The Impact of Random Tax Policy on Investment in Wind Power

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Abstract

This thesis considers the impact of tax policy uncertainty on firm level investment in wind power. I model the decision to invest as an irreversible investment with both price uncertainty and policy uncertainty. Tax policy uncertainty is introduced by a production tax credit that makes random discrete jumps. The model suggests that uncertain tax policy may decrease the time to investment and increase the level of investment as firms concentrate investment in high tax credit periods. Policy uncertainty may also lead to decreased government tax revenues and have negative effects on the wind power industry. Further work remains to solve the model numerically to determine the precise effects of policy uncertainty.

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Chapter 1

Introduction

2008 was a record-breaking year for the U.S. wind industry. The U.S. wind-power market added a staggering 8,358 MW of new capacity last year, nearly a 50% increase in generating capacity, bringing the total domestic wind energy capacity to 25,170 MW. The United States led the world in wind capacity additions for the fourth consecutive year and now claims the largest wind energy capacity in the world, surpassing Germany for the first time. Furthermore, for the third straight year, the U.S. added a record amount of wind capacity, installing more than three times the capacity added in 2006 and approximately 60% more than the previous record set in 2007 (Global Wind Energy Council, 2009). Figure 1.1 illustrates the impressive growth in wind power over the past 10 years.

Clearly wind power has been at the forefront of the booming renewable energy investment in recent years. Moreover, though it currently only supplies about 1% of the nation's energy consumption, wind power now represents a mainstream energy source in the United States. In 2008, wind power contributed 42% of all new U.S. electric generating capacity and, for the fourth consecutive year, wind power was the second-largest resource in terms of new generating capacity added, behind only new natural gas additions.[1] Additionally, wind power has recently received increased attention in the press due to President Obama's campaign pledge to support renewable energy and oil billionaire T. Boone Pickens' national media campaign promoting his plan to replace oil imports with domestic natural gas and wind.

Considering the rapid growth in the wind-power industry, wind energy has demonstrated the potential to supply a significant portion of our electricity generation needs. A recent report from the Department of Energy, published last year, suggests that it is feasible

[1] In 2008 wind power was actually the largest source of new net generating electricity capacity due to the retirement of old natural gas plants (Global Wind Energy Council, 2009).

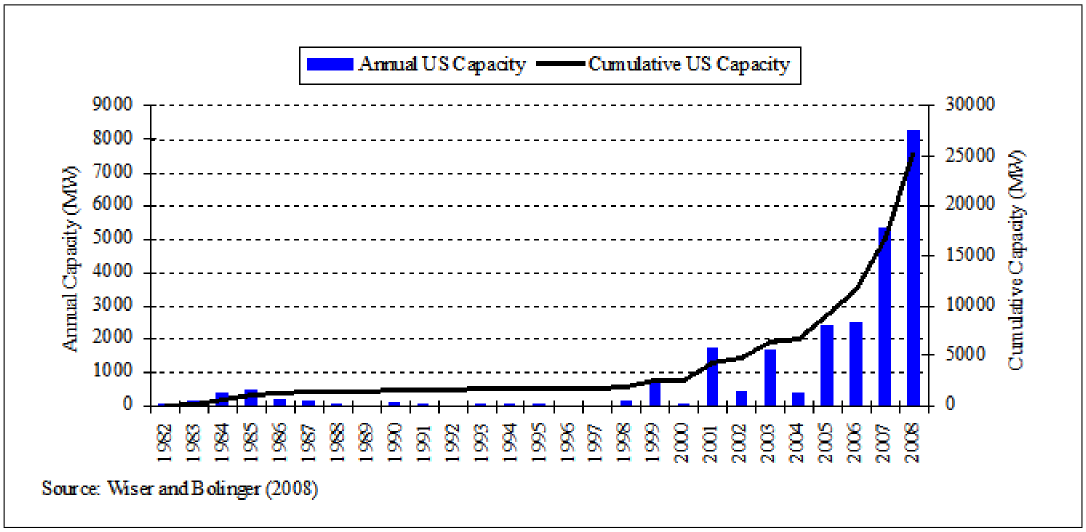

FIGURE 1.1: Annual and Cumulative Growth in U.S. Wind Capacity

for wind energy to provide 20% of U.S. electricity by 2030 (Lindenberg, Smith, O'Dell, DeMeo, and Ram, 2008). If wind energy is to achieve such ambitious growth, policy will certainly play an important role.

Tax policy is constantly an issue of debate in the political arena and as a result, tax policy provides a key source of uncertainty for the cost of capital to U.S. firms. In particular, over the past decade the wind-power industry has faced considerable uncertainty over taxes. The production tax credit (PTC) for wind power has lapsed three times since its introduction in 1992, meaning that three times the PTC expired before being renewed. During these lapse periods it was not certain when or if the PTC would be renewed. Also, the short-term renewal cycle of the production tax credit presents the wind-power industry with significant uncertainty even when the PTC does not lapse. Given that the PTC provides a substantial incentive to investors in wind power and has been a significant driver of the recent growth of the U.S. wind industry, it is essential to understand the impact of uncertain tax policy on investment in wind power.

That tax policy may be uncertain is not a new concept. Auerbach and Hines Jr (1988) recognize that changes in tax rates may substantially affect investor incentives and depart from the approach in which investors never anticipate tax changes. They consider a discrete-time model in which there is a chance that tax policy will change next period and find a solution by linearizing the model around steady-state values of investment and capital stock. Bizer and Judd (1989) develop a general equilibrium model in which taxpayers understand that tax policy is uncertain. Focusing on the efficiency cost of uncertain tax policy, Bizer and Judd (1989) find that the impact of uncertain policy depends on which taxes (income taxes or investment incentives) are affected by randomness.

Nonetheless, overall there has been little work on the impact of uncertain tax policy on investment behavior. A notable exception is Hassett and Metcalf (1999), from which this thesis draws heavily. Hassett and Metcalf (1999) introduce random tax policy in an investment tax credit that follows a jump process and focus on uncertainty in the form of mean preserving spreads. They find that increased uncertainty can decrease the time to investment and increase the amount of investment.

The literature focused on the effect of tax policy uncertainty on wind power investment is even smaller. Grobman and Carey (2002) construct a Markov decision process model to consider the uncertainty over the enactment or repeal of a PTC and show that even a small chance of an introduction of a PTC in the future can lead to a significant decrease in investment in the current period. Barradale (2008) suggests that fluctuations in wind energy investment are due to the effect policy uncertainty has on power purchase agreement negotiations. I discuss these papers in more detail in Section 3.3.

This thesis extends the literature by considering the impact of uncertain tax policy on the decision to invest in wind power. In particular, I focus on changes in tax policy for production tax credits since wind is currently eligible for a PTC in the United States. My approach differs from that of Grobman and Carey (2002) in that I concentrate on the firm's decision to invest in wind power only. In Grobman and Carey's model the agent can choose to invest in either wind power or a portfolio of conventional technologies while in my model the firm chooses to invest in wind power or not to invest at all. Moreover, I present a continuous-time model of investment instead of a discrete-time model.

In the investment literature uncertainty is frequently incorporated by assuming some parameter follows a continuous-time stochastic process such as Brownian motion. Similarly, I allow the output price, p, to be exogenous and follow geometric Brownian motion.[2] I also introduce randomness in tax policy. Unlike prices, tax policy parameters tend to make infrequent but discrete jumps, and therefore are not well described by stochastic processes that are everywhere continuous. Furthermore, changes in tax policy are likely to be mean-reverting; that is, when tax levels are high, it is more likely that they will be reduced in the future, and when tax levels are low, it is more likely that they will be increased in the future. As a result, I allow changes in tax policy to follow a Poisson

[2] One might argue that it is not realistic for price to follow a process such as Brownian motion that is unbounded above and instead price should be modeled as a mean-reverting process. The advantage of using Brownian motion is that it leads to a tractable analytic solution and intuitive investment decision rules. Furthermore, Metcalf and Hassett (1995) find that cumulative investment behavior under geometric Brownian motion is comparable to cumulative investment behavior under geometric mean reversion. Also, it often requires over 50 years of data to statistically determine whether an economic variable follows a random walk or a mean-reverting process (Dixit and Pindyck, 1994).

jump process. I then determine the value of the option to invest, V, in terms of the diffusion process for price and the Poisson process for changes in tax policy.

In the next chapter I review the history of U.S. energy tax policy and the federal production tax credit. This chapter provides the reader with a context for the discussion of tax policy uncertainty towards wind energy. In Chapter 3 I review the literature on uncertain tax policy. In Chapter 4 I sketch a simple model of investment under price uncertainty and present my model of investment with tax policy uncertainty. I present investment and policy implications based on the model in Chapter 5. Finally, in Chapter 6 I conclude.

Chapter 2

History of Energy Tax Policy

2.1 Energy Tax Policy Prior to 1970

Prior to the 1970s, U.S. federal energy tax policy largely favored increasing the domestic supply of oil and gas. Tax incentives such as the expensing of intangible drilling costs (IDCs) and the percentage depletion allowance reduced the production costs of oil and gas, led to a quicker depletion of domestic energy resources than normal, and diverted investment toward oil and gas production and away from other economic activities. Furthermore, there were no policies promoting energy conservation or the development of alternative fuels. Instead, low oil prices encouraged increased oil consumption and slowed the growth of alternative energy sources. 1970 marked the peak for oil and gas production; in that year oil and gas contributed 71.1% of total energy production (Lazzari, 2008).

2.2 Energy Tax Policy During the 1970s

Federal energy tax policy shifted course dramatically in the 1970s. As Lazzari (2008) notes, several factors led to this change. Increasing federal budget deficits made the significant expenditures on tax incentives for oil and gas more difficult to rationalize, growing concern for the environment led to greater criticism of pollution and environmental degradation, and issues of equity and fairness became more relevant as large oil companies made the wealthy even richer. Most important, however, in causing the change in energy tax policy were the oil crises of the 1970s.

The oil embargo of 1973, known as the first oil crisis, led to soaring oil prices as the Organization of Arab Petroleum Exporting Countries limited the supply of crude oil to the West. Roughly five years later the second oil shock rocked world markets when Ayatollah Khomeini took control of Iran in early 1979. Both of these oil crises triggered a meaningful shift in energy tax policy toward greater emphasis on conservation and the development of alternative energy sources. As a result, the long-standing tax preferences for the oil industry - expensing of IDCs and percentage depletion - were reduced while taxes on fossil fuels were increased. The infamous "gas guzzler" tax, which taxes vehicles that fail to achieve a minimum fuel efficiency and is still in effect today, was introduced in 1978 and a windfall profit tax (WPT) was imposed on the oil industry in 1980.[3] In addition to the new taxes on conventional fossil fuels the U.S. introduced several tax incentives for energy conservation and the development of alternative energy. Among the new energy incentives were conservation tax credits for homeowners and businesses and a variety of subsidies for unconventional fuel technologies.

2.3 Energy Tax Policy During the 1980s and 1990s

In the 1980s the Reagan Administration scaled back many of the tax incentive programs for alternative energy that were enacted during the previous decade in addition to the enduring subsidies for oil and gas. As part of his free-market view, Reagan promoted less distortionary tax policy and opposed using the tax code to support the supply of conventional fuels as well as the development of alternative fuels. Accordingly, the Reagan Administration let several energy tax credits for conventional fuels expire, reduced the incentives available for oil and gas, and repealed the WPT. Nevertheless, despite the administration's efforts to create a neutral tax law, several distortionary taxes remained leaving certain sectors favored under the federal energy tax policy.

The 1990s marked a renewed attempt to use energy tax policy to support conservation and alternative energy. The Revenue Provisions of the Omnibus Reconciliation Act of 1990 increased the gasoline tax, doubled the gas-guzzler tax, and expanded available tax credits for unconventional fuels. Notably, it also provided tax incentives oil and gas. Another major energy tax law, the Energy Policy Act of 1992, introduced the section 45 production based tax credit for wind and closed-loop biomass that is still available today, further increased the production tax credit for unconventional fuels, and again provided tax breaks for oil and gas. Thus despite the growing focus on the environment and alternative energy, no energy tax laws were passed without providing some form of tax relief for the oil and gas industries.

[3] The gas guzzler tax contains a spectacular loophole in that it excludes the light truck category, which consists of sport utility vehicles, minivans, and pickup trucks (i.e., the true "gas-guzzlers"). The light truck category represented 54% of new automobile sales in 2004 (U.S. Census Bureau, 2006).

2.4 Recent Energy Tax Policy Issues

After several years of failed attempts to enact comprehensive energy legislation, the Energy Policy Act of 2005 was the most extensive energy tax law passed since 1992. The law contains over $14 billion in tax incentives, some of which are new incentives and some of which are extensions of previous tax credits (Metcalf, 2007). In particular, the Energy Policy Act of 2005 extended the section 45 production tax credit through the end of 2007 and added new renewable technologies to those that are eligible to receive the credit. The law also increased the expanded the investment tax credit for renewables, increased the investment tax credit for solar, and provided a production tax credit to nuclear power plants (Metcalf, 2007).

Signed into law earlier this year, the American Recovery and Reinvestment Act of 2009 contains over $40 billion in energy tax incentives and attempts to make federal tax incentives more effective for renewable energy sources (Bolinger, Wiser, Cory, and James, 2009). The law contains the following important provisions: it extends the PTC through 2012 for wind and through 2013 for other technologies eligible for the PTC; it gives entities eligible for the PTC the option to choose a 30% ITC (the tax incentive currently available to solar) instead of the PTC; and it provides entities eligible for the ITC the option to choose an equivalent cash grant in place of the ITC (Bolinger et al., 2009). Bolinger et al. (2009) assess the choice between the PTC, ITC, and cash grant for several different PTC-eligible technologies and find that wind receives slightly more value from the PTC. However, the relative value of each incentive depends highly on installed project costs and expected capacity factor so ultimately the choice of the PTC, ITC, or cash grant must be made on a project-by-project basis.

2.5 History of the Production Tax Credit

The federal production tax credit was first introduced in the Energy Policy Act of 1992 in an effort to stimulate growth in renewable energy power production. The PTC currently provides an income tax credit of 2.0¢per kWh (adjusted annually for inflation) for electricity generated during the first 10 years of a wind plant's operation. The credit was 1.5¢per kWh in 1992. In order to receive the credit, the plant must only come online before the PTC expiration date; that is, if the PTC expires after a plant begins operation the plant will still receive the credit for its first 10 years of electricity generation. Also, only projects in the U.S. that sell their output to an unrelated entity qualify for the credit, which may be reduced if an eligible project also qualifies for other federal tax credits. While initially available only for wind and closed-loop biomass,

Legislation	Date Enacted	PTC Eligibility Window (for wind)
Energy Policy Act of 1992	10/24/1992	1994-June 1999
Ticket to Work and Work Incentives Improvement Act of 1999	12/19/1999	July 1999-2001
Job Creation and Worker Assistance Act	03/09/2002	2002-2003
Working Families Tax Relief Act	10/04/2004	2004-2005
Energy Policy Act of 2005	08/08/2005	2006-2007
Tax Relief and Health Care Act of 2006	12/20/2006	2008
Emergency Economic Stabilization Act of 2008	10/03/2008	2009
American Recovery and Reinvestment Act of 2009	02/17/2009	2010-1012

TABLE 2.1: Legislative History of the PTC

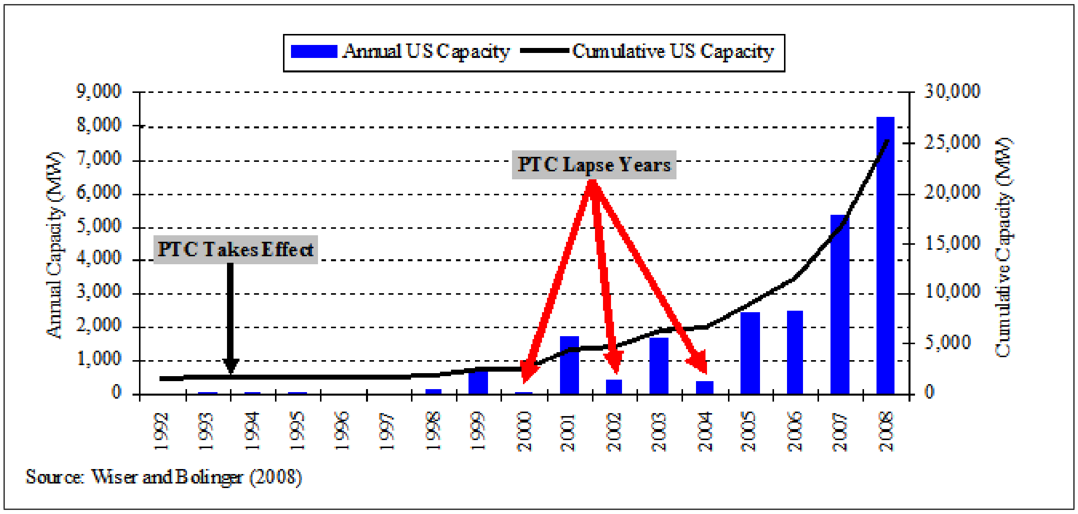

FIGURE 2.1: PTC Lapse Years

the PTC is currently available for a variety of renewable energy technologies: wind, closed- and open-loop biomass, geothermal, landfill gas, municipal solid waste, qualified hydropower, and hydrokinetic and wave energy.[4]

As Table 2.1 illustrates, the PTC has lapsed on three occasions since its enactment (i.e. three times the PTC has been renewed after it has already expired). The first PTC lapse lasted from June 1999 to mid-December 1999, the second lasted for the first few months of 2002, and the most recent PTC lapse lasted for most of 2004. Figure 2.1 shows that there have been pronounced declines in wind capacity additions in the same three years in which the PTC lapsed: 2000, 2002, and 2004. Though the first PTC lapse actually occurred during late 1999 it had a considerable impact on wind development in 2000.

After the first expiration date in 1999, the PTC has typically been renewed for one or two year periods. Nevertheless, earlier this year the PTC was extended for a period of three years until the end of 2012 as part of the American Recovery and Reinvestment

[4]Closed-loop biomass refers to any plant material that is grown for the exclusive purpose of being used to produce energy in a biomass generator. Alternatively, open-loop biomass means agricultural livestock waste and other waste material from agricultural or forest-related sources.

Act of 2009. Prior to this most recent renewal the PTC was set to expire at the end of 2009 following a one year renewal in late 2008.

Chapter 3

Literature Review

3.1 The Real Options Approach to Investment

In place of the orthodox theory of investment, this thesis considers investment decisions using the real options approach to investment proposed by Dixit and Pindyck (1994). Such divergence from conventional theory begets two very natural questions; what is the conventional theory and why should it be modified?

First, the orthodox theory of investment is what we know familiarly as the net present value (NPV) rule. This method states that if the net present value of an investment is positive, it is optimal to invest. The net present value of the investment is calculated by taking the difference between the present value of the expected flow of profits and the present value of the expected flow of expenditures. While this approach provides an initial framework for dealing with investment decisions, its usual implementation misses a key factor.

Before examining why the net present value rule is not the best way to make investment decisions it may be useful to establish the important characteristics of investment decisions. First, investment is typically irreversible. This means that the cost of investment is largely irrecoverable and effectively a sunk cost. Even if a firm decides after the fact that the decision to invest was incorrect it cannot recover its initial expenditure. Second, there is significant uncertainty over both the future flow of profits and the future flow of costs of the investment. And third, investors have the ability to choose when to invest and therefore can wait for new information before investing.

The orthodox theory of investment, or the net present value rule, however, is based on several assumptions that do not agree with the characteristics just described. First, the rule assumes that investment is reversible and that expenditures can be recovered if conditions do not turn out as expected. Or, if irreversibility is satisfied, the net present value rule implies that there is no flexibility over the timing of the investment, that it is a now or never proposition.

Because most investment decisions are irreversible and firms have the option to delay investment, it is more appropriate to model the investment opportunity as an option to invest, similar to a financial call option. When firms choose to invest they exercise their option to invest and give up the opportunity to wait for new information. As a result there is a certain opportunity cost to investment, namely the lost option value. This opportunity cost can be large and can significantly affect the decision to invest. As an example, assuming that the value of a project evolves stochastically, Dixit and Pindyck (1994) show that given reasonable parameter values the optimal investment rule is to invest when the value of the project is twice as large as the cost of investment. Thus the simple NPV rule, which says that the firm should invest as long as the project value is at least as large as the cost, is extremely mistaken.

3.2 The Theory of Uncertain Tax Policy

As noted in Chapter 1, that tax policy is uncertain is not a new concept. Nonetheless, the topic is often misunderstood. A common misconception is that uncertainty discourages investment. This view is not consistent with Hartman (1972) and Abel (1983), which find that increased uncertainty leads to an increase in the firm's optimal capital stock. However, investment is modeled as reversible in these papers. Pindyck (1988) shows that when investment is irreversible, increasing uncertainty delays investment and leads to lower levels of investment. This is because increasing uncertainty increases the opportunity cost of investing. Importantly, this finding follows from allowing capital costs to follow a continuous time stochastic process. If tax policy uncertainty leads to capital costs following a continuous-time stochastic process we can draw conclusions about the impact of policy uncertainty on investment. However, if tax policy follows a discrete jump process, Hassett and Metcalf (1999) show that the implications for investment are quite different.

Auerbach and Hines Jr (1988) recognize that changes in tax rates may substantially affect investor incentives and depart from the approach in which investors never anticipate tax changes. They use a discrete-time model in which there is a chance that tax policy will change next period to analyze U.S. corporate investment incentives from 1956–1986. Auerbach and Hines Jr obtain a solution by linearizing the model around steady-state values of investment and capital stock and find that investment adjustment costs play an important role in the investment decision. When adjustment costs are high, future tax incentives are of little importance to firms because the cost of changing investment rates is substantial and they feel locked in to current investments. But, when adjustment costs are low but nonzero, the anticipation of changes in investment incentives has a considerably greater impact on current period investment.

Bizer and Judd (1989) develop a general equilibrium model in which taxpayers understand that tax policy is uncertain. Like Auerbach and Hines Jr (1988), they find it unrealistic that investors never anticipate tax changes. Similarly, they argue that it is equally unrealistic that investors perfectly predict future tax policy. Focusing on the efficiency cost of uncertain tax policy, Bizer and Judd (1989) find that the impact of uncertain policy depends on how taxes are affected by randomness. The randomization of capital income taxes will increase government revenue at relatively low efficiency cost. Alternatively, the randomization of investment incentives will decrease government revenue at significant welfare cost. Their findings also suggest that tax analysis of this type is not well served by using an aggregate effective tax rate that summarizes the diverse tax code. Instead, it is more valuable to focus on specific components of the tax code.

More recently, Hassett and Metcalf (1999) study the effects of uncertain tax policy on both firm level investment behavior and aggregate investment. They consider policy uncertainty for investment tax credits (ITCs) by allowing the ITC to randomly make discrete jumps between a high and low level. Hassett and Metcalf find that when tax policy changes follow such a discrete jump process, increased uncertainty (in the form of mean preserving spreads) can in fact decrease the time to investment and increase the amount invested conditional on investing. At the aggregate level, uncertainty leads to a higher capital stock initially and after time. Hassett and Metcalf also note that when capital costs follow geometric Brownian motion, thus simulating continuous policy uncertainty, they find that increasing uncertainty has the opposite effect, increasing the time to investment and decreasing the aggregate capital stock. This result confirms the work of Pindyck (1988), which also used a continuous-time stochastic process to model uncertainty.

3.3 Uncertain Tax Policy and Wind Power Investment

Few attempts have been made to rigorously investigate the effect of tax policy uncertainty on investment in wind power. Grobman and Carey (2002) study the effect of uncertain tax policy on investment in wind power by considering policy uncertainty regarding the enactment or repeal of a PTC. They investigate the effect of uncertain tax policy on a single technology from a group of substitutable technologies. In this model firms can substitute between wind power and classical energy investments, knowing that the uncertainty of the PTC affects only wind. A firm anticipating the enactment of a PTC can shift investment to classical technologies (or renewables if they are subsidized) and defer investment in wind power until after the PTC is introduced. Similarly, a firm anticipating the repeal of a PTC can increase current investment in wind power and compensate by decreasing investment in classical technologies.

Specifically, Grobman and Carey use a Markov decision process model to determine how a welfare-maximizing agent will choose to invest under uncertainty. A Markov decision process is a discrete-time stochastic process with a set of states, each with a set of actions from which the decision maker must choose. In each period the agent can invest in 150 MW blocks of two technologies, gas-powered generation and wind power. They find that a small chance of an introduction of a PTC in the future can lead to a significant decrease in wind-power investment in the current period as firms wait for the credit to be enacted. For example, when the PTC is not in effect and the probability of moving from a PTC state to a no-PTC state is 0, increasing the probability of transitioning from the no-PTC state to the PTC state from 0 to .25 reduces investment in wind power (in megawatts) by one-sixth. Similarly, uncertainty over an extension of a PTC may increase current investment in wind power as firms rush to finish projects before the PTC expires.

Barradale (2008) takes a different approach and suggests that fluctuations in wind energy investment are due to the effect policy uncertainty has on power purchase agreement (PPA) negotiations.[5] Wind is a non-dispatchable resource, meaning that it cannot increase or decrease output on demand, so wind plants are typically under PPA because it is more difficult to bid into spot electricity markets. Barradale considers three scenarios in which a power producer is negotiating with a PPA price with a utility.

In the first case, when a PTC is available, the power producer and the utility sign a PPA at a price equal to the producer's net cost of generating wind power, which is given by the producer's nominal cost less the amount of the PTC. In the second case, when no PTC is available, the producer and utility again sign a PPA at the producer's net cost of production, which in this scenario is equal to the producer's nominal cost of production. However, in the third case, when the renewal of the PTC is uncertain, the power producer and the utility fail to reach an agreement. The producer assumes the PTC will not be renewed and demands a price equal to its nominal cost of production while the utility prefers to wait, not wanting to overpay if the PTC is actually renewed. Based on this analysis Barradale argues that uncertainty over the renewal of the PTC during

[5] Power purchase agreements are long-term contracts through which power producers sell electricity to utilities. Power producers may also bid into regional spot markets.

"off" years drives the drastic dips in wind-power investment because of the bargaining positions of the power producer and the utility.

In the literature the focus is largely on how uncertain tax policy affects the decision to invest. Uncertainty in this context refers to random changes in the level of taxes, whether they be PTCs, ITCs, or income taxes. This type of analysis assumes that when tax credits or subsidies are in effect, the firm is able to fully exploit such incentives. However, Carlson and Metcalf (2008) show that firms are often unable to take advantage of all available energy-related tax credits due to limitations within the corporate income tax and the Alternative Minimum Tax (AMT). Firms may be unable to use all of their energy tax credits because they do not earn enough positive income or because they bump up against the AMT. That firms are limited in their ability to take advantage of energy-related tax credits is yet another source of tax policy uncertainty.

There are reasons to believe that tax policy uncertainty may have a negative impact on the nascent renewable energy sector and the wind sector in particular. Wiser, Bolinger, and Barbose (2007) note that the policy uncertainty resulting from the short-term extensions of the PTC has led to a boom-and-bust cycle in wind power development as firms are forced to hastily finish projects before expiration deadlines arrive. This boom-and-bust cycle may have significant negative effects for the growth of wind power in the U.S. for several reasons: short-term uncertainty over the availability of the PTC may disrupt long-term planning of project development and reduce R&D expenditure; during boom periods when the PTC is expected to soon expire equipment and supply costs are inflated as demand exceeds the short-run supply; policy uncertainty may lead to under-investment in the already limited domestic wind turbine manufacturing industry.

In particular, the explosive growth of wind power in the United States and around the world has created a long waiting list for wind turbines. News articles report that the average lead time for delivery of turbines has increased dramatically in recent years from around six months to upwards of 24 months. Long delays between the order and the delivery of turbines provide an additional cost to wind energy developers since delays increase project development costs and prevent developers from taking advantage of the PTC. This is another part of the cost of uncertain tax policy.

Also, the boom-and-bust cycle fuels the common misconception that the severe reduction in investment when the PTC is not in effect is evidence that wind power is not viable without the PTC. This is not accurate. As the model in Section 4.2 will demonstrate, firms choose to delay investment until the PTC is available and therefore cause a lull in investment during periods when the PTC is not available.

Chapter 4

A Model of Investment with Uncertainty

4.1 Investment with No Tax Uncertainty

In this section I present a simple model of investment in which the only uncertainty in the model is due to price uncertainty. This will provide a framework for developing a model of investment with uncertain tax policy and also help us find a solution to the model with tax uncertainty. I make the important assumption that investment is irreversible so there is an opportunity cost to investing (and similarly an option value to not investing) that tells us the optimal investment rule is to invest when the output price, p, is greater than a certain price threshold. This price threshold is larger than it would be under a deterministic scenario.

When investment is irreversible there is a value to waiting. Firms can delay investment to wait for additional information. As a result, a model of irreversible investment must combine dynamics with uncertainty. In order to model price uncertainty I allow the price to evolve randomly over time, thus incorporating dynamics with uncertainty. Specifically, I allow the price to follow a stochastic process.

In such a process, the price in the current state determines only the probability distribution of the price in future state, not the actual value. A simple example of a stochastic process is a discrete-time discrete-state random walk. In this case the price begins at some known value and at times $t = 1, 2, 3, ...$, takes a jump of size 1 either up or down, each with a given probability. If the probability of an upward jump differs from the probability of a downward jump, we call the stochastic process a random walk with drift. An important characteristic of such a process is that the jumps are independent of each other – that is the probability distribution of the price in the future depends only on where it is now. The probability of an upward or downward jump in each period is independent of what has happened in past periods.[6] Taking the limit of a random walk by letting the jump size and the time interval go to zero results in a continuous-time stochastic process known as Brownian motion.

I assume the price of energy, p_t, follows geometric Brownian motion, a simple generalization of a Wiener process[7]:

$$dp_t = \mu_p p_t dt + \sigma_p p_t dz_p \tag{4.1}$$

with drift μ_p and volatility σ_p. Here dz_p is the increment of a Wiener process, i.e., $dz_p = \epsilon\sqrt{dt}$ where $\epsilon \sim \mathcal{N}(0,1)$. Note that absolute changes in p_t are lognormally distributed when p_t follows geometric Brownian motion. That is, changes in the logarithm of the price are normally distributed.

I also consider a constant production tax credit, π, that provides a production based income tax credit for electricity generated during the first y years of operation of a wind plant. The expected discounted value of the stream of profits from the investment consists of three parts: the stream of profits during the period in which the production tax credit is providing benefit, the stream of profits after the production tax credit expires, and the cost of the investment itself. The firm maximizes the expected return from the investment:

$$V = \max_{K,T} E\left[\int_T^{T+y} (p_s + \pi) K_T e^{-\rho s} ds + \int_{T+y}^{\infty} p_s K_T e^{-\rho s} ds - p_k K_T e^{-\rho T}\right] \tag{4.2}$$

where p_k is the price of capital, ρ is the discount rate, and K is the amount of capital used for the project. For simplicity, I set $K = 1$:

$$V = \max_{T} E\left[\int_T^{T+y} (p_s + \pi) e^{-\rho s} ds + \int_{T+y}^{\infty} p_s e^{-\rho s} ds - p_k e^{-\rho T}\right]. \tag{4.3}$$

I assume that $\mu_p < \rho$ to ensure the existence of an optimal solution in finite time. Alternatively, if $\mu_p > \rho$, it would always be optimal to wait and there would never be investment.

[6] This important property is known as the Markov property. Again, it implies that only current information is relevant in predicting the future path of a process.

[7] The continuous-time stochastic process known as Brownian motion is often called a Wiener process in honor of the mathematician Norbert Wiener. Wiener received his B.A. in Mathematics from Tufts exactly 100 years ago at the tender age of fourteen. Among his many contributions to pure mathematics, Wiener built on the initial work of Einstein to make formal the construction of Brownian motion.

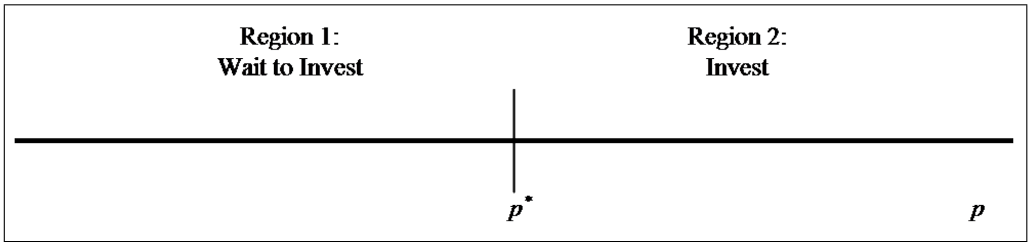

FIGURE 4.1: Investment Regions with No Tax Uncertainty

This is an optimal stopping problem in continuous time in which I solve for the price level, p^*, at which it is optimal to invest. Continuation (not investing) is optimal when $p < p^*$ and stopping (investment) is optimal when $p > p^*$ (See Figure 4.1). The opportunity to invest yields no profit up until the investment is made so the only return from holding the option to invest is its capital appreciation. Thus, the Bellman equation for the dynamic optimization problem is given by

$$\rho V dt = E\left[dV\right]. \tag{4.4}$$

Equation (4.4) simply says that over a short period of time, dt, the total expected return on the investment opportunity, $\rho V dt$, is equal to the expected capital appreciation of the investment opportunity.

Applying Ito's Lemma, I obtain

$$E\left[dV\right] = \mu_p p V_p dt + .5 V_{pp}({\mu_p}^2 p^2 dt^2 + \sigma_p^2 p^2 dt). \tag{4.5}$$

Substituting this expression into (4.4), dividing through by dt, and letting dt go to zero, I am left with the following second order partial differential equation:

$$\rho V = \mu_p p V_p + .5\sigma_p^2 p^2 V_{pp}. \tag{4.6}$$

The general solution to the differential equation (4.6) is given by $V = A_1 p^{\beta_1} + A_2 p^{\beta_2}$ where the β's are the roots of the quadratic equation, $Q(x) = \mu_p x + .5\sigma_p^2 x(x-1) - \rho$. Because $Q(0) = -\rho < 0$ and $Q(\pm\infty) = \infty$, I define the β's such that $\beta_1 < 0 < \beta_2$ (See Figure 4.2). Also, $Q(1) = \mu_p - \rho < 0$ because of the assumption that $\mu_p < \rho$ so we know that β_2 is positive and $\beta_2 > 1$. By a limiting argument I determine that A_1 is equal to zero. Because p follows a geometric Brownian motion, if p is ever zero then it remains at zero forever; that is, zero is an absorbing barrier for p. Formally, this implies that the following boundary condition must be satisfied: $V(0) = 0$. Thus, when p goes to zero V must also go to zero, but because $\beta_1 < 0$, the first term goes to infinity when p tends to zero therefore I set $A_1 = 0$. Hence the solution to this differential equation

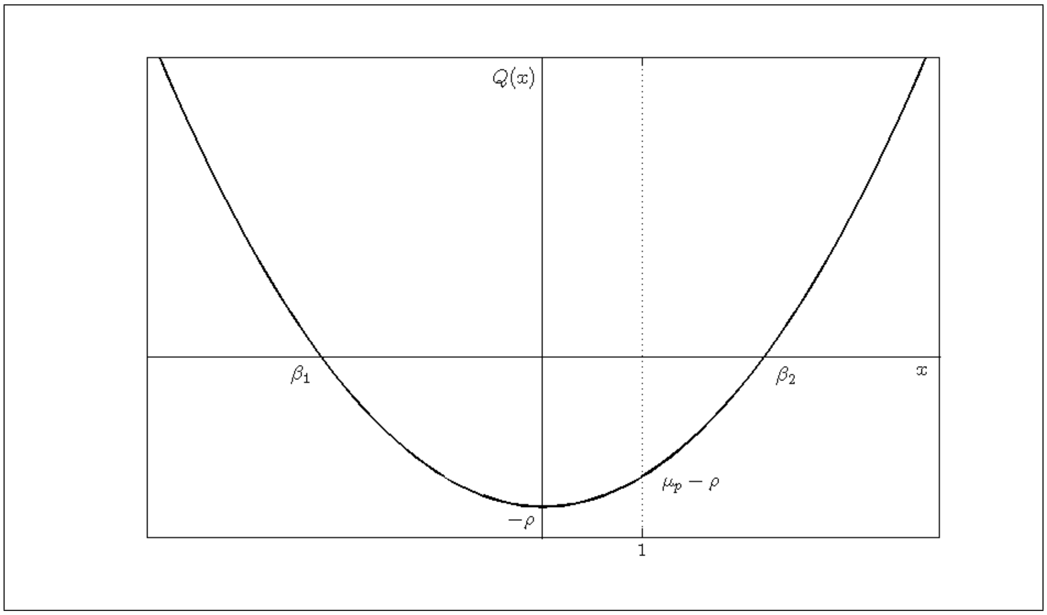

FIGURE 4.2: The Fundamental Quadratic

is $V = Ap^{\beta}$ where β is the positive root of $Q(x) = \mu_p x + .5\sigma_p^2 x(x-1) - \rho$. I invoke two additional boundary conditions to solve the system: value matching and smooth pasting. The value matching condition says that the value function must equal the net payoff obtained upon investing:

$$Ap^{\beta} = \frac{p}{\rho - \mu_p} + \Omega - p_k \tag{4.7}$$

where $\Omega = \frac{\pi}{\rho}\left(1 - e^{-\rho y}\right)$ and (4.7) is evaluated at p^*. Equation (4.7) has another useful interpretation if we rewrite it as

$$\frac{p}{\rho - \mu_p} + \Omega - Ap^{\beta} = p_k. \tag{4.8}$$

When the firm invests it obtains the payoff from the project but it gives us the option to invest, which has value Ap^{β}. Hence the firm's payoff, net of the opportunity cost of investing, is given by $p/(\rho - \mu_p) + \Omega - Ap^{\beta}$. The critical value p^* is where this net gain is equal to the tangible cost of investment, p_k.

Taking the first derivative of (4.7) with respect to p and evaluating at p^* yields the smooth pasting condition:

$$\beta A p^{\beta - 1} = \frac{1}{\rho - \mu_p}. \tag{4.9}$$

If V were not continuous and smooth at the critical point p^*, one could do better by exercising the option to invest at a different point.[8]

Solving this system of two equations and two unknowns I obtain

$$p^* = \frac{\beta}{\beta - 1}(\Omega - p_k)(\mu_p - \rho) \tag{4.10}$$

where p^* is the trigger price at which it is optimal to invest. Also, $A^* = \frac{1}{\beta(\rho - \mu_p)} p^{*1-\beta}$. Hence, the optimal time to invest is when

$$p > p^* = \frac{\beta}{\beta - 1}(\Omega - p_k)(\mu_p - \rho). \tag{4.11}$$

It is useful to take a moment to determine how β, and hence the optimal investment rule, changes when certain parameters of interest vary. In particular, I show how β is affected by changes in σ. Since β is the positive root of $Q(x)$ we have

$$\beta = \frac{\frac{1}{2}\sigma^2 - \mu_p + \sqrt{(\mu_p - \frac{1}{2}\sigma^2)^2 + 2\rho\sigma^2}}{\sigma^2}. \tag{4.12}$$

I totally differentiate $Q(x)$ to obtain

$$\frac{\partial Q}{\partial x}\frac{\partial \beta}{\partial \sigma} + \frac{\partial Q}{\partial \sigma} = 0. \tag{4.13}$$

where derivatives are evaluated at the positive root, β. First, $\frac{\partial Q}{\partial x} > 0$ at β (See Figure 4.2). Also, $\frac{\partial Q}{\partial \sigma} = \sigma\beta(\beta - 1) > 0$ at β. Consequently, it must be true that $\frac{\partial \beta}{\partial \sigma} < 0$ so we see that as uncertainty (σ) increases, β decreases and therefore $\frac{\beta}{\beta - 1}$ increases. Therefore greater uncertainty leads the investment rule to be scaled up by a larger factor.

Note also the limiting behavior of β when σ approaches infinity and when σ approaches zero. As $\sigma \to \infty$, $\beta \to 1$ so $p^* \to \infty$ and the investment is never made. Conversely, as $\sigma \to 0$, $\beta \to \infty$ so $\frac{\beta}{\beta - 1} \to 1$ and hence the investment rule converges to the simple deterministic net present value rule where one should invest if the expected present discounted value of the returns exceeds the cost of investment.

[8] For a more detailed exposition of the value matching and smooth pasting conditions, see Dixit and Pindyck (1994).

4.2 Investment with Tax Uncertainty

4.2.1 The Poisson Model

I now consider the investment decision when there is uncertainty over tax policy. There are now two sources of uncertainty in the model, price uncertainty and tax policy uncertainty. Though it is relatively straightforward to introduce tax uncertainty, the model becomes significantly more complicated to solve.

Once again I assume the price of energy, p_t, follows geometric Brownian motion:

$$dp_t = \mu_p p_t dt + \sigma_p p_t dz_p \tag{4.14}$$

with drift μ_p and volatility σ_p and where dz_p is the standard increment of a Wiener process. I also consider a production tax credit $\pi_t \in \{\pi_0, \pi_1\}$ that increases the output price from p_t to $p_t + \pi_t$ for a period of y years. While it seems sensible to model price as a continuous stochastic process it is less realistic to model tax policy changes with a continuous diffusion process such as geometric Brownian motion. Historic observations suggest that tax policy typically remains constant for several years and then changes abruptly. Thus, tax policy changes are better described by a process that makes infrequent and discrete jumps. Furthermore, tax parameters are likely to be mean-reverting since policy-makers are more likely to reduce taxes when they are high and increase taxes when they are low.[9]

Thus a model of tax policy changes should incorporate infrequent yet discrete jumps and mean reversion. A stochastic model that has these characteristics is the Poisson (jump) process. The Poisson process is a stochastic process that is subject to jumps of fixed or random size, for which the arrival times of the jumps follow a Poisson distribution. In this case the jumps are of fixed size given that $\pi_t \in \{\pi_0, \pi_1\}$.

The PTC evolves according to the following equation:

$$d\pi_t = \begin{cases} \Delta\pi & \lambda_1 dt \\ 0 & 1 - \lambda_1 dt \quad \pi = \pi_0 \\ & \\ -\Delta\pi & \lambda_0 dt \\ 0 & 1 - \lambda_0 dt \quad \pi = \pi_1 \end{cases}$$

[9]One might argue that tax policy is endogenous and should therefore respond to the firm's output price. A model using this covariance between tax policy and profitability is described in Hasset and Metcalf (1999).

where $\Delta\pi = \pi_1 - \pi_0 > 0$. The production tax credit, π_t, switches randomly between π_0 and π_1, where λ_1 is the mean arrival rate of an increase in the PTC when $\pi_t = \pi_0$ and λ_0 is the mean arrival rate of a decrease in the PTC when $\pi_t = \pi_1$. Here λ_1 and λ_0 are positive. Thus, when $\pi_t = \pi_0$ the probability of an increase in the PTC during a time interval of infinitesimal length dt is given by $\lambda_1 dt$. Similarly, when $\pi_t = \pi_1$ the probability of a decrease in the PTC during a time interval of length dt is given by $\lambda_0 dt$.

The expected discounted value of the stream of profits from the investment consists of three parts: the stream of profits during the period in which the production tax credit is providing benefit, the stream of profits after the production tax credit expires, and the cost of the investment itself. The firm determines the optimal investment rule by maximizing the expected return from the investment:

$$V = \max_{K,T} E\left[\int_T^{T+y}(p_s+\pi_T)F(K_T)e^{-\rho s}ds + \int_{T+y}^{\infty} p_s F(K_T)e^{-\rho s}ds - p_k K_T e^{-\rho T}\right] \tag{4.15}$$

that is, I solve for the critical values p_0 and p_1, at which it is optimal to invest. Again, in the interest of clarity, I set $F(K_T) = K_T = 1$:

$$V = \max_{T} E\left[\int_T^{T+y}(p_s+\pi_T)e^{-\rho s}ds + \int_{T+y}^{\infty} p_s e^{-\rho s}ds - p_k e^{-\rho T}\right]. \tag{4.16}$$

In this problem there are three regions of interest (see Figure 4.3). I let p_1 be the boundary between regions 1 and 2 and p_0 be the boundary between regions 2 and 3. In region 1 ($p < p_1$) the firm does not invest, regardless of the price level. In region 2 ($p_1 < p < p_0$) the firm invests only if the high PTC, π_1, is in place. Finally, in region 3 ($p > p_0$) the firm invests regardless of the level of the PTC. I let V^1 be the value function when the high PTC, π_1, is in effect and V^0 be the value function when the low PTC, π_0, is in effect. To solve this investment problem I solve for the value function, conditional on the level of the PTC, in each region and then invoke value matching and smooth pasting conditions. Value matching requires that the value functions be equal at the boundaries between regions and smooth pasting requires that the first derivative of the value functions be equal at the boundaries.

Region 1, or the continuation region, consists of the values of p for which it is not optimal to invest regardless of the level of the tax credit. In region 1 the Bellman equations are[10]:

$$\rho V^0 dt = E\left[dV^0\right] \tag{4.17}$$

$$\rho V^1 dt = E\left[dV^1\right]. \tag{4.18}$$

[10] These also follow from an arbitrage argument by assuming that there is some asset that is perfectly correlated with p.

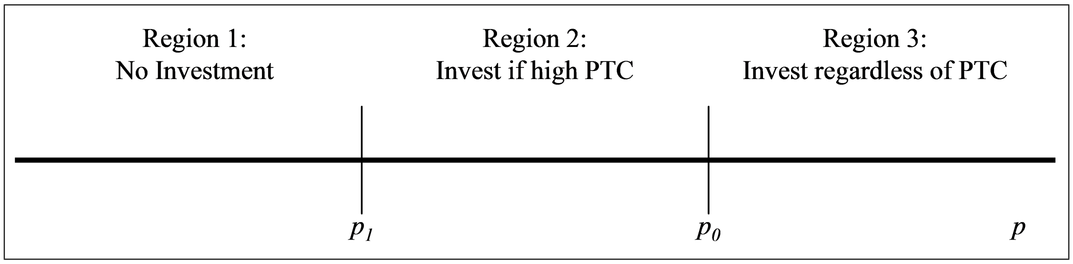

FIGURE 4.3: Investment Regions with Tax Uncertainty

The intuition behind the Bellman equations is that the normal return per unit time required to hold the asset, V, discounted at rate ρ, must be equal to the expected capital appreciation of V. Because the Ito process described in (4.14) is continuous but not differentiable, I apply Ito's Lemma:

$$E\left[dV^0\right] = \mu_p V_p^0 dt + .5V_{pp}^0({\mu_p}^2 p^2 dt^2 + \sigma_p^2 p^2 dt) + \lambda_1(V^1 - V^0)dt \tag{4.19}$$

$$E\left[dV^1\right] = \mu_p V_p^1 dt + .5V_{pp}^0({\mu_p}^2 p^2 dt^2 + \sigma_p^2 p^2 dt) - \lambda_0(V^1 - V^0)dt. \tag{4.20}$$

The expectation on the right hand sides of (4.19) and (4.20) are composed of two distinct parts. The first part is the expected gain in V^0 and V^1 as prices evolve stochastically. The second part is the capital gain (loss) if the production tax credit is increased (decreased). The increase occurs with probability λ_1 while the decrease occurs with probability λ_0.

By substituting these two expressions into (4.17) and (4.18), dividing through by dt, and letting dt go to zero, I obtain the following system of partial differential equations.

$$\rho V^0 = \mu_p p V_p^0 + .5\sigma_p^2 p^2 V_{pp}^0 + \lambda_1(V^1 - V^0) \tag{4.21}$$

$$\rho V^1 = \mu_p p V_p^1 + .5\sigma_p^2 p^2 V_{pp}^1 - \lambda_0(V^1 - V^0) \tag{4.22}$$

Letting $Z = V^1 - V^0$ and $X = \lambda_0 V^0 + \lambda_1 V^1$, I obtain the following independent partial differential equations.

$$\rho Z = \mu_p p Z_p + .5\sigma_p^2 p^2 V_{pp} - (\lambda_1 + \lambda_0)Z \tag{4.23}$$

$$\rho X = \mu_p p X_p + .5\sigma_p^2 p^2 X_{pp} \tag{4.24}$$

The general solution to the differential equation (4.23) is given by $Z = C_1 p^{\gamma_1} + C_2 p^{\gamma_2}$ where the γ's are the roots of the quadratic equation, $R(x) = \mu_p x + .5\sigma_p^2 x(x-1) - (\lambda_1 +$

$\lambda_0 + \rho$). Because $R(0) = -(\lambda_1 + \lambda_0 + \rho) < 0$ and $R(\pm\infty) = \infty$, I define the γ's such that $\gamma_1 < 0 < \gamma_2$. By a limiting argument I determine that C_1 is equal to zero. Because p follows a geometric Brownian motion, if p is ever zero then it remains at zero forever; that is, zero is an absorbing barrier for p. Thus, when p goes to zero Z must also go to zero, but because $\gamma_1 < 0$, the first term goes to infinity when p tends to zero therefore I set $C_1 = 0$.

The general solution to the differential equation (4.24) is given by $X = B_1 p^{\beta_1} + B_2 p^{\beta_2}$ where the β's are the roots of the quadratic equation, $Q(x) = \mu_p x + .5\sigma_p^2 x(x-1) - \rho$. Because $Q(0) = -\rho < 0$ and $Q(\pm\infty) = \infty$, the roots are defined such that $\beta_1 < 0 < \beta_2$. Using a similar limiting argument as p approaches zero I set $B_1 = 0$.

Thus in the first region I obtain the following expressions for V^0 and V^1:

$$V^0 = \frac{Bp^\beta - \lambda_1 C p^\gamma}{(\lambda_0 + \lambda_1)} \tag{4.25}$$

$$V^1 = \frac{Bp^\beta + \lambda_0 C p^\gamma}{(\lambda_0 + \lambda_1)} \tag{4.26}$$

where γ is the positive solution to $R(x) = \mu_p x + .5\sigma_p^2 x(x-1) - (\lambda_1 + \lambda_0 + \rho)$ and β is the positive solution to $Q(x) = \mu_p x + .5\sigma_p^2 x(x-1) - \rho$.

In region 2 ($p_1 < p < p_0$) the firm invests only if the high production tax credit, π_1, is in place. Therefore if the low PTC, π_0, is in place, it is optimal to wait whereas if the high PTC is in place the firm invests and receives the termination payoff. I modify the equations from above to obtain:

$$\rho V^0 dt = E\left[dV^0\right] \tag{4.27}$$

$$V^1 = \frac{p_T}{\rho - \mu_p} + \Omega^1 - p_k \tag{4.28}$$

where $\Omega^1 = \frac{\pi_1}{\rho}\left(1 - e^{-\rho y}\right)$.

Applying Ito's Lemma, substituting into (4.27), dividing through by dt, and letting dt go to zero, I am left with

$$\rho V^0 = \mu_p p V_p^0 + .5\sigma_p^2 p^2 V_{pp}^0 + \lambda_1(V^1 - V^0). \tag{4.29}$$

Substituting (4.28) into (4.29), I obtain the following non-homogenous partial differential equation.

$$-\lambda_1\left(\frac{p}{\rho-\mu_p}+\Omega^1-p_k\right)=\mu_p p V_p^0+.5\sigma_p^2 p^2 V_{pp}^0-(\lambda_1+\rho)V^0 \qquad (4.30)$$

I first solve the homogenous part of the equation and then use any particular solution to solve the non-homegenous differential equation. The solution to the homogenous part of (4.30) is given by $V^0 = D_1 p^{\delta_1} + D_2 p^{\delta_2}$ where δ_1 and δ_2 are the solutions to the quadratic $S(x) = \mu_p x + .5\sigma_p^2 x(x-1) - (\lambda_1+\rho)$. Here $\delta_1 < 0 < \delta_2$ because $S(0) = -(\lambda_1+\rho) < 0$ and $S(\pm\infty) = \infty$. No limiting argument is used to eliminate terms because in the middle region the price is bounded both above and below. It can be shown that a particular solution to the non-homogenous part of (4.30) is given by:

$$V^0 = \Phi - p\Theta \qquad (4.31)$$

where

$$\Phi = \frac{\lambda_1(\Omega^1 - p_k)}{\lambda_1+\rho} \qquad (4.32)$$

and

$$\Theta = \frac{\lambda_1}{(\rho-\mu_p)(\mu_p-\lambda_1-\rho)}. \qquad (4.33)$$

Hence the general solution to (4.30) is given by:

$$V^0 = D_1 p^{\delta_1} + D_2 p^{\delta_2} - p\Theta + \Phi. \qquad (4.34)$$

In region 3 ($p > p_0$) the firm invests regardless of the level of the production tax credit. Therefore the value functions are

$$V^0 = \frac{p_T}{\rho-\mu_p} + \Omega^0 - p_k \qquad (4.35)$$

$$V^1 = \frac{p_T}{\rho-\mu_p} + \Omega^1 - p_k \qquad (4.36)$$

where $\Omega^0 = \frac{\pi_0}{\rho}(1-e^{-\rho y})$.

4.2.2 Value Matching and Smooth Pasting

I invoke value matching and smooth pasting conditions to solve the system of six equations in six unknowns. Value matching conditions imply that the value functions V^0 and V^1 must be equal, or match, at the boundaries of the regions. Thus I obtain (4.37) by equating (4.25) with (4.34) and evaluating at p_1. Similarly, I obtain (4.38) by equating (4.26) with (4.28) and evaluating at p_1 and I obtain (4.39) by equating (4.34) with (4.35) and evaluating at p_0. In addition to value matching, smooth pasting conditions must be invoked to complete the system. Smooth pasting requires that the derivatives of the value functions be equal at the boundaries. I differentiate (4.37) and (4.38) with respect to p and evaluate at p_1 while I differentiate (4.39) with respect to p and evaluate at p_0.

The value matching equations along with the smooth pasting equations produce a system of six equations in six unknowns: p_0, p_1, B, C, D_1, and D_2.

$$\frac{Bp_1^{\beta} - \lambda_1 C p_1^{\gamma}}{\lambda_1 + \lambda_0} = D_1 p_1^{\delta_1} + D_2 p_1^{\delta_2} - p_1\Theta + \Phi \tag{4.37}$$

$$\frac{Bp_1^{\beta} + \lambda_0 C p_1^{\gamma}}{\lambda_1 + \lambda_0} = \frac{p_1}{\rho - \mu_p} + \Omega^1 - p_k \tag{4.38}$$

$$D_1 p_0^{\delta_1} + D_2 p_0^{\delta_2} - p_0\Theta + \Phi = \frac{p_0}{\rho - \mu_p} + \Omega^0 - p_k \tag{4.39}$$

$$\frac{B\beta {p_1}^{\beta-1} - \lambda_1 C\gamma {p_1}^{\gamma-1}}{\lambda_1 + \lambda_0} = D_1\delta_1 p_1^{\delta_1 - 1} + D_2\delta_2 p_1^{\delta_2 - 1} - \Theta \tag{4.40}$$

$$\frac{B\beta {p_1}^{\beta-1} + \lambda_0 C\gamma {p_1}^{\gamma-1}}{\lambda_1 + \lambda_0} = \frac{1}{\rho - \mu_p} \tag{4.41}$$

$$D_1\delta_1 p_1^{\delta_1 - 1} + D_2\delta_2 p_1^{\delta_2 - 1} - \Theta = \frac{1}{\rho - \mu_p} \tag{4.42}$$

4.3 Relationship between the Two Cases

Though the investment decision in the presence of uncertain tax policy is significantly more complicated than the case in which there is no tax uncertainty, the more complex case reduces to the simple model by equating the levels of the low and high production tax credits.

Setting $\pi_0 = \pi_1$ implies that $V^0 = V^1$ since the tax credit switches randomly between the same level and is therefore constant. This also means that the three regions converge to just two, namely the two regions in the simple model with no tax uncertainty. When $V^0 = V^1 = V$, $Z = V^1 - V^0 = 0$, and since we also know $Z = Cp^{\gamma}$, it must be true that $C = 0$. Also, letting $\lambda_1 = \lambda_0 = \lambda$, we have $X = \lambda_0 V^0 + \lambda_1 V^1 = 2\lambda V$. Additionally, we know $X = Bp^{\beta}$ so $V = (B/2\lambda)p^{\beta}$. Setting this equal to the value function from the no tax uncertainty case, where $V = Ap^{\beta}$, and noting that the β's are the same (both are the positive solutions to $Q(x)$), I determine that $B = 2\lambda A$.

Furthermore, $\pi_0 = \pi_1$ implies that $p_1 = p_0 = p^*$ where p^* is the trigger price from the case with no tax uncertainty. Thus, when $\pi_0 = \pi_1$, I have expressions for p_1, p_0, B, and C in terms of only constants. Using those expressions and equations (4.37) and (4.42), I can solve for D_1 and D_2 since I have a system of two linear equations in two unknowns[11]:

$$\begin{bmatrix} l_1 & l_2 \\ k_1 & k_2 \end{bmatrix} \times \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} M \\ N \end{bmatrix}$$

where $l_1 = p^{*\delta_1}$, $l_2 = p^{*\delta_2}$, $k_1 = \delta_1 p^{*\delta_1 - 1}$, $k_2 = \delta_2 p^{*\delta_2 - 1}$, $M = Ap^{*\beta} + p^*\Theta - \Phi$, and $N = \Theta + 1/(\rho + \mu_p)$. Thus, when $\pi_0 = \pi_1$, I know that $p_1 = p_0 = p^*$, $C = 0$, $B = 2\lambda A$, and I can solve for D_1 and D_2 in terms of known constants.

4.4 Using Numerical Methods to Solve the Nonlinear System

Given the system of six equations in six unknowns developed in Section 4.2.2 ((4.37) - (4.42)), I am left with the basic task of solving the equations numerically. Subtracting the two sides from each equation, I obtain the traditional expression

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0} \tag{4.43}$$

where $\boldsymbol{f}$ is the 6-dimensional vector-valued function whose components are the individual equations from Section 4.2.2 to be satisfied simultaneously. The goal is to find the 6-dimensional solution vector $\boldsymbol{x}$. Unfortunately, solving a set of N nonlinear equations in N unknowns simultaneously is significantly more difficult that finding roots in one dimension. In fact, Press, Teukolsky, Vetterling, and Flannery (1986) make the following statement: "There are *no* good, general methods for solving systems of more than one nonlinear equation."

Nonetheless, the *Newton-Raphson method* or *Newton's method*, which is commonly used for root-finding in one-dimension, can be easily generalized to multiple dimensions. In one dimension, Newton's method uses the local derivative at a current point x_i to find a better estimate of the root x_{i+1} by extending the tangent line at x_i until it crosses the x-axis. The point at which the tangent line intersects the x-axis gives the estimate x_{i+1}.

[11] Note that a solution to this system exists because $\delta_1 < 0 < \delta_2$ so the left-hand matrix is invertible.

The method derives from the linear terms of the Taylor series expansion of a function in the neighborhood of a point,

$$f(x+\delta) \approx f(x) + f'(x)\delta + \frac{f''(x)}{2!}\delta^2 + \dots \tag{4.44}$$

When δ is small enough, terms of order two or more are insignificant, so $f(x+\delta) = 0$ implies

$$\delta = -\frac{f(x)}{f'(x)}. \tag{4.45}$$

Thus Newton's method tells us that a better estimate of x is

$$x_{i+1} = x_i + \delta = x_i - \frac{f(x_i)}{f'(x_i)}. \tag{4.46}$$

Hence it is crucial that the initial guess be close to the desired root. If the initial guess is far from a root Newton's method can give inaccurate estimates since higher order terms in the series will in fact be important.

With this in mind, Section 4.3 demonstrates how to obtain an initial guess for the system of six equations when the low production tax credit is equal to the high production tax credit. As I show in that section, when the low PTC is equal to the high PTC, the model reduces to the simple case in which there is no uncertainty over tax policy. Section 4.1 provides a closed-form solution for the simple case which I can therefore use to find an exact 6-dimensional solution vector, $\boldsymbol{x}$.

However, I am interested in solving the six equation system when the low PTC is not equal to the high PTC. To do this I set the tax credits equal, use the initial guess, and find the solution vector (in this case the solution is exactly equal to the guess). I then decrement the low PTC, use the previous solution to the system as the initial guess, and solve for the new solution vector. I continue this looping process until the low PTC reaches some specified value. In my case I begin by setting $\pi_1 = \pi_0 = .02$ and decrement the low PTC (π_0) until it equals zero. When $\pi_1 = .02$ and $\pi_0 = 0$ the model simulates the case in which the PTC switches on and off randomly with the high PTC being equal to the current level of the federal PTC.

I initially attempted to implement this procedure using the MATLAB program *fsolve*, which is designed to solve systems of nonlinear equations. After decrementing the low PTC the *fsolve* program failed to converge to a root. I then attempted to implement C. T. Kelley's *nsoli* routine which uses a modified Newton's method. The *nsoli* routine successfully solved the system until the low PTC began to approach zero, at which point the program also failed to converge to a root. Solving this system numerically is left for future research.

Chapter 5

Implications

5.1 Implications for Investment Behavior

The model presented in Section 4.2 is similar to the model described in Hassett and Metcalf (1999) but differs by introducing a production tax credit instead of an investment tax credit. This is an important distinction since the two credits are structured differently - the PTC offers a production based income tax credit and provides tax relief over a longer period while the ITC reduces the capital cost of a project by a given percentage in the year that the project begins operations.[12] To see how the system would differ if a ITC was considered, note that π_T would vanish from the first integral in (4.16) and $(1 - \pi_T)$ would enter the last term to reduce the capital cost. Despite the differences, I expect the two models to generate qualitatively similar results because both models introduce tax policy uncertainty through a Poisson process.

Recall from Section 4.1 that greater price uncertainty increases the trigger price, p^*, and thus increases the value of the option to invest. The increased value of the investment opportunity causes the firm to wait longer to invest and decreases the amount of investing that the firm does. In the model described in Section 4.2 we must consider both the effect of price uncertainty and the effect of random tax policy.

Hasset and Metcalf (1999) show that greater uncertainty over tax policy, in the form of mean preserving spreads, decreases the median time to investment and increases the amount of investment when the firm actually decides to invest. Also, investment tends to take place mostly when the high tax credit is in effect. These results are illustrative of the nature of the stationarity and discrete nature of the Poisson process. They also suggest

[12] Note that the federal ITC is realized in the year that the project begins operations but it vests linearly over a five-year period. Thus the investor must hold the project for a full five years in order to realize the entire value of the ITC. Theoretically this will effect the liquidity of a project.

a similar impact on investment behavior when random tax policy affects a production tax credit.

When the production tax credit follows the Poisson process described in Section 4.2, there are two distinct states: a "good" state when the high PTC is in effect and a "bad" state when the low PTC is in effect. Holding price constant, the value of the investment opportunity is clearly greater in the high tax credit state. For this reason there is a much smaller value to waiting to invest in the high tax credit state because the stationarity of the Poisson process dictates that the firm can only transit to the low credit state. In other words, when in the "good" state the firm has nowhere to go but to the "bad" state. Knowing this the firm responds by concentrating investment in the high tax credit states and not waiting long to do so. The discrete nature of the Poisson process further magnifies this result. When the difference in the tax credits is greater so is the incentive to take advantage of the high tax credit state. Equation 4.22 shows that as the tax credits diverge and the difference between the value functions $V^1 - V^0$ increases, the value of the option to invest in the high PTC or "good" state (V^1) decreases. Thus the value of waiting in the "good" state decreases when the difference in the tax credits is greater. There is, however, still a value to waiting while in the high tax credit state because firms want to combine low capital cost and high output price at the time of investment. Alternatively, Equation 4.21 shows that as the tax credits diverge and the difference between the value functions $V^1 - V^0$ increases, the value of the option to invest in the low PTC or "bad" state (V^0) increases. The value of waiting to invest is greater in the "bad" state since it is only possible for the tax credit to transition to a higher level.

When firms concentrate investment in high tax credit periods and, alternatively, invest sparingly in low tax credit periods, they produce a boom-and-bust cycle of investment. This is precisely what has characterized the wind power industry in recent years. Such wild swings in investment can be inefficient and even detrimental to the wind energy industry. The boom and bust cycle of investment discourages long-term investment and planning, particularly in the domestic wind turbine industry, and leads to higher capital costs during boom periods when short-run demand exceeds supply. The underdeveloped U.S. turbine manufacturing sector further amplifies the increase in capital costs during boom periods since U.S. firms are forced to import turbines. I leave the price of capital fixed in my model but a useful contribution would be to examine the effect of allowing capital costs to vary.

5.2 Implications for Policy

Because the randomness of the production tax credit leads firms to concentrate investment in the high tax credit states, the government loses significant tax revenues as a result of uncertain tax policy. In effect, the uncertainty acts as an additional subsidy for investment since firms always have the option to choose when to invest. This is an important result for policy makers that interested in fiscal responsibility and efficiently stimulating investment in alternative energy.

Table 19-1 of the FY2009 Budget of the United States Government shows that the total revenue cost of the section 45 production tax credits and the section 48 investment tax credits is expected to be over $5 billion from 2009-2013. Even though recent government interventions have dwarfed this figure by orders of magnitude, $5 billion is not a trivial amount. The PTC imposes a considerable cost to taxpayers in the form of decreased federal tax revenue and uncertainty over the PTC only serves to increase the cost. In addition, the National Renewable Energy Laboratory has estimated that an extension of the PTC through 2020 could result in wind providing 20% of the domestic energy supply in 2020 (Short, Blair, Denholm, and Heimiller, 2006). A fixed tax policy for investment works to minimize tax revenue losses and could lead to wind supplying a substantial fraction of the nation's energy. If the government is unable to commit to a fixed tax policy it gives up revenue and risks inhibiting the growth of wind energy.

Uncertainty aside, the production tax credit is not an especially efficient tax incentive. In fact, I argue that tax incentives in the form of the PTC are not preferable if energy tax policy is to effectively and efficiently lead to energy independence from fossil fuels. As discussed in Chapter 3, U.S. energy tax policy has long promoted domestic oil and gas production. These policies encourage the consumption of fossil fuels and discourage conservation. In addition, the energy subsidies for alternative energy sources appear to be misguided and motivated by politics rather than economics. Huge energy tax incentives are available for ethanol while the incentives for solar energy are not great enough to make solar cost-competitive with natural gas (Hassett and Metcalf, 2007). Providing tax credits for individual conventional and alternative energy sources is expensive and often magnifies distortions in the energy markets rather than correcting them.

If the ultimate concern is carbon emissions, a better approach would be to give carbon emissions a price. One could do this by levying a tax directly on carbon emissions or by implementing an emissions trading (cap-and-trade) system under which the total emissions are limited to a given level. According to Hassett and Metcalf (2007), a tax of $12 per metric ton of carbon dioxide in place of the production tax credit would make wind and biomass competitive with natural gas. And, in contrast with the alternative energy subsidies that cost the federal government billions of dollars, a carbon tax would generate revenue, which could then be used to reduce other distortionary taxes. Hassett and Metcalf (2007) argue that a carbon tax is preferable to a cap-and-trade system because the efficiency costs of a carbon tax are expected to be lower than the costs of a cap-and-trade system and because carbon permits are likely to be given away, in which case the government loses substantial revenue. Nonetheless both approaches are preferred over the currently unfocused and imprecise U.S. energy tax policy. If such changes are too sweeping for members of Congress, policymakers could consider increasing the gasoline tax. Among OECD countries, the U.S. has the lowest tax rate on gasoline (Metcalf, 2007). Though not as efficient as a comprehensive carbon policy it would be an improvement and a step towards levying a cost on the negative externalities of driving motor vehicles. Policymakers would also do well to remove the loophole in the gas guzzler tax and to provide incentives for fuel-efficient vehicles and conservation investments.

Chapter 6

Conclusion

When tax policy is fixed and output price follows a continuous-time stochastic process, increasing uncertainty delays firm level investment because of the opportunity cost of investing. This follows directly from Dixit and Pindyck (1994) and others. But history tells us that taxes are rarely fixed for long. There is, in fact, considerable uncertainty with respect to taxes, especially for the federal production tax credit. When tax policy follows a stationary and discrete Poisson jump process the model suggests that increasing uncertainty will decrease the time to investment and increase the amount of capital purchased. In a high-PTC state the value of waiting to invest is small and the loss from a transition to a low-PTC state is substantial. Conversely, in a low-PTC state the value of waiting to invest is large because the gain from a transition to a high-PTC state is also large. As a result firms will concentrate investment in the high tax credit periods, producing a boom-and-bust cycle of investment.

The boom-and-bust cycle of investment in the wind industry that results from tax policy uncertainty has several negative consequences. When the growing demand for wind power is compressed into frenzied boom periods wind project costs increase because delays in the supply chain lead to higher supply and project development costs. Furthermore, long-term planning and investment in wind projects and in the domestic turbine industry may be deterred.

The federal production tax credit is also a significant expenditure for the U.S. government and tax policy uncertainty only brings an increased loss in tax revenues as firms concentrate investment in periods with high tax credits. If policymakers want to promote alternative energy and move away from fossil fuels, a better energy tax policy would consist of a carbon tax or a cap-and-trade system for carbon emissions. Compared to the current assortment of energy subsidies, these approaches would be more effective, more efficient, and instead of costing billions of dollars they could generate significant government revenue which could be used to reduce other distortionary taxes.

The model presented in Section 4.2 of this thesis provides an initial step towards understanding the effect of uncertain tax policy on investment in wind power but it also leaves much for future research. An obvious next step will be to solve the model numerically and to then perform Monte Carlo simulations to calculate the exact effects of the tax policy uncertainty. But even this is only an early step in understanding the effects of PTC uncertainty. Important changes can be made to the model so that it better describes the dynamics of the wind-power industry. Note that in my model the price of capital is constant. Recent history tells us that there are significant adjustment costs when wind-power investment swings wildly from booms to busts. A useful contribution would be to build a model in which the price of capital responds to the boom-and-bust cycle. Further research could also incorporate the work of Barradale (2008) to examine the role of long-term PPAs in a model of investment with uncertainty. Alternatively, one might develop a model of investment that incorporates the findings of Carlson and Metcalf (2008) that firms are limited in their ability to take advantage of the federal PTC.

Appendix A

Ito's Lemma

I use Ito's Lemma several times in solving the models in Section 4 so I provide a brief explanation of the result that is necessary to differentiate functions of Ito processes. Ito's Lemma is easiest to understand as a Taylor series expansion. First suppose that $x(t)$ follows an Ito process (generalized Brownian motion)

$$dx = a(x,t)dt + b(x,t)dz$$

where $dz = \epsilon\sqrt{dt}$ with $\epsilon \sim \mathcal{N}(0,1)$ and consider a function $F(x,t)$ that is twice differentiable in x and once in t. Here $a(x,t)$ and $b(x,t)$ are known, nonrandom functions. To find the total differential of the function, dF, we typically only consider first-order changes in x and t and therefore write

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial t}dt.$$

Including higher-order terms, we can write

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial t}dt + \frac{1}{2!}\frac{\partial^2 F}{\partial x^2}(dx)^2 + \frac{1}{3!}\frac{\partial^3 F}{\partial x^3}(dx)^3 + \ldots.$$

Recall that in ordinary (deterministic) calculus the higher-order terms vanish in the limit. Now note that

$$\begin{aligned}(dx)^2 &= a^2(x,t)(dt)^2 + 2a(x,t)b(x,t)dzdt + b^2(x,t)(dz)^2 \\ &= a^2(x,t)(dt)^2 + 2a(x,t)b(x,t)(dt)^{3/2} + b^2(x,t)dt.\end{aligned}$$

As $dt \to 0$ terms of dt of order greater than one go to zero faster than dt so we see that

$$(dx)^2 = b^2(x,t)dt.$$

Therefore when $x(t)$ follows an Ito process the differential dF is given by

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}(dx)^2.$$

An appropriate version of Ito's Lemma can also be applied to a combination of an Ito process and a jump process.

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