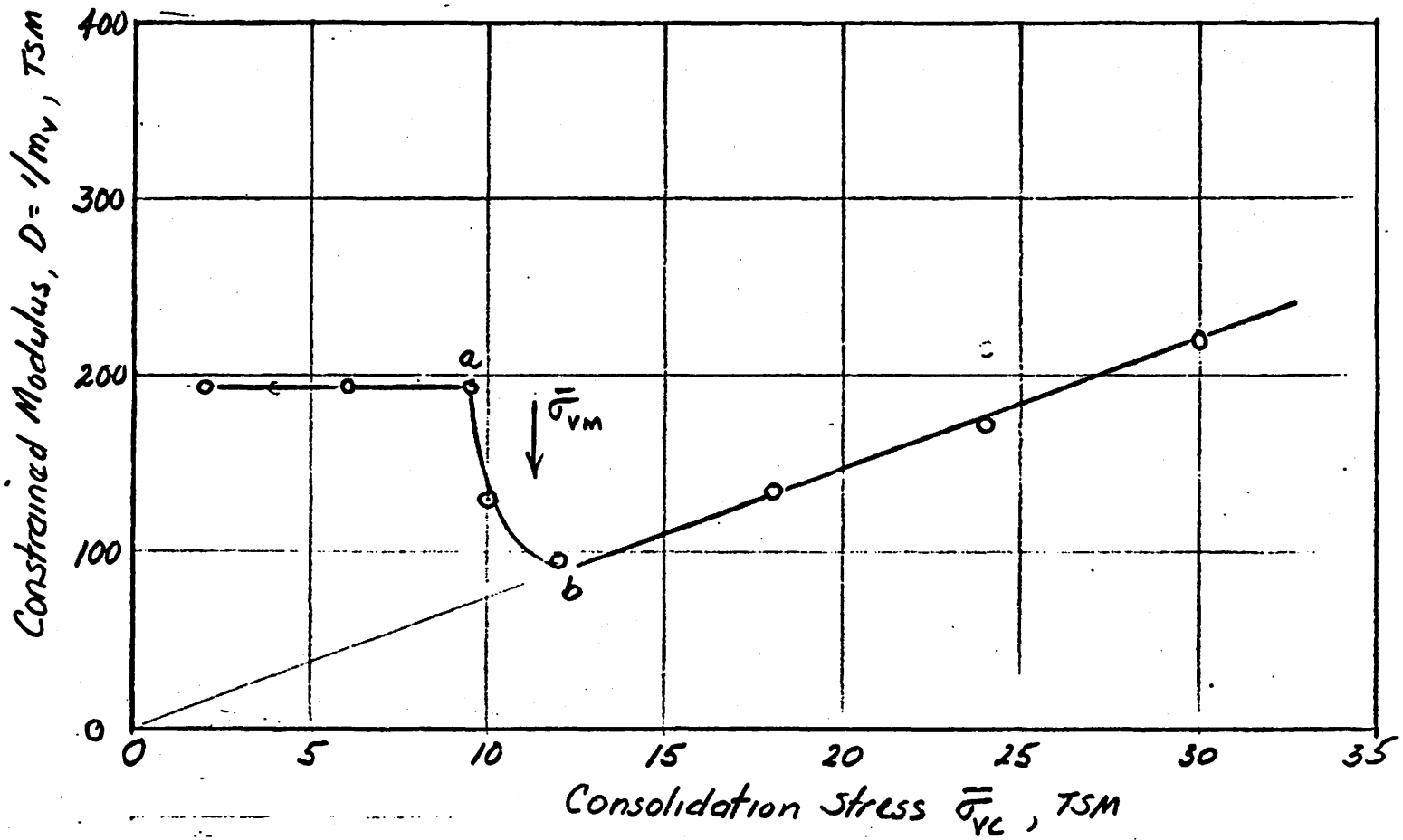


Figure 2-5

CONSTRAINED MODULUS VS CONSOLIDATION STRESS FOR COMPRESSION CURVE, FIG. 2-1

Reconstruction of In situ Compression Curve using Schmertmann's Method

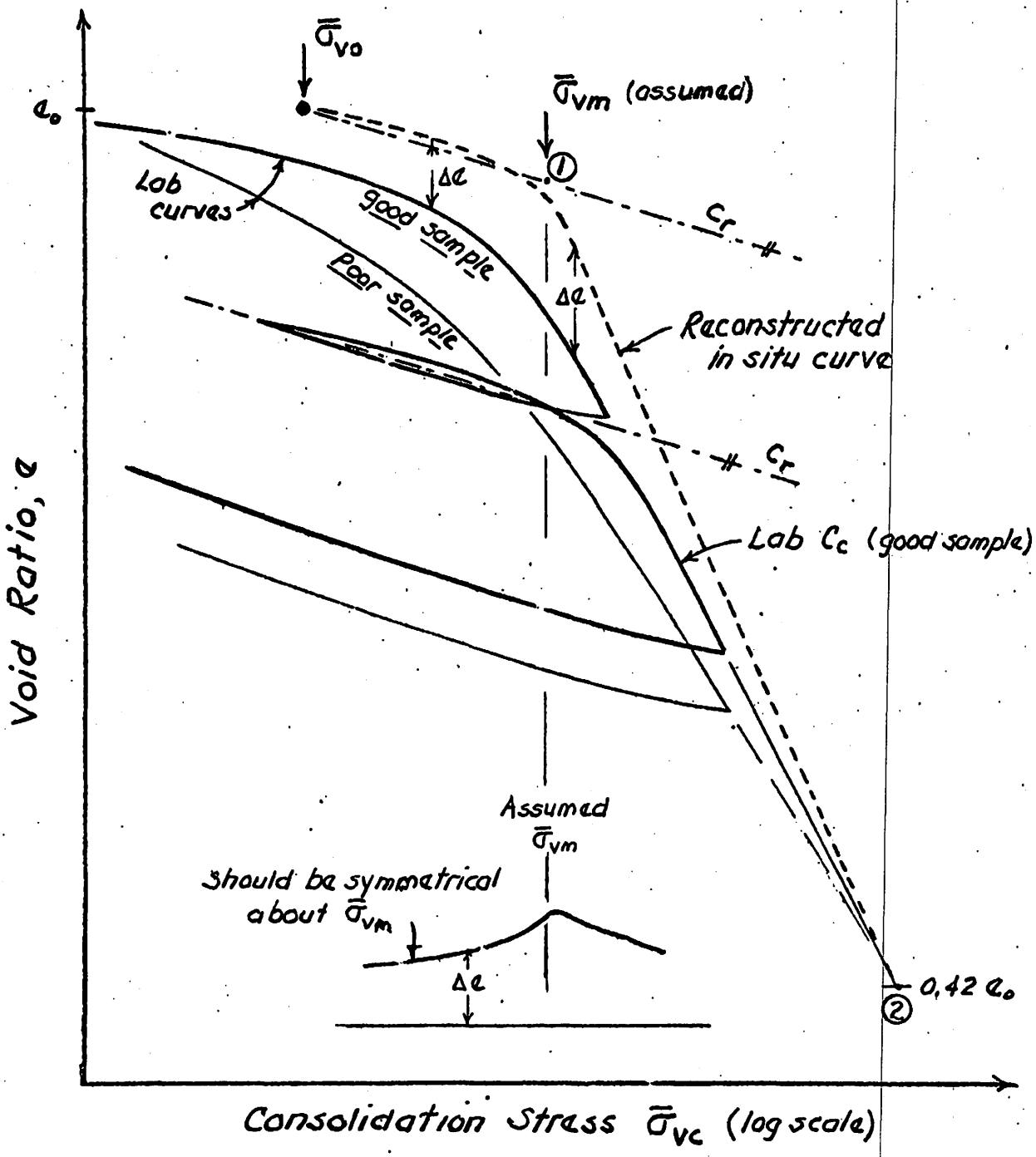
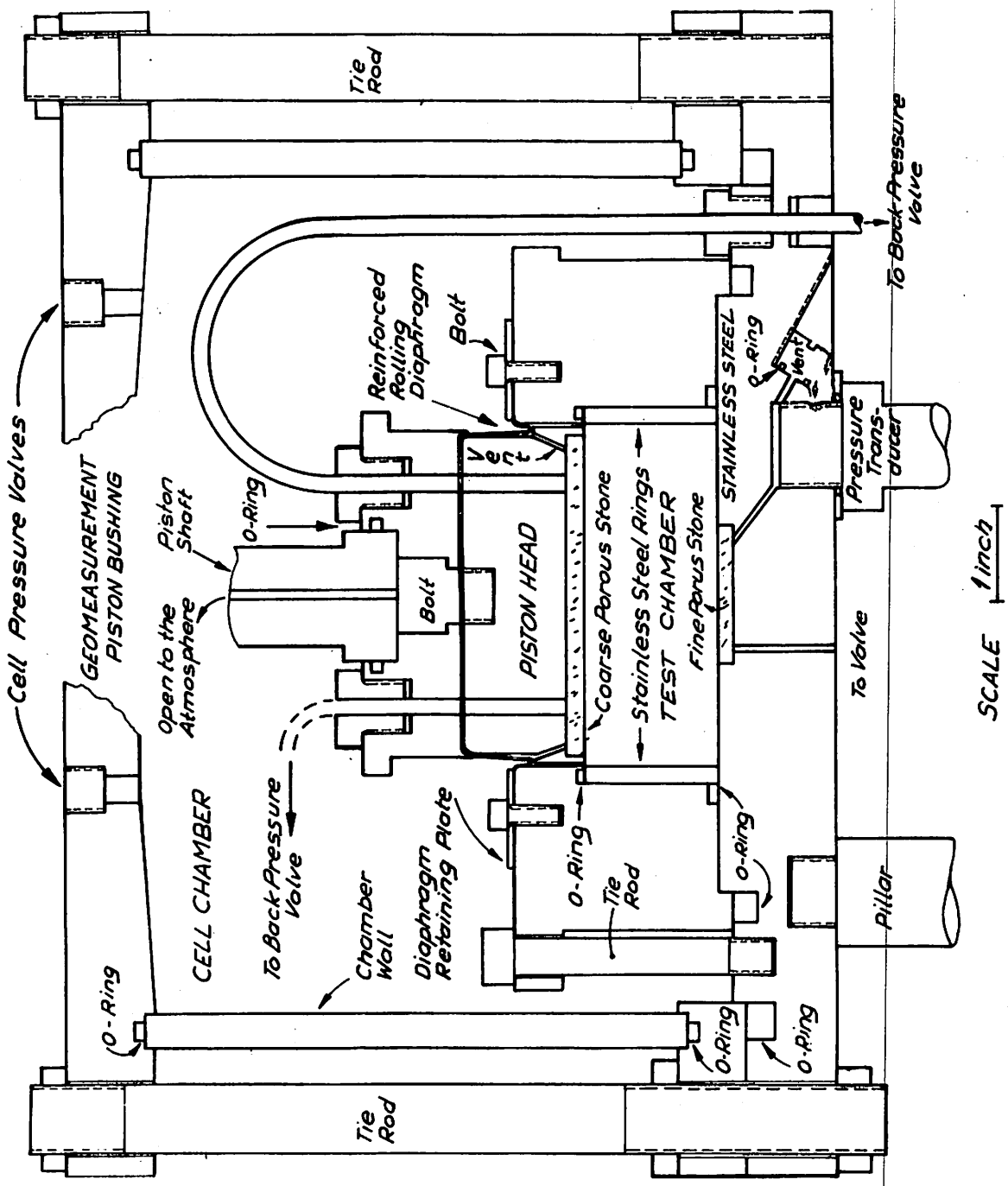


Figure 2-6



From Wissa & Heiberg (1969)

Also see Wissa et al (1971)

M.I.T. GENERAL PURPOSE CONSOLIDOMETER

Figure 2-7

Effect of Secondary Compression on Location of Compression Curve

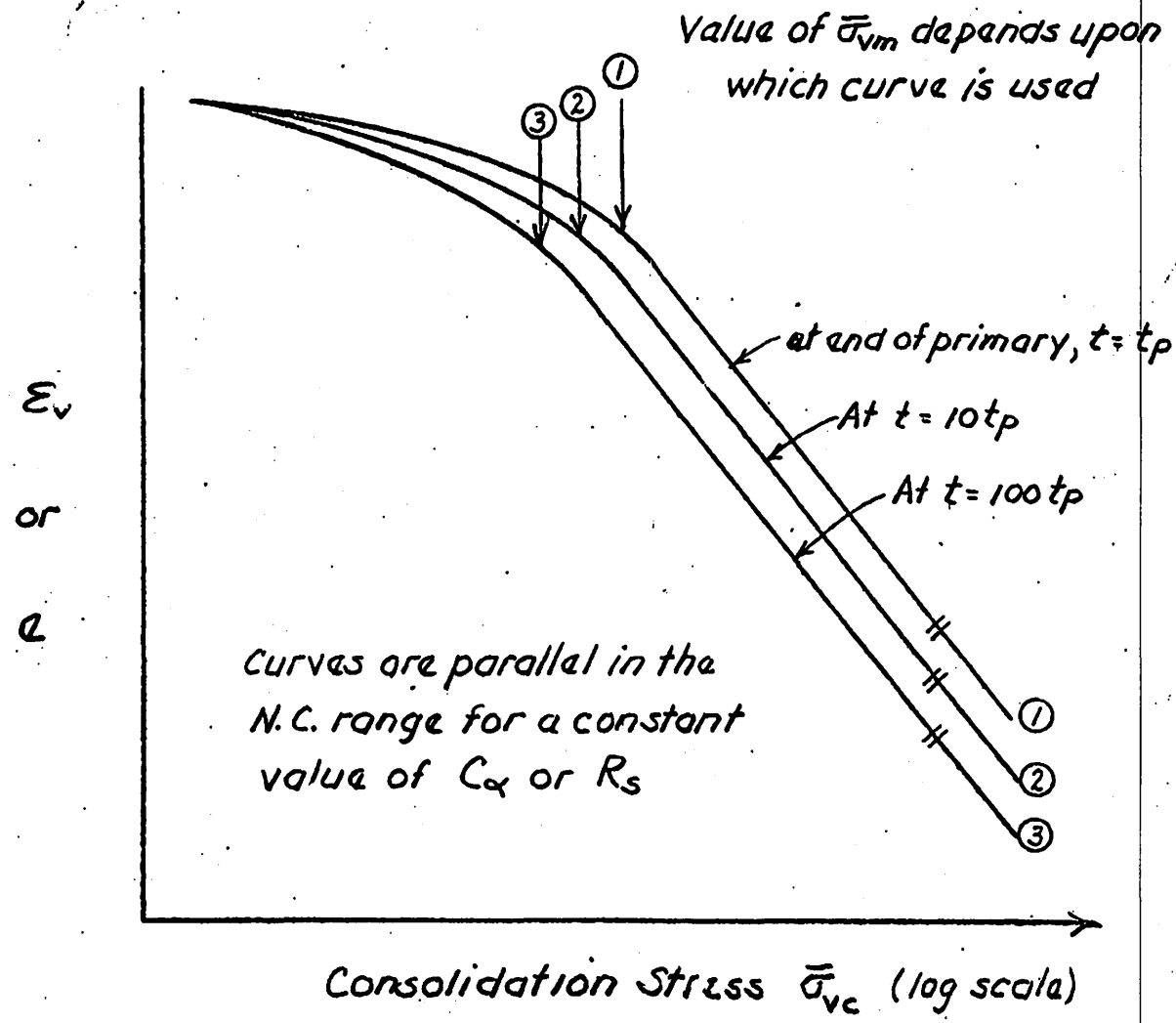
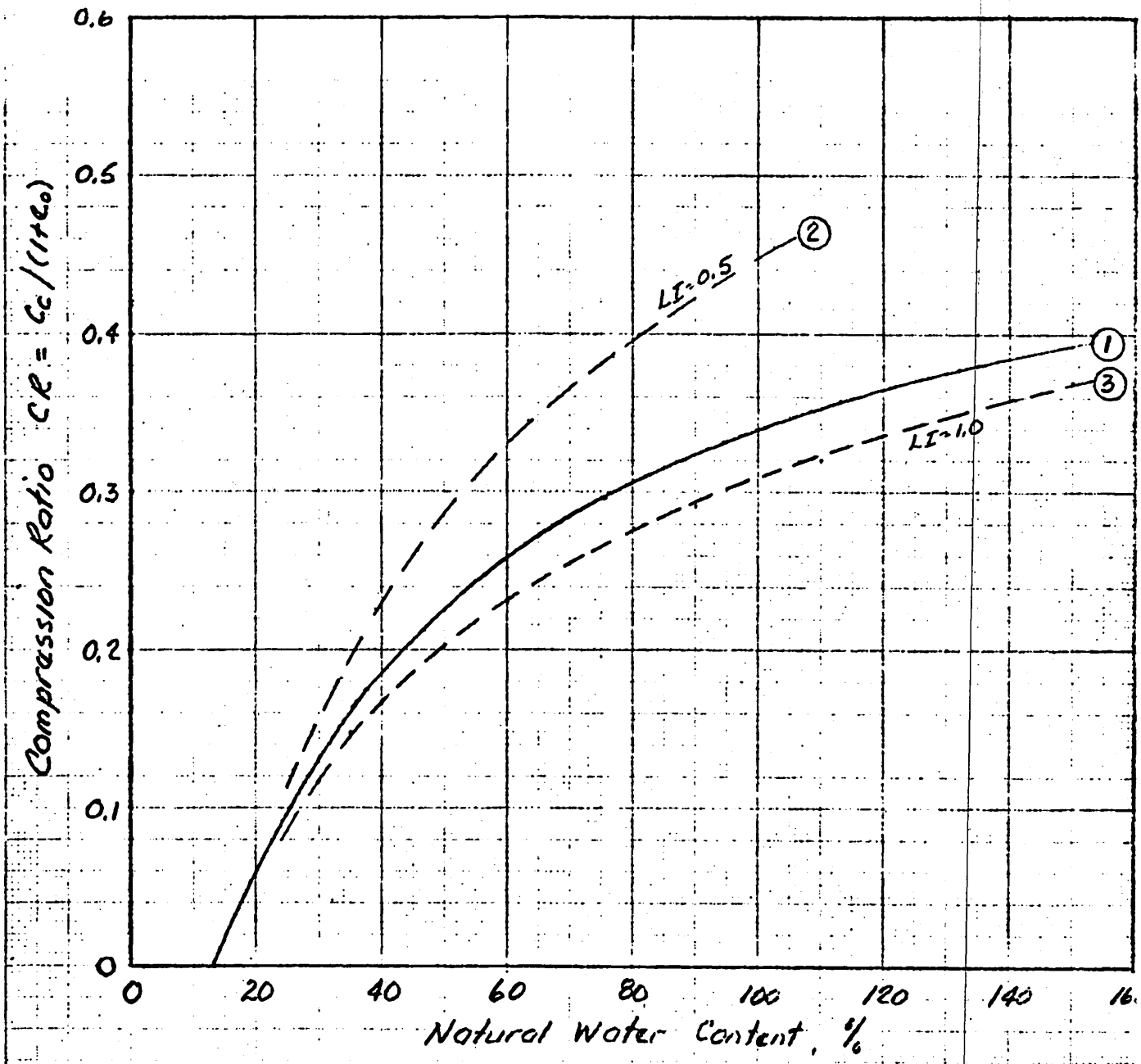


Figure 2-8



① Nishida relationship $C_c = 0.54(e_0 - 0.35)$, for $G_s = 2.65$

② ③ MPMR (1958) relationship of C_c vs e_0 for variable w_L converted to CR vs w_N for variable L.I. assuming $G_s = 2.65$ and $P.I. = 0.73(w_L - 20)$

COMPRESSION RATIO VS NATURAL WATER CONTENT

Figure 2-9

CCL 1/26/71

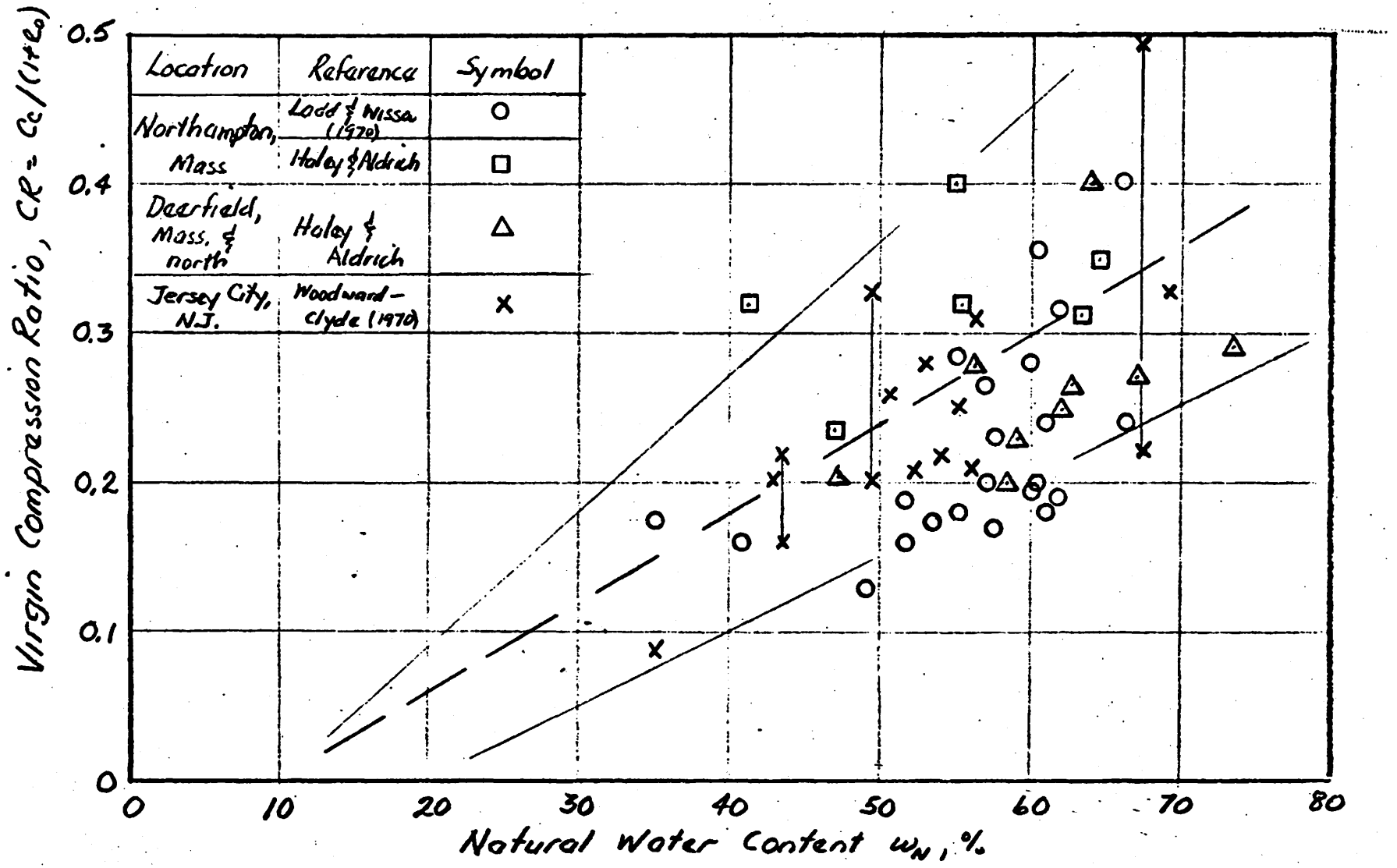
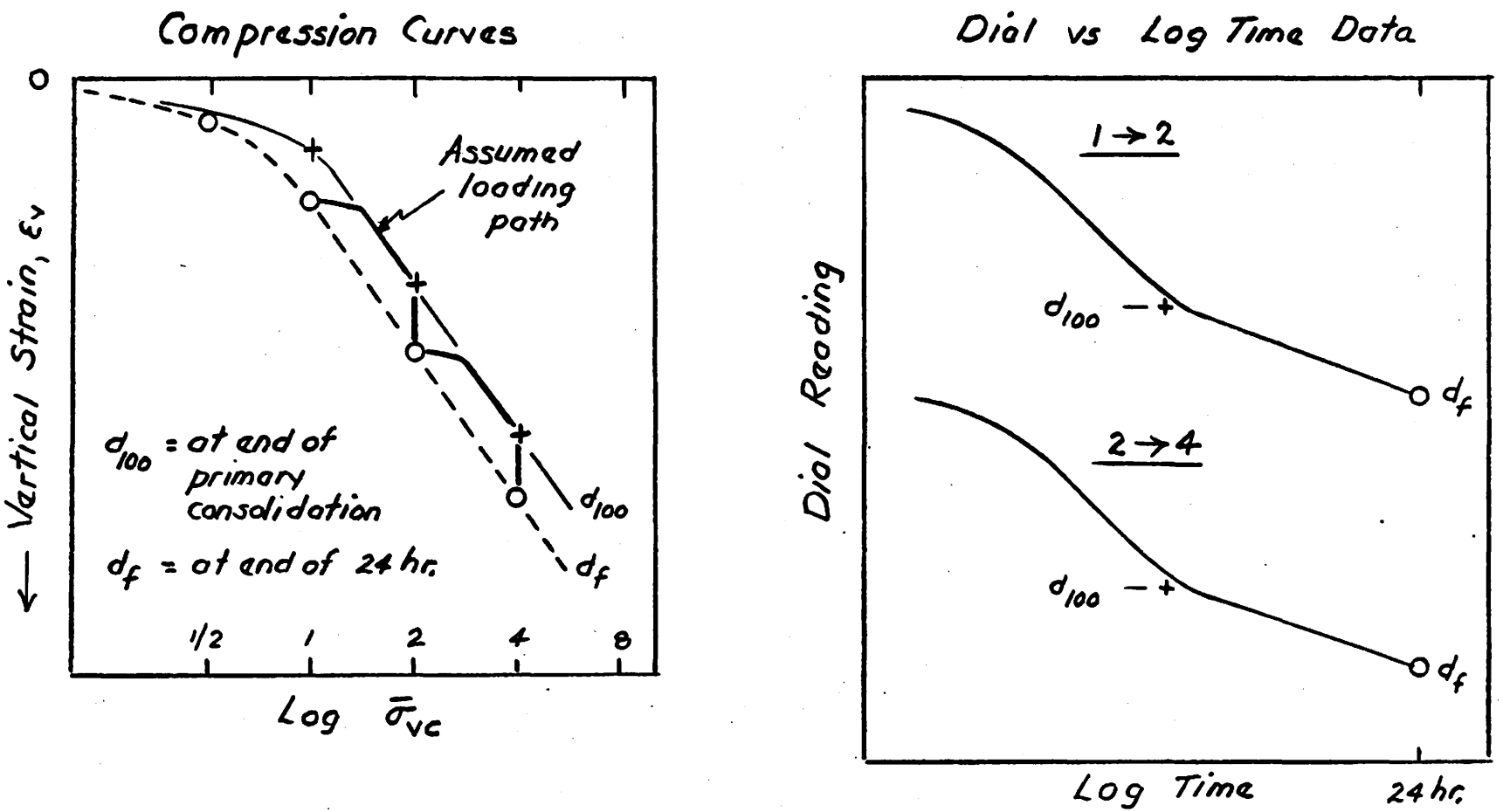


Figure 2-10

VIRGIN COMPRESSION RATIO VS NATURAL WATER CONTENT FOR VARVED CLAYS

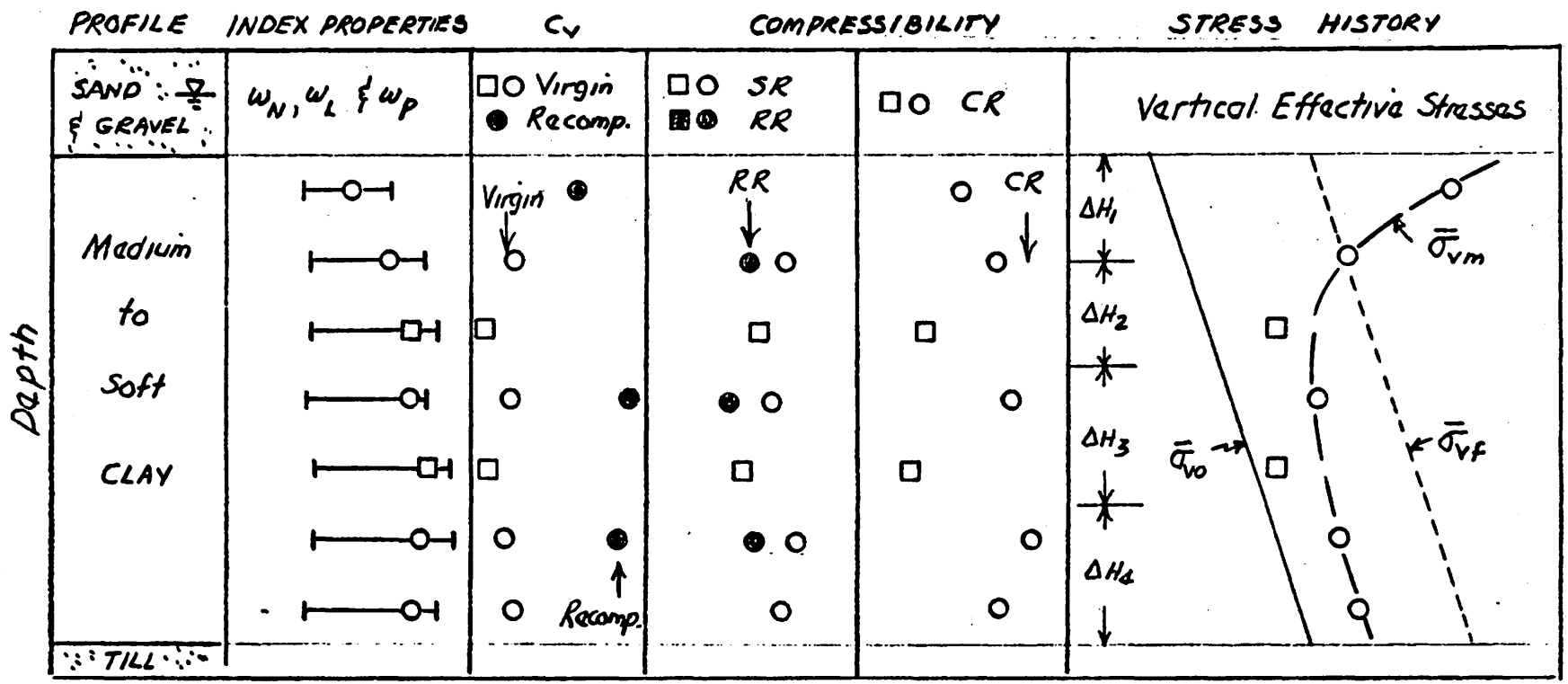
CCL 3/10/73



- 44a -

Figure 2-11

CORRECTION OF OEDOMETER TEST DATA TO OBTAIN END OF PRIMARY CONSOLIDATION COMPRESSION CURVE



$$\rho_{cf} = \Sigma \Delta H \left[RR \log \frac{\bar{\sigma}_{vm}}{\bar{\sigma}_{vo}} + CR \log \frac{\bar{\sigma}_{vf}}{\bar{\sigma}_{vm}} \right]$$

COMMENTS:

A. Stress History
1. Use t_p compression curve
2. Consider geology in selecting $\bar{\sigma}_{vm}$
3. Lab values of $\bar{\sigma}_{vm}$ most likely to be too low

B. Compressibility and Coef. of Consolidation
1. Use strain - log $\bar{\sigma}_{vc}$ data
2. Plot up all values of CR, RR, SR & c_v
3. Select design values based on consideration of all data plus effects of sample disturbance

Fig. 2-12

PRACTICAL CONSIDERATIONS IN SETTLEMENT ANALYSES

III ONE-DIMENSIONAL CONSOLIDATION
(Coefficient of Consolidation)

3.1 INTRODUCTION

Estimates of the rate of consolidation based on the Terzaghi theory require a determination of the coefficient of consolidation, c_v, which is equal to:

Eq. (3-1) $$c_v = \frac{k(1+e)}{a_v \gamma_w} = \frac{k}{m_v \gamma_w}$$

where k = coefficient of permeability

a_v = coefficient of compressibility = $-\Delta e/\Delta \bar{\sigma}_{vc}$

m_v = coefficient of volume change = $\varepsilon_v/\Delta \bar{\sigma}_{vc}$

The Terzaghi theory makes the following assumptions:

(1) The soil is saturated and homogeneous

(2) The pore water and soil particles are incompressible relative to the soil skeleton

(3) The flow and compression are one-dimensional, with uniform stresses at any given depth

(4) The soil obeys Darcy's law and has a constant k

(5) The void ratio is an unique function of vertical consolidation stress $\bar{\sigma}_{vc}$ with a constant value of a_v

(6) Vertical strains are small in comparison with the thickness of the soil layer.

The resulting equation is:

Eq. (3-2) $$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$

where u = excess pore water pressure

z = vertical height to nearest drainage boundary

t = time

The solution of Eq. (3-2) leads to certain characteristic shapes of average degree of consolidation $\bar{U}_v$ vs time factor T_v that can be used to obtain c_v from dial reading versus time data from incremental oedometer tests. The most widely used curve fitting procedures, described in Fig. 3-1, are

the square root time (Taylor, 1948) and log time (A. Casagrande) methods.

If the soil exactly followed the Terzaghi theory, c_v from the square root time and log time methods would be identical and the primary compression ratio,

$$r = \frac{(d_s - d_{100})}{(d_o - d_f)} = \frac{\text{Primary consolidation dial change}}{\text{Total dial change for the increment}}$$

would equal unity. In fact, the primary compression ratio is always less than unity ($r = 0.7 \pm 0.2$ typically for normally consolidated clays) because of secondary compression and perhaps because of an immediate deformation due to gas compression. Moreover, the value of c_v also depends on the method of determination. For normally consolidated clays and standard test procedures, one typically gets:

$$c_v\ (\sqrt{t}) = (2 \pm 0.5)\ c_v\ (\log t)$$

This probably results from the fact that the square root method places emphasis on the early stages of consolidation whereas the log time method emphasizes the latter stages of consolidation. Scott (1961) devised a procedure for evaluating c_v for intermediate time intervals that often yields values that are approximately equal to the average of the above two methods.

The controlled gradient (Lowe, et. al., 1969) and constant rate of strain (Smith and Wahls, 1969; Wissa, et. al., 1971) consolidation tests required development of special theories in order to obtain c_v data. Data from such tests will be discussed in Section 3.3.

3.2 STRESS HISTORY AND SAMPLE DISTURBANCE

Figure 3-2 presents compression and coefficient of consolidation data from a constant rate of strain oedometer

test on a sample of varved clay*. The figure illustrates three important trends that are believed to be typical of consolidation data on slightly overconsolidated clays:

(1) c_v decreases significantly as the consolidation stress approaches the maximum past pressure.

(2) c_v is approximately constant in the normally consolidated region (some clays may show a slight increase at higher consolidation stresses)

(3) c_v from the reload cycle is much larger than that obtained from the initial loading up to the maximum past pressure.

These trends can be explained by looking at Eq. (3-1). For normally consolidated clay, both k and m_v are decreasing with increasing stress, but the ratio k/m_v remains essentially constant. During recompression, the permeability decreases slightly while m_v increases considerably, thus the large decrease in c_v as $\bar{\sigma}_{vc}$ approaches $\bar{\sigma}_{vm}$. Finally, c_v during the initial loading to $\bar{\sigma}_{vm}$ is too small because sample disturbance has increased m_v.

Until the advent of the controlled gradient and constant rate of strain consolidation tests that yield continuous values of c_v, it was not possible to obtain the complete picture of how c_v changed so markedly during recompression. For example, Lowe et. al. (1969) performed a controlled gradient test wherein the load was increased after allowing the sample to undergo appreciate secondary compression. The initial values of c_v were 5 to 10 times larger than that for virgin compression because the values of m_v were decreased by a factor of 5 to 10. Such data are impossible to obtain with a conventional incremental test.

* The fact that the clay is varved is not important since one would expect the same trends with most clays.

MPMR (1958) suggest the following guidelines for the effects of disturbance on the values of c_v in the normally consolidated region:

Good Undisturbed/Poor Undisturbed/Remolded Sample = 3:2:1; that is c_v from a high quality undisturbed sample will be 1.5 times larger than that from a poor quality sample and 3 times larger than that from a remolded sample. The writer believes that these guidelines will generally represent an upper limit and that some disturbance will often have a relatively minor effect on c_v for virgin compression.

The coefficient of consolidation for swelling decreases rapidly from a very high value as clay is rebounded from the maximum past pressure, as illustrated by the data in Fig. 3-3. The decrease in c_v can be explained in large part by a corresponding increase in m_v, although excessive side friction in the oedometer ring may influence the results.

In summary, the following points should be emphasized:

(1) c_v for recompression is substantially larger than that for virgin compression. A factor of 5 to 10 may be typical of many clays.

(2) c_v for swelling is very large initially, but it decreases rapidly with continued rebound.

(3) Sample disturbance will generally have a relatively minor effect on c_v in the normally consolidated region.

(4) Sample disturbance will have a large effect on c_v during initial loading; an unload-reload cycle starting from a stress greater than the insitu $\bar{\sigma}_{vm}$ is recommended.

(5) Controlled gradient and constant rate of strain consolidation tests are much superior to incremental tests in studying the variation in c_v with stress history.

3.3 EFFECT OF TEST PROCEDURES

3.3.1 Load Increment Ratio and Duration

Standard test procedures generally employ 24 hour increments with a doubling of the load using 0.75 ± 0.25 in. high specimens having double drainage. The influence on c_v of varying the sample height, Load Increment Ratio, and/or the load duration have been studied for a variety of clays (see, for example, Taylor, 1942; Leonards and Ramiah, 1959; Newland and Allely, 1960; Simons, 1965). It can be assumed that such variations in test procedure will generally lead to insignificant (from a practical viewpoint) changes in c_v provided that:

(1) the resulting dial vs time data yield the characteristic Terzaghi shape, i.e. a Type I curve (see Section IV). This means that there must be appreciable primary consolidation during the increment.

(2) the increments are for the same stress history, i.e. for recompression to the same OCR or for normally consolidated clay.

Data are now becoming available to compare controlled gradient and constant rate of strain c_v data with data from standard incremental tests. Lowe et. al (1969), Smith and Wahls (1969), Wissa et al (1971) all show good agreement in the normally consolidated region when compared to an incremental c_v based on the average of the sq. root and log time methods.

3.3.2 Temperature

Variations in temperature have a substantial effect on the coefficient of consolidation, but experimental data (Lewis, 1950; Finn in ASTM, 1951; Simons, 1965) show that the change

can be explained, for all practical purposes, by the variation in the viscosity of water with temperature. The variation in c_v with temperature would therefore be:

Temperature (^oC)	Viscosity (millipoises)	Relative c_v
5	15.2	0.66
10	13.1	0.76
15	11.5	0.87
20	10.0	1.00 Reference
25	9.0	1.11
30	8.0	1.25

3.3.3 Miscellaneous

Porous stones need to be periodically replaced (or covered by filter paper) since they tend to become clogged with clay. Newland and Allely (1960) show that improper stones will delay early consolidation leading to a "negative correction" (i.e. d_s greater than d_o in Fig. 3-1).

MPMR (1958) discuss the effects of gas on the rate on consolidation and conclude that gas will:

(1) not affect the final amount of consolidation
(2) cause a large immediate deformation (up to 30 to 50% of the total for some organic soils)
(3) slow down the rate of consolidation because of expansion of the gas as the pore pressures dissipate.

3.4 EMPIRICAL CORRELATIONS

Navdocks DM-7 (1961) presents a correlation between c_v and liquid limit for normally consolidated clays, that is reproduced in Fig. 3-4. It encompasses a wide range of values for a given liquid limit and is, in the writer's opinion, fairly reliable.

3.5 DEVIATIONS FROM TERZAGHI THEORY

One frequently encounters the problem of estimating the rate of consolidation of two or more layers having quite different values of c_v. Barden and Younan (1969) review various methods for handling this case. The simplest, though theoretically incorrect, approach is to transform the thicknesses in accordance with the sq. root of c_v, (Leonards, 1962, p.578):

Layer H_1 with $c_v(1)$
Layer H_2 with $c_v(2)$ less than $c_v(1)$
Use $H = H_2 + H_c$ and $c_v(2)$ where

Eq. (3-3) $$H_c = H_1 \sqrt{\frac{c_v(2)}{c_v(1)}}$$

Navdocks DM-7 (1961), Fig. 6-9 gives a more complicated transformation procedure and compares it with Gray's (1945) exact solutions. Abbott (1960) presents a finite difference method.

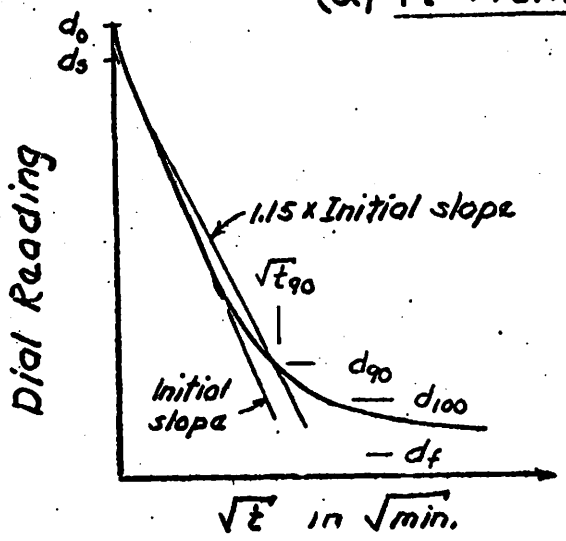
Schiffman (1958) considers the effects of ramp loading and Taylor (1948, p.291) presents a simple approximate method for handling settlements during the loading period (after loading, consider that an instantaneous load was applied at the middle of the loading period).

Schiffman and Gibson (1964) treat consolidation with varying permeability and compressibility, Davis and Raymond (1965) develop a theory for a constant C_c (or CR) rather than constant m_v, Gibson et. al. (1967) consider large strains, and Raymond (1969) presents a theory in which void ratio is proportional to log permeability and log stress.

The above references only illustrate the types of theoretically advances and are by no means complete.

Calculation of Coefficient of Consolidation and Primary Compression Ratio

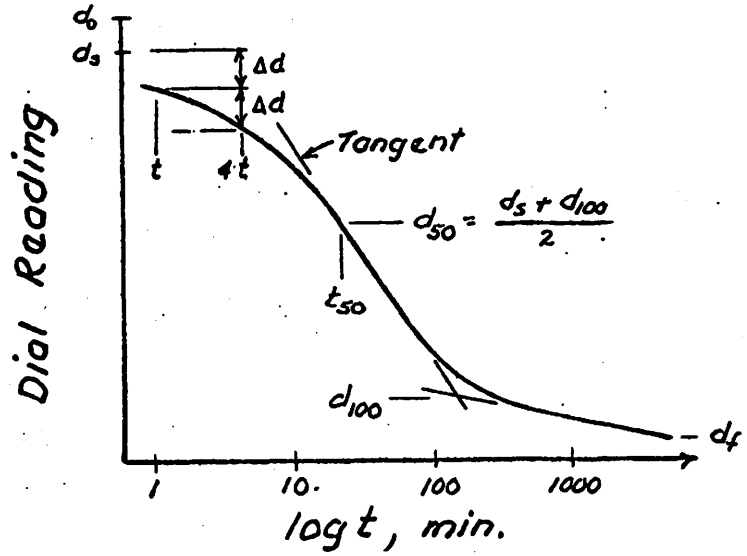
(a) $\sqrt{t}$ Method



$$c_v = \frac{0.848\ H_d^2}{t_{90}}$$

$$r = \frac{\frac{10}{9}(d_s - d_{90})}{(d_o - d_f)}$$

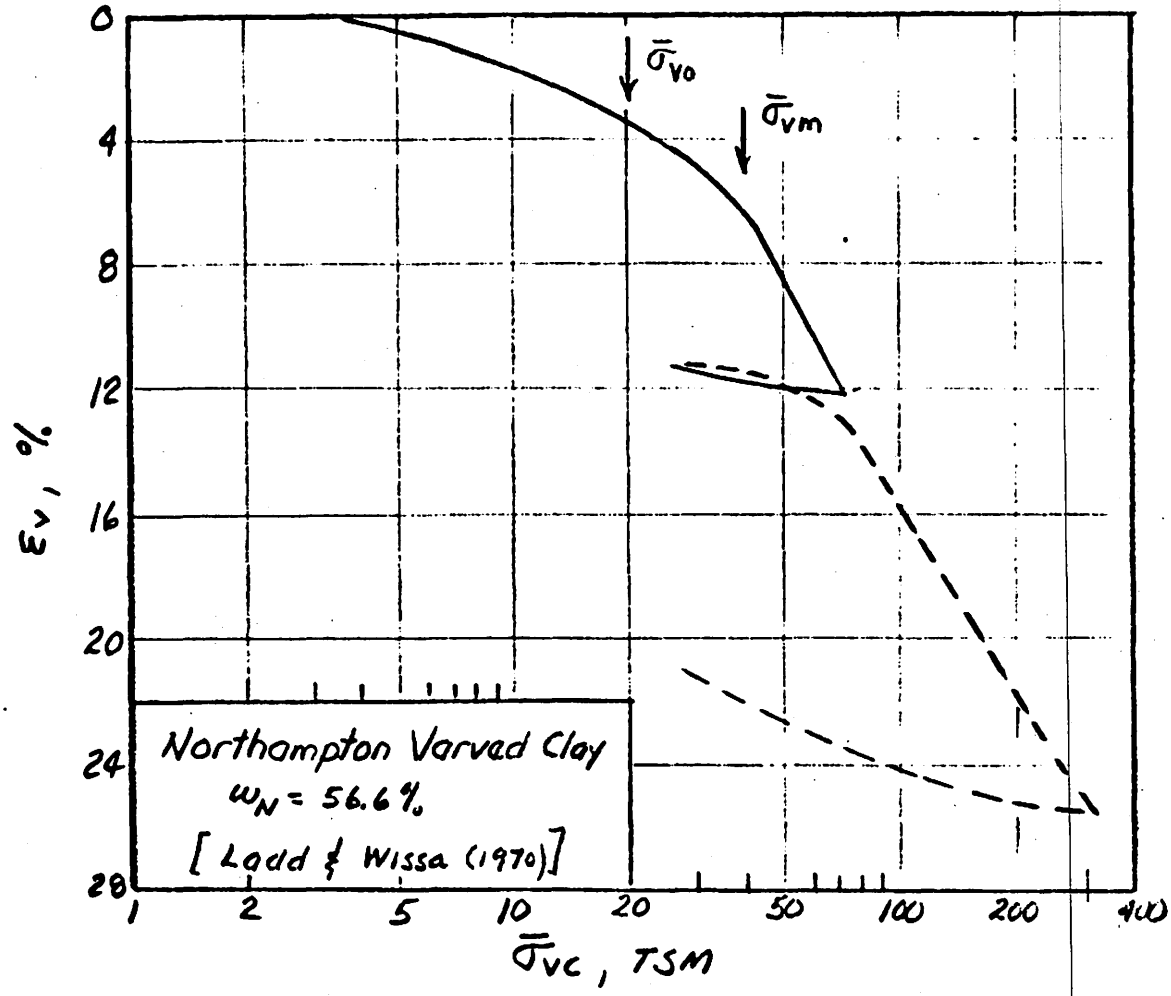
(b) Log t Method

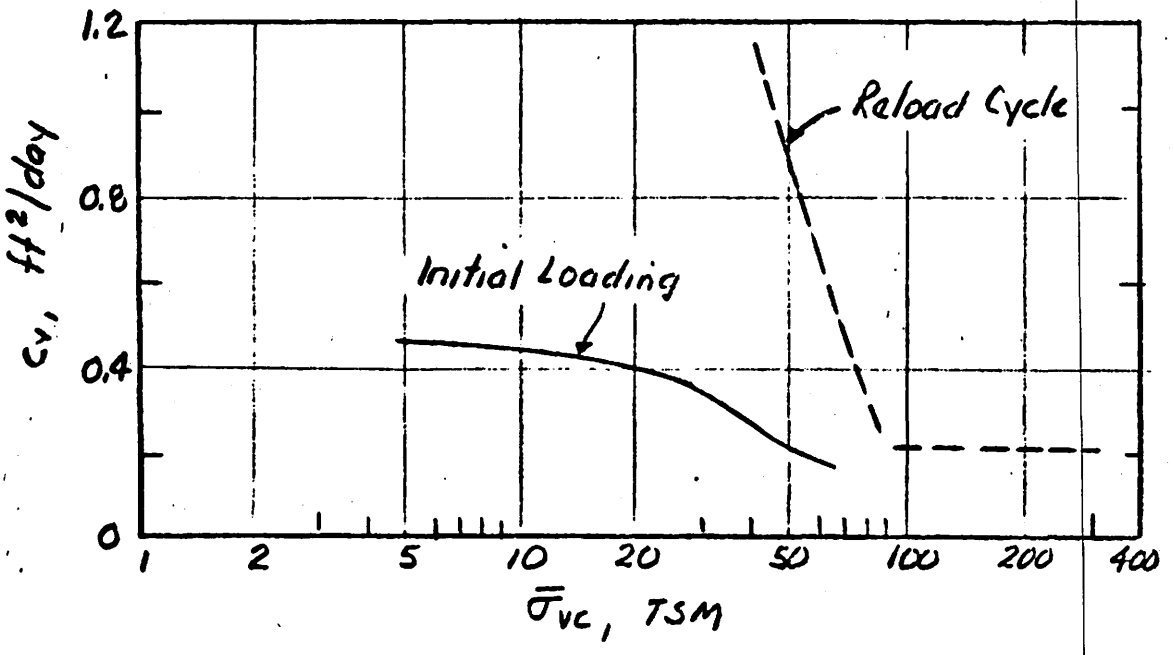


$$c_v = \frac{0.197\ H_d^2}{t_{50}}$$

$$r = \frac{(d_s - d_{100})}{(d_o - d_f)}$$

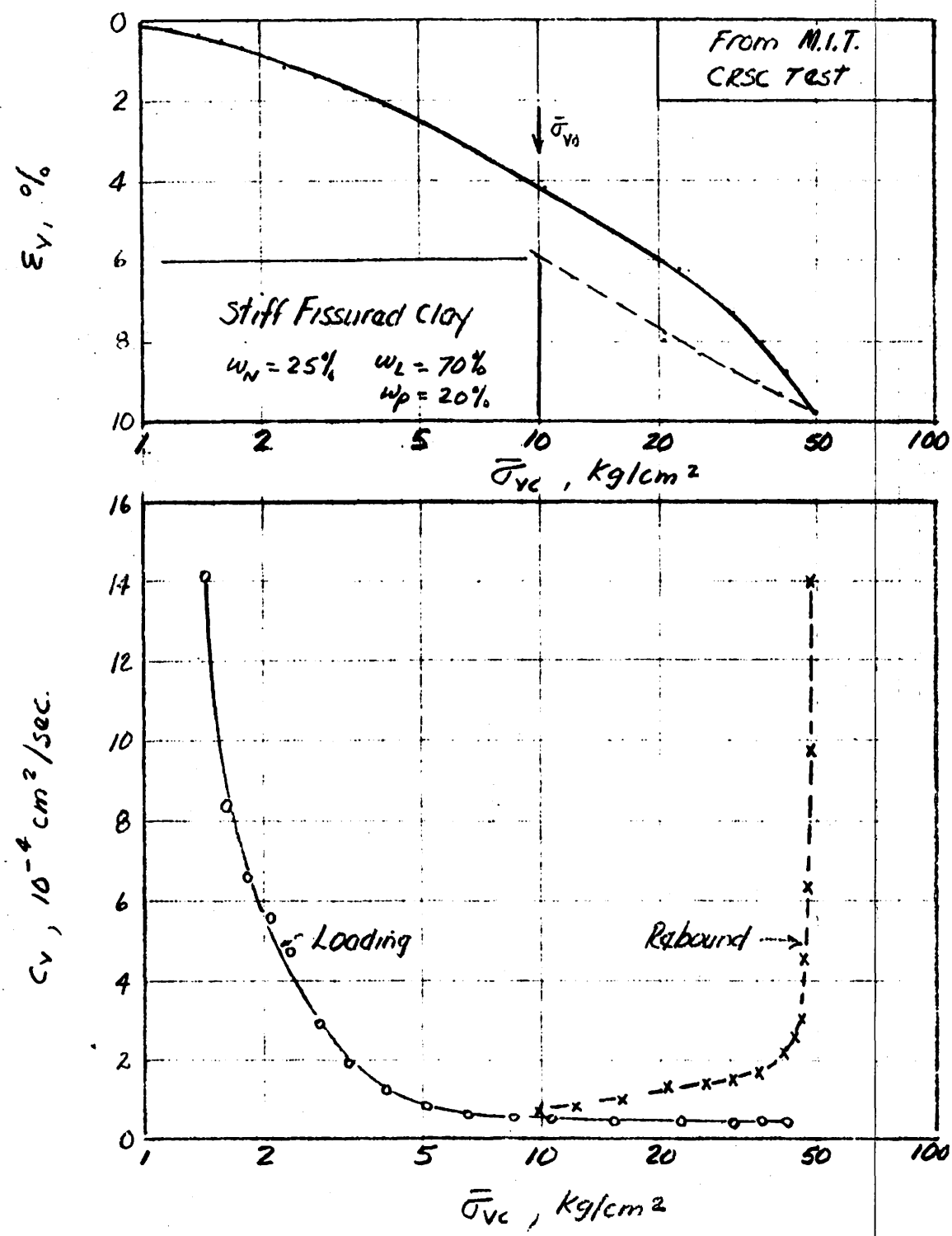
Figure 3-1





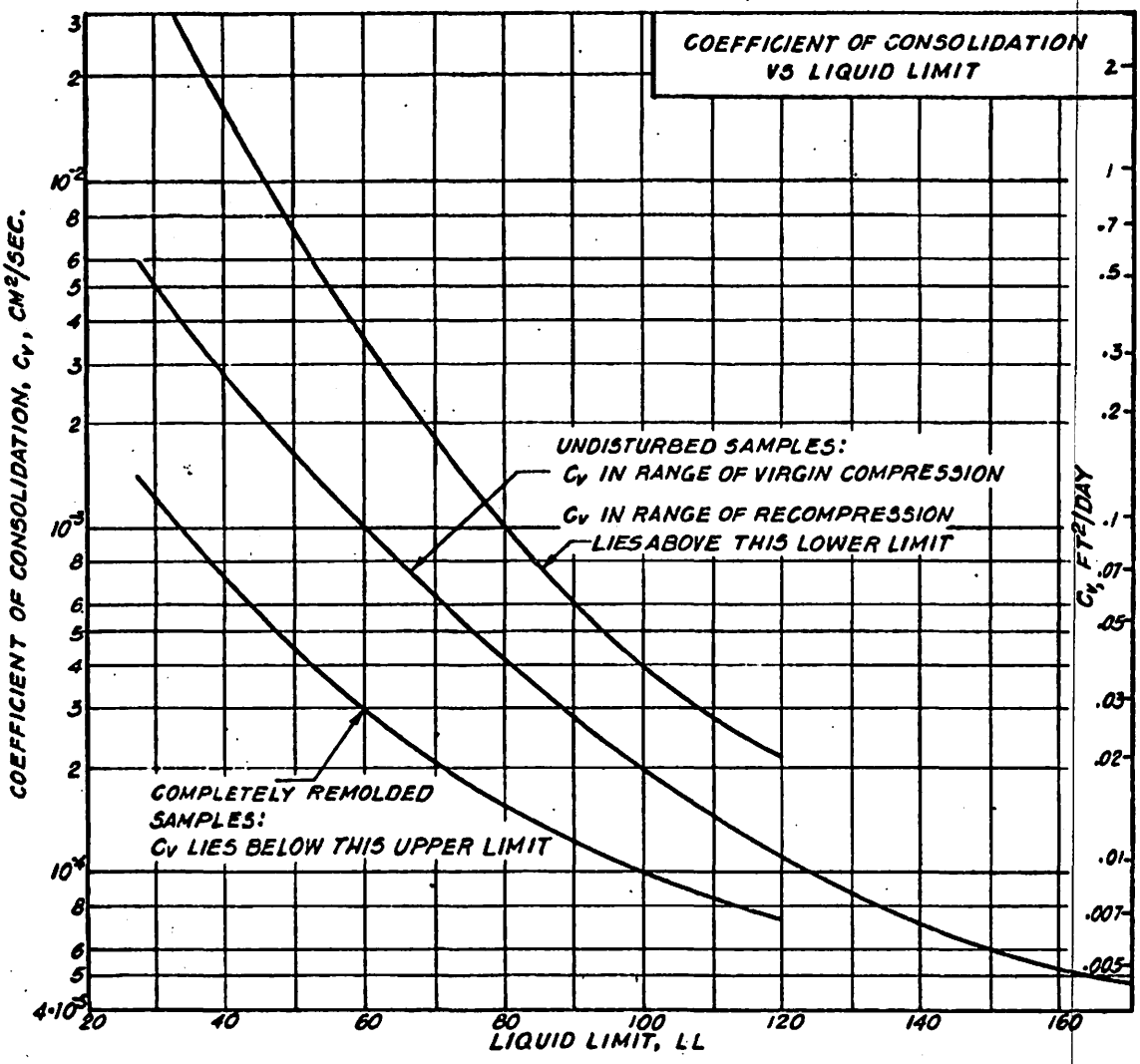
VARIATION IN COEFICIENT OF CONSOLIDATION DURING CRSC TEST ON VARVED CLAY

Figure 3-2



COEFFICIENT OF CONSOLIDATION DURING REBOUND

Figure 3-3



From Navdocks DM-7 (1961)

EMPIRICAL CORRELATION FOR COEFFICIENT OF CONSOLIDATION

Figura 3-4

IV. SECONDARY COMPRESSION

4.1 INTRODUCTION

Secondary compression was defined as the decrease in volume that occurs at essentially constant effective stress, i.e., after all excess pore pressures have essentially dissipated during primary consolidation. Throughout this section the assumption is made that secondary compression does not start until after primary consolidation is completed. This rather arbitrary division is required in order to develop a working hypothesis that can be conveniently applied to practice. Therefore, during primary consolidation the rate of settlement is controlled by the hydrodynamic time lag (Terzaghi theory), whereas after completion of primary, the rate of settlement is controlled by other factors.

It will further be assumed that during secondary compression there is a straight line relationship between settlement and log time. This assumption is generally a very good approximation of real behavior, at least over one or two time cycles for most clays. The rate of secondary compression is defined as (see Fig. 4-1):

$$R_s = \Delta e / \Delta \log t = \text{change in void ratio per log cycle of time} \quad (4\text{-}1)$$

$$C_\alpha = \Delta \epsilon_v / \Delta \log t = \text{change in vertical strain per log cycle of time.} \quad (4\text{-}2)$$

Since strain is based on the original height of the sample

$$C_\alpha = R_s/(1+e_o) \text{ since } \Delta \epsilon_v = \Delta e/(1+e_o)$$

Figure 4-1 shows two types of curves, I and III (Leonards and Altschaeffl, 1964). With the Type I curve, which results with a "large" Load Increment Ratio, the soil follows the Terzaghi theory reasonably well during dissipation of excess pore pressure. With the Type III curve, which results with a "small" Load Increment Ratio, the soil does not follow the Terzaghi theory during dissipation of excess pore pressures.

4.2 INFLUENCE OF STRESS HISTORY AND SOIL TYPE

Figure 4-2 shows the variation in the rate of secondary compression, C_α, with stress history for several undisturbed cohesive soils. The data are from initial loading of high quality undisturbed samples or from a reloading cycle after rebound from just beyond the maximum past pressure. The magnitude of C_α is a very low value at stresses less than one-half of $\bar{\sigma}_{vm}$. It then increases rapidly, reaching a maximum at stresses 1.5 to 2 times $\bar{\sigma}_{vm}$. In the normally consolidated range C_α remains almost constant or or decreases slightly for soils with a constant CR (No. 1 and 2 in Fig. 4-2). For soils exhibiting a marked decrease in CR with increasing stress, C_α also decreases.

The rate of secondary compression is plotted versus the virgin compression ratio CR for a variety of cohesive soils in Fig. 4-3. For soils with the same compression ratio, those falling below the A-line on Casagrande's plasticity chart generally have a higher rate of secondary compression. Remolded soils exhibit a smaller value of C_α than undisturbed soils for the same compressibility.

Empirical correlations have also been developed relating C_α to natural water content and void ratio. Navdocks DM-7 (1961) presents:

$$C_\alpha (\%) = 0.015\ w_N (\%) \text{ for } w_N < 200\%$$

$$C_\alpha (\%) = 2.6 + 0.002\ w_N (\%) \text{ for } w_N > 200\%$$

Kapp et al (1966) developed the following relationship for New York marsh deposits:

$$C_\alpha (\%) = 0.625\ e_o \text{ for } e_o = 1 - 4$$

$$C_\alpha (\%) = 2.5 + 0.10\ e_o \text{ for } e_o = 4 - 15$$

4.3 MECHANISM OF SECONDARY COMPRESSION

Barden (1969) reviews concepts of the mechanisms that have been proposed to explain secondary compression. The most commonly proposed mechanisms can be divided into three broad groups:

(1) Mechanism based on a "structural viscosity" due to the viscosity of "adsorbed" water (e.g., Taylor, 1942; Leonards and Altschaeffl, 1964);

(2) Mechanism based on jumping bonds, such as is hypothesized in the rate process theory (e.g., Mitchell, 1964);

(3) Mechanism based on micropores where water flows out of extremely small pores during secondary compression (e.g., De Jong and Verruijt, 1965).

In the writers opinion, there is no reason to believe that the mechanisms responsible for secondary compression are necessarily different than those responsible for primary consolidation. During primary consolidation there is a large rearrangement of particles

and sliding of contacts, etc., the rate of which is controlled by how fast the water can be squeezed out. When the excess pore pressures become negligible, many of the particle contacts are still rather unstable and then they continue to move until finding a "home." With increasing time, more and more particles have moved to equilibrium positions and thus the rate of volume change decreases. One would expect such a process to follow an exponential decay.

This rather simple picture can readily explain why the rate of secondary compression increases with compressibility. The more compressible the soil, the greater the tendency for a larger number of particles to be unstable at the end of primary compression.

4.4 WORKING HYPOTHESIS

As a working hypothesis for engineering practice, the following assumptions can be for cases where the Load Increment Ratio is of sufficient magnitude to cause some primary consolidation:

(1) C_α is independent of time, at least over the time span of interest;

(2) C_α is independent of the thickness of the soil layer. Of course, the thicker the layer, the longer the time required for primary consolidation, but the strain per log cycle of time remains constant.

(3) C_α is independent of Load Increment Ratio, as long as some primary consolidation occurs.

(4) C_α is independent of consolidation stress for normally consolidated clays with a constant compression ratio CR.

Behavior resulting from the working hypothesis is illustrated in Fig. 4-4, which shows the effects of varying drainage height, consolidation stress, and Load Increment Ratio for a normally consolidated clay with a constant CR.

The working hypothesis is admittedly an oversimplification of actual behavior. However, there are data to support, at least as a first approximation, the assumptions. Some of the references are listed below:

(1) C_α independent of thickness:

Newland and Allely (1960)

Leonards and Girault (1961)

(2) C_α independent of Load Increment Ratio:

Newland and Allely (1960)

Leonards and Girault (1961)

Wahls (1962)

(3) C_α independent of consolidation stress for constant CR:

Newland and Allely (1960)

Leonards and Girault (1961)

Lo (1961)

Wahls (1962)

Jonas (1964)

Schiffman, Ladd and Chen (1964).

4.5 EFFECT OF SECONDARY COMPRESSION ON AMOUNT AND RATE OF COMPRESSION WITH "NORMALLY LOADED" CLAYS

Compression curves for varying load durations are plotted in Fig. 4-5 for a normally "loaded" clay with a constant compression

ratio and rate of secondary compression. The curves labelled 0, 1, 2 and 3 correspond to the end of primary consolidation and after 1, 2 and 3 cycles of secondary compression (the vertical distance between adjacent lines equals C_α). These data could have been obtained from:

(1) Four incremental tests with consolidation times $t_c = t_p$, $10\ t_p$, $100\ t_p$, etc.;

(2) One incremental test with $t_c = 1000\ t_p$, but strains at $t_c = t_p$, $10\ t_p$, etc. are also plotted;

(3) Similar data could be obtained by constant rate of strain tests, but it would be difficult to assess the amount of secondary compression corresponding to each test.

Note that secondary compression does not alter the virgin compression index when C_α is a constant and if the data for a given curve all correspond to the same consolidation time.

Figure 4-5 also shows the development of a maximum past pressure as secondary compression occurs. This has been termed a quasi-preconsolidation pressure $\bar{\sigma}_{vm}(Q)$ by Leonards and Altschaeffl (1964) and the critical pressure P_c by Bjerrum (1967). The longer the load is left on, the greater is the stress increment required to bring the clay to the compression curve corresponding to the end of primary consolidation.

Equation (4-3) in Fig. 4-5 shows that the magnitude of $\bar{\sigma}_{vm}(Q)/\bar{\sigma}_{vo}$ depends on the number of cycles of secondary compression and the ratio C_α/CR. Leonards and Altschaeffl (1964) summarize laboratory data that show $\bar{\sigma}_{vm}(Q)/\bar{\sigma}_{vo} = 1.4$ for a variety of clays (number of cycles of secondary not specified). Bjerrum (1967, 1971)

presents data summarized in Table 4-1 for natural marine clays where the ratio varies from 1.15 to 2.0. Thus quasi-preconsolidation pressures should be considered in practice.

Figure 4-6 illustrates the effect of Load Increment Ratio on the amount and rate of compression for a normally loaded clay that has developed a quasi-preconsolidation pressure due to two cycles of secondary compression. The basic assumption is made (Ladd and Preston, 1965; Bjerrum, 1967) that the soil follows the "end of primary," t_p line, during dissipation of excess pore pressure, once the stresses exceed $\overline{\sigma}_{vm}(Q)$.

For a large stress increase a Type I curve results wherein the Terzaghi theory can be used to estimate the rate of primary consolidation. At the end of primary, the rate of compression is governed by C_α.

For a small stress increase, but one sufficient to cause some compression along the primary line, a Type III curve results. Although there is a dissipation of excess pore pressures and Δu still equals $\Delta\sigma_v$ (Leonards and Girault, 1961), the Terzaghi theory does not adequately describe the rate of pore pressure dissipation. Nor can meaningful values of c_v be obtained from the curve fitting procedures (attempts to do so result in very low values of c_v, perhaps because c_v decreases rapidly during the increment, as shown by Lowe, et al, 1969). However, after dissipation of pore pressures, the rate of compression is controlled by the same value of C_α as obtained with the large Load Increment Ratio.

Figure 2-8 showed how secondary compression affected the location of the compression curve and the resulting value of maximum past pressure. As the consolidation stress approaches the $\overline{\sigma}_{vm}$, the

rate of secondary compression increases (see Fig. 4-2) and the curves for varying load duration become more widely separated. But once $\bar{\sigma}_{vc}$ is well into the virgin compression region where C_α is constant (for constant CR), the compression curves for different loading times become parallel.

4.6 OTHER FACTORS INFLUENCING THE RATE OF SECONDARY COMPRESSION

4.6.1 Sample Disturbance

Section 4.2 showed that C_α is generally proportional to the compressibility of the soil. This fact, and data comparing undisturbed and remolded samples, result in the following:

(1) For recompression up to the maximum past pressure disturbance increases compressibility and therefore increases C_α;

(2) In the normally consolidated region disturbance decreases compressibility and therefore decreases C_α.

In order to obtain reliable C_α data for recompression, one should perform an unload-reload cycle from a stress beyond the in situ $\bar{\sigma}_{vm}$. High quality samples are needed in order to define the maximum value of C_α in the normally consolidated range (Fig. 4-2).

4.6.2 Temperature

The rate process theory (Mitchell, 1964) predicts an increasing rate of creep (equal to secondary compression for an oedometer test) with increasing temperature. Somewhat limited experimental data support the hypothesis that C_α will increase with increasing temperature

(Lo, 1961; Murayama and Shibata, 1961; Schiffman et al (1964), although Plum and Esrig (1969) question the results. It is concluded that increasing temperature will probably increase the magnitude of C_α, however, the effect is likely to be small compared to other factors influencing C_α.

4.6.3 Non-K_o Stress Conditions

It is widely believed that secondary compression will only occur if shear stresses are applied to the soil, as opposed to volumetric stresses per se. This erroneous view neglects the fact that there are shear stresses acting at particle contacts even when no external shear stresses are applied to the soil, i.e., when a uniform pressure is applied. Moreover, there are experimental data showing that the rate of secondary compression for isotropic consolidation in a triaxial cell, where shear stresses are minimal, is about equal to that from oedometer tests, where the shear stresses are much larger since $K_o = 0.5 - 0.6$ (Schiffman et al, 1964).

However, as the applied shear stresses approach failure, the rate of secondary compression (based on vertical strain) does indeed increase. Bishop (1966) and Bishop and Lovenbury (1969) present "creep" data from well controlled oedometer and drained triaxial compression tests on precompressed London clay and a soft plastic clay from Pisa. The normally consolidated Pisa clay showed as a first approximation:

q/q_f	C_α (% Axial Strain/Δ log t)
Oedometer	0.8
50%	0.7
75%	1.3
85%	2.4

4.6.4 Effect of Surcharge

Surcharges are frequently used to reduce subsequent settlements due to secondary compression, especially with sand drain installations where primary consolidation is over rather quickly. The design variables are the amount of surcharge and the length of time that it is left on relative to the time required for primary compression. Data showing the reduction in C_α with the amount of surcharge are shown in Fig. 4-7. The amount of surcharge is expressed as

$$\text{A.O.S.} = \frac{(\bar{\sigma}_{vs} - \bar{\sigma}_{vf})}{\bar{\sigma}_{vf}}, \ \%$$

where

$\bar{\sigma}_{vs}$ = in situ stress with the surcharge

$\bar{\sigma}_{vf}$ = in situ final stress without the surcharge.

The time at which the surcharge is removed, t_s, is also important; generally speaking, it should be left on until primary consolidation is essentially completed. See Johnson (1970) for a further treatment of using surcharges to reduce secondary compression.

4.7 RECOMMENDED PRACTICE FOR HANDLING SECONDARY COMPRESSION

4.7.1 Recompression of Overconsolidated Clay

If the stress increase results in a consolidation stress that is still less than one-half of the in situ $\bar{\sigma}_{vm}$ (i.e., OCR $\geq$ 2), the rate of secondary compression is likely to be too small to be of practical significance (see Fig. 4-2). At higher stresses, C_α will increase significantly and data such as shown in Fig. 4-2 should be obtained

from a laboratory $\bar{\sigma}_{vm}$ somewhat greater than the in situ $\bar{\sigma}_{vm}$.

4.7.2 Loading Overconsolidated Clay Into the Normally Consolidated Range

The amount of settlement due to primary consolidation is estimated with Eq. 2-1. The amount of subsequent settlement due to secondary compression is obtained with Eq. (4-4):

$$\rho_s = \Sigma HC_\alpha \log (t/t_p) \tag{4-4}$$

In order to develop a plot of settlement versus time, first plot settlement due to primary consolidation and then simply "tack on" the rate of secondary which has a slope of ΣHC_α per log cycle of time.

The value of C_α that is selected for the analysis should take into account any major changes in C_α with consolidation stress, such as shown by lines 3 and 4 in Fig. 4-2.

4.7.3 Stress Increase on a Normally Loaded Clay With a Quasi-Preconsolidation Pressure

The amount of settlement due to primary consolidation is estimated with a modified version of Eq. 2-1 (Bjerrum, 1967):

$$\rho_{cf} = \Sigma H \, CR \log \frac{\bar{\sigma}_{vf}}{\bar{\sigma}_{vm}(Q)} \tag{4-5}$$

which assumes that the settlement occurring as the stress increases from $\bar{\sigma}_{vo}$ to $\bar{\sigma}_{vm}(Q)$ is negligible.

If the stress increase is large (Type I curve), the Terzaghi theory can be used to predict the rate of primary consolidation and settlements due to secondary compression can be handled as proposed in Section 4.7.2.

If the stress increase is small, so that a Type III curve results, there is no accepted theory that can be used to predict the rate of primary consolidation. The writer proposes the following:

(1) Compute ρ_{cf} with Eq. (4-5);

(2) Assume that it occurs by time t_p based on a c_v for a large stress increase;

(3) Plot ρ_{cf} at t_p on a plot of settlement versus log time;

(4) Add on the rate of secondary compression based on C_α for normally consolidated clay;

(5) Sketch in the primary settlement curve with a Type III shape.

If the stress increase is so small that the primary compression line is not reached, i.e., $\bar{\sigma}_{vf}$ is less than $\bar{\sigma}_{vm}(Q)$, settlements due to primary consolidation will be negligible. The rate of secondary compression will also be reduced compared to soil that has been loaded out to the primary compression curve. Bjerrum (1967) suggests a method for handling this case.

The above approach is simple in theory but very difficult in practice because of the difficulties associated with a reliable estimate of the quasi-preconsolidation pressure. Its determination from oedometer tests requires:

(1) Very high quality samples;

(2) A well defined compression curve in the vicinity of $\bar{\sigma}_{vm}(Q)$;

(3) A compression curve corresponding to the end of primary consolidation.

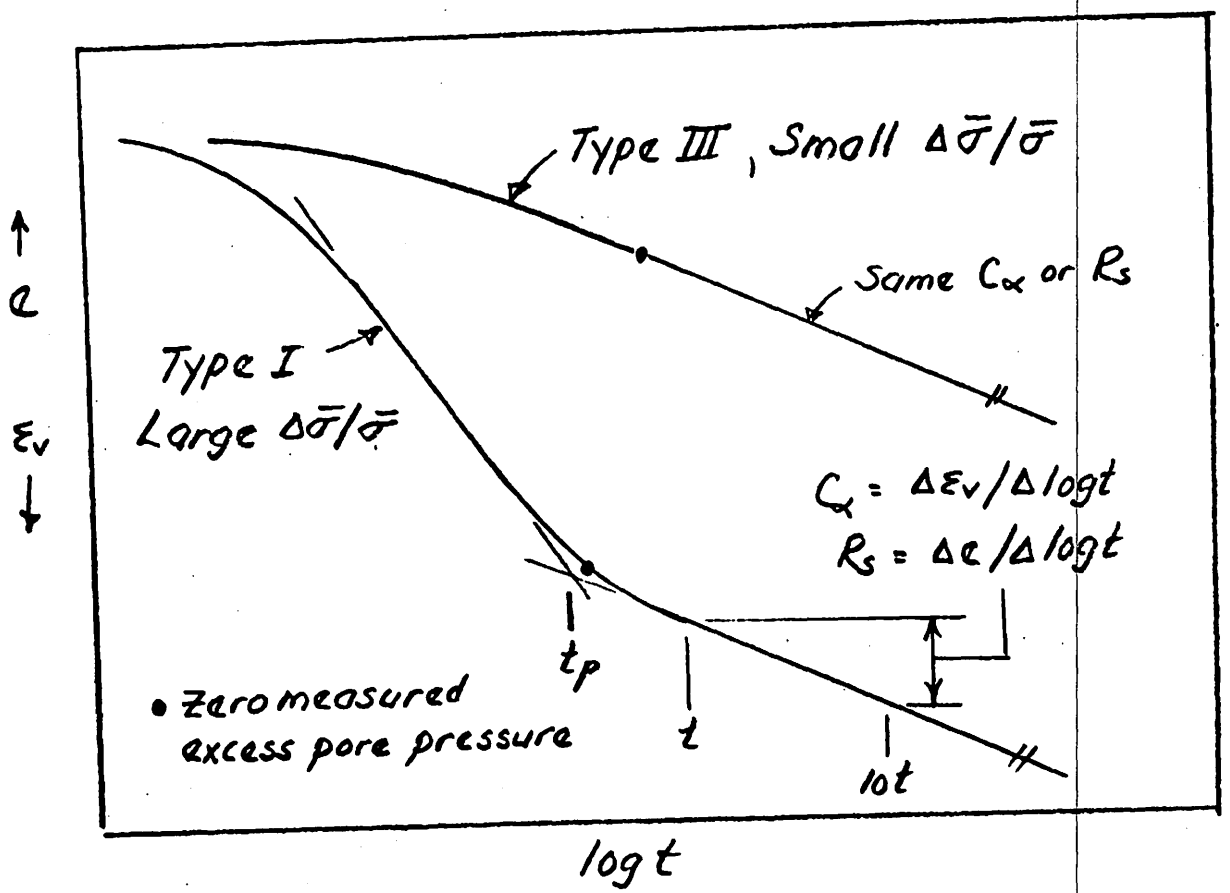
The controlled gradient and constant rate of strain consolidation tests can fulfill the last two requirements, but the first is generally beyond our control.

Obviously, laboratory results must be combined with an excellent knowledge of the geologic history of the soil before arriving at a realistic value of $\bar{\sigma}_{vm}(Q)$.

QUASI-PRECONSOLIDATION PRESSURES OF SEVERAL NORMALLY LOADED MARINE CLAYS (from Bjerrum, 1971)

Type of Clay		Index Properties				$\frac{\bar{\sigma}_{vm}(Q)}{\bar{\sigma}_{vo}}$	$s_u/\bar{\sigma}_{vo}$ from Field Vane
		w_N %	w_L %	w_P %	PI %		
Korsör, Denmark		210	250	150	100	2.0	——
Bangkok, Thailand		145	143	63	80	1.9	0.59
Kimola, Finland		59	62	34	28	1.6	0.45
Plastic Drammen, Norway		52	61	32	29	1.5	0.37
Vaterland, Norway		38	45	27	18	1.4	0.33
Lean Drammen, Norway		30	33	22	11	1.2	0.24
Quick Clays	Mastemyr, Norway	44	29	22	7	1.25	0.21
	Manglerud, Norway	40	27	19	8	1.15	0.13

Table 4-1



Eq (4-3) ρ_s = Settlement due to secondary compression

$$= \Sigma H C_\alpha \log \frac{t_c}{t_p}$$

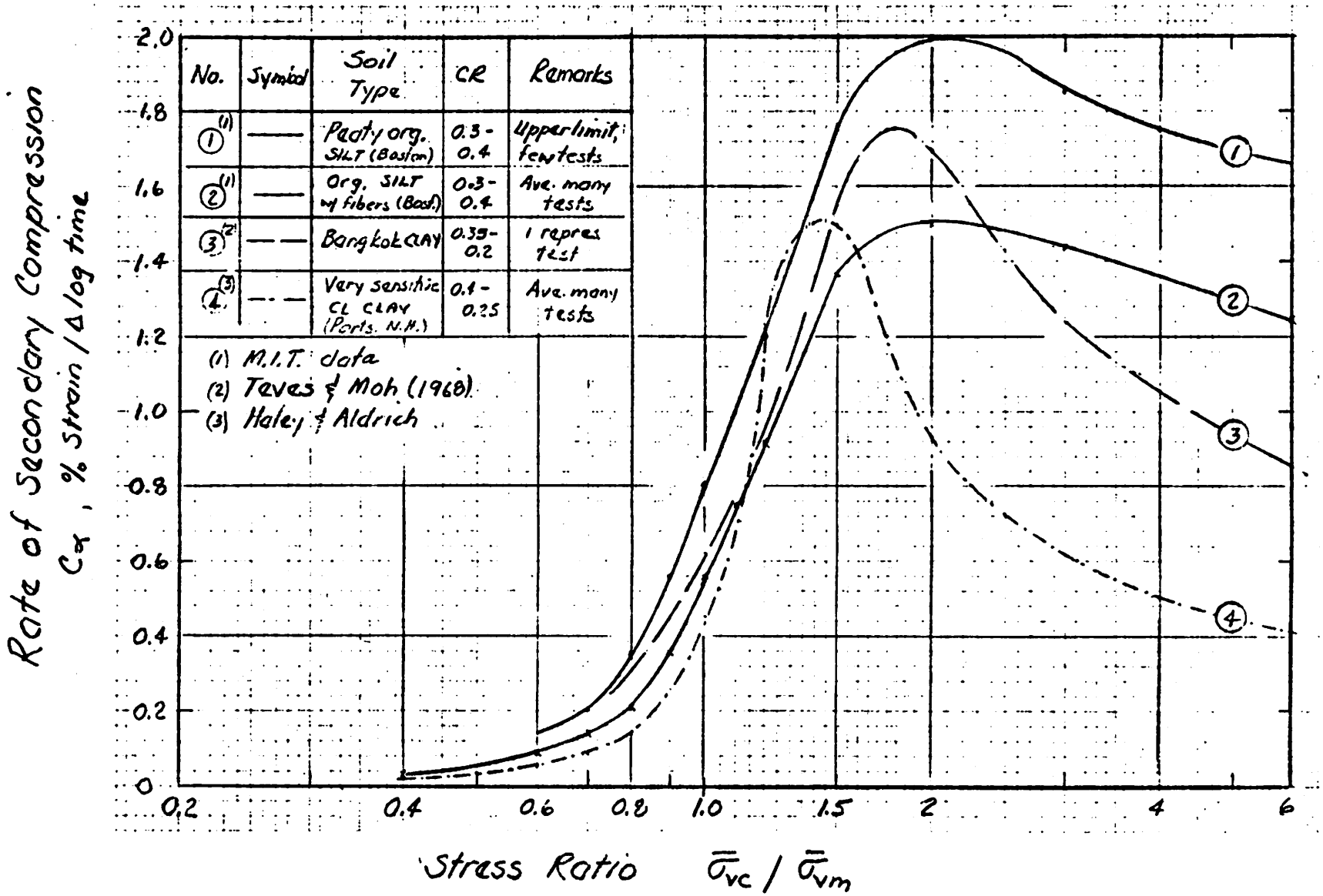
t_c = time since middle of loading period

t_p = time required for primary consolidation

VOLUME CHANGE VS LOG TIME DURING LOAD INCREMENT ON SATURATED CLAY

Figure 4-1

CCL 3/24/70



$\bar{\sigma}_{vc}$ = Consolidation stress in oedometer test

$\bar{\sigma}_{vm}$ = In situ maximum past vertical stress

RATE OF SECONDARY COMPRESSION VS STRESS HISTORY

Figure 4.

C.C. Ladd 3/24/70

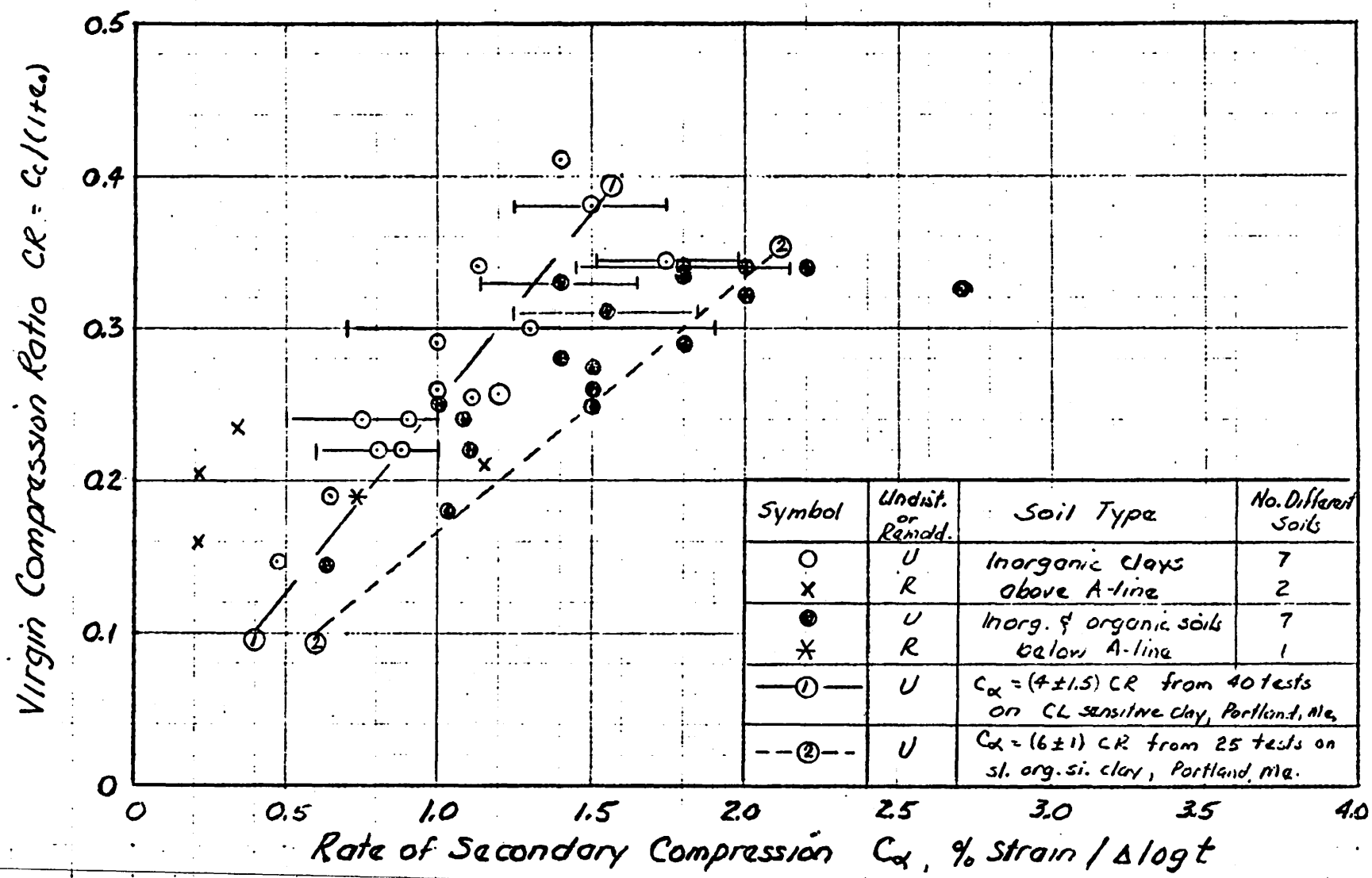


Figure 3

RATE OF SECONDARY COMPRESSION VS COMPRESSIBILITY FOR NORMALLY CONSOLIDATED COHESIVE SOILS

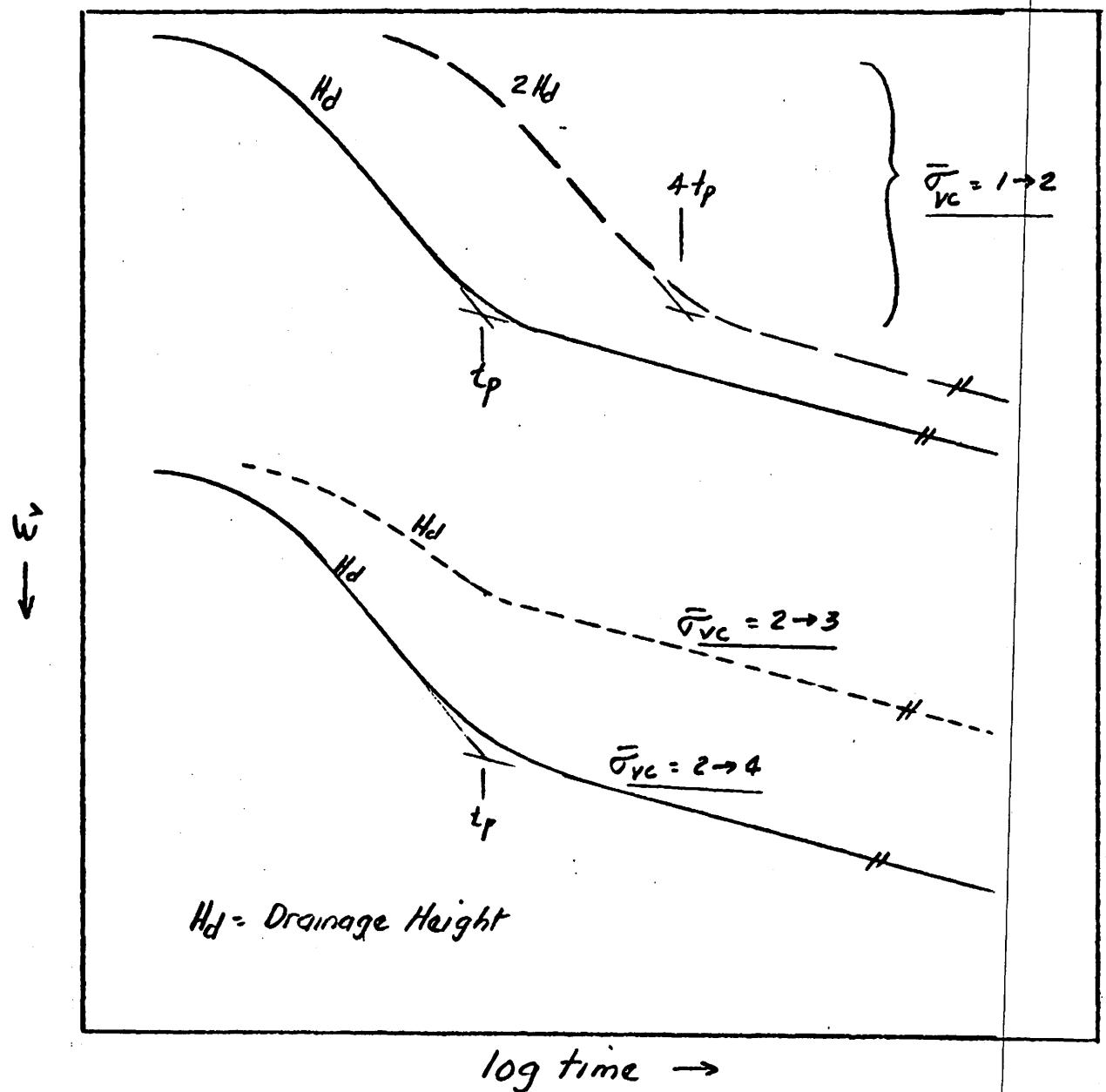
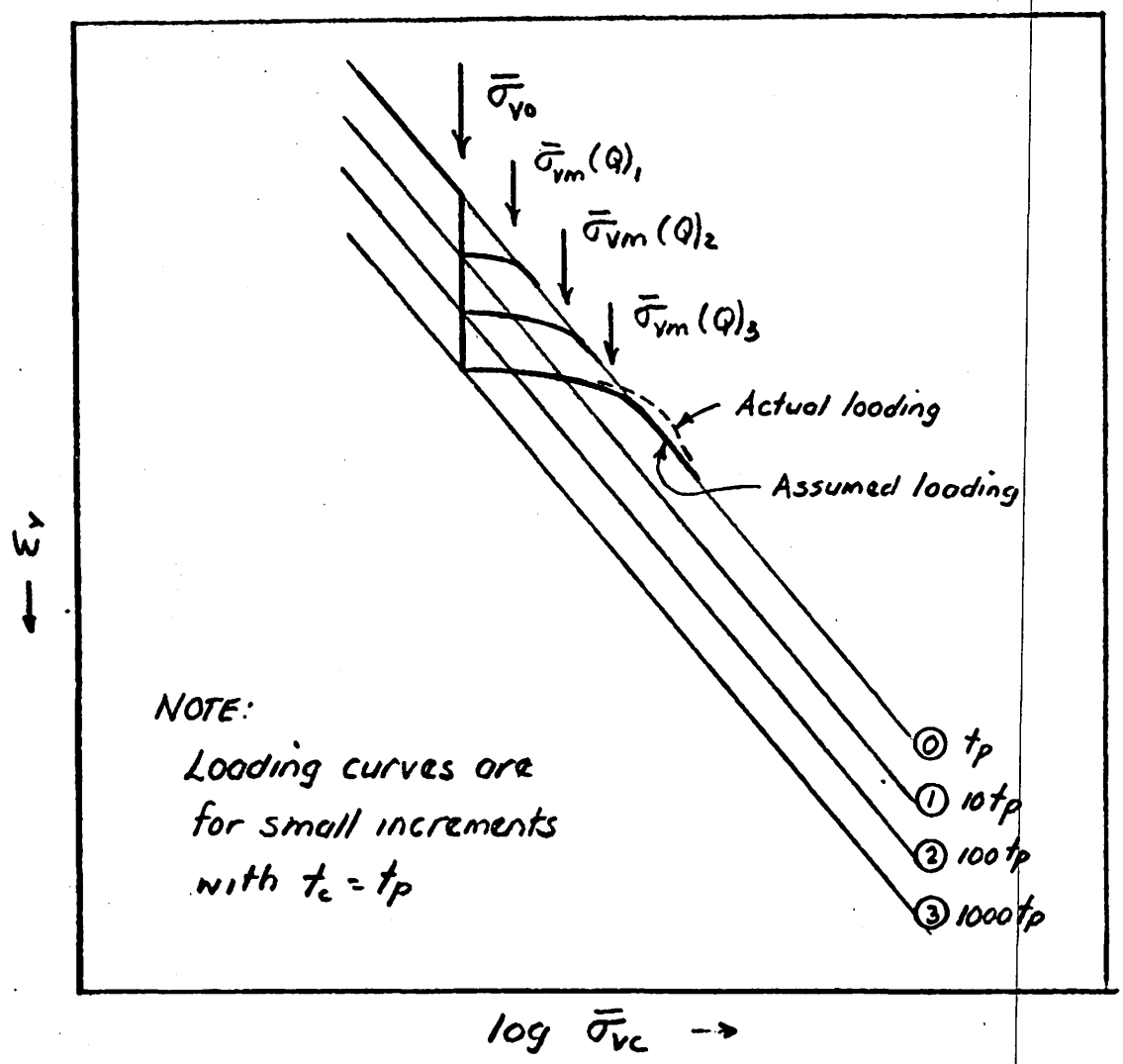


ILLUSTRATION OF WORKING HYPOTHESIS

Figure 4-4



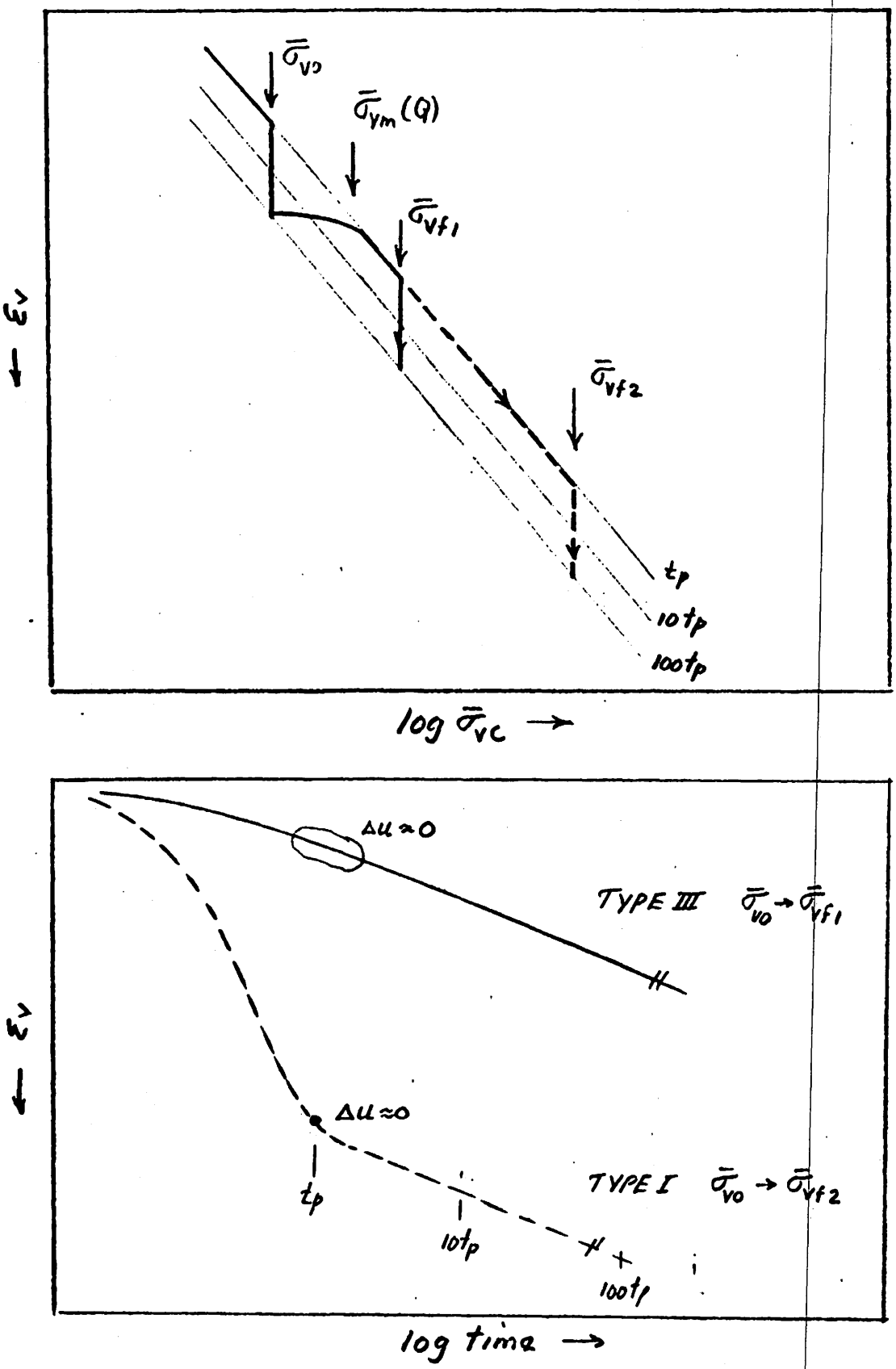
Eq.(4-3) $$\log\left(\frac{\bar{\sigma}_{vm}(Q)}{\bar{\sigma}_{vo}}\right) = \frac{C_\alpha}{CR}\log\left(\frac{t_c}{t_p}\right)$$

For $C_\alpha/CR = 0.05$

t_c/t_p	$\bar{\sigma}_{vm}(Q)/\bar{\sigma}_{vo}$
10	1.12
100	1.26
1000	1.41

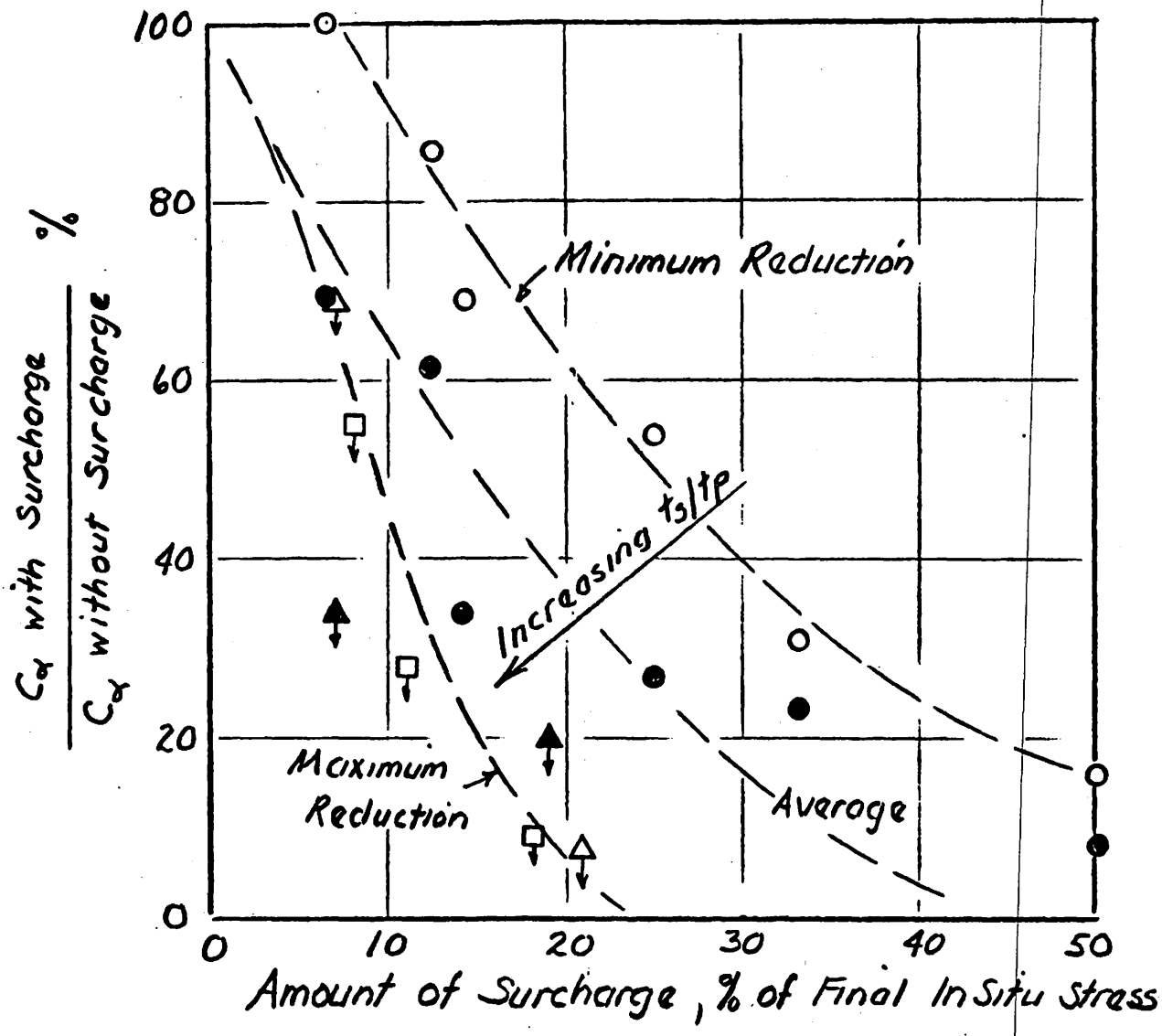
QUASI-PRECONSOLIDATION OF NORMALLY LOADED CLAY

Figure 4-5



EFFECT OF LOAD INCREMENT RATIO ON AMOUNT AND RATE OF COMPRESSION

Figure 4-6



Soil Type	Symbol	t_s/t_p	C_α, % (N.C. region)	Reference
Norwegian Marine CH Clay, $S_t = 6$, $S_u = 1-2.5$ TSM	○	0.8-1.0	1.4	Simons (1965)
	●	2-6	0.6-0.7	NGI Pub. 62
N.J. Varved Clay CL to CH	□	1 ±	2 ±	Johnson (1970)
Medium CH Clay with thin layers of sand	△	1-2	2.5	MPMR (1958)
	▲	>10 ±	3.5	

t_s = time of surcharge removal

t_p = end of primary

REDUCTION IN RATE OF SECONDARY COMPRESSION DUE TO SURCHARGING

Figure 4-7

V THREE DIMENSIONAL LOADING

5.1 Introduction

For the general case of a fairly rapid rate of loading on a saturated clay, the problem of estimating the amount and rate of settlement is far more complicated than for the one-dimensional case. As illustrated in Fig. 1-4:

(1) There are initial settlements due to undrained shear deformations. With soft clays, a significant portion of the clay may be stressed to failure, thus deformations due to contained plastic flow are very important. Undrained creep may also be significant.

(2) Consolidation settlements do not necessarily follow the K_o line. Moreover, the rate of settlement due to two or three dimensional flow becomes much more complicated.

(3) There will also be settlements due to secondary compression, the rate of which is dependent on how close the soil is to failure.

If the load is applied very slowly, so that no excess pore pressures develop, initial settlements are eliminated and the final total settlement will be less. However, the pattern of deformation and drainage is still very complicated. Figure 5-1 shows stress paths for several elements below the centerline of a circular footing during drained loading. At shallow depths (point 1), the applied stress ratio ΔK is greater than K_o and the soil undergoes vertical and lateral compression. At point 2 and below, ΔK is less than K_o and the soil undergoes vertical compression and lateral expansions.

The problem of estimating the amount and rate of settlement for a case of three-dimensional loading might be divided into the following steps:

(1) Develop the soil profile, with particular emphasis on identification of the compressible strata and the location of drainage layers.

(2) Determine the stress history (net overburden stress $\bar{\sigma}_{vo}$ and maximum past pressure $\bar{\sigma}_{vm}$) of the compressible strata. Chapter II discussed the problems associated with this step.

(3) Ascertain the loading conditions. This includes the geometry and magnitude of the load and its probable rate of application.

(4) Compute the increment of vertical stress due to the load. DM-7 presents charts for computation of vertical stresses for a variety of loading geometries. It also shows the influence on vertical stress of a two layer system. Lysmer and Duncan (1969) present a more comprehensive collection of charts and tables. The writer believes that the Boussinesq solutions are generally more appropriate than the Westergaard solutions, which assume zero horizontal strain. Vertical stresses from Boussinesq are independent of Poisson's ratio.

(5) Prepare a plot of $\bar{\sigma}_{vo}$, $\bar{\sigma}_{vm}$ and $\bar{\sigma}_{vf} = \bar{\sigma}_{vo} + \Delta\sigma_v$ versus depth or elevation within the compressible strata. In particular, compare the relative magnitudes of $\bar{\sigma}_{vm}$ and $\bar{\sigma}_{vf}$ to ascertain whether it is a case of recompression, virgin compression, or a combination of both.

(6) How one has to decide whether the problem is best treated as a case of drained loading or one of undrained loading followed by consolidation by estimating the average degree of consolidation at the end of loading. At this stage one can simply assume one-dimensional drainage unless the width of the loaded area B is less than one half of the height of the drainage path H_d. If B/H_d is less than 0.5, or if the deposit is a varved clay, the influence of horizontal drainage may be quite important (see Section 5.4).

Cases of drained three-dimensional loadings on saturated cohesive soils are probably quite rare, except for the situation where a relatively thin clay layer is encountered within a sand stratum. This case can be treated as if it were one-dimensional after accounting for the stress attenuation with depth below the loaded area.

In most practical cases some drainage will occur during loading. However, no theories are available which account for both initial and consolidation settlements occuring at the same time. Thus one must generally treat them separately, and then combine the results as if they were independent phenomena. Section 5.2 considers initial settlements, where the two most important variables are the value of the undrained Young's or shear modulus of the soil and the extent of local yielding within the foundation clay. Several methods of estimating the final consolidation settlement are presented in Section 5.3, while Section 5.4 discusses the effects of horizontal drainage on the rate of consolidation.

5.2 INITIAL SETTLEMENTS

5.2.1 Yielding

Figure 5-2 shows the factor of safety at which yielding first occurs as a function of the initial shear stress ratio, f. The relationship between the initial shear stress ratio and overconsolidation ratio is presented in Figure 5-3 for several clays.

For normally consolidated clays:

f = 0.6 - 0.8

F.S. at firstyield = 4 - 8

For slightly overconsolidated clays (OCR = 1.5 - 2):

f = 0.2 - 0.4

F.S. at first yield = 2 - 3

For heavily overconsolidated clays:

f = 0 $\pm$ 0.15

F.S. at first yield = 2 $\pm$ 0.4

Figure 1-4 presented stress paths for undrained loading followed by consolidation for a normally consolidated and a heavily overconsolidated clay where yielding did not occur. The effect of local yielding on the stress paths for an element of slightly overconsolidated clay located beneath the centerline of a loaded area is shown in Figure 5-4. When the TSP-μ_o reaches point a, the ESP hits the failure envelope (K_f line) and the undrained strength of the clay element is fully mobilized. Further loading of the tank causes a horizontal total stress path (i.e. $\Delta\sigma_v = \Delta\sigma_h$) because the clay element cannot take any further increase in shear stress. After yielding, the increment in excess

pore pressure equals $\Delta\sigma_v$, as shown in part (c) of the figure. During consolidation the TSP-μ_o goes along the 45 degree line from point b to d while the ESP follows some indeterminate path from c to d.

5.2.2 Initial Settlements Without Yielding

Computation of initial settlements without yielding can be computed using elastic displacement theory with an equation in the form of:

$$\rho_i = \frac{qBI}{E_u} \qquad (5\text{-}1)$$

where q = applied stress
B = width of loaded area
E_u = undrained Young's modulus of the soil
I = influence factor that depends on the geometry of the problem and Poisson's ratio = 0.50.

References for I include: Davis and Poulos (1968); Davis and Taylor (1961); Poulos (1967); and Ueshita and Meyerhof (1967). Lysmer and Duncan (1969) present a very useful collection of charts and tables for values of I from many sources.

See Poulos & Davis (1974) Book

The principal difficulty in using Eq. 5-1 lies in a proper evaluation of the undrained modulus E_u. Ladd (1964) discusses the many problems associated with determining E_u from laboratory tests. UU type tests will almost always greatly underestimate E_u because of sample disturbance. CU type tests are preferred, but many variables must be considered in using such tests. Because of these difficulties

empirical correlations have been proposed, such as E_u/s_u, where s_u is the undrained shear strength. Table 5-1 presents values of E_u/s_u backcalculated from several case studies. $E_u/s_u = 1500 \pm 500$ for low plasticity clays with OCR = 1 to 3. For clays of higher plasticity or more highly precompressed and for organic clays, the ratio is much lower.

The writer has found that values of E_u/s_u measured in consolidated-undrained direct-simple shear tests often yield reasonable estimates of the in situ values. Figure 5-5a plots E_u/s_u versus the applied shear stress level from $\overline{CK_oU}$DSS tests on several normally consolidated clays, where E_u is the secant modulus. These data show that the stress level is a very important variable which must be considered in selecting an appropriate value of E_u/s_u. Figure 5-5b shows how E_u/s_u varies with the overconsolidation ratio of the soil.

5.2.3 Initial Settlements With Yielding

Finite element programs have been developed to compute undrained deformations with contained plastic flow. The FEAST-3 program developed at M.I.T. considers the variables summarized in Fig. 5-6 (D'Appolonia and Lambe, 1970) Simon et al (1972) present a more versatile program which considers embankment stiffness.

The effect of yielding on initial settlement is illustrated in Fig. 5-7, where:

ρ_i = initial settlement

ρ_e = "elastic" settlement computed assuming no yielding.

For a normally consolidated clay with f = 0.75, yielding doubles the initial settlement at an applied stress q equal to one-half of the ultimate bearing capacity q_{ult} ; at 80 percent of the failure load, the settlement is increased by a factor 3.6. For a slightly overconsolidated clay with f = 0.25 (OCR = 1.8 to 2.0 from Fig. 5-3), yielding does not occur until q/q_{ult} exceeds 0.5. At 90 percent of the failure load, yielding increases the initial settlement by a factor of two.

D'Appolonia et al (1971) present charts, such as shown in Fig. 5-8, that can be used to estimate initial settlements, where:

$$\rho_i = \rho_e / S_R$$

S_R = function of the initial shear stress ratio and the factor of safety.

The procedure for using Fig. 5-8 to estimate ρ_i is summarized as follows:

(1) Perform an undrained stability analysis to obtain the ultimate bearing capacity and hence the applied stress ratio q/q_{ult} at the working load.
(2) Estimate the value of E_u taking into consideration the soil type, stress history, and general stress level.
(3) Compute the elastic settlement ρ_e using Eq. 5-1.
(4) Estimate the value of f, which is primarily a function of the overconsolidation ratio of the soil (see Fig. 5-3).
(5) Enter Fig. 5-8 with the appropriate values of f and q/q_{ult} to obtain the settlement ratio S_R.
(6) The initial settlement $\rho_i = \rho_e / S_R$

5.3 CONSOLIDATION SETTLEMENTS

5.3.1 Skempton-Bjerrum (1957) Method

A semi-empirical method is given where:

$\rho_{cf} = \mu\rho_{oed}$

ρ_{oed} = computed settlement based on oedometer tests assuming $\Delta\sigma_v$ during consolidation equals the applied $\Delta\sigma_v$

μ = correction factor to account for the fact that the actual $\Delta\sigma_v$ during consolidation is different from the applied $\Delta\sigma_v$.

Skempton and Bjerrum (1957) present a chart, redrawn in Fig. 5-9, relating μ to the pore pressure parameter A for circular and strip loads for several values of H/B. The correction factor μ decreases with decreasing A, i.e. the ratio ρ_{cf}/ρ_{oed} decreases as the clay becomes more overconsolidated. This occurs because the excess pore pressure after undrained loading is a smaller fraction of the applied $\Delta\sigma_v$ as the OCR increases. This concept is illustrated in Fig. 1-4, which shows that $\Delta u_i/\Delta\sigma_v$ is much smaller for the heavily overconsolidated clay than for the normally consolidated clay. Again, referring to Fig. 1-4, the authors assume that consolidation due to dissipation of Δu_i occurs along the K_o line, whereas consolidation actually occurs along the ESP from point b to c. See Lambe (1964) for a further discussion of this important concept.

Figure 5-9 suggests that the assumption that $\Delta u_i = \Delta\sigma_v$ will lead to a serious overestimate of the final consolidation settlement for overconsolidated clays. Skempton and

Bjerrum (1957) present several case studies which support their method. For example, four case studies involving overconsolidated clays with measured ρ_{cf} = 0.5 to 2.8 in. showed:

$$\rho_{oed}/\rho_{cf} = 1.85 \text{ (1.54 to 2.55)}$$

$$\mu\rho_{oed}/\rho_{cf} = 0.95 \text{ (0.78 to 1.28)}$$

In contrast, four case studies with normally consolidated clays with measured ρ_{cf} = 7 to 19 in. showed:

$$\rho_{oed}/\rho_{cf} = 1.12 \text{ (0.98 to 1.28)}$$

$$\mu\rho_{oed}/\rho_{cf} = 0.99 \text{ (0.79 to 1.14)}$$

Hwang et al (1972) developed a finite element method for solving the Biot Theory of consolidation wherein the initial excess pore pressures are computed in terms of the pore pressure parameter A. The results for H/B = 2 and Poisson's ratio = 0 and 0.4, are plotted in Fig. 5-9. Their data generally support the Skempton-Bjerrum approach.

5.3.2 Lambe Stress Path Method

Lambe's (1964, 1967) stress path method is now widely known and being increasingly used. However, there are still few well documented case studies comparing the various methods.

Information, such as that shown in Fig. 5-10, is needed to show the importance of ΔK on the magnitude of vertical strains during consolidation.*

---

* See Simmons and Som (1969) for an interesting study of the effect of ΔK on axial and volumetric strains of undisturbed overconsolidated London Clay.

The concept is extremely valuable in understanding what is happening during loading and consolidation, even when stress path tests are not used.

5.3.3 Davis-Poulos (1963, 1968) Method

Stress path tests, such as recommended by Lambe, are run but the data are used to compute elastic constants that are then used in conjunction with the theory of elasticity to estimate consolidation settlements.

Davis and Poulos (1968) also present charts showing the relationships among initial settlement, final consolidation settlement, and the conventional I-D consolidation settlement for a variety of loading geometries and values of Poisson's ratio. These relations assume an ideal linear elastic soil skeleton, and thus must be used with caution. In particular, the derived relationship

$$E_u = \frac{3\bar{E}}{2(1+\bar{\nu})}$$

is likely to be in serious error with sensitive clays. However, the charts may give an upper limit of the effects of non-I-D conditions.

5.3.4 Discussion

Several theoretical methods are available for the computation of the final consolidation settlement for three dimensional loadings based on the assumption that the soil is a linear elastic isotropic material (Davis and Poulos, 1968; Christian et al, 1972; Hwang et al, 1972). These

methods require a knowledge of Young's modulus and Poisson's ratio in terms of effective stresses. Because soil is not a linear elastic isotropic material, the measured values of these "elastic" constants are highly dependent upon the test procedures used. In the writer's opinion, any approach using elastic constants is probably liable to greater errors than a semi-empirical approach such as proposed by Skempton and Bjerrum (1957). However, as shown in Fig. 5-9, the correction factor μ is highly dependent on the pore pressure parameter A. In turn, the value of A is greatly influenced by the stress history (OCR) of the soil, the stress level (percentage of failure stress), and the stress system (value of intermediate principal stress and rotation of principal planes). The values of A are best studied via stress path tests or, preferably, based on prior field data.

5.4 RATE OF CONSOLIDATION

See Schiffman, Chen, and Jordan (1969) for a review, analysis and comparison of various three-dimensional consolidation theories.

Davis and Poulos (1972) present approximate procedures for handling circular and strip loading; and Hwang et al (1972) developed a finite element program for a strip load on an elastic layer with isotropic permeability. Christian et al (1972) developed a finite element program CONSOL that can be used to study plane strain and axisymmetrical consolidation with anisotropic permeabilities.

Christian et al (1972) made parametric studies of the effects of goemetry and the ratio k_h/k_v on the rate of consolidation under a strip load. Some of the results are summarized below for a strip load of width B placed on a stratum of thickness H with single drainage at the top:

B/H	$\bar{U}$ at Centerline	$k_v/k_h=1$	$\frac{\text{Time Computed with Horizontal Drainage}}{\text{Time Computed with 1-D Drainage Only}}$ % 5	10	16
1/2	50%	46	21	14	10
	90%	71	43	32	26
1	50%	71	36	24	17
	90%	80	52	39	32

These numbers show that horizontal flow is very important for a layer with top drainage only, especially when $k_h/k_v = 10 \pm 5$, as is probably the case for many varved clays.

The effect is less pronounced for a layer with both top and bottom drainage; as illustrated by the data in Fig. 5-11. For the case shown:

k_h/k_v	Effect Compared to 1-D Drainage Only
1	None
10	Double rate
100	Increase rate by factor of ten

5.5 MISCELLANEOUS

Johnson (1970a) and (1970b) presents an excellent treatment and review of the use of preloading and of the use of vertical sand drains.

Ladd and Wissa (1970) present fairly extensive data on varved clay deposits, especially those encountered in the Connecticut Valley.

Ladd et al (1972) present a case study involving embankments constructed on a very sensitive deposit of marine clay.

CASE STUDIES OF INITIAL SETTLEMENT

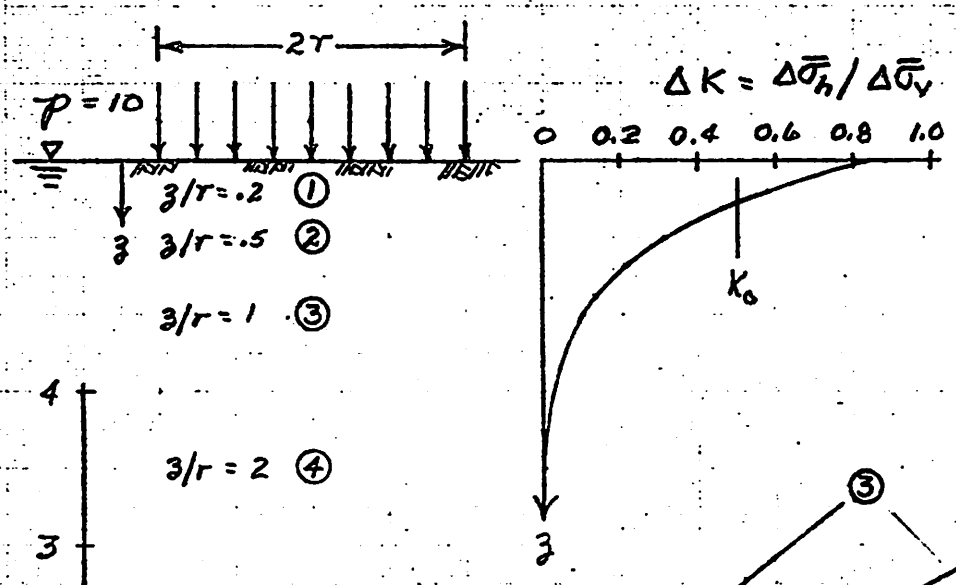
Case No.	Location and Structure	Clay Properties			$(E_u)_{field}$	E_u/S_u	Source of S_u	Reference
		PI %	S_t	OCR	(t/m^2)			
1	Oslo - 9 story bldg.	15	2	3.5	7600	1200	CIU	Simons (1963)
2	Asrum I - Circular load Test	16	100	2.5	990	1000 1200	Field Vane CIU	Höeg et al (1969)
3	Asrum II - Circular load Test	14	100	1.7	880	1000 1100	Field Vane CIU	Höeg et al (1969)
4	Mastemyr - Circular load Test	14		1.5	1300	1200 1700	Field Vane Bearing Capacity	Clausen (1969)
5	Portsmouth - Highway embank.	15	10	1.3	3000	2000 1700	Field Vane Bearing Capacity	Haley & Aldrich, Inc. Ladd (1969)
6	Boston - Highway embank.	24	5	1.5 1.0	10,000 13,000	1600 1200 2500 1500	Field Vane CK_oU Field Vane CK_oU	MIT
7	Drammen - Circular load Test	28	10	1.4	3200	1400 1100	Field Vane CK_oU	NGI
8	Kawasaki - Circular load Test	38	6±3	1.0	2200	400	Field Vane CIU	MIT
9	Venezuala - Oil tanks	37	8±2	1.0	5000	800	CIU	Lambe (1962)
10	Maine - Rectangular load Test	33 2	4	1.5-4.5	100-200	80-160	UU & Bearing Capacity	Ladd et al (1969)

From D'Appolonia, Poulos and Ladd (1971)

Table 5-1

CCL 5/25/16

Stress Paths Below a Circular Footing



Stresses from theory of elasticity for halfspace with:

Poisson's ratio = 1/3

$K_o = 0.5$ $\gamma_b = 1.00$

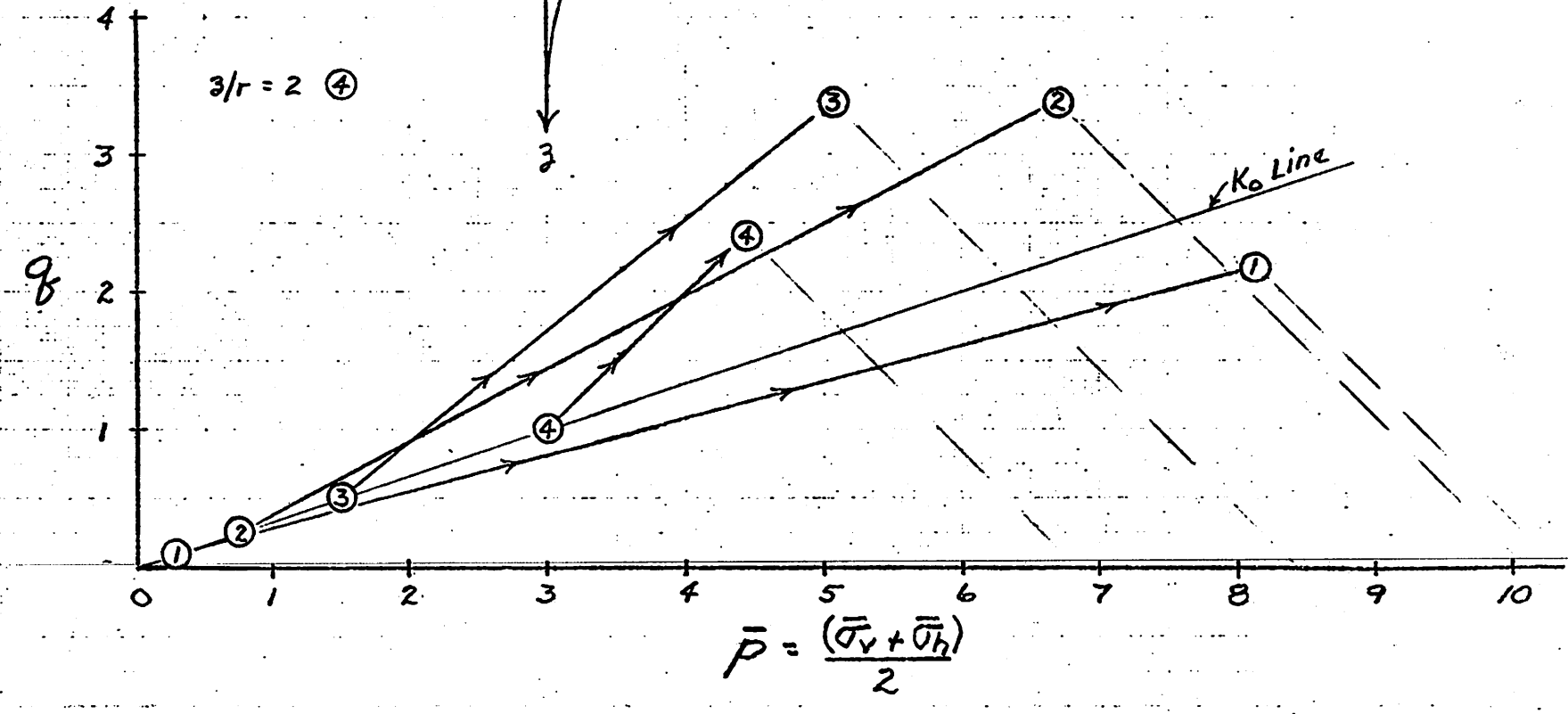
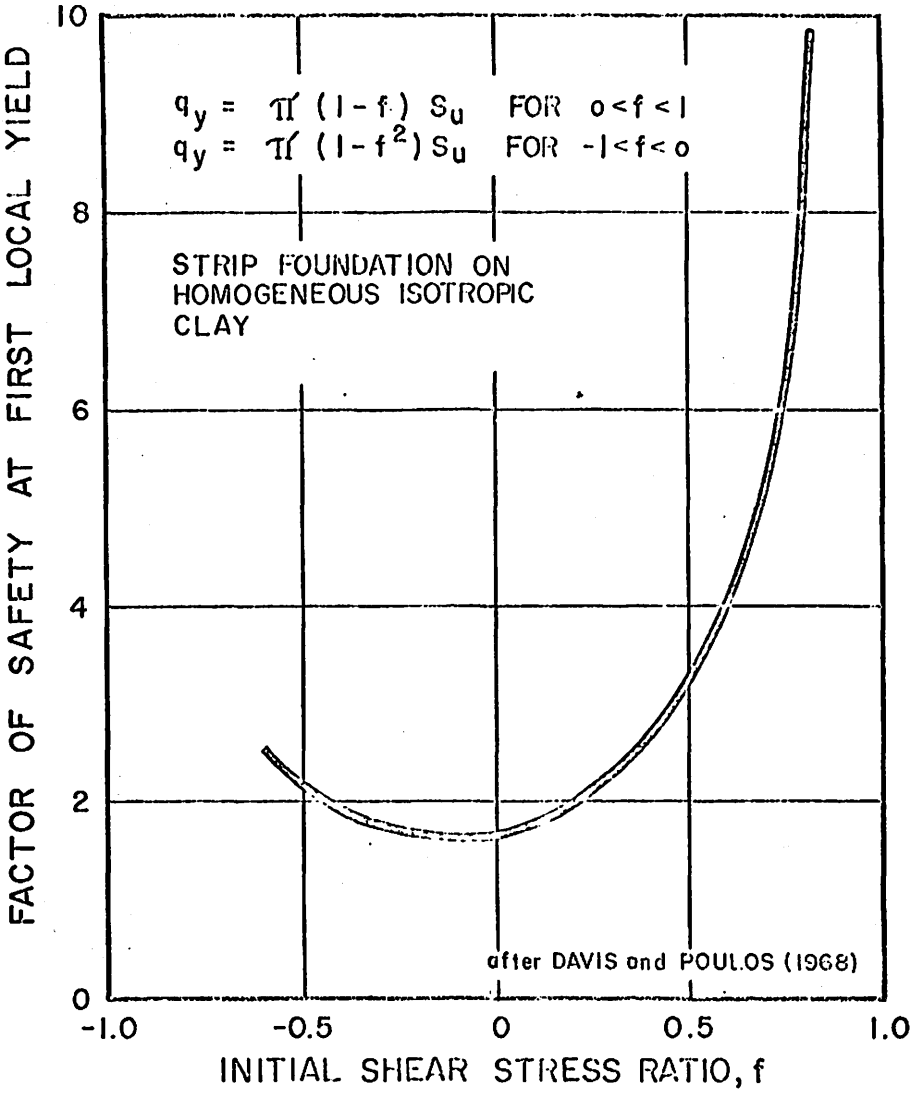


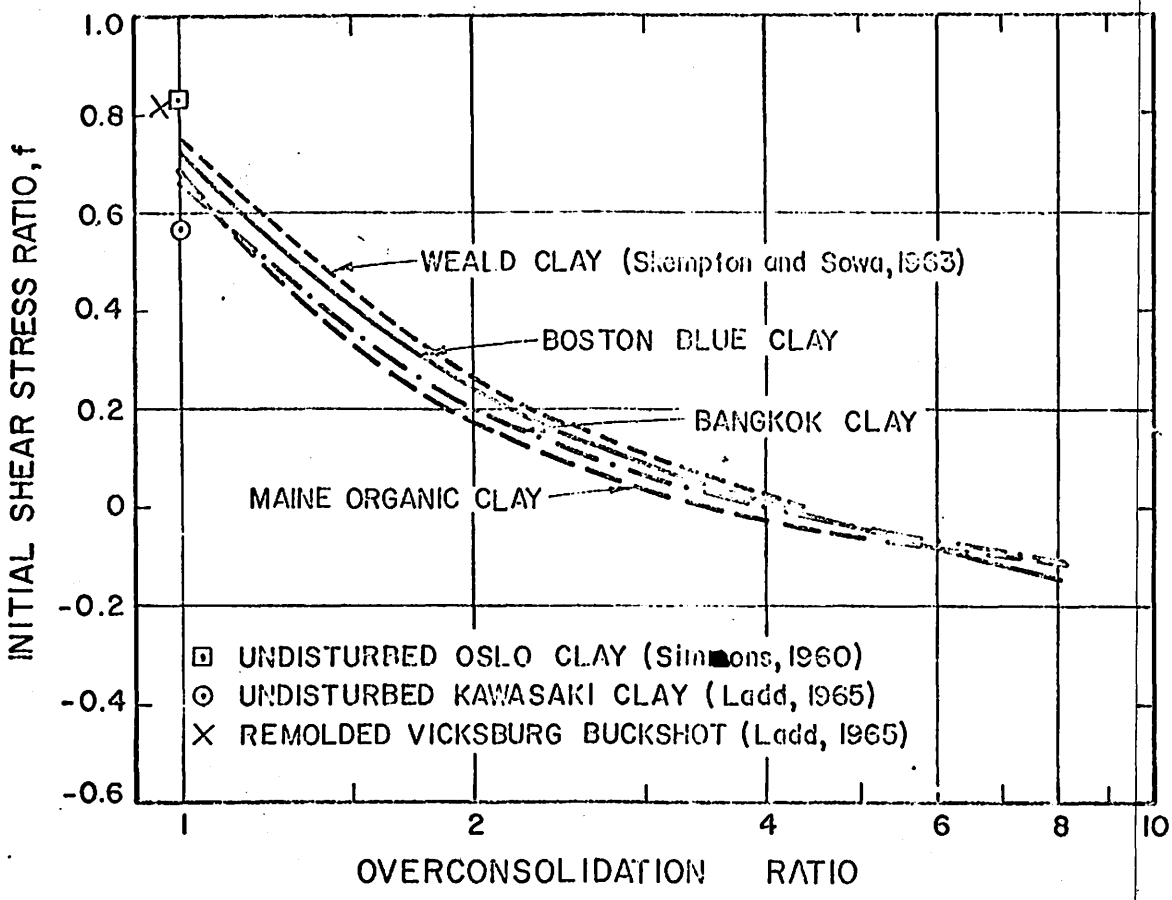
Figure 5-1



FACTOR OF SAFETY AT WHICH FIRST LOCAL YIELD OCCURS UNDER A STRIP FOUNDATION

$$f = \frac{(1-K_o)\,\bar{\sigma}_{vo}}{2 s_u}$$

Figure 5-2



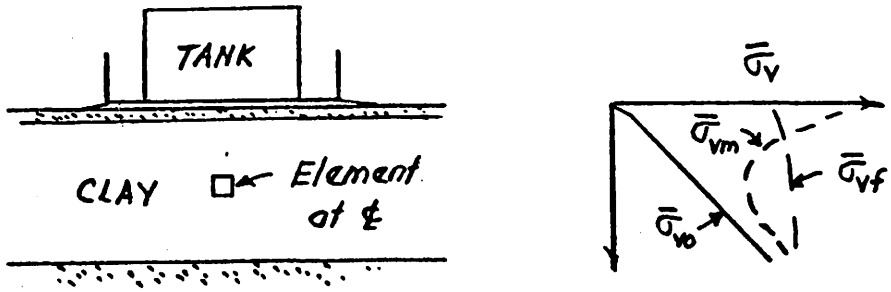
Note: The relationships presented for the Bangkok clay and the Maine organic clay based on values of K_o estimated from the correlations of Booker and Ireland (1965).

RELATIONSHIP BETWEEN INITIAL SHEAR STRESS AND OVERCONSOLIDATION RATIO

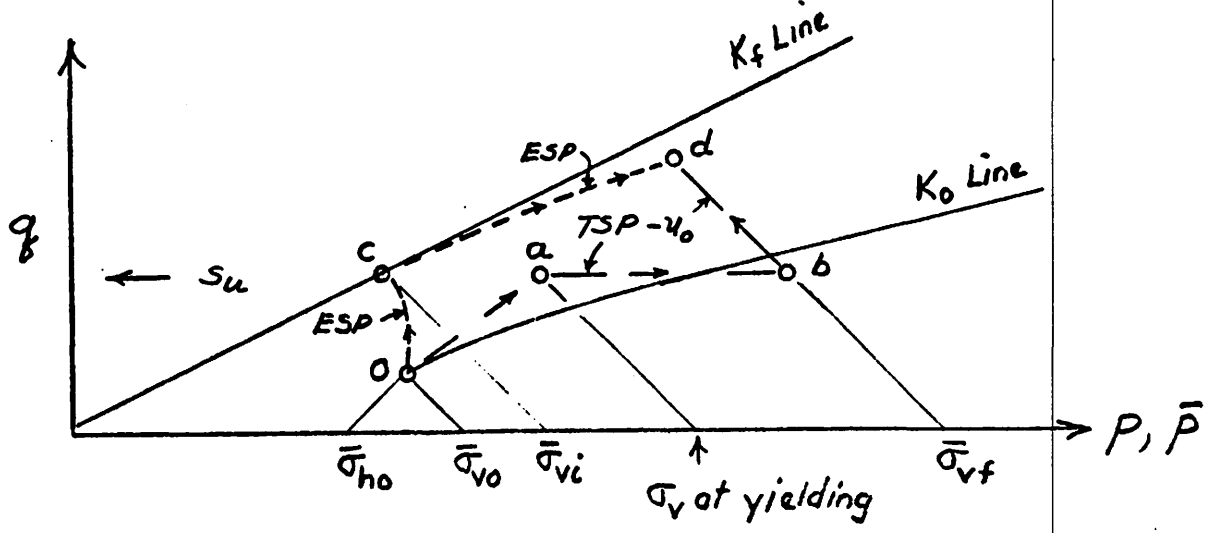
From D'Appolonia, Poulos and Ladd (1971)

Figure 5-3

(a) Field Situation



(b) Stress Paths



Undrained Loading

— — TSP-u_o o → a (yielding) → b

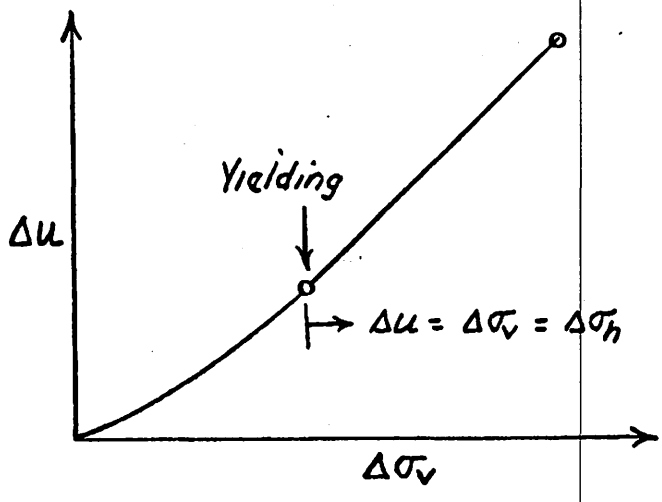
- - - - - ESP o → c (yielding)

Consolidation

— — TSP-u_o b → d

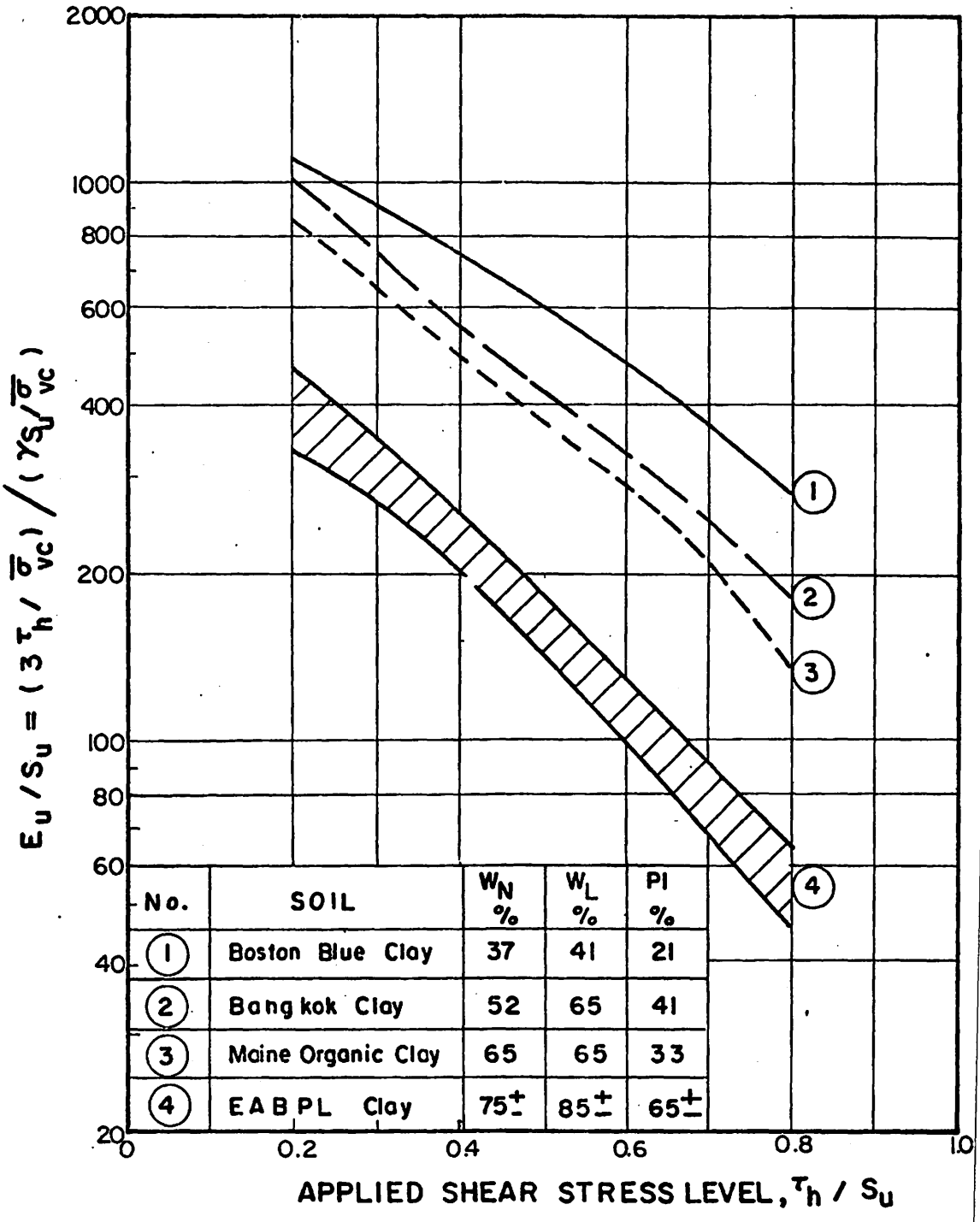
- - - - - ESP c → d

(c) Δu vs $\Delta\sigma_v$ During Loading



STRESS PATHS FOR A YIELDED ELEMENT OF SLIGHTLY OVERCONSOLIDATED CLAY

Figure 5-4



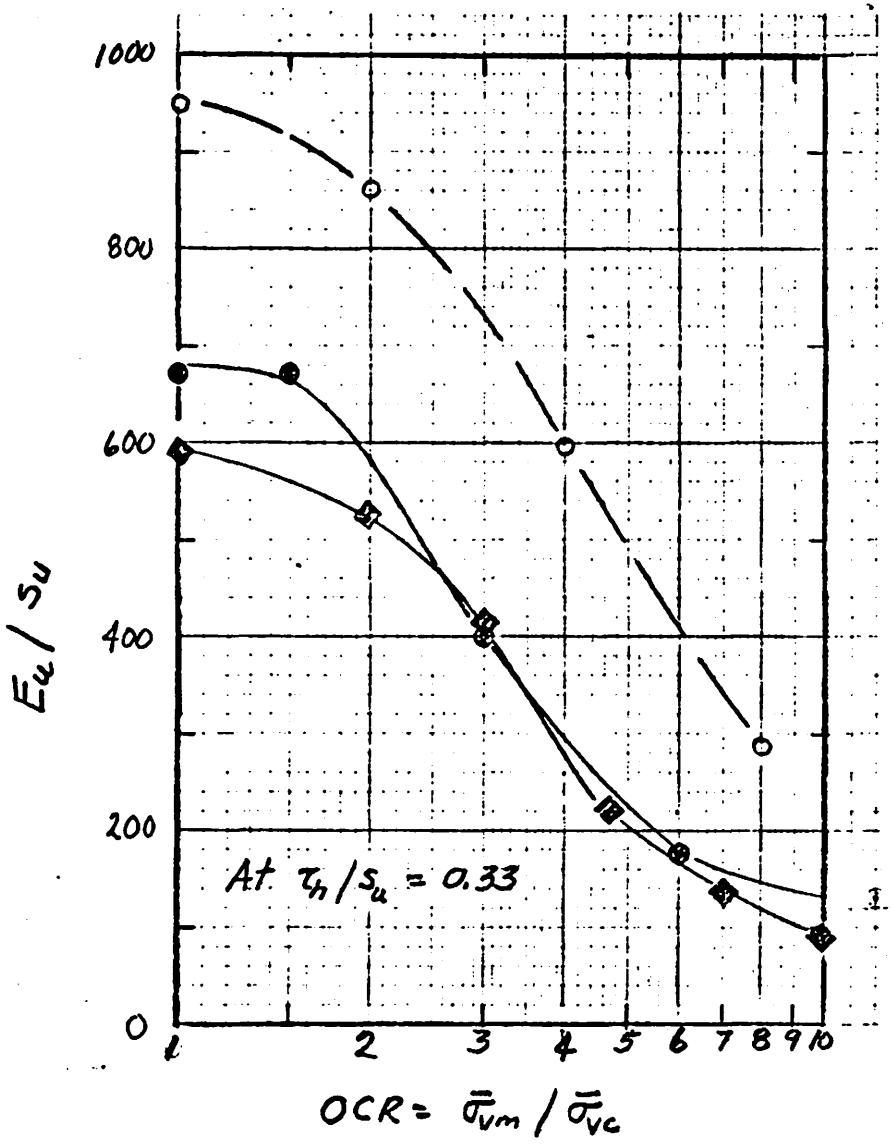
No.	SOIL	W_N %	W_L %	PI %
1	Boston Blue Clay	37	41	21
2	Bangkok Clay	52	65	41
3	Maine Organic Clay	65	65	33
4	EABPL Clay	75^{+}_{-}	85^{+}_{-}	65^{+}_{-}

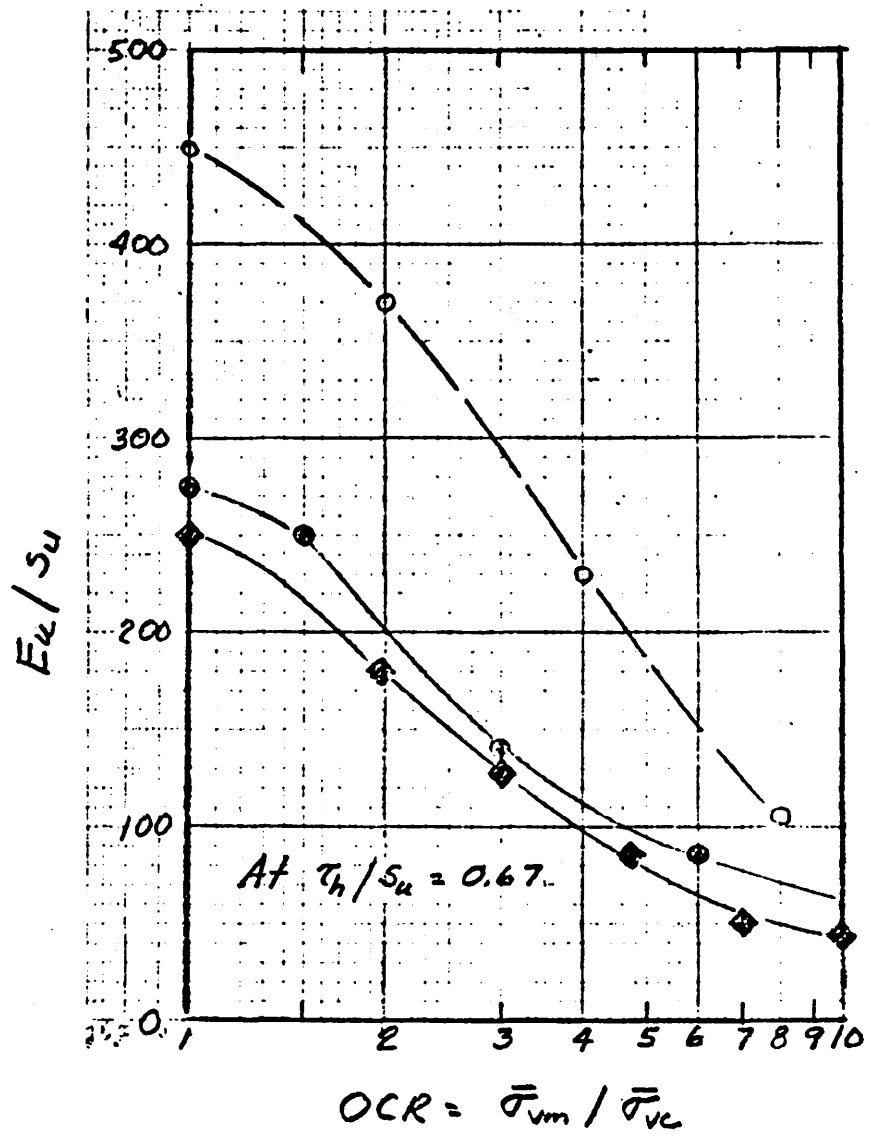
τ_h = Horizontal Shear Stress

γ = Shear Strain

E_u / S_u VERSUS APPLIED SHEAR STRESS LEVEL FROM $\overline{CK_oUDSS}$ TESTS ON SEVERAL NORMALLY CONSOLIDATED CLAYS

Figure 5-5a



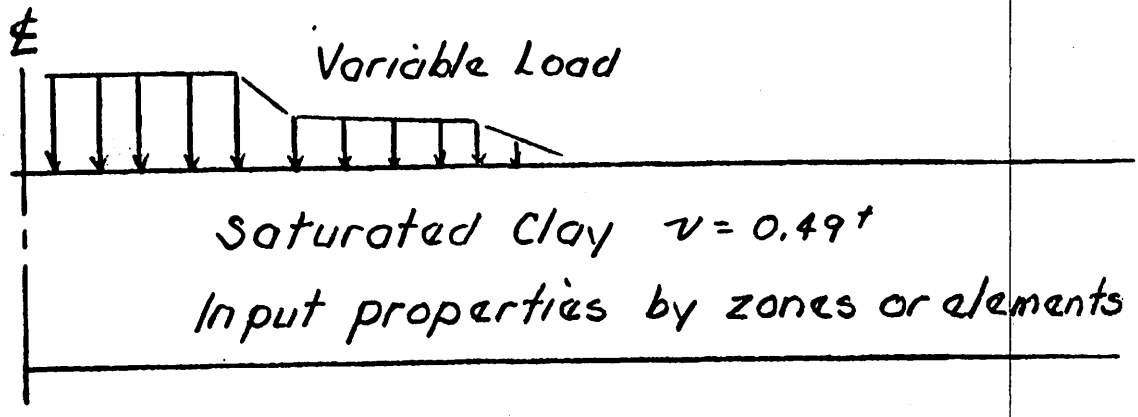


Symbol	Soil
○	Boston Blue Clay
●	Bangkok Clay
◆	Maine Organic Clay

Fig. 5-5(9)

E_u/s_u vs OCR from $\overline{CK_oU}$ Direct-Simple Shear Tests

FEAST-3 Program for Undrained Loading



Initial Stresses	Undrained Strength	Bi-Linear Stress-Strain	Pore Pressure
γ_b or $\bar{\sigma}_{vc}$	$s_u(V)$	E_u	a & k
K_o	$K_s = \frac{s_u(H)}{s_u(V)}$	E_Y	$\Delta u = \Delta\sigma_{oct}$
	$s_u(\beta)$ function		$+ a\,\Delta\tau_{oct}^{k}$

Relative Importance of Variables

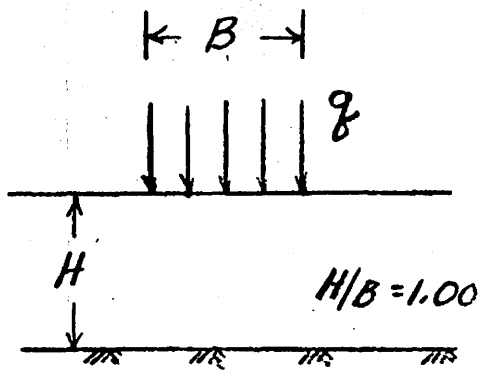
E_u : Deformation proportional to $1/E_u$

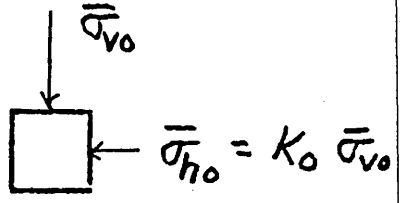
K_o & $s_u(V)$: Very important at low OCR (high f) since yielding is a controlling factor. Less important as OCR increases (f decr.)

K_s : Important at low factors of safety

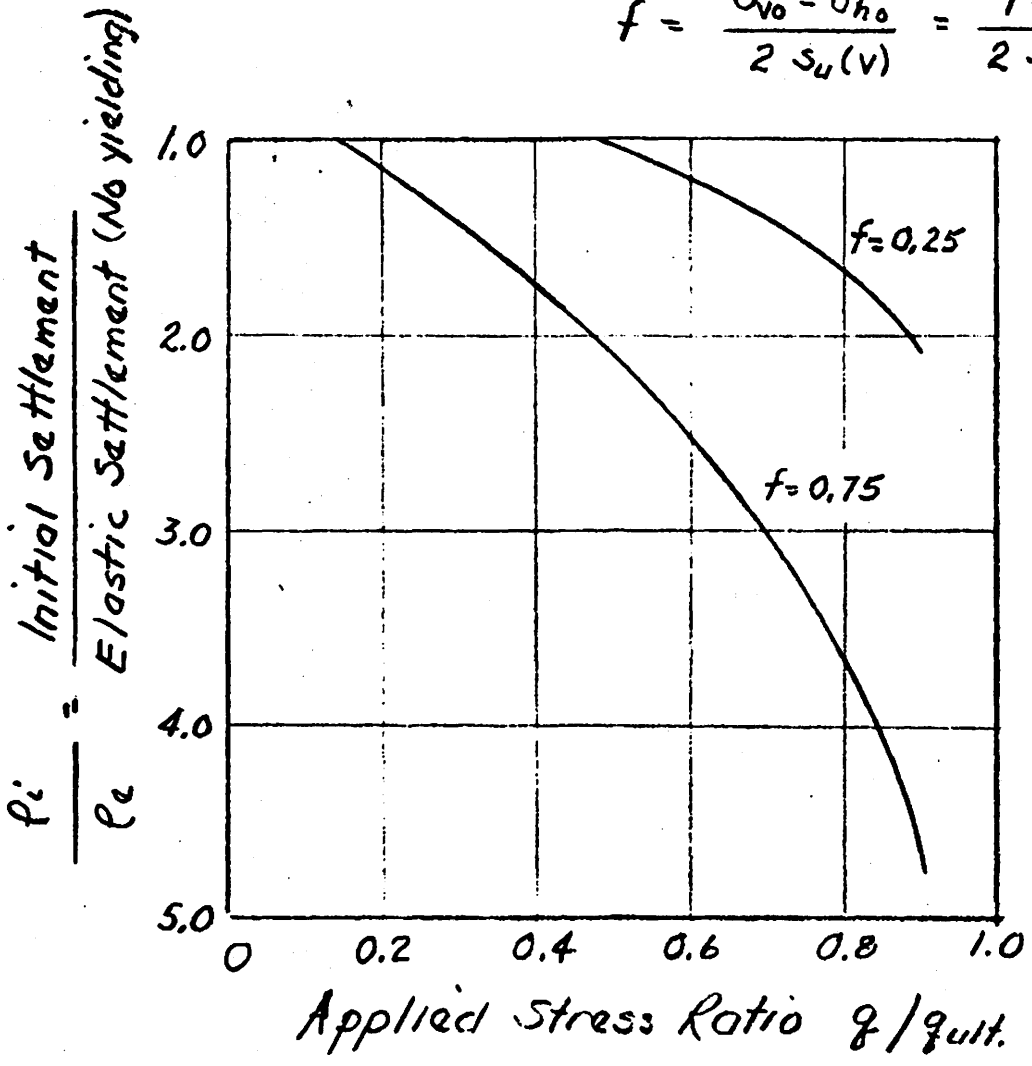
Figure 5-6

EFFECT OF YIELDING ON INITIAL SETTLEMENT





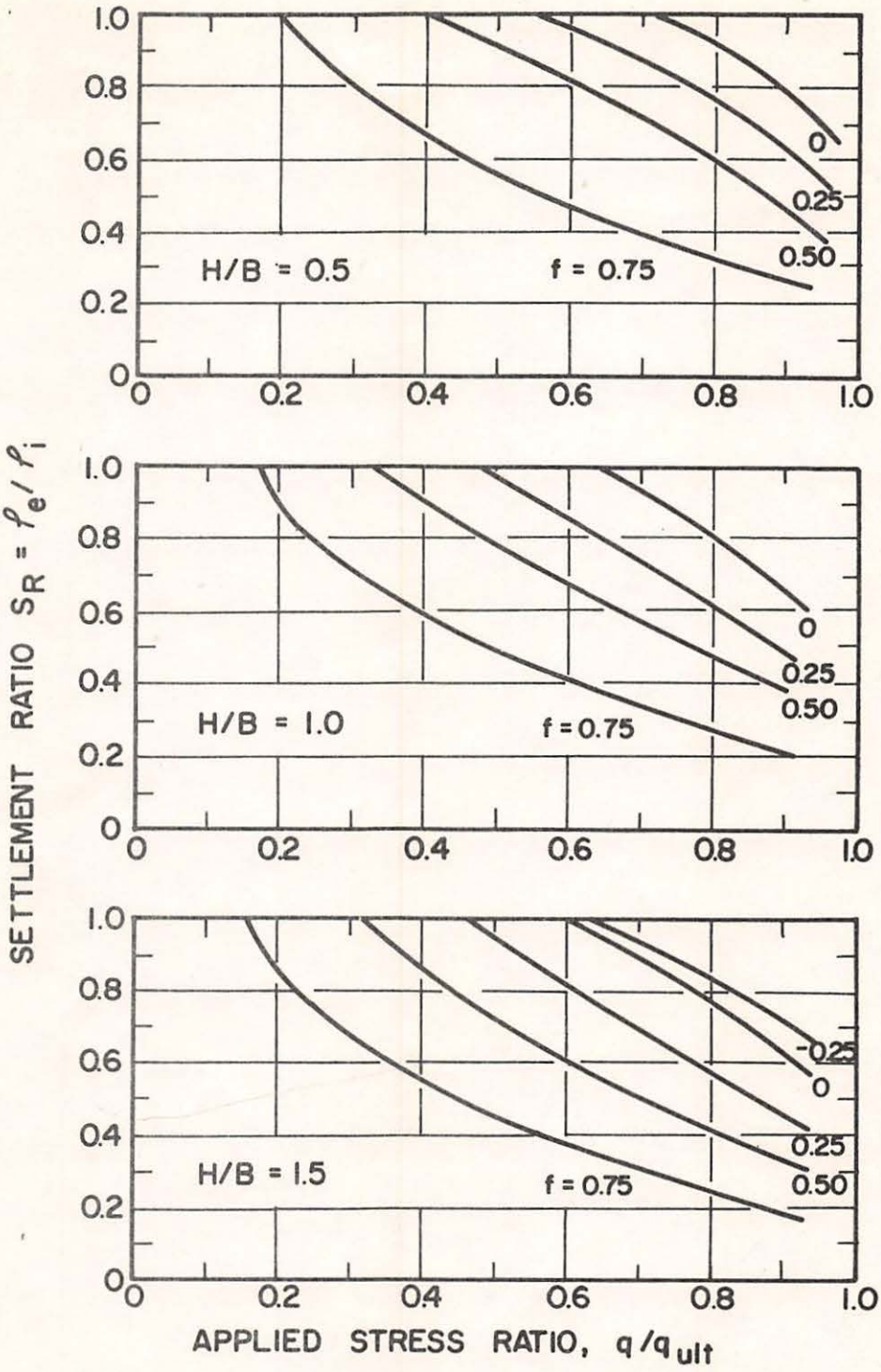
$$f = \frac{\bar{\sigma}_{vo} - \bar{\sigma}_{ho}}{2\, s_u(v)} = \frac{1 - K_o}{2\, s_u(v)/\bar{\sigma}_{vo}}$$



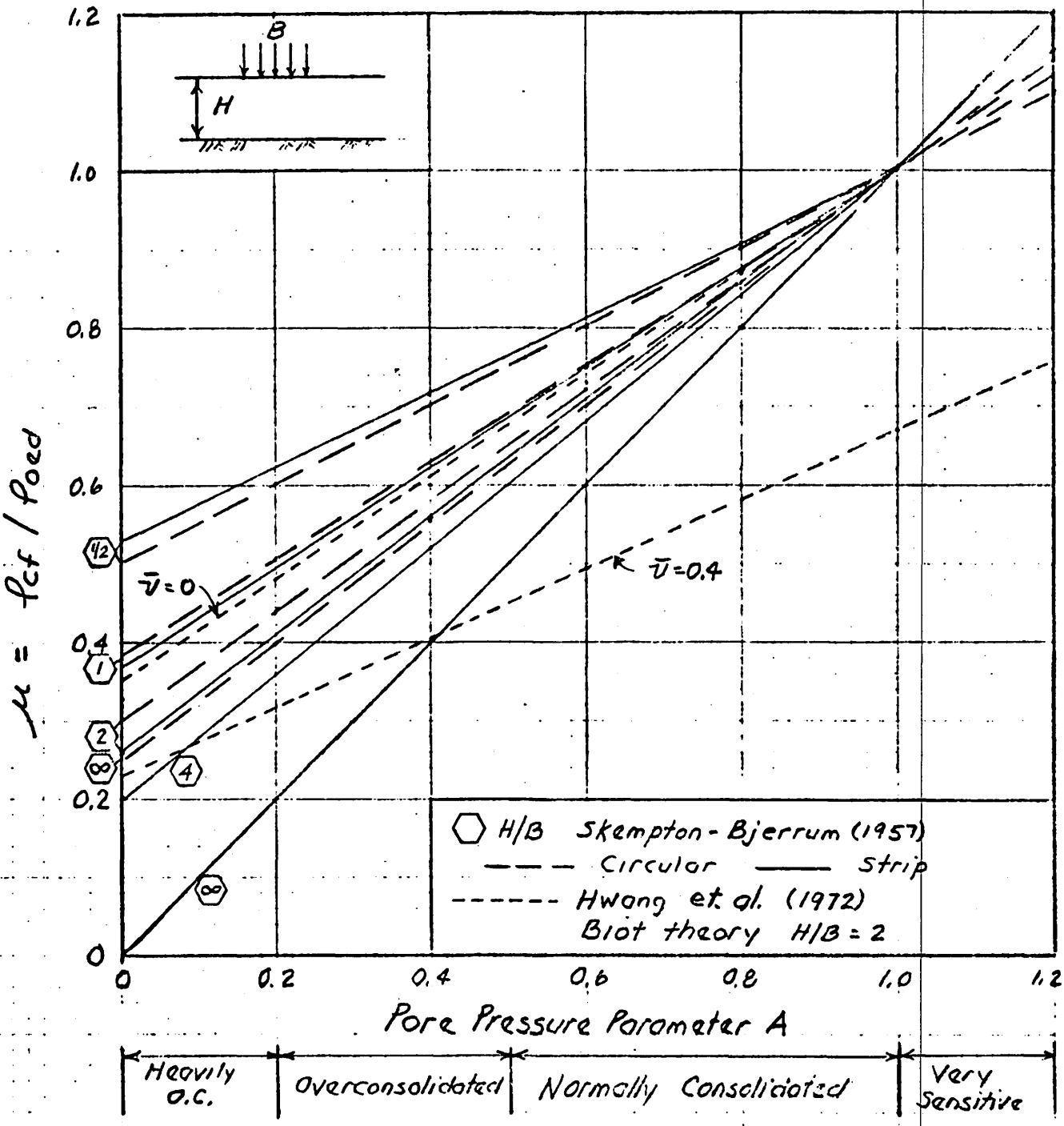
NOTES: (1) Settlement at centerline

(2) $K_s = s_u(H)/s_u(v) = 1.0$

Figure 5-7



SETTLEMENT RATIO VS. APPLIED STRESS RATIO FOR STRIP LOAD ON ISOTROPIC HOMOGENEOUS FOUNDATION (From D'Appolonia, Poulos and Ladd, 1971) FIGURE 5-8



Meyerhoff (1953) Geot. Vol. 8 No. 2 p 101

$\mu = 1 - \frac{3(1-A)}{4+B/H}$ for circular or square areas

INFLUENCE OF A PARAMETER AND GEOMETRY ON THREE DIMENSIONAL VS ONE DIMENSIONAL FINAL CONSOLIDATION SETTLEMENT

Fig. 5-9 ccc ρ Notes

Fig. 5-9

Influence of Stress Path on Vertical Strain During Consolidation of Boston Blue Clay

(from Guertin, 1967)

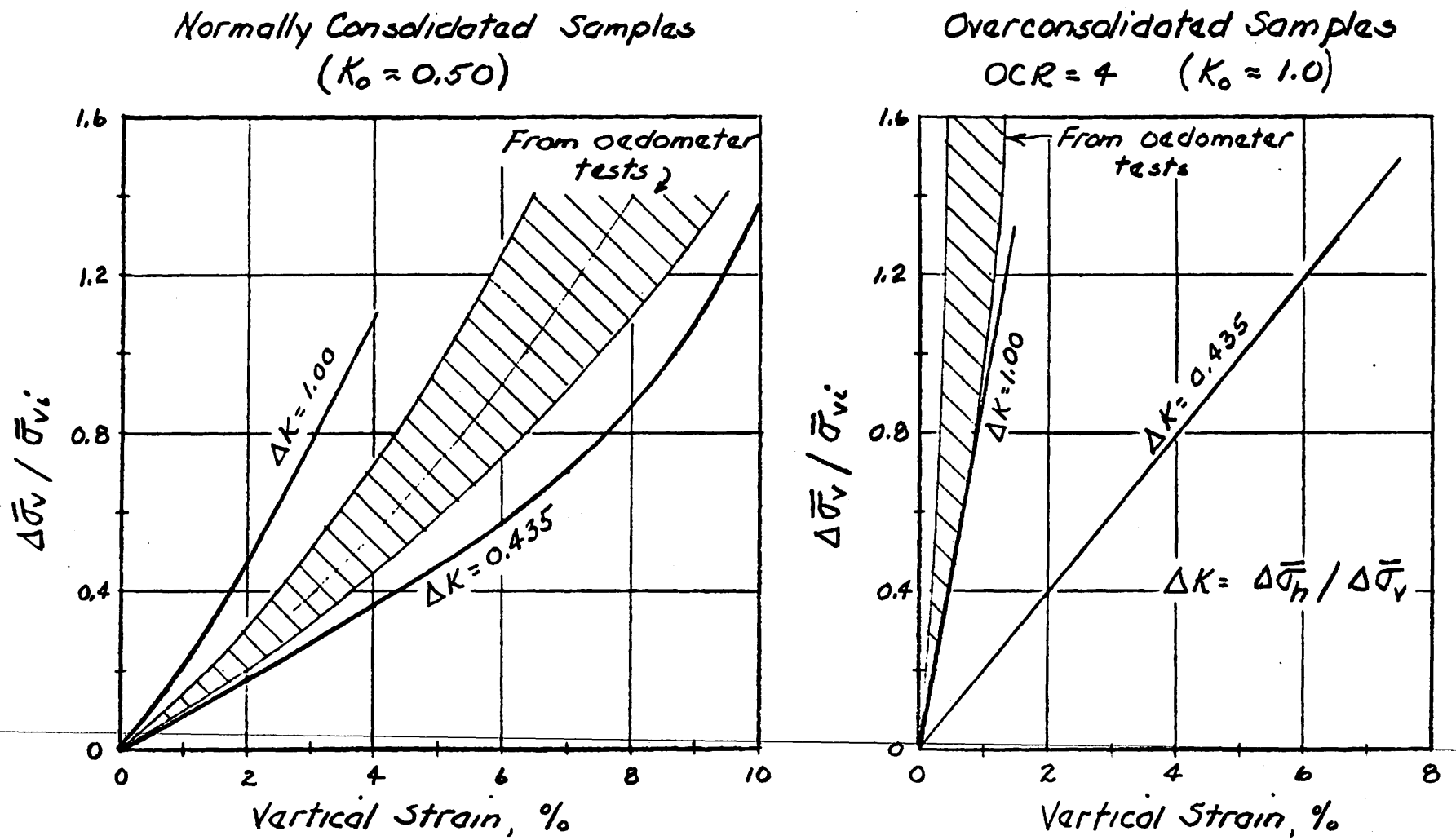
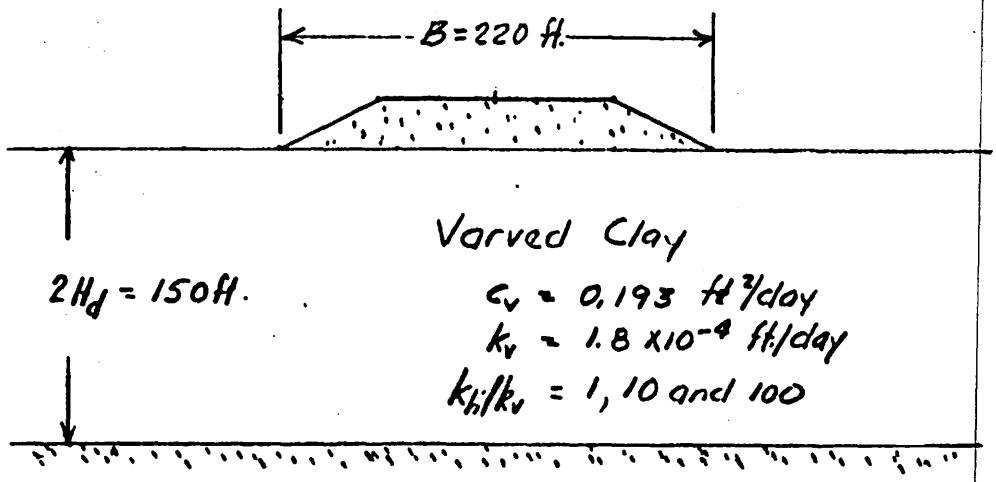
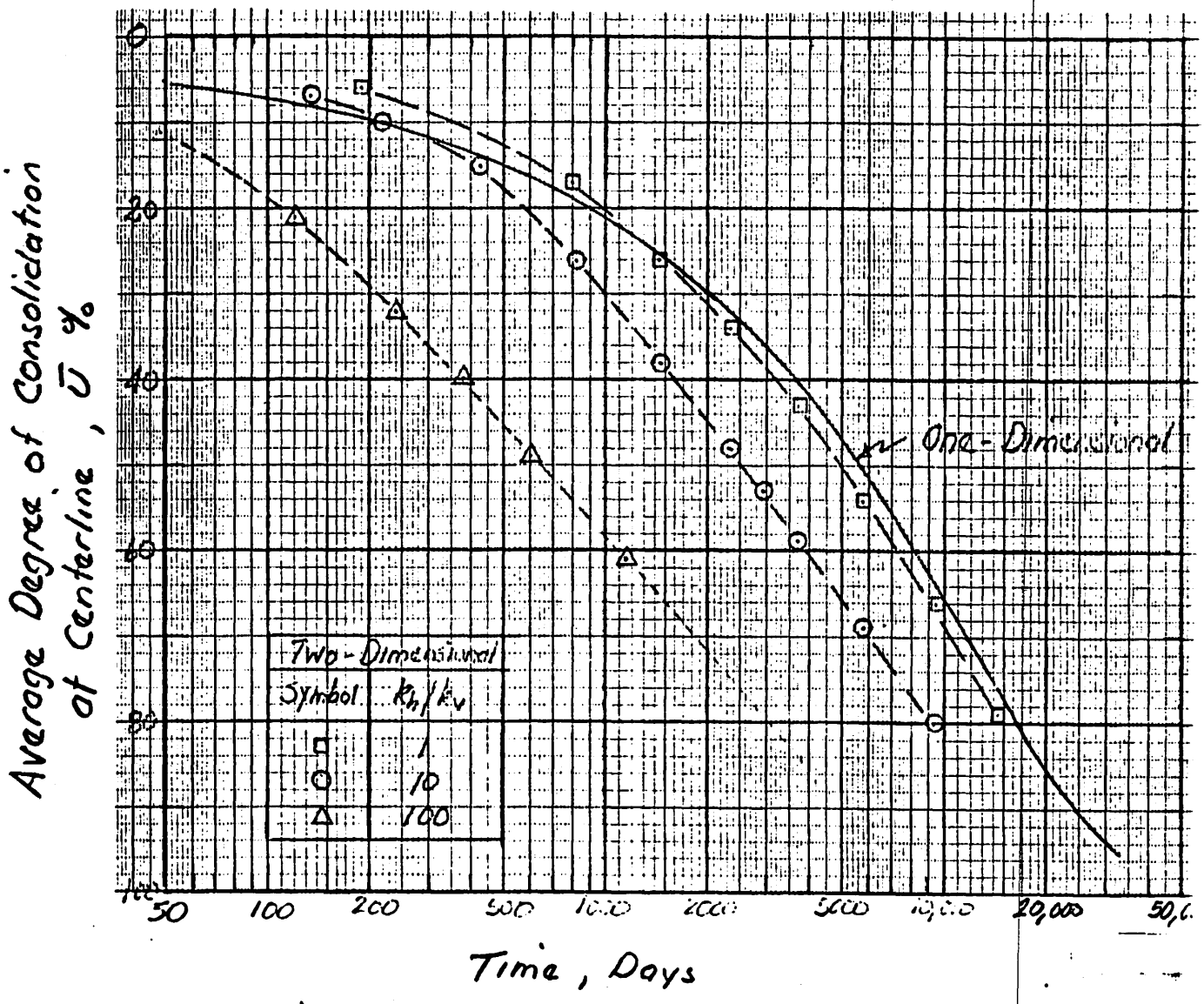


Figure 5-10





Note: Two-dimensional cases obtained by M.I.T.'s finite element program CONSOL

EFFECT OF LATERAL DRAINAGE ON RATE OF CONSOLIDATION (from Ladd & Wissa, 1970)

Figure 5-11.

APPENDIX A NOTATION

Note: Suffix f indicates final or failure condition
Prefix Δ indicates a change
A bar over a stress indicates an effective stress

STRESSES AND STRESS RATIOS

A	Skempton's pore pressure parameter = $(\Delta u-\Delta\sigma_3)/(\Delta\sigma_1-\Delta\sigma_3)$
Au	A for unloading = $(\Delta u-\Delta\sigma_h)/(\Delta\sigma_v-\Delta\sigma_h)$
B	Skempton's pore pressure parameter = $\Delta u/\Delta\sigma_c$
f	Initial shear stress ratio = $(1-K_o)/(2s_u/\bar{\sigma}_{vo})$
K	$\bar{\sigma}_h/\bar{\sigma}_v$
K_o	Coefficient of earth pressure at rest = K for no lateral yield
OCR	Overconsolidation Ratio = $\bar{\sigma}_{vm}/\bar{\sigma}_{vc}$
p	$\frac{1}{2}(\sigma_v+\sigma_h) = \frac{1}{2}(\sigma_1+\sigma_3)$
$\bar{p}$	p in terms of effective stress
q	$\frac{1}{2}(\sigma_v-\sigma_h)$
q_y	q at first yield
q_{ult}	Ultimate bearing capacity
u	Pore pressure
u_i, u_o	Initial pore pressure
τ	Shear stress
τ_{oct}	Octahedral shear stress
σ, $\bar{\sigma}$	Total, effective normal stress

σ_c	Confining stress
$\bar{\sigma}_c$	Consolidation (K=1) stress
$\bar{\sigma}_h$, $\bar{\sigma}_v$	Horizontal, vertical $\bar{\sigma}$
$\bar{\sigma}_{hc}$, $\bar{\sigma}_{vc}$	$\bar{\sigma}_h$, $\bar{\sigma}_v$ at consolidation
$\bar{\sigma}_{ho}$, $\bar{\sigma}_{vo}$	$\bar{\sigma}_h$, $\bar{\sigma}_v$ initially or in situ
$\bar{\sigma}_{vm}$	Maximum past pressure
$\bar{\sigma}_{vm}(Q)$	Quasi-preconsolidation pressure
$\bar{\sigma}_{ps}$	Isotropic $\bar{\sigma}$ after "perfect sampling".
σ_{oct}	Octahedral normal stress
σ_1, σ_2, σ_3	Principal stresses

CONSOLIDATION, SETTLEMENT AND PERMEABILITY PARAMETERS

a_v	Coefficient of compressibility $= -\Delta e/\Delta\bar{\sigma}_{vc}$
m_v	Coefficient volume change $= \varepsilon_v/\Delta\bar{\sigma}_{vc} = a_v/(1+e_o)$
C	Compression index $= \Delta e/\Delta \log \bar{\sigma}_{vc}$
C_c	Virgin compression index
C_r	Recompression index
C_s	Swelling index
CR	Compression Ratio $= C_c/(1+e_o)$
RR	Recompression Ratio $= C_r/(1+e_o)$
SR	Swelling Ratio $= C_s/(1+e_o)$
c_v	Coefficient of consolidation $= k_v/(m_v\gamma_w)$
c_h	Coefficient of consolidation for horizontal flow

C_α	Rate of secondary compression = $\Delta \varepsilon_v / \Delta \log t$
k	Coefficient of permeability
k_h, k_v	k for horizontal, vertical flow
R_s	Rate of secondary compression = $\Delta e / \Delta \log t$
S_R	S_R = Settlement Ratio = ρ_e / ρ_i
T_v	Time factor for vertical drainage
T_h	Time factor for horizontal drainage
U	Degree of consolidation
U_z	U at given depth
$\overline{U}$	Average degree of consolidation
$\overline{U}_h$, $\overline{U}_v$	$\overline{U}$ for horizontal, vertical drainage
ρ	Settlement
ρ_c	Consolidation settlement = $\overline{U}\rho_{cf}$
ρ_{cf}	Final consolidation settlement
ρ_e	Elastic settlement for no yielding
ρ_i	Initial settlement
ρ_s	Settlement due to secondary compression

STRENGTH AND STRESS-STRAIN PARAMETERS

$\overline{c}$	Effective cohesion intercept
$\overline{\phi}$	Effective friction angle
E	Young's modulus
E_u	E for undrained loading in terms of total stress
$\overline{E}$	E in terms of effective stresses

s_u	Undrained shear strength
$s_u(H)$	s_u for loading in horizontal direction
$s_u(V)$	s_u for loading in vertical direction
K_s	Anisotropic strength ratio = $s_u(H)/s_u(V)$
S_t	Sensitivity
ν	Poisson's ratio in terms of total stresses
$\bar{\nu}$	Poisson's ratio in terms of effective stresses

MISCELLANEOUS

B	Width of loaded area
e	Void ratio
e_o	Initial void ratio
H	Height or depth of compressible layer
H_d	Drainage height
ESP	Effective stress path, q vs $\bar{p}$
TSP	Total stress path, q vs p
$(T-u_o)SP$	Total minus u_o stress path, q vs $p-u_o$
N.C.	Normally consolidated
O.C.	Overconsolidated
t	time
t_c	Consolidation time
t_p	Time for primary consolidation
z	Depth
ε	Strain

ε_h	Horizontal strain
ε_v	Vertical strain
ε_{vol}	Volumetric strain
w	Water content
w_N	Natural water content
w_L	Liquid limit
w_p	Plastic limit
PI	Plasticity Index
γ_t	Total unit weight
γ_b	Bouyant unit weight
γ_w	Unit weight of water

APPENDIX B

REFERENCES

Note: ASCE = American Society of Civil Engineers
ASTM = American Society for Testing and Materials
ICSMFE = International Conference on Soil Mechanics and Foundation Engineering
JSMFD = Journal of Soil Mechanics and Foundations Division
RCSSCS = Research Conference on the Shear Strength of Cohesive Soils, Boulder, Colorado

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