CALCULATING PLANETARY ORBITS — 1680

	ORBITAL TRAJECTORY	LOCATION VS. TIME	MEAN DIST. FROM SUN
KEPLER	ellipse	area rule	from observations
BOULLIAU	ellipse	a geometric construction	from observations
HORROCKS	ellipse	area rule	via 3/2 power rule
STREETE	ellipse	Boulliau's construction	via 3/2 power rule
WING	ellipse	oscillating equant -------------- a geometric construction	from observations
MERCATOR	ellipse	a geometric construction	from observations

Principal Results from the Registered Version De Motu Corporum in Gyrum

- **A sufficient condition for Kepler's area rule to hold exactly**
- **A necessary and sufficient condition for Kepler's 3/2 power rule to hold exactly for a multiple bodies moving uniformly in concentric circles**
- **A necessary condition for bodies to be moving exactly in ellipses in which all departures from uniform motion in a straight line are directed toward a focus of the ellipse**
- **A sufficient condition for Kepler's 3/2 power rule to hold exactly for multiple bodies moving in confocal ellipses**
- **A solution for the closed-circuit motion of a projectile under a $1/r^2$ centripetal force that in principle can be applied even to comets**
- **A solution for vertical fall under a $1/r^2$ centripetal force that allows the difference between this rule and uniform acceleration in free fall to be determined**
- **A solution for Galilean motion under resistance forces that vary linearly with velocity which, in principle, allows the differences between Galileo's solutions for free fall and projectile motion and the corresponding motions in resisting media to be calculated**

Upshot: ***Kepler-Horrocks orbital rules have prima facie claim to being at least essentially exact, while Galilean free fall and parabolic projection correlatively have claim only to being approximate (and not in the mean)***

Loose-Ends in the Registered Version of De Motu Corporum in Gyrum

- What is the basis of the reasoning from the phenomenon of the 3/2 power rule to the inverse-square for the planets in the Scholium to Theorem 2 and then to the ellipse in the Scholium to Problem 3?

- Is there any independent evidence for an inverse-square centripetal tendency extending throughout the space around the Sun other than the yet to be substantiated potential evidence from the trajectories of comets?

- What evidence is there that, contrary to the findings of Galileo and Huygens, terrestrial gravity is inverse-square? (Also, what evidence is there that air resistance varies linearly with velocity?)

- Insofar as at least three centers of inverse-square forces have been identified – the Sun, Jupiter, and Saturn – (and perhaps a fourth – the Earth), and at least Jupiter and Saturn are in motion around the Sun, how can the motions of the planetary satellites be referred to the planets as their centers? Indeed, to what point in space, taken to be at rest, should all the orbital motions be referred?

- How far do the centripetal tendencies toward Jupiter and Saturn extend outward from them, all the way to the Sun? If so, why isn't the Sun, contrary to Copernicanism, itself in motion as well?

- The deceleration from resistance depends on the weight of the body, as does Huygens's centrifugal tension in a string retaining a body in circular motion; yet the centripetal forces of Theorems 2 and 4 and Problem 5 appear to be independent of the weight of the body, in the manner of Galilean motion. What justifies this difference?

De motu sphæricorum Corporum in fluidis.

Def. 1. Vim centripetam appello qua corpus attrahitur vel impellitur versus punctum aliquod quod ut centrum spectatur.

Def. 2. Et vim corporis seu corpori insitam qua id conatur perseverare in motu suo secundum lineam rectam.

Def. 3. Et resistentiam quæ est medij regulariter impedientis.

Def. 4. Exponentes quantitatum sunt aliæ quævis quantitates proportionales expositis.

~~Hypoth~~ Lex 1. Sola vi insita corpus ~~motu~~ uniformiter in linea recta semper pergere si nil impediat.

~~Hypoth~~ Lex 2. ~~mot. quietis vel~~ Mutationem ~~motus~~ status movendi vel quiescendi proportionalem esse vi impressæ et fieri secundum lineam rectam qua vis illa imprimitur.

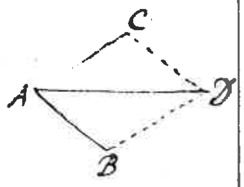
~~Hypoth~~ Lex 3. Corporum dato spatio inclusorum eosdem esse motus inter se sive spatium illud quiescat sive moveat id perpetuò et uniformiter in directum absq motu circulari.

~~Hypoth~~ Lex 4. Mutuis corporum actionibus commune centrum gravitatis non mutare statum suum motus vel quietis. Constat ex ~~[illegible]~~ Lege 3

! ~~Hypoth~~ Lex 5. Resistentiam medij esse ut medij illius densitas et corporis moti sphærica superficies & velocitas conjunctim.

Lemma 1. Corpus viribus conjunctis diagonalem parallelogrammi eodem tempore describere quo latera separatis.

Si corpus dato tempore vi sola M ferretur ab A ad B et vi sola N ab A ad C, compleatur parallelogrammum ABDC et vi utraq feretur id eodem tempore ab A ad D. Nam quoniam vis M agit secundum lineam AC ipsi BD parallelam, hæc vis per Legem 2 nihil mutabit celeritatem accedendi ad lineam illam BD vi altera impressam. Accedet igitur corpus eodem tempore ad lineam BD sive vis AC imprimatur sive non, atq adeò in fine illius temporis reperietur alicubi in linea illa BD. Eodem argumento in fine temporis ejusdem reperietur alicubi in linea CD, et proinde in utriusq lineæ concursu D reperiri necesse est.



Lemma 2. Spatium quod corpus urgente quacunq vi centripeta ipso motus initio describit, esse in duplicata ratione temporis.

De Motu Sphaericorum Corporum in Fluidis

Law 1. A body always goes uniformly in a straight line by its innate force alone if nothing impedes it.

Law 2. **A change of the state of motion or rest is proportional to the impressed force and occurs along the straight line in which that force is impressed.**

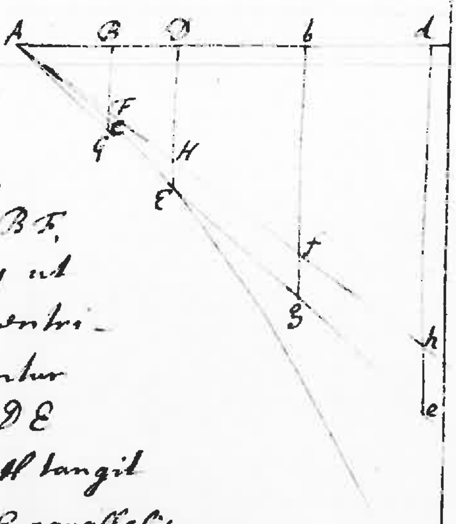
Law 3. **The relative motions of bodies contained in a given space are the same whether that space is at rest or whether it moves perpetually and uniformly in a straight line without circular motion.**

Law 4. **The common center of gravity does not alter its state of motion or rest through the mutual actions of bodies. This follows from Law 3.**

Law 5. The resistance of a medium is as the density of that medium and as the spherical surface of the moving body and its velocity conjointly.

Lemma 1. A body describes by the action of combined forces the diagonal of a parallelogram in the same time as it would describe the sides by the action of separate forces.

Lemma 2. The space described by a body urged by any centripetal force at the beginning of its motion is as the square of the time.



Exponantur tempora per lineas AB, AD datis Ab Ad proportionales, et urgente vi centripeta æquabili exponentur ſpatia deſcripta per areas rectilineas ABF ADH perpendiculis BF, DH et rectâ quavis AFH terminatas ut expoſuit Galileus. ~~[illegible]~~ Urgente autem vi centripeta inæquabili~~[illegible]~~ exponantur ſpatia deſcripta per areas ABC, ADE curva qualis ACE quam recta AFH tangit in A, comprehenſas. Age rectam AE parallelis BF, bf, dh occurrentem in G, g, e, et ipſis bf, dh occurrat AFH producta in f et h. Quoniam area ABC major eſt area ABF minor area ABG et area curvilinea ADEC major area ADH minor area ADEG erit area ABC ad aream ADEG major quam area ABF ad aream ADEG minor quam area ABG ad aream ADH hoc eſt major quam area Abf ad aream Ade minor quam area Abg ad aream Adh. Diminuantur jam lineæ AB, AD in ratione ſua data uſq dum puncta ABD coeunt et linea Ae conveniet cum tangente Ah, adeoq ultimæ rationes Abf ad Ade et Abg ad Adh evadent eædem cum ratione Abf ad Adh. Sed hæc ratio eſt dupla rationis Ab ad Ad ſeu AB ad AD ergo ratio ABC ad ADEC ultimis illis intermedia jam fit dupla rationis AB ad AD id eſt ratio ultima evaneſcentium ſpatiorum ſeu prima naſcentium dupla eſt rationis temporum.

Lemma 3. Quantitates differentiis ſuis proportionales ſunt continuè proportionales. Ponatur A ad A−B, ut B ad B−C & C ad C−D &c et dividendo fiet A ad B ut B ad C et C ad D &c

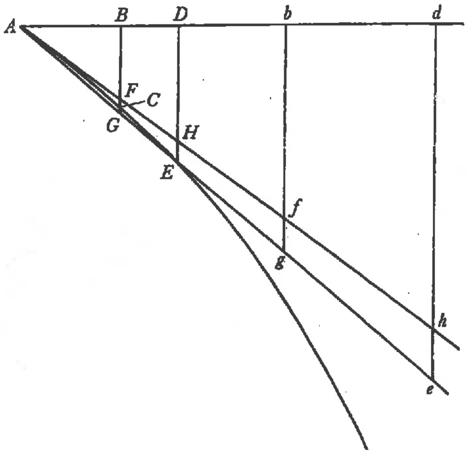
Lemma 4. Parallelogramma omnia circa datam Ellipſin deſcripta, eſſe inter ſe æqualia. Conſtat ex Conicis.

De motu corporum in mediis non reſiſtentibus

Theorema 1. Gyrantia omnia radiis ad centrum ductis areas temporibus proportionales deſcribere.

Dividatur tempus in partes æquales, et prima temporis

Lemma 2: *The space described by a body urged by any centripetal force at the beginnning of its motion is as the square of the time.*



Sketch of proof:

Let the times be represented by the lines AB, AD; the spaces described under a uniform centripetal force, by the areas ABF and ADH; and the spaces described under a non-uniform centripetal force by ABC and ADE, where AH is tangent to the curve ACE at A.

Draw a straight line AGE, with the areas under it, e.g. ABG, representing the spaces described under a uniform centripetal force (greater than that corresponding to AFH); and extend the line ABD to ABDbd with AB:AD as Ab:Ad.

Since the areas under ACE, such as ABC, are greater than those under AFH, such as ABF, and less than those under AGE, such as ABG, the spaces described in times less than or equal to AD under the non-uniform force are greater than or equal to those under the uniform force corresponding to AFH and less than or equal to those under the uniform force corresponding to AGE

Let the lines AB and AD decrease in their given ratio until the points A, B, and D meet and the line Ae coincides with the tangent Ah; then the ultimate ratios of Abf to Ade and Abg to Adh become the same as the ratio of Abf to Adh. But the latter is as the square of the ratio of AB to AD, and therefore the ratio ABC to ADEC, intermediate between the ultimate ratios will also be as the square of the ratio of AB to AD.

Cæterum totum cœli Planetarij spatium vel quiescit (ut vulgo creditur) vel uniformiter movetur in directum et perinde Planetarum commune centrum gravitatis (per ~~Hyp.~~ Legem 4) vel quiescit vel una movetur. Utroque in casu motus Planetarum inter se (per ~~Hyp.~~ Legem 3) eodem modo se habent, et eorum commune centrum gravitatis respectu spatij totius quiescit, atque adeo pro centro immobili Systematis totius Planetarij haberi debet. Inde vero Systema Copernicæum probatur a priori. Nam si in quovis Planetarum situ computetur commune centrum gravitatis hoc vel incidet in corpus Solis vel ei semper proximum erit. Eo Solis a centro gravitatis errore fit ut vis centripeta non semper tendat ad centrum illud immobile et inde ut planetæ nec moveantur in Ellipsibus exacte neque bis revolvant in eadem orbita. Tot sunt orbitæ Planetæ cujusque quot revolutiones, ut fit in motu Lunæ et pendet orbita unaquæque ab omnium Planetarum motibus conjunctis, ut taceam eorum omnium actiones in se invicem. Tot autem motuum causas simul considerare et legibus exactis calculum commodum admittentibus motus ipsos definire superat ni fallor vim omnem humani ingenij. Omitte minutias illas et orbita simplex et inter omnes errores mediocris erit Ellipsis de qua jam egi. Siquis hanc Ellipsin ex tribus observationibus per computum trigonometricum (ut solet) determinare tentaverit, hic minus caute rem aggressus fuerit. Participabunt observationes illæ de minutijs motuum irregularium hic negligendis adeoque Ellipsin de justa sua magnitudine et positione (quæ inter omnes errores mediocris esse debet) aliquantulum deflectere facient, atque tot dabunt Ellipses ab invicem discrepantes quot adhibentur observationes trinæ. Conjungendæ sunt igitur et una operatione inter se conferendæ observationes quamplurimæ, quæ se mutuo contemperent et Ellipsin positione et magnitudine mediocrem exhibeant.

Prob. 4 Posito quod vis centripeta sit reciproce proportionalis quadrato distantiæ a centro, et cognita vis illius quantitate, requiritur Ellipsis quam corpus describet de loco dato cum data celeritate secundum datam rectam emissum.

Vis centripeta tendens ad punctum S ea sit quæ corpus π in circulo πχ centro S intervallo quovis Sπ descripto gyrare faciat. De loco P secundum lineam PR emittatur corpus P, et mox

Moreover, the whole space of the planetary heavens is either at rest (as is commonly believed) or moves uniformly in a straight line, and hence the common center of gravity of the planets (by Law 4) is either at rest or moves along with it. In either case the motions of the planets among themselves (by Law 3) are the same, and their common center of gravity is at rest with respect to the whole space, and thus can be taken for the immobile center of the whole planetary system.

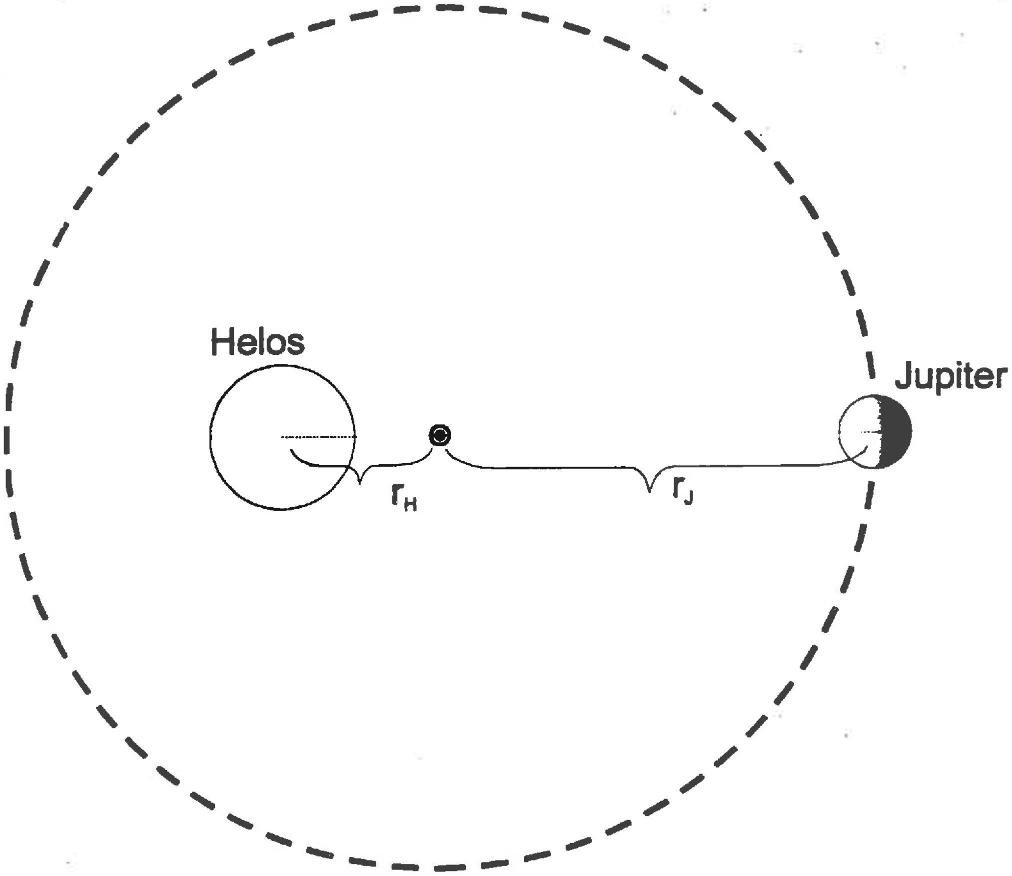
Hence indeed the Copernican system is proved *a priori*. For if in any position of the planets their common center of gravity is computed, this either falls in the body of the Sun or will always be close to it.

By reason of the deviation of the Sun from the center of gravity, the centripetal force does not always tend to that immobile center, and hence the planets neither move exactly in ellipses nor revolve twice in the same orbit. Each time a planet revolves it traces a fresh orbit, as in the motion of the Moon, and each orbit depends on the combined motions of all the planets, not to mention the actions of all these on each other. But to consider simultaneously all these causes of motion and to define these motions by exact laws admitting of easy calculation exceeds, if I am not mistaken, the force of any human mind.

Omit these minutiae, and the simple orbit and mean among all the deviations will be the ellipse that I have already discussed. If any one tries to determine this ellipse by trigonometrical computation from three observations (as is customary), he will have proceeded without due caution. For those observations will share in the minute irregular motions here neglected and so make the ellipse deviate a little from its just magnitude and position (which ought to be the mean among all the deviations), and so will yield as many ellipses differing from one another as there are trios of observations to be employed. Therefore there are to be joined together and compared with one another in a single operation a great number of observations, which temper each other mutually and yield the mean ellipse in both position and magnitude.

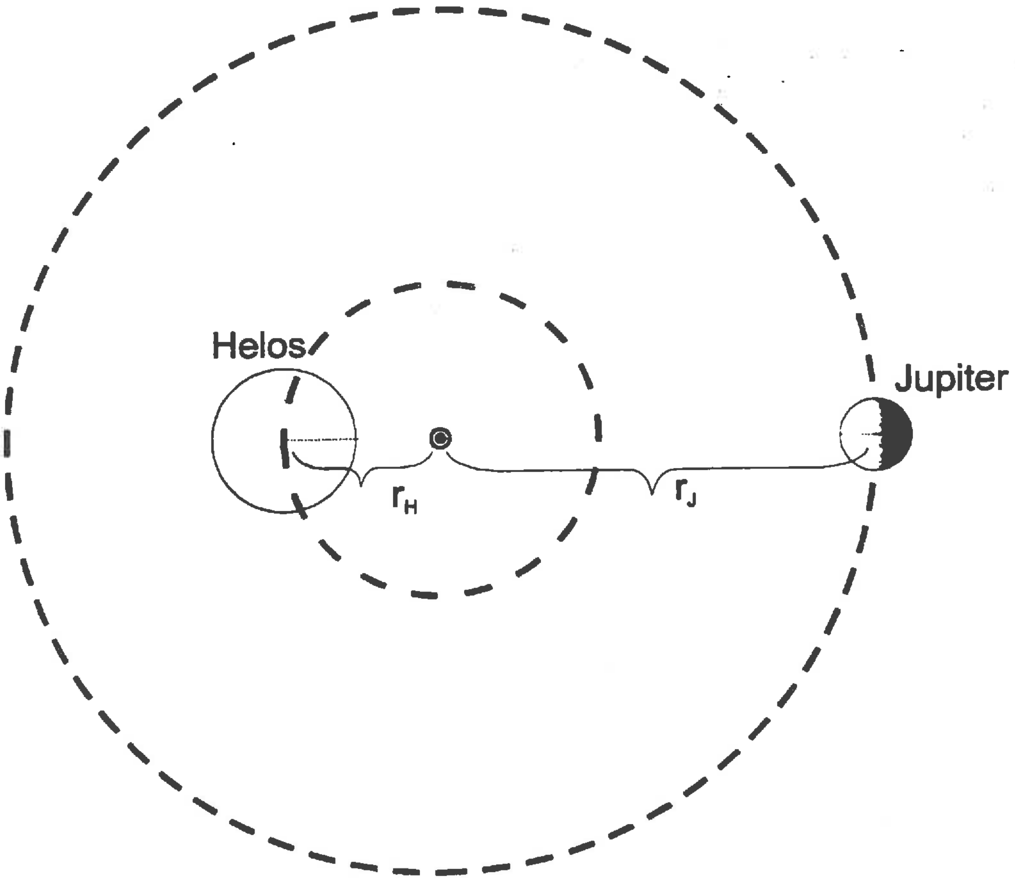
Newton, ca. December 1684
First published by Rouse Ball in 1893

JUPITER INTERACTING WITH THE SUN



$$\frac{r_H}{r_J} = constant$$

DETERMINATION OF r_H



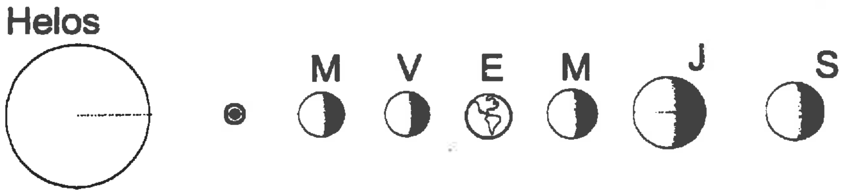
$$\dot{V}_{J_{cent}} \propto \frac{r_J}{P_J^2} \propto \frac{[a^3/P^2]_H}{r_{JH}^2}$$

$$\dot{V}_{H_{cent}} \propto \frac{r_H}{P_H^2} \propto \frac{[a^3/P^2]_J}{r_{JH}^2}$$

$$\frac{r_H}{r_J} = \frac{[a^3/P^2]_J}{[a^3/P^2]_H}$$

GENERALIZING TO THE "PROOF"

THE WORST CASE

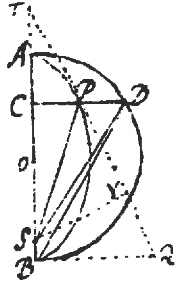


MAX $r_H \leq$ 6 times r_H from Jupiter-Sun case

"...For if in any position of the planets their common center of gravity is computed, this either falls in the body of the Sun or will always be close to it...."

Prob. 5. Poſito quod vis centripeta ſit reciprocè proportionalis quadrato diſtantiæ a centro, ſpatia definire quæ corpus recta cadendo datis temporibus deſcribit.

Si corpus non cadit perpendiculariter deſcribet id Ellipſin puta APB cujus umbilicus inferior puta S congruet cum centro. Id ex jam demonſtratis conſtat. Super Ellipſeos axe majore AB deſcribatur ſemicirculus ADB et per corpus decidens tranſeat recta DPC perpendicularis ad axem, actiſq; DS, PS, erit area ASD areæ ASP atq; adeò etiam tempori proportionalis. Manente axe AB minuatur perpetuò latitudo Ellipſeos, et ſemper manebit area ASD tempori proportionalis. Minuatur latitudo illa in infinitum et orbita APB jam coincidente cum axe AB et umbilico S cum axis termino B deſcendet corpus in recta AC et area ABD evadet tempori proportionalis. Definietur itaq; ſpatium AC quod corpus de loco A perpendiculariter cadendo tempore dato deſcribit ſi modò tempori proportionalis capiatur area ABD et a puncto D ad rectam AB demittatur perpendicularis DC. Q.E.F.



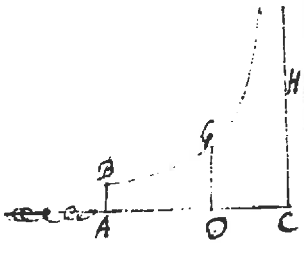
Schol. Hactenus motum corporum in medijs non reſiſtentibus expoſui; id adeo ut motus corporum cæleſtium in æthere determinarem. Ætheris enim puri reſiſtentia quantum ſentio vel nulla eſt vel perquam exigua. Valide reſiſtit argentum vivum, longè minus aqua, aer verò longè adhuc minus. Pro denſitate ſua quæ ponderi fere proportionalis eſt atq; adeo (pene dixerim) pro quantitate materiæ ſuæ craſsæ reſiſtunt hæc media. Minuatur igitur aeris materia craſsa et in eadem circiter proportione minuetur medij reſiſtentia uſq; dum ad ætheris tenuitatem perventum ſit. Celeri curſu equitantes vehementer aeris reſiſtentiam ſentiunt, at navigantes excluſis e mari interiore ventis nihil omninò ex æthere præterfluente patiuntur. Si aer liberè interflueret particulas corporum et ſic ageret, non modo in externam totius ſuperficiem, ſed etiam in ſuperficies ſingularum partium, longè major foret ejus reſiſtentia. Interfluit æther liberrimè nec tamen reſiſtit ſenſibiliter. Cometas infra orbitam Saturni deſcendere jam ſentiunt Aſtronomi ſaniores quotquot diſtantias eorum ex orbis magni parallaxi præterpropter colligere norunt: hi igitur celeritate immenſa in omnes cæli noſtri partes indifferenter feruntur, nec tamen vel crinem ſeu vaporem capiti circumdatum reſiſtentia ætheris impeditum et abreptum amittunt. Planetæ verò jam per annos millenos in motu ſuo perſeverarunt, tantum abest ut impedimentum sentiant.

Demonstratis igitur legibus reguntur motus in coelis. Sed et in aere nostro, si resistentia ejus non consideratur, innotescunt motus projectilium per Prob. 4. et motus gravium perpendiculariter cadentium per Prob. 5. posito nimirum quod gravitas sit reciprocè proportionalis quadrato distantiae a centro terrae. Nam virium centripetarum species una est gravitas; et computanti mihi prodijt vis centripeta qua luna nostra detinetur in motu suo menstruo circa terram, ad vim gravitatis hic in superficie terrae, reciprocè ut quadrata distantiarum a centro terrae quamproximè. Ex horologij oscillatorij motu tardiore in cacumine montis praealti quam in valle liquet etiam gravitatem ex aucta nostra a terrae centro distantia diminui, sed qua proportione nondum observatum est.

Caeterum projectilium motus in aere nostro referendi sunt ad immensum et revera immobile caelorum spatium, non ad spatium mobile quod una cum terra et aere nostro convolvitur et a rusticis ut immobile spectatur. Invenienda est Ellipsis quam projectile describit in spatio illo verè immobili et inde motus ejus in spatio mobili determinandus. Hoc pacto colligitur grave, quod de aedificij sublimis vertice demittitur, inter cadendum deflectere aliquantulum a perpendiculo, ut et quanta sit illa deflexio et quam in partem. Et vicissim ex deflexione experimentis comprobata colligitur motus terrae. Cum ipse olim hanc deflexionem Clarissimo Hookio significarem, is experimento ter facto rem ita se habere confirmavit, deflectente semper gravi a perpendiculo versus orientem et austrum ut in latitudine nostra boreali oportuit.

De motu corporum in medijs resistentibus.

Prob. 6. Corporis sola vi insita per medium similare resistens delati motum definire.



Asymtotis rectangulis ADC, CH describatur Hyperbola secans perpendicula AB, DG in B, G. Exponatur tum corporis celeritas tum resistentia medij ipso motus initio per lineam AC data longitudine, elapso autem ~~elapso~~ tempore aliquo per lineam indefinitam DC, et tempus exponi potest per aream ABGD atque spatium eo tempore descriptum per lineam AD. Nam celeritati proportionalis

Thus far I have explained the motions of bodies in non-resisting mediums, in order that I might determine the motions of celestial bodies in the aether. For I think that the resistance of pure aether is either non-existent or extremely small. Quicksilver resists strongly, water far less, and air still less. These mediums resist according to their density, which is almost proportional to their weights and hence (I may almost say) according to the quantity of their solid matter. Therefore as the solid matter of air is diminished so is the resistance of the medium and in about the same proportion up to the point that it attains the tenuity of air.... If air were to penetrate the parts of bodies freely and so were to act not only on the external surface of the whole but also on the surfaces of its [internal] individual parts, its resistance would be far greater. Aether penetrates very freely and does not resist. All those sounder astronomers think that comets descend below the orb of Saturn, who know how to compute their distances from the parallax of the Earth's orbit, more or less; these therefore are indifferently carried through all parts of our heaven with an immense velocity, and yet they do not lose their tails nor the vapour surrounding their heads, which the resistance of the aether would resist and tear away. Planets have in fact now persevered in their motion for thousands of years, so far are they from experiencing any impediment.

Motion in the heavens, therefore, is ruled by the laws demonstrated. But if the resistance of our air is not taken into account, the motions of projectiles in it are known from Problem 4 and the motions of bodies falling perpendicularly from Problem 5, assuming indeed that gravity is reciprocally proportional to the square of the distance from the center of the Earth. For one kind of centripetal force is gravity, and from my computations it appears that the centripetal force by which our moon is kept in its monthly motion about the Earth is to the force of gravity on the surface of the Earth reciprocally as the squares of the distances from the center of the Earth, more or less. From the slower motion of pendulum clocks on the summits of high mountains than in valleys it is clear also that gravity diminishes with increase of distance from the center of the Earth, but in what proportion has not yet been determined.

The "Moon Test" Revisited

1669:

- From Galileo's *Dialogue*:
  Radius of Earth = 3500 It. miles
  = 17,500,000 It.ft
- Earth – Moon distance = 60 Earth radii
- Period of Moon = 27d7h43m
  = 2.36058e6 sec
- Moon angular velocity = 2.6617e-6 rad/sec
- Moon $r\omega^2$ = 7.43888e-3 It. ft/sec/sec
- Moon's tendency to fall toward Earth =
  3.71944e-3 It. ft in first second
- Tendency to fall at surface of Earth =
  16 It. ft in first second
- Ratio of tendencies = 4302
- Compared to Earth-Moon distance squared:
  19.5% difference

1684:

- From Picard's 1 deg longitude measurement (342,360 Paris ft, vs. Cassini's 342,366):
  Radius of Earth = 19,615,783 Paris ft
- Earth – Moon distance = 60 Earth radii
- Period of Moon = 27d7h43m
  = 2.36058e6 sec
- Moon angular velocity = 2.6617e-6 rad/sec
- Moon $r\omega^2$ = 8.33833e-3 Paris ft/sec/sec
- Moon's tendency to fall toward Earth =
  4.16916e-3 Paris ft in first second
- Tendency to fall at surface of Earth =
  15 Paris ft 1 inch in first second
- Ratio of tendencies = 3617.9
- Compared to Earth-Moon distance squared:
  0.5% difference

The motions of projectiles in our air, moreover, are to be referred to the immense and indeed motionless space of the heavens, not to the moving space which is revolved along with our Earth and our air, and is naively regarded as immobile. The ellipse which the projectile describes in that motionless space is to be found and thence its motion in a moving space is to be determined. With this agreed, it will be gathered that the heavy body which is let fall from the top of a tall building will be deflected a little from the perpendicular in falling, so that the amount of its deflection and the direction thereof [may be determined]. And conversely the motion of the Earth may be gathered from the deflection as established by experiments. When I myself formerly communicated this deflection to the celebrated Hooke, he confirmed that it was so by an experiment three times repeated, the heavy body always deflect-ting from the perpendicular towards the east and south as in our northern latitude it should.

272 NEWTON TO PAGET(1)

DECEMBER 1684

The letter has not been found. For answer see Letter 273

NOTE

(1) We learn of this letter from one (Letter 273) which follows next. In it Newton sends two messages to Flamsteed: first, an offer of the perusal of certain papers; and from Flamsteed's reply (see Letter 273, note (8) and Letter 275, note (1)) we gather that they are the papers on motion, that led to the writing of the *Principia*; secondly, a request for the determination of the distances and periods of the satellites of Jupiter.

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273 FLAMSTEED TO NEWTON

27 DECEMBER 1684

From the original in the University Library, Cambridge.
In reply to Letter 272; for answer see Letter 274

Honored Sr *The Observatory December:* 27. 1684.

Mr Halley delivered me a note wherein you desire me to give you ye places of 2 fixed Stars(1) in ye foot of Perseus, & Mr Paget soone after a letter from you wherein you kindly offerede me ye perusall of your papers for which I humbly thanke you. Mr Pagit promised to be here on Tuesday last but the severity of that dayes weather I suppose prevented him else I had sent you by him the places of the Stars, which I now transmit(2) with ye distances whence I derive them. Octob: 23. 1680 at 9h.p.m. I measured the distance of ye bright star of ♈ from ye brighter of ye 2 marked ζ in Bayer 25°· 03′· 40″ from ye lesser, *O*, 23°· 09′· 15″. and at 9h· 50′ betwixt Aldebaran & ζ. 18°· 02′· 35″. but from it to *O*. 19°· 36′· 35″ allowing about 15″ for ye contraction by refraction I state theire distances & find theire places to ye begining of ye yeare 1681(3) as followes:

Lucida ♈ a ζ, 25°· 03′ 50″
Palilicium a ζ, 18 · 02 · 50 } Long ∗ ♉ 28·40·16. Lat Sept. 11° 17′⅕

Lucida ♈ ab *O*, 23 · 09 · 30
Palilicium ab *O*, 19 · 36 · 50 } Long ∗ ♉ 26·41·48. Lat Sept. 12·08⅓

} tho Tychonick differ of longitude betwixt these ∗∗ is 4½ too big his latitudes well.

Long diff: 1· 58 · 28 ipsarum inde distantia 2°· 06′· 46″ I had proceeded to calculate ye places of ye comet from your observations but that I find you make ye observed distance betwixt ye 2 stars but 1°· 46′· 06″. which(4) is one third of a degree lesse then my calculus gave it, which tho I was very well satisfied in, yet for greater assurance the next night after I had made it I measur'd yt distance with my sextant but had a very bad assistant: yet by severall trialls concluded it could not be more yn 2°· 07′·10″. nor above ½ a minute lesse, which agrees altogeather with ye calculation. I would entreat you therefore examine your notes againe & then I shall either give you ye places of ye comet calculated from your observations or if you desire it rather, from ye french who give its right Ascentions & Declinations on ye 27 & 28 of Feb. & 8th of March, our stile. from which they are easily deducible. Monsieur Cassini saw & observed it neare ye same stars. he gives a map of them in his booke(5) & the comets path amongst them, whereby I should have concluded that you mistooke one of your stars for another that lies more to ye north & is lesse yn either ζ or O, but that ye distance is too small for your measure as mine is too big.

In your letter to Mr Paget you further require my determinations of ye distances of ♃s satellits from him & theire periods. I gave theire utmost elongations in Num 96. of Mr Oldenburg's *transactions*.(6) but haveing since that time made many observations of theire eclipses I find they require them something larger, & that they are as exactly in sequialte proportion to theire periods as it is possible for our sences to determine. I give you here the times(7) of theire revolutions to ♃s in his meane distance for you well know yt when hee is on his Aphelium they are made swifter, when Perihelius slower; & with them theire utmost elongations from ♃s center in his semidiameters & ye millesme parts of them

		Revolutio 1	Revolutiones 5	Elong. a ♃ sd ♃. mill	partibus qualium extima est 100.000	
		di. h ′ ″	dies. h ′ ″		♃ semid.	4·015
Revolutionum tempora ad Jovem.	1.	1 · 18 · 28 · 36:	8 · 20 · 23 · 00	5 · 578	1 elong.	22·377
	2.	3 · 13 · 17 · 54	17 · 18 · 29 · 30	8 · 876	2	35·642
	3.	7 · 03 · 59 · 36	35 · 19 · 58 · 00	14 · 159	3	56·855
	4.	16 · 18 · 05 · 03	83 · 18 · 25 · 15	24 · 903	4	100·

In ye *transactions* of November(8) last I published ye Eclipses of the Satellits for all ye next yeare any where visible. I find I can answer all the Eclipses of ye 1st that have beene carefully observed within lesse yn 2 minutes of time, the 4th has not faild me much more nor the 3d above thrice as much, but the 2d will not be brought to so neare a rule but that it will exorbitate much more. yet is not the error halfe so greate as wee frequently find in ye best lunar tables.(9) I use theire motions altogeather æquable onely allowing Roemers æquation of light,(10) without which allowance the error of my tables would be above 10′ minutes of time. now it seemes strange ye moones motion should be so perplexed with inæqualitys & these, for ought I can perceave yet, except in ye 2d, wholly free from them & I have some reason to thinke ye errors I meet with in my Numbers for ye 2d Satellit may partly proceed from my haveing allowed its orbit to lie in the same plaine with ye orbits of ye other 3, whereas I have some observations yt will scarce allow it to lie otherwayes yn neare ym, but I have not had leasure to examine this matter fully nor could I ever get the observations I proposed for this triall, the heavens either proveing cloudy, or businesse interfereing when they were to be made. I have corrected all their motions, Cassinis old numbers being now 8 degrees false, in the first,(11) which is nearely an hours motion. If you have occasion for them I shall get a breviate of them transcribed for you, & be glad of such an occasion to serve you & shew how much I am obliged by your kind concession of ye perusall of your papers, tho I beleive I shall not get a sight of them till our common freind Mr Hooke & the rest of the towne have beene first satisfied.(12)

I have no opinion of ye supernumerary satellits of ♄. I feare they were discovered as the former when hee was neare his station when his motion in ye Ephemerides being contrary to yt in ye heavens makes our over curious observers to make satellits of small fixed stars & when they cannot find them againe, the planet haveing left theire neighbourhood, they pretend that these dissappeare for a part of theire revolutions: this is the truth of the case in some of them for I can not find the 2 new ones with a 24 foot glasse hitherto. but I designe to trie againe ere long if perhaps I may have better successe.

I forgot to adde one particular concerneing ye orbits of ye satellits: that I make its plane inclined to ye plane of ♃s orbite 2°· 40′ and the North intersection in $3^{\text{gr.}}$ of ♒ at present; tis retrograde but the motion very slow.(13) I have not yet defined the quantity per Annum. it will be requisite to have a many more yeares observations before yt be done. if you desire to be satisfied in any further particulars, let me have your desires proposed in a line or two, you can not propose more readily then you shall be willingly answered by Sr

Your very affectionate freind & humble Servant

JOHN FLAMSTEED

P.S.

Pray when you see Mr Crompton give him my humble service & let him know I cannot send him his Watch till after the holidayes that all faults will be amended it & hee will have no cause to complaine of it hereafter. J.F.

To
Mr Isaack Newton.
at his Chamber in
Trinity Colledge in
Cambridge
these
present.
Cambridge.

NOTES

(1) The stars marked *A* and *B* in figure of No. 253, p. 357. The proper placing of these stars is a prominent item in the ensuing correspondence, see Letters 281 and 282.

(2) The figures are given in the next few lines: later they are revised (see Letter 283, p. 429).

(3) The date 1682 was first written; then Flamsteed altered the '2' to '1', but not very clearly. Put into Newton's notation (p. 425) the particulars are:

Star *A* Longitude ♉ 26° 41′ 48″ Latitude 12° 8′ 20″ N,
Star *B* „ ♉ 28° 40′ 16″ „ 11° 17′ 12″ N.

'Difference of longitude 1° 58′ 28″: therefore their distance 2°. 06′. 46″.'

(4) The distance 1° 46′ 06″ is given by Newton in No. 253, a copy of which was presumably sent to Flamsteed.

(5) This is probably a reference to Cassini's *Observations sur la Comète qui a paru au mois de Décembre 1680 et en Janvier 1681* (Paris, 1681).

(6) *Phil. Trans.* **8** (1673), 6094.

(7) The entries in the first and second columns of the subjoined table give the times of one and of five revolutions.

(8) *Phil. Trans.* **13** (1683), 404. Flamsteed gave the corresponding figures for the years 1685, 6 and 7 in subsequent communications to the *Philosophical Transactions*.

(9) It is unfortunate that we have no particulars of what Newton wrote in the letter Flamsteed received through Paget, for it appears that Newton is answering specific queries throughout the present letter. The lunar theory became a topic of great importance in their subsequent correspondence, and it would be interesting to know if its first introduction here arose spontaneously, or in answer to a remark of Newton's.

(10) See note (13), p. 299.

(11) That is, in the first satellite. To make the meaning clear, the punctuation of this sentence has been corrected by the insertion of commas and the removal of a stop after 'false'.

(12) Flamsteed seems to be alluding to the papers on motion which Paget brought to Halley. See Letter 275, note (1).

(13) See Letter 293, and note (6), p. 450.

274 NEWTON TO FLAMSTEED

30 DECEMBER 1684(1)

From the original in the Bodleian Library, Oxford.
In reply to Letter 273; for answer see Letter 275

Sr

I thank you heartily for your kind information about those things I desired. In my observations about ye Comet I was only carefull of ye proportions of ye distances from small stars to those between ye starrs. Afterwards I believe I made some mistake in ye reduction to minutes & seconds, for ye observations in many cross distances fitted one another well. Your information about ye Satellits of Jupiter gives me very much satisfaction. The orbit of Saturn is defined by Kepler too little for ye sesquialterate proportion.(2) This Planet so oft as he is in conjunction with Jupiter ought (by reason of Jupiters action upon him) to run beyond his orbit about one or two of ye suns semidiameters or a little more & almost all the rest of his motion to run as much or more within it. Perhaps that might be ye ground of Keplers defining it too little. But I would gladly know if you ever observed Saturn to err considerably from Keplers tables(3) about ye time of his conjunction with Jupiter. The greatest error I conceive should be either ye year before conjunction when Saturn(4) is 3 or 4 signes from ye Sun *in consequentia* or the yeare after when Saturn is as far from ye sun *in antecedentia*. You seem to insinuate as if Saturn had not yet any more Satellits that [*sic*] one discovered by Hugenius. I should be glad to know if it be so. And one more favour I would beg, yt if you have any observation(5) of the greatest elongation of any of ye Satellites of Jupiter from his center (chiefly that of ye outmost satellite) in degrees & minutes you would communicate it to me wth ye time of ye observation. For I would glad know ye proportion of ye orbits of ye Satellites to yt of Jupiter as exactly as I can. And if you have any such observation of ye greatest elongation of ye satellite of ♄ I would also beg ye favour of that. Sr I am

Your much obliged Friend
to serve you
Is. Newton.

Trin. Coll. Dec 30
1680.(6)

A good new yeare to you.

For Mr John Flamsteed at
the Observatory in
Greenwich neare
London

NOTES

(1) The correct year of the date is determined by a comparison with the dates and contents of Letters 273 and 275. The '1680' clearly written at the foot of the letter is a slip. Above the 0 someone has put a 6, and following it Flamsteed has written 'should be 1686 or 1685'.

(2) Kepler's third planetary law.

(3) The Rudolphine Tables, see note (11) of Letter 57, vol. I, p. 149.

(4) Except in this sentence every reference to Saturn throughout the letter has been lightly underlined, as also the opening sentence.

(5) Flamsteed has written in the margin 'sent him hereon ye elongations of all & diameters which he has employed in his *Principia*'. See note (5) of Letter 315, p. 494, for the table of elongations given in *Principia*, III, 403, to which Flamsteed refers. At pp. 427–9 Newton summarized in Propositions 22 and 23 his work upon the inequalities of the Moon, and its extension to those of the satellites of ♃ and ♄.

(6) See note (1) above.

275 FLAMSTEED TO NEWTON

5 JANUARY 1684/5

From the original in the University Library, Cambridge.
In reply to Letter 274; for answer see Letter 276

The Observatory Jan 5 168$\frac{5}{4}$

Worthy Sr

I am heartily glad that my comunications are usefull to you & I intreate you whenever you thinke that any thinge in my power may serve you, you would freely command it. if you desire to have ye places of the comet calculated to ye dayes I mentioned tis a short businesse: I shall doe it from ye French observations which seeme sufficiently accurate & send them you. if you will give me leave to guesse at your designe I beleive you are endeavoring to define ye curve yt comet described in ye æther from your Theory of motion(1) & if my assistance in so usefull a designe might helpe I would lend it most willingly. As for ye motion of ♄ I have found it about 27′ slower in ye Antonicall(2) appearances since I came here, then Keplers numbers, & ♃^{s} about 14 or 15′ swifter as you will find by ye account of theire Conjunctions which I published in ye last yeares *transactions*,(3) the error in Jupiter is not allwayes the same, by reason ye place of his Aphelion is amisse in Kepler. nor is ye fault in ♄ alwayes the same but lesse in ye Quadratures as it ought to be, yet the differences in both are regular & may be easily answered by a small alteration in ye Numbers as is found in ♄ by our New Tables(4) which Mr Halley made at my request & Instigation. I have corrected ♃ my selfe so yt hee has of late yeares answered my calculus in all places of his orbit but I have not beene strict enough to affirme that there is no such exorbitation as you suggest of ♄, but after ye next terme if not sooner I will inquire diligently. tho to confesse my thoughts freely to you I can scarce thinke there should be any such influence since the distance of ye planets from each other in those positions is neare four Radij of the *Orbis annuus* so that, in such yeilding matter as our æther, I can not conceave that any impression made by ye one planet upon it can disturbe ye motion of the other: but if you thinke that when they approach so neare each other theire attractive powers exert themselves more vigorously, & either draw them nearer or thrust them further from each other then they would doe, were the planets at a greater distance, I can onely say that it seemes unlikely such small bodies as they are compared with ye Sun, the largest & most vigorous Magnet of our susteme, should have any influence upon each other at so great a distance. the largest magnets that have yet beene got out of our earth for ought I can understand have no influence either upon one another or a Needle at ye distance of 100 yards & You will easily conclude that ye space of forty Thousand diameters of our earth which is neare ye distance of ye 2 planets when in ☌ of ye ☉ beares a much greater proportion to theire diameters then 100 yards does to ye diameter of an ordinary Magnet.(5) but these are onely suggestion & perhaps I mistake all this while the reason of your suspicion that ye one planet operates on ye other. I therefore must beg your pardon for this freedom & assure you that I am wholly unprejudiced by any præconceived opinion & ready to examine any hint that may be usefull or assist in ye restitution of theire Motions. I know Keplers distances of ♄ agree not with ye sesquialter proportion & that ♃s too ought to be amended & both must be altered before wee set upon ye enquiry whether ♃s motion had any influence on ♄s ye yeare before or after ye great conjunction, which I shall willingly trie as soone as our cold weather goes of & I can get a little leasure.

Such observations as you desire of ♃s Satellites you will find printed in Mr Oldenburgs *Transactions*(6) Num. 96. pag 6095 from which I determined then the elongations of each satellite of ♃ in his semidiameters something lesse, then since I have made them on ye examination of theire Eclipses, but because it mey be some trouble to you to worke those observations I will endeavor to ease you as much as I can by giveing you ye elongations of each Satellite in minutes and seconds as they appeare from ye Sun when ♃ is in his Meane distance from him with the same at our earth when hee is in his greatest & nearest distances, as also ye semidiameters of ♃ at our earth in those positions

Juxta Tabulas Carolinas distantiæ a Terra.
♃ diam e propriis observationibus

					Satell.	1	2	3	4
						′ ″	′ ″	′ ″	′ ″
♃ˢ Aphelij in	☍.☉ˢ	420145	50′· 36″	- -		2 · 14	3 · 44⅓	5 · 54½	10 · 11
	☌ ☉	620238	34 · 16	et Satellitum Elongationes		1 · 30⅔	2 · 32	4 · 00	6 · 54
♃ Perihelij in	☍ ☉	394932	53 · 50	inde deductæ extimæ e		2 · 22⅓	3 · 58½	6 · 17	10 · 50
	☌ ☉	594985	35 · 44	propriis observationibus deductæ.		1 · 34½	2 · 38⅕	4 · 10	7 11½

Elongationes autem a ♃ᵉ medio
e sole spectatæ satellitis -
ab iisdem derivatæ - - - - - - -

		′ ″	
	1ⁱ	1 · 48	et Solis e Jove medio visi semidiameter. 3′· 06″
	2ⁱ.	3 · 01	
*(7)	3ⁱʲ.	4 · 16	
*	4ⁱ.	8 · 13½	

These I have transcribed from a little Tract(8) I wrote about 11 yeares agone at Derby concerneing ye diameters of ye planets but never published: the observations from whence they were deduced are published in ye aforementioned place in ye *Transactions* but because ye minutes and seconds answering ye parts of my screw were not necessary to be added there they were omitted. I shall give you ym therefore here by which you may examine my determinations
A.D. 1672. Derbiæ

Martij 19. hor $7\frac{1}{6}$ limbus ♃^{s} rem. a 4to satellite 1601 = 9′· 34″ ♃^{s} semid
27. . 8 limbus ♃^{s} rem a 4to rep.(9) 1591 = 9· 30 perh(10) 64 = 0′· 22″,
28. . 8 Eadem distantia ---------- 1598 = 9 · 33

on ye 27th of March at 8h. ♃ was in ♍ 9° 22′, his distance from our Earth 460900 such as ye earths meane distance from ye ⊙ is 100000. the Satellit then wanted about $\frac{2}{3}$ of a semidiameter of its utmost elongation which was therefore now 9′· 17″ whence ye elongations in ye other distances before recited were derived, whereby tis evident, thus its distance from ♃ is not more then $2\frac{2}{3}$ semidiameters of ye Sun.

The observations whereby ye elongations of ye 3d, 2d, & 1st were limited I made April 4. and 11th. 1673. I rely cheifly on ye latter, tho the former agree very well with them. The Measures were these

1673 April 27(11) hor $7\frac{1}{2}$ ♃^{s} limbus rem. a 3° satellite 947 = 5′· 40″ ♃^{s} semidiameter
a 2° ---- 622 = 3 · 43 auferenda
66 = 0′· 24″
a 1° ---- 405 = 2 · 25

the third satellit by my tables then wanted $\frac{2}{3}$ of a semidiameter of ♃ of its utmost elongation. the second neare $\frac{1}{2}$. & the first $\frac{1}{8}$. which, added to ye observed distances, correct by ye subtraction of ♃^{s} semidiameter because they were measured from his remoter limbe, give ye present utmost elongations of ye
3: 5′· 31″
2. 3 · $29\frac{1}{2}$
1 2 · 05.
♃^{s} distance from our earth was now 449863 such partes as the earths meane distance from ye Sun is 100000. hence ye elongations in ye Tablet were derived, and by these you may examine them.

Sr I have not had the happinesse of Mr Pagetts company this Christmas tho hee promised it me; the hard weather perhaps prevented him as it did me from goeing to London so I have not yet had the happinesse of ye perusal of your papers. I am very well pleased however to heare that you intend to oblige us with ye publication of them next terme when I hope to have ye use of them [not](12) being obliged to any but your selfe for it. Sr I am ever

Your affectionate Freind & Servant

JOHN FLAMSTEED

I wish you an happy yeare pray give the same to Mr Crompton from me when you see him & let him know I am mindfull of his concerne & my promise.

To
Mr Isaack Newton at
his Chamber in Trinity
Colledge in Cambridge
these
present

*(7) 3 d. 4. 46 rectius 4.47
4. 8. $13\frac{1}{2}$ $8.12\frac{2}{3}$

NOTES

(1) At a meeting of the Royal Society on 10 December 1684 'Mr HALLEY gave an account, that he had lately seen Mr NEWTON at Cambridge, who had shewed him a curious treatise, *De Motu*; which, upon Mr HALLEY's desire, was, he said, promised to be sent to the Society to be entered upon their register. Mr HALLEY was desired to put Mr NEWTON in mind of his promise for the securing his invention to himself till such time as he could be at leisure to publish it. Mr PAGET was desired to join with Mr HALLEY' (Birch, IV, 347).

In August 1684, when Halley visited Newton at Cambridge, he had learnt that Newton had proved that for a body moving in an ellipse under an attractive force directed to a focus the law of force was that of the inverse square. In November Paget brought to Halley from Newton a paper containing the demonstration. Thereupon Halley paid Newton a second visit at Cambridge, and afterwards reported the facts to the Royal Society, as recorded above. See Letters 278 and 285, pp. 415 and 431.

There are various conjectures as to the paper brought by Paget to Halley, but probably it was the Tract *De Motu*, printed in Rigaud's *Historical Essay on the first publication of Sir Isaac Newton's Principia* (Oxford, 1838) and again in Rouse Ball's *Essay on Newton's Principia* (London, 1893). Certainly the paper was a forerunner of the *Principia*; and we have Newton's own testimony that he had embarked on the latter in November or December 1684; for he writes 'The Book of Principles was writ in about 17 or 18 months, whereof about two months were taken up with journeys, & the MS was sent to ye R.S. in spring 1686 [and presented on 28 April 1686]; & the shortness of the time in which I wrote it, makes me not ashamed of having committing some faults' (in a rough draft among the Macclesfield papers, printed by Rigaud, *op. cit.* p. 92 and Rouse Ball, *op. cit.* p. 59). For full discussions of the points in doubt, see Rigaud, pp. 15–16, 77; Edleston, p. lv; Rouse Ball, pp. 30–2 and More, *Newton*, pp. 301, 302.

Flamsteed guessed right; the theory was applied to a comet's path in the *Principia*, Book III.

(2) The meaning of the word is doubtful. It may be a slip for 'anatonicall', from the Latin *anatonus* = ἀνάτονος, 'stretching upwards'.

(3) *Phil. Trans.* 13 (1683), 244–58.

(4) Writing to Abraham Sharp on 11 February 1709/10, Flamsteed remarked: 'I must add concerning Saturn that, whereas Sir Isaac Newton suggested to me that all the planets increased in their bulk continually by an accession of matter from the æther about them, this now seems not probable. Mr Halley had told him that the motions of Saturn were slower this last 100 years much than formerly. I have tables of Saturn by me of his making, presented to Sir Jonas Moor, wherein he makes Saturn's motion in 100 years 26 minutes slower than 'tis in the *Caroline tables*. Now if the planets grow slower in their motions they must consequently remove further from the sun, and there is no reason for their removing further from the sun except they increase in bulk and weight: but I do not find that Saturn moves any slower now than he did almost 2000 years agone, which makes me think our earth and the other planets have gained little or nothing from the tayls of comets, and that the fumes from them have filled our orbit from the sun as far as the orb of Venus with that matter which causes the light we see in the moonless nights about the time of the vernal equinox, of which Mr Ffatio has given an account' (from an original draft in Greenwich Observatory library: for the whole letter see Baily, *Flamsteed*, p. 274).

Sir Jonas Moore died in 1679 so these might be the tables referred to above.

(5) Cf. Flamsteed's remarks on magnetic attraction in Letter 250.

(6) *Phil. Trans.* 8 (1673), 6094–5.

(7) Newton's mark indicating a footnote that he placed immediately below the signature at the end of the letter. He has also lightly dotted in a 4 over the figure 1 of the third entry 4'. 16"; thus '4'.

(8) See a MS. tract which he wrote on the apparent diameters of the planets (Greenwich Observatory MSS. vol. 41, 220). 'In this year [1673] also, as I remember, I wrote a small tract in English concerning the true diameters of all the planets, and their visible [*sic*], when at their nearest distance from our Earth, or their greatest remove from it: which I sent to Mr. Newton in the year 1685, who has made use of it in the 4th book of his *Principia*' (memorandum by Flamsteed), see Baily, *Flamsteed*, p. 33.

(9) A contraction for *repetito* (repeated)?

(10) Meaning Jupiter's semidiameter at *perihelion* (below it is taken as 24"). The '64' and (below) '66' presumably refer to readings on his micrometer screw.

(11) An error for April 11.

(12) A short word has been overlaid by the wax that sealed the letter.

276 NEWTON TO FLAMSTEED(1)

[12 JANUARY 1684/5]

From the original in the Bodleian Library.
In reply to Letter 275; for answer see Letter 277

[12 January 1684/5]

Sr

Whilst I was concerned that you should be so long without ye sight of those papers I received a letter from Mr Paget by wch I understood he has been laid up sick of an ague. I am writing to him to transmit ye papers to you as soon as he has a convenient opportunity. In my last(2) I made an allowance for ye distance of Jupiter & Saturn one from another diminishing their virtue in a duplicate proportion of ye distance. But yet I spake there but at randome not knowing their virtues till I had your numbers for Jupiter, by wch I understand his vertue is less then I supposed. But I am still at a loss for Saturn. I have not at all minded Astronomy of some years till on this occasion wch makes me more to seek. I cannot meet wth Hygen's book(3) of Saturn. Mercator(4) & another or two wch I have consulted leave me as wise as I was. I find Saturns ring is to his body in breadth as 9 to 4 & Hygens makes ye ring in Saturns nearest distance 68″ long at most, that is in his meane distance from ye sun about 1′: But it is ye dimension of ye orbit of ye Satelles about him that I want. Now I am upon this subject I would gladly know ye bottom of it before I publish my papers. I believe you can tell me what Hygenius measures are, or if there have been any other since assigned more exactly. For by Hygenius large measures of ♃ I suspect he may have assigned ye apparent diameter of ye ring of ♄ too large. Your information(5) about ye error of Keplers tables for ♃ & ♄ has eased me of several scruples. I was apt to suspect there might be some cause or other unknown to me, wch might disturb ye sesquialtera proportion. For ye influences of ye Planets one upon another seemed not great enough tho I imagined ♃'s influence greater then Your numbers determin it. It would ad to my satisfaction if you would be pleased to let me know the long diameters of ye orbits of ♃ & ♄ assigned by your self & Mr Halley in your new tables, that I may see how the sesquiplicate proportion fills ye heavens together wth another small proportion wch must be allowed for. I thank you for your kind offer of calculating the places of ye Comet from ye French observations to ye days you mentioned. I do intend to determin ye lines described by ye Comets of 1664 & 1680 according to ye principles of motion observed by ye Planets, & should be glad of your help as to those places of ye latter, if I shal not give you too much trouble.(6) Sr I am

Your most obliged Friend to serve you

I. Newton.

For Mr John Flamsteed at ye
Observatory in Greenwich
neare
London

NOTES

(1) At the top of the page Flamsteed has written: 'Mr Paget was not Master till Apr. 1682 therefore this wrote about 85 or Jan 85/6.' The name 'Mr Paget' occurring on the third line is underlined. The postmark is JA/14, from which the date at the head of the letter has been conjectured.

A SMALL "TWO-BODY" CORRECTION
TO KEPLER'S 3/2 POWER RULE

If Jupiter and the Sun interact, then

$$P_J^2 \propto r_{JH}^3 \quad \frac{1}{1 + \frac{[a^3/P^2]_J}{[a^3/P^2]_H}}$$

...It would add to my satisfaction if you would be pleased to let me know the long diameters of the orbits of Jupiter and Saturn by yourself and Mr. Halley in your new tables, that I may see how the sesquiplicate proportion fills the heavens together with another small proportion which must be allowed for....

Newton to Flamsteed
12 January 1684/5

(2) See Letter 274.

(3) *Systema Saturnium* (1659).

(4) *Institutionum Astronomicarum libri duo* (London, 1676). See Letter 8, note (2), vol. I, p. 16.

(5) See Letter 275.

(6) See note (1), Letter 275.

277 FLAMSTEED TO NEWTON

27 JANUARY 1684/5

From the original in the University Library, Cambridge.
In reply to Letter 276

The Observatory Jan: 27. 1684/5

Sr

I receaved your papers from Mr Paget before your last(1) without date came to hand. but a benifice haveing beene bestowed upon me in the meane time I have not had leasure to peruse it yet. being provideing for a short Jorney to see it. however I have not failed to examine Hugens but can not find any thing for your satisfaction in him. but remembring yt Mr Halley had corrected ye Motion of ♄s Satellit in our *Transactions*(2) I turnd to them & found that hee states the utmost Elongation of ye Hugenian satellit from ♄s center. Nine diameters of ye body or 4 of the ring whose diameters I shall give you as deduced from my Derby observations. by which:

	of ♄	of ye Ring
in ye remotest distance.	19″·10‴	43″·06‴
The diameters in ye meane ---------	22 ·15	50 ·02
in ye Nearest --------	26 ·36	59 ·51. which

is lesse yn Hugens make it very sensibly. Hence:

The greatest elongation of ye Satellit from ♄ in his Meane distance is 3′·20″ but the semidiameter of ye ⊙ viewd from ♄ is at the same distance 1′·41″. So yt ye radius of the Orbe of this satellit is but 2 semidiameters of the Sunn. I am calld away excuse this brevity of Sr

Your affectionate Freind & Servant

JOHN FLAMSTEED

I have considered what would be ye change of ye visible place of ♄ admitting him thrust out of his orbite & find it would scarce be sensible tho observed in ye Quadratures but it would be something sensible in ♃ if hee be so thrust out of his path:

J. F.

To
Mr Isaac Newton
at his Chamber in Trinity College
Cambridge these
present.

PENY POST
S
TV
PAYD (3)

NOTES

(1) Letter 276.

(2) *Phil. Trans.* 13 (1683), 82.

(3) The triangular mark indicates the London Penny Post which was introduced by William Dockwra (see *D.N.B.*) in 1683 and later taken over by the Government. The above form of the postmark was used during the period 1684–1711. S (Southwark), TV (Tuesday).

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278 NEWTON TO ASTON(1)

23 FEBRUARY 1684/5

From a copy in the Letter book of the Royal Society

[Sir]

The designe of a Philosophick Meeting here Mr Paget when last with us pusht forward, and I concurred with him, and engaged Dr More(2) to be of it, and others were spoke too partly by me, partly by Mr Charles Montague(3) but that which chiefly dasht the buisiness was the want of persons willing to try experiments, he whom we chiefly relyed on, refusing to concern himself in that kind him self, And more what to add further about this buisiness I know not, but only this that I should be very ready to concurre with any persons for promoting such a designe so far as I can doe it without engaging the loss of my own time in those thinges.

I thank you for entring in your Register my Notions about Motion.(4) I designed them for you before now but the examining severall thinges has taken a greater part of my time then I expected, and a great deale of it to no purpose. And now I am to goe into Lincolnshire(5) for a Month or six weeks. Afterwards I intend to finish it(6) as soon as I can conveniently &c

Is. Newton.

Cambridge Febr. 23th. 1684/5

De motu corporum in mediis regulariter cedentibus.

Definitiones.

Def. 1. Tempus absolutum est quod sua natura absq relatione ad aliud quodvis æquabiliter fluit. Tale est, cujus æquationem investigant Astronomi, alio nomine dictum Duratio.

Def. 2. Tempus relativum spectatum est quod ~~alicujus sensibilis transitu seu rei alterius fluxione~~ respectu ~~fluxionis~~ seu transitus rei alicujus sensibilis consideratur ut æquabile. ~~seu transitu mensuratur~~. Tale est tempus dierum mensium et aliarum periodorum cælestium, ~~ex hypothesi quod hæ periodi sunt æquabiles quod profecto cum hoc nomine vulgi~~ ~~quæ vulgus~~ ~~[illegible]~~ ~~et æquabiles considerat ea~~ ~~mente vulgi~~ ~~esse creditur~~ apud vulgus.

Def. 3. Spatium absolutum ~~dictum~~ est quod sua natura absq relatione ad aliud quodvis semper manet immobile. Ut partium temporis ordo immutabilis est sic etiam partium spatii. Moveantur hæ de locis suis et movebuntur de seipsis. Nam tempora et spatia sunt suiipsorum et rerum omnium loca. In tempore quoad ordinem successionis, in spatio quoad ordinem situs locantur universa. De illorum essentia est ut sint loca et loca primaria moveri absurdum est. ~~[illegible]~~. Porro vi illata moveatur una pars spatii et vi ~~eadem~~ tanta ad omnes in infinitum partes applicata movebitur totum, quod rursus absurdum est.

Def. 4. Spatium relativum est quod respectu rei alicujus sensibilis ~~alterius~~ consideratur ut immobile: uti spatium aeris nostri respectu terræ. Distinguuntur autem hæc spatia ab invicem ipso facto per descensum gravium quæ in spatio absoluto rectam petunt centrum in relativo absolute gyranti deflectunt ad latus.

Def. 5. Corpora ~~[illegible] et extensa et~~ in sensus omnia incurrunt ~~[illegible]~~ ut res ~~[illegible]~~ mobiles quæ se mutuo penetrare nequeunt. ~~Def. 6. Centrum corporis cujusq est quod vulgo dicitur centrum gravitatis~~.

Axis corporis est linea quævis recta per centrum transiens.

Def. 7. Locus corporis est pars spatii in quo corpus existit, estq pro genere spatii vel absolutus vel relativus

Def. 8. Quies corporis est perseverantia ejus in eodem loco, estq

6 Densitas corporis est quantitas seu copia materiæ collata cum quantitate ~~spatij quod~~ occupati spatij.

7. Per pondus intelligo quantitatem seu copiam materiæ movendæ abstracta ~~materiæ abstracta consideratione~~ gravitationis consideratione quoties de gravitantibus non agitur. Quippe ~~copia potentia~~ pondus gravitantium proportionale est quantitati materiæ, et per se ~~innotescit~~ exponere et designare licet. Analogia vero sic colligitur. Pendulis æqualibus numerentur oscillationes corporum duo

1 analoga rum ejusdem ponderis ~~[illegible]~~ et copia materiæ ~~[illegible]~~ in utroque erit reciprocè ut numerus oscillationum eodem tempore factarum. Experimentis autem in auro, argento, plumbo, ~~sale communi~~ vitro, arena, sale communi, aqua, ligno, tritico diligenter factis ~~[illegible]~~ incidi semper in eundem oscillationum numerum. [Ob hanc analogiam et defectu vocis commodioris expono et designo quantitatem materiæ per pondus, etiam in ~~non gravitantibus~~ corporibus quorum gravitatio non consideratur]

8 Locus

9 Quies

10 Motus

11 Velocitas

12 Quantitas motus est quæ oritur ex velocitate et pondere corporis translati conjunctim: Motus additione corporis alterius tanto cum motu fit duplus et dupli catata velocitate quadruplus

Def 14 Corporis vis exercita est qua id conatur ~~singulis momentis~~ conservare status sui movendi vel quiescendi illam quam singulis momentis amittit partim, ~~amittat~~ estque status illius ~~[illegible]~~ mutationi seu singulis momentis parti amissæ proportionalis. nec obluctatio vel propriè resistentia ~~[illegible]~~ ~~obluctatio~~ corporis dicitur. Hujus ~~una~~ ~~refractio~~ species est vis centrifuga gyrantium.

vel absoluta vel relativa pro genere loci.

Def. 9. Motus corporis est translatio ejus de loco in locum, estque itidem vel absolutus vel relativus pro genere loci. Distinguitur autem ipso facto motus absolutus a relativo in gyrantibus, per conatum recedendi a centro, quippe qui ~~in~~ gyratione nudè relativa nullus est, in relative quiete permagnus esse potest, ut in corporibus cælestibus quæ Cartesianorum juxta stant ~~[illegible]~~ conantur tamen a sole recedere. Conatus ille ~~qui~~ certus semper et determinatus arguit certam aliquam et determinatam esse motus realis quantitatem in singulis corporibus, a relationibus quæ innumeræ sunt, totidemque motus relativos constituunt minime pendentem. Porro motum et quietem absolute dictos non pendere a situ et relatione corporum ad invicem manifestum est ex eo quod hæ nunquam mutantur nisi vi in ipsum corpus motu~~m~~ vel quiescens impressa, tali autem vi semper mutantur; at relativa mutari possunt vi solummodo impressa in altera corpora ad quæ fit relatio et non mutari vi impressa in utraque sic ut situs relativus conservetur.

Def. 10. ~~Celeritas motus~~ Velocitas est quantitas ~~momentanea~~ translationis quoad longitudinem itineris certo tempore confecti. Iter vero est quod corporis puncto medio describitur a Geometris dicto centro gravitatis. Loquor de motu progressivo.

ibi conjunctim:

Def. 11. Quantitas motus est quæ oritur ex velocitate et quantitate corporis translati conjunctim. Æstimatur autem quantitas corporis ex copia materiæ corporeæ quæ gravitati suæ ~~[illegible]~~ proportionalis esse solet. Pendulis æqualibus numerentur oscillationes corporum duorum ejusdem ponderis, et copia materiæ in utroque erit reciprocè ut numerus oscillationum eodem tempore factarum.

Def. 12. ~~Vis corporis seu~~ Corporis vis insita innata et essentialis ~~[illegible], innata~~, est potentia qua id ~~conatur~~ perseverat in statu suo quiescendi vel movendi uniformiter in linea recta, estque corporis quantitati proportionalis, exercetur vero proportionaliter mutationi status et quatenus exercetur dici potest ~~corporis~~ vis exercita ~~est~~ cujus species est vis centrifuga gyrantium. ~~Hujus generis~~

Def. 13. Vis motus seu corpori ex motu ~~adventitia est~~ qua corpus totam quantitatem sui motus conservare conatur. ~~Ea~~ vulgo dicitur impetus estque motui proportionalis, et pro genere motus vel absoluta est vel relativa. ~~Ad absolutam referenda est vis centrifuga gyrantium~~

Def. 14. Vis ~~[illegible]~~ ~~impressa~~ corpori illata et impressa est qua corpus urgetur mutare statum suum movendi vel quiescendi, estque diversarum specierum ut pulsus seu pressio percutientis, pressio continua, vis centripeta, resistentia medii etc. ~~[illegible]~~.

Def. 15. Vim centripetam appello qua corpus impellitur vel attrahitur versus punctum aliquod quod ut centrum spectatur. Hujus generis est gravitas tendens ad centrum terræ, vis magnetica tendens ad centrum magnetis et vis cælestis cohibens Planetas ne abeant in tangentibus ~~orbitarum~~ orbitarum.

Def. 17. ~~Per~~ medii Resistentiam ~~est~~ in sequentibus intelligo vim medii regulariter impedientis. ~~Ali~~

~~resistentiam [illegible] non considero~~. Sunt et aliæ vires in corporum elasticitate, mollitie, tenacitate &c pendentes quas hic non considero.

Def. 10. Momenta quantitatum sunt ipsarum principia generantia vel alterantia fluxu continuo: ut temporis præsens præteriti et futuri, motus præsens præteriti et futuri, vis centripeta aut alia quævis momentanea impetus, punctum lineæ, linea superficiei, superficies solidi et angulus contactus anguli rectilinei.

Def. 13. Exponentes temporum spatiorum motuum celeritatum et virium sunt quantitates quævis proportionales exponendis.

Hæc omnia fusius explicare visum est ut Lector, præjudiciis quibusdam vulgaribus liberatus, ~~clarus~~ et distinctis principiorum Mechanicorum conceptibus imbutus accederet ad sequentia. Quantitates autem absolutas et relativas ab invicem sedulo ~~accurate~~ distinguere necesse fuit ~~[illegible]~~ eo, quod phænomena omnia pendeant ab absolutis, Vulgus autem qui cogitationes a sensibus abstrahere nesciunt semper loquuntur de relativis, usque adeo ut absurdum foret vel sapientibus ~~aut~~ vel etiam Prophetis apud hos aliter loqui. Unde et Sacræ literæ et Scripta Theologorum de relativis semper intelligenda sunt, et crasso laboraret præjudicio qui inde de rerum naturalium motibus absolutis philosophicas disputationes moveret~~et~~. ~~Perinde est ac si quis in [illegible] magnitudine non apparente sed absoluta [illegible] inter duo lumina [illegible] contenderet. [illegible]~~

Leges motus.

Lex 1. Vi insita corpus omne ~~[illegible]~~ perseverare in statu suo quiescendi vel movendi uniformiter in linea recta ~~in linea recta~~ nisi quatenus viribus impressis ~~[illegible]~~ cogitur statum illum mutare. Motus autem uniformis hic est duplex, progressivus secundum lineam rectam quam corpus centro suo æquabiliter motu describit et circularis circa axem suum quemvis qui vel quiescit vel motu uniformi latus semper manet ~~[illegible]~~ positionibus suis prioribus ~~[illegible]~~ parallelus.

Lex 2. Mutationem motus proportionalem esse vi impressæ et fieri secundum lineam rectam quâ vis illa imprimitur. Hisce duabus Legibus jam receptissimis Galilæus invenit projectilia gravitate uniformiter et secundum lineas parallelas agente in medio non resistente lineas Parabolicas describere. Et suffragatur experientia nisi quatenus motus projectilium ~~a~~ resistentia aeris ~~[illegible]~~ aliquantulum retardatur. A6.

Lex 3. Corpus omne tantum pati reactione quantum agit in alterum. ~~Ergo~~ [illegible] premit vel trahit alterum, ab eo tantum premitur vel trahitur. Si vesica aere plena premit vel ferit alteram sibi consimilem cedet utraque æqualiter introrsum. ~~Si magnes trahit ferrum ipse vicissim tantum trahitur~~. Si corpus impingens in alterum vi sua mutat motum alterius et ipsius motus (ob æqualitatem pressionis mutuæ) vi alterius tantum mutabitur. Si magnes trahit ferrum ipse vicissim tantum trahitur et sic in aliis. Constat vero hæc Lex per Def 12 et 14 in quantum vis corporis ad status sui conservationem exercita sit eadem cum vi in corpus alterum ad illius statum mutandum impressa, et vi priori proportionalis sit mutatio status prioris posteriori ea posterioris.

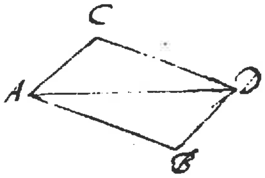
Lex 4. Corporum dato spatio inclusorum eosdem esse motus inter se sive spatium illud absolute quiescat sive moveat id perpetuo et uniformiter in directum absq motu circulari. E.g. Motus rerum in navi perinde ſe habent ſive navis quieſcat ſive moveat ea uniformiter in directum.

Lex 5. Mutuis corporum actionibus commune centrum gravitatis non mutare ſtatum ſuum motus vel quietis. Hæc lex et duæ ſuperiores ſe mutuò probant.

~~Lex 6. Reſiſtentiam medij eſſe ut medij illius denſitas et ſphærici corporis moti ſuperficies et velocitas conjunctim. Hanc legem exactam eſſe non affirmo. Sufficit quod ſit vero proxima Corpora vero ſphærica eſſe ſuppono in ſequentibus, ne opus ſit circumſtantias diverſarum figurarum conſiderare~~

Lemmata

Lem. 1. Corpus viribus conjunctis diagonalem parallelogrammi eodem tempore deſcribere quo latera ſeparatis.



Si corpus dato tempore vi ſola M ferretur ab A ad B et vi ſola N ab A ad C, compleatur parallelogrammum ABDC et vi utraq feretur id eodem tempore ab A ad D. Nam quoniam vis M agit ſecundum lineam AC ipſi BD parallelam, hæc vis nihil mutabit celeritatem accedendi ad lineam illam BD vi altera impreſſam. Accedet igitur corpus eodem tempore ad lineam BD ſive vis AC imprimatur ſive non, atq adeò in fine illius temporis reperietur alicubi in linea illa BD. Eodem argumento in fine temporis ejuſdem reperietur alicubi in linea CD, et proinde in utriusq lineæ concurſu D reperiri neceſſe eſt.

Lem. 2. Spatium quod corpus urgente quacunq vi centripeta ipſo motus initio deſcribit, eſſe in duplicata ratione temporis.

Exponantur tempora per lineas AB, AD datis Ab Ad proportionales, et urgente vi centripeta æquabili exponentur ſpatia deſcripta per areas rectilineas ABF ADH perpendiculis

De Motu Corporum in Mediis Regulariter Cedentibus

The aim of explaining all these things at length is that the reader may be freed from certain vulgar prejudices and imbued with the distinct principles of mechanics may agree in what follows to distinguish carefully from each other quantities which are both absolute and relative, a thing very necessary since all phenomena depend on absolute quantities. But ordinary people who fail to abstract thought from sensible appearances always speak of relative quantities, so much so that it would be absurd for wise men or even Prophets to speak to them otherwise. Hence both the sacred writings and theological writings are always to be understood in terms of relative quantities, and he who would on this account bandy words with philosophers concerning the absolute motions of natural things would be labouring under a gross misapprehension.

Def. 4. Relative space is that which is regarded as immobile in relation to any sensible thing: such as the space of our air in relation to the Earth. However, **these spaces are in fact distinguished from each other through the descent of heavy bodies which in absolute space seek the centre directly but in relative space rotating absolutely are deflected to one side.**

Def. 9. The motion of a body is its translation from one place to another, and is consequently either absolute or relative according to the kind of place. **But absolute motion is in fact distinguished from relative in circular motions by the endeavour to recede from the centre,** which in an entirely relative circular motion is zero, but in a circular motion reative to bodies at rest may be very large, as in the celestial bodies which the Cartesians believe to be at rest, although they endeavour to recede from the Sun. The fact that this endeavour is certain and determinate argues some certain and determinate quantity of real motion in individual bodies in no wise dependent on the relations [between bodies] which are innumerable and make up as many relative motions. For example, **that motion and rest absolutely speaking do not depend on the situation and relation of bodies between themselves is evident from the fact that these are never changed except by force impressed on the body moved or at rest, and are always changed after such a force; but the relative can be changed by forces impressed only on other bodies to which the relation belongs, and is not changed by a force impressed on both so that their relative situation is preserved.**

From … *in mediis regulariter cedentibus*

Def. 11. The quantity of motion is that which arises from the velocity and the quantity of a body in translation [*corporis translati*] jointly. Moreover, the quantity of a body is to be reckoned [*aestimatur*] from the amount [*copia*] of the corporeal matter, which is usually proportional to its gravity [*gravitati*]. The oscillations of two equal pendulums with bodies of equal heaviness [*ponderis*] are counted, and the amount [*copia*] of matter in each will be reciprocally as the number of oscillations made in the same time.

Def. 12. [~~*Vis corporis seu*~~ ^*Corporis vis insita, innata, et essentialis*^] The internal, innate, and essential force of a body is the power by which it ~~*conatur*~~ perseveres in its state of rest or of moving uniformly in a straight line. It is proportional to the quantity of the body, and is truly [*vero*] exercised [*exercetur*] proportionally to the change ~~brought about~~ ^of state^, ^and insofar as it is exercised it can be said to be the exercised force of the body, of which one kind is the centrifugal force of rotating [*gyrantium*] bodies^.

{Canceled: *Def. 13.* The force of a motion or of a body [*Vis motus seu corpori*] from motion at its approach [*ex motu sua adventitia*] is that by which a body endeavors to preserve the ^total^ quantity of its motion. It is commonly called impetus and is proportional to its motion, and according to its kind is absolute or relative. ~~The centrifugal force of rotating bodies is of the absolute kind~~.}

Def. 14. The force brought against and impressed on a body [*Vis corpori illata et impresssa*] is that by which a body is urged to change its state of moving or rest ^and is of diverse kinds such as impulse or pressure of percussion, continuous pressure, centripetal force, resistance of a medium, etc.^

Def. 16. I call centripetal force that by which a body is impelled or attracted [*attrahitur*] towards a certain point regarded as its center. Of this kind is gravity [*gravitas*] tending toward the center of the earth, magnetic force tending toward the center of a loadstone, and the celestial force restraining [*cohibens*] the Planets from going off [*abeant*] along the tangents of their orbits.

De Motu Corporum in Mediis Regulariter Cedentibus

Law 3. As much as any body acts on another so much does it experience in reaction. Whatever presses or pulls another thing by this equally is pressed or pulled. If a bladder full of air presses or carries another equal to itself both yield equally inwards. If a body impinging on another changes by its force the motion of the other then its own motion (by reason of the equality of the mutual pressure) will be changed by the same amount by the force of the other. If a magnet attracts iron it is itself equally attracted, and likewise in other cases. In fact this law follows from Definitions 12 and 14 in so far as the force exerted by a body to conserve its state is the same as the impressed force in the other body to change the state of the first, and the change in the state of the first is proportional to the first force and the second to the second force.

Law 4. The relative motion of bodies enclosed in a given space is the same whether that space rests absolutely or moves perpetually and uniformly in a straight line without circular motion. For example, the motions of objects in a ship are the same whether the ship is at rest or moves uniformly in a straight line.

Law 5. The common center of gravity of bodies does not change its state of rest or motion by reason of the mutual actions of the bodies. This law and the two above mutually confirm each other.

A Page Inserted in …*in mediis regulariter cedentibus*, all in Newton's hand

6. The density of a body is the quantity or amount [*copia*] of matter compared with the quantity of space occupied.

7. By the heaviness [*pondus*] of a body I understand the quantity ^or amount [*copiam*]^ of matter ^moved^ apart [*abstracta*] from considerations of gravitation [*gravitationis*] as often as it is not said [*non agitur*] of gravitating bodies [*de gravitantibus*]. To be sure, the heaviness [*pondus*] of a gravitating body [*gravitantium*] is proportional to its quantity of matter, ^and the agreement [*analogia*] legitimates [*licet*] setting forth [*exponere*] and designating each by the other. The agreement is actually to be gathered [*colligitur*] as follows.^ The oscillations of two equal pendulums of the same heaviness [*ponderis*] are counted and the amount [*copia*] of matter in each will be reciprocally as the number of oscillations made in the same time. Moreover, experiments carefully [*diligenter*] made on gold, silver, lead, glass, sand, common salt, water, wood, and wheat always led to the same number of oscillations. On account of this agreement ^and lacking a more convenient word^ I set forth and designate quantity of matter by heaviness [*pondus*] even when gravitation [*gravitatio*] is not being considered.

8. Place

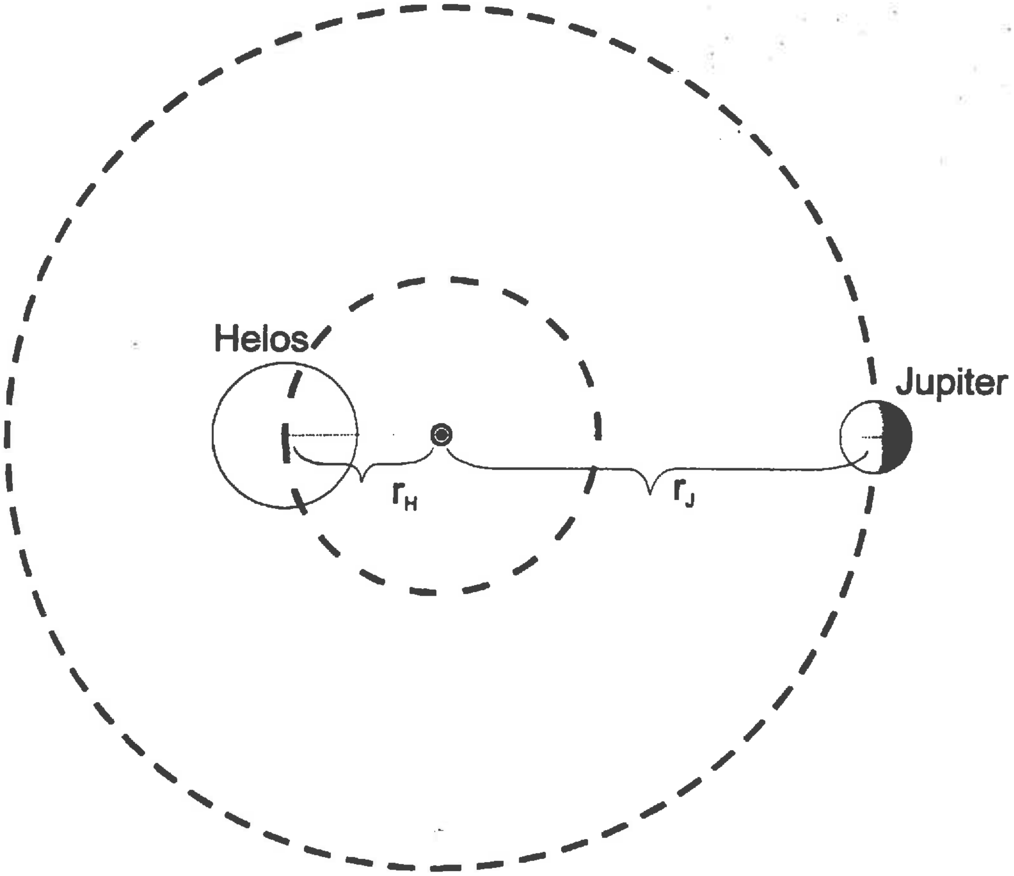
9. Rest

10. Motion

11. Velocity

12. Quantity of motion is that which arises from the velocity and quantity of matter of the body in translation [*corporis translati*] jointly. The motion by addition of another body of the same motion is double and with doubled velocity quadruple.

JUPITER INTERACTING WITH THE SUN



$$r_J W_J = r_H W_H$$

$$\frac{r_H}{r_J} = \frac{[a^3/P^2]_J}{[a^3/P^2]_H}$$

$$\frac{[a^3/P^2]_J}{[a^3/P^2]_H} = \frac{W_J}{W_H}$$

TWO PATHS TO THE LAW OF GRAVITY

$$\frac{[a^3/P^2]_J}{[a^3/P^2]_H} = \frac{W_J}{W_H} = \frac{M_J}{M_H}$$

$$\dot{V}_{J_{cent}} \propto \frac{[a^3/P^2]_H}{r^2_{JH}} \propto \frac{M_H}{r^2_{JH}}$$

$$F_{J_{cent}} \propto M_J \frac{M_H}{r^2_{JH}}$$

$$F_{H_{cent}} \propto M_H \frac{M_J}{r^2_{JH}}$$

i.e.

$$F \propto \frac{m\ M}{r^2}$$

$$F_{J_{cent}} \propto M_J \frac{[a^3/P^2]_H}{r^2_{JH}}$$

$$F_{H_{cent}} \propto M_H \frac{[a^3/P^2]_J}{r^2_{JH}}$$

$$F_{H_{cent}} = F_{J_{cent}}$$

$$\frac{[a^3/P^2]_J}{[a^3/P^2]_H} = \frac{M_J}{M_H}$$

i.e.

$$F \propto \frac{m\ M}{r^2}$$

De motu Corporum.

Liber Secundus.

Fixas in supremis mundi partibus immotas persistere, et Planetas his inferiores circa Solem revolvi, Terram pariter moveri cursu annuo, diurno verò circa axem proprium, et Solem, seu focum Universi, in communi centro quiescere, antiquissima fuit Philosophantium sententia. Sic enim senserant olim Philolaus, Aristarchus Samius, Plato ætate maturiore, ~~Solebant Mathematici~~, Pythagoreorum turba, et his antiquior Anaximander et Romanorum Rex ille sapientissimus Numa Pompilius. Is in symbolum orbis rotundi et ignis Solaris in centro, templum erexit Vestæ forma rotunda, et ignem perpetuum in medio asservari sanxit. Ab Ægyptiis autem astrorum antiquissimis observatoribus propagatam esse hanc sententiam verisimile est. Etenim ab illis et a gentibus conterminis ad Græcos gentem magis ~~et~~ philologicam quam philosophicam, philosophia omnis antiquior juxta et sanior manasse videtur: Et sacra Vestæ ingenium Ægyptiorum sapiunt mysteria captum vulgi superantia sacris ritibus et Hyeroglyphicis tegentium. Subinde docuerunt Anaxagoras Democritus et alii nonnulli Terram in centro mundi immotam stare & astra omnia in occasum, aliqua celerius alia tardius moveri, idque in spatiis liberrimis. Namque orbes solidi postea ab Eudoxo, Calippo, Aristotele introducti sunt, declinante indies philosophia primitus introducta, et novis Græcorum commentis paulatim prævalentibus. Cum his orbibus male consistunt Phænomena Cometarum. Hos inter corpora cælestia a multis olim numeratos Chaldæi rerum astronomicarum peritissimi pro stellis errantibus habuere, quasi semel singulis revolutionibus in orbium valde excentricorum partes infimas descendendo se nobis per vices ~~[illegible]~~ conspiciendos exhiberent. Eosdem postea ~~[illegible]~~ in regiones infra Lunam necessario detrusit ista orbium Solidorum Hypothesis, & [illegible] eidem ~~[illegible]~~ vicissim per nuperas Astronomorum observationes in cælos Luna superiores restitutis confracti sunt iidem orbes ~~est eadem~~ et ex æthere ~~et penitus explosa~~ deturbati.

Quibus vinculis Antiqui ~~Chaldæi, Pythagorei, cæterique~~ Planetas in spatiis liberis retineri, deque cursu rectilineo perpetuo retractos in orbem regulariter agi docuerunt non constat. In hujus rei explicationem orbes solidos excogitatos fuisse opinor. ~~Hi recentiores~~ Philosophi recentiores aut vortices esse volunt, ut Keplerus et Cartesius, aut aliud aliquod, ut Hookius et e nostratibus aliqui et Borellus, impulsus sive attractionis principium, ~~[illegible]~~

1 Cælos esse fluidos.

Archimedes in Arenario. Aristot. lib. 2 de cælo. Plutarch. lib. 3 de placitis Philos. 4 in Numa ~~[illegible]~~

II Principium motus circulari in spatiis liberis.

From *De Motu Corporum, Liber Secundus* [Add. 3990]

18. Another agreement [*analogia*] *between forces and bodies. It is proved for heavenly bodies* [*coelestibus*]

A second agreement between the forces and the attracted bodies is akin to the one just described. Since the action of the centripetal force upon the planets decreases in the duplicate ratio of the distance, and the periodic time is increased in the sesquialteral ratio, it is manifest that if equal planets were equally distant from the Sun, their actions would be equal and their periodic times would be equal, and that if unequal planets were at equal distances, their ^collective [*collectitiae*]^ actions would be as the ~~[*pondera*]~~ ^bodies^ of the planets. For actions that were not as the ~~[*pondera*]~~ ^bodies to be moved^ could not draw those ~~[*pondera*]~~ ^bodies^ equally back from the tangents of the Orbits and cause revolutions to be completed in equal times in Orbits that are also equal. But neither could the motions of the satellites of Jupiter be so regular if the circumsolar force were not exerted equally upon Jupiter and all the satellites in proportion to their weights [*ponderum*]. And the same is true of Saturn and its satellite and also of the earth and our Moon, as (from prop. 35, corol. 2 and 3) is manifest ~~and soon will be made more fully clear~~. At equal distances, therefore, there is an equal action of centripetal force upon all the planets in proportion to their ~~[*ponderum*]~~ ^bodies or quantities of matter in the bodies^, and thus also upon all the particles of that ~~[*ponderis*]~~ ^quantity^ of which the planets are composed. For if the action were greater upon particles of one kind of matter, and less upon those of another, than in proportion to the ~~[*ponderum*]~~ ^quantity of matter^, the action upon the planets would also be greater or less not only in proportion to the ~~[*ponderum*]~~ ^~~[*corporum*]~~ quantity^, but also in accordance with the kind of matter, which would be found more abundantly [*copiosius*] in one body and more sparingly in another.

19. *It is proved for terrestrial bodies* [*terrestribus*]

I have actually tested this proportion with the greatest exactness as possible in different kinds of bodies that exist on our Earth. The action of a circumterrestrial force that is proportional to the bodies to be moved will move them in equal times with equal velocity (by law 2) and will make all bodies that are let fall descend through equal spaces in equal times and will also make all bodies suspended by equal cords oscillate in equal times. If the action is greater, the times will be smaller, and if the action is smaller, the times will be greater. Others have long since observed that all bodies descend in equal times (at least if the very small resistance of air is removed), and it is possible to discern the equality of the times to the highest degree of accuracy in pendulums. I have tested this with gold, silver, lead, glass, sand, common salt, wood, water, and wheat. I got two equal wooden boxes. I filled one with wood and I suspended the same weight [*pondus*] of gold (as exactly as I could) at the center of oscillation of the other. The boxes, hanging by equal eleven-foot cords, made pendulums exactly like one another with respect to their weight [*pondus*], shape, and air-resistance. Then when placed close to each other, they kept swinging back and forth together with equal oscillations for a very long time. Accordingly, the amount [*copia*] of matter in the gold ^(by Prop. ____)^ was to the amount of matter in the wood as the action of the motive force [*vis motricis*] upon all the gold to this action upon all the wood – that is, as the weight [*pondus*] of one to the weight [*pondus*] of the other. And so for all the others. In these experiments, in bodies of the same weight [*pondus*], a difference of matter that would be even less than a thousandth of the whole could have been clearly noticed. ~~Because of this agreement, I have throughout designated the quantity of matter in each individual body by the word *pondus*, using the name of the measure for the thing measured, as is the common custom.~~

20. The unanimity of the agreements [*Analogiarum consensus*]

And since the action of centripetal force upon the attracted [*attractum*] body, at equal distances, is proportional to the matter in this body, it is reasonable also to grant [*rationi etiam consentaneum est*] that it is proportional as well to the matter in the attracting [*trahente*] body. For the action is mutual, and causes the bodies by a mutual endeavor [*conatu mutuo*] (by Law 3) to approach each other, and accordingly the action in one body must necessarily be in conformity with the action in the other. One body can be considered as attracting and the other as attracted, but this distinction is more mathematical than natural. The attraction is really that of either of the two bodies towards the other, ^and thus is of the same kind in each of the bodies^.

21. And their coincidence [*Et coincidentia*]

And hence it is that the attractive force is found in both bodies. The Sun attracts [*trahit*] Jupiter and the other Planets, Jupiter attracts its Satellites and similarly the Satellites act on one another and on Jupiter, and all the Planets act on one another. And although, in a pair of Planets, the action of each on the other can be distinguished and can be considered as paired actions by which each attracts [*trahi*] the other, yet inasmuch as these are actions between two bodies, they are not two but a simple operation between two termini. Two bodies can be drawn [*trahi*] to each other by the contraction of a single rope between them. The cause of the action is two-fold, namely the disposition of each of the two bodies; the action is likewise two-fold, insofar as it is upon two bodies; but insofar as it is between two bodies it is a simple and single action. There is not, for example, one operation by which the Sun attracts [*trahit*] Jupiter and another operation by which Jupiter attracts the Sun, but a single operation by which the Sun and Jupiter endeavor to approach each other. By the action by which the Sun attracts Jupiter, Jupiter and the Sun endeavor to approach each other (by Law 3), and by the action by which Jupiter attracts the Sun, Jupiter and the Sun also endeavor to approach each other. Moreover, the Sun is not attracted [*attrahitur*] by a twofold action towards Jupiter, nor is Jupiter attracted by a twofold action towards the Sun, but there is one action between them by which both approach each other. Iron attracts [*trahit*] a Loadstone [*magnetum*] just as much as a Loadstone attracts iron. For any iron in the vicinity of a Loadstone attracts other iron also. But the action between the Loadstone and the iron is simple, and natural philosophers consider it as simple; the operation of the iron upon the Loadstone is the very operation of the Loadstone between itself and the iron, by which both endeavor to approach each other. This is manifest from the fact that if the loadstone is removed, nearly the whole force of the iron ceases. In this way conceive that a simple operation, arising from the concurring [*conspirante*] nature of two Planets, is exerted between them; then this operation will be the same with respect to both and thus, being proportional to the matter in one of them, will be proportional to the matter in the other.

22. The forces of small bodies are insensible

Someone will perhaps say that by this law all bodies must attract [*trahere*] each other, which is contrary to experience in terrestrial bodies. But my answer is that there is no experience at all in terrestrial bodies…. Not even whole mountains would suffice for sensible effects. At the foot of a hemispherical mountain three miles high and six miles wide, a pendulum attracted by the force of the mountain will not deviate two minutes from the perpendicular. It is possible [*licet*] to observe these forces only in the huge bodies of the Planets, but we can discuss lesser bodies as follows.

~~*23. Forces tend toward all terrestrial bodies*~~

~~**Let ABCD designate the globe of the earth cut by two planes that are parallel and equally distant from the center on both sides. Since the middle part AHEDIB is pressed equally on both sides by the weights [*ponderibus*] of the outermost parts AHEF and BIDC and since, because of the equality of the pressures, the middle part remains in equilibrium, it is manifest that if either part were somewhat raised by some force applied from outside and were slowly withdrawn, the middle part HI would yield to the urging weight of the other part BIDC and would tend toward the withdrawn part. And accordingly, if the part FHG and the sum of the parts HI and IC were forcefully [*violenter*] held back at some certain distance from each other and then were let go simultaneously, both bulks [*moles*] and the part FH and the bulk [*moles*] HC would rush toward each other and thus they have the power of mutual gravitation. The bulk [*moles*] HC gravitates towards the part FH equally as much as the part FH gravitates towards the bulk [*moles*] HC, because both fall towards each other.**~~

{Inserted by Newton on the facing verso sheet: ‸~~It is not legitimate [*non licet*] to imagine that the gravitation of each of the two takes places toward another place distinct from the two bodies. For the imagined space is similar and does not have any~~ ‸specific‸ ~~point towards which gravitation takes place more than any other. If the whole earth were moved out of its place, there is no doubt that its parts would even then gravitate towards its center and would not seek the middle (which is now put out of the center) of its former place. For the properties [*affectiones*] and operations of bodies depend on the bodies and thus will not remain in spaces out of which bodies are moved, but will accompany bodies when they are transferred. Magnetic force follows a magnet, electric force follows amber, and centripetal force a Planet. And similarly the forces with which the bodies FH and HC fall towards each other accompany these bodies when they have been drawn apart….~~‸}

~~**… The parts therefore urge each other equally by their weights, that is, are attracted towards each other equally (as the third Law requires) and thus if drawn apart from each other and let go would fall towards each other with velocities that would be reciprocally as the bodies…. It is possible to test and observe all this in a magnet….**~~

#Fingere non licet gravitationem utriusque fieri in ~~aliquod spatium~~ locum ali
a corporibus distinctum. Nam spatium imaginarium similare est neque u
habet punctum specificum in quod gravitatio magis fiat quam in aliud quodvis.
Terra tota de loco suo moveretur non dubium est quin partes ejus
omnium gravitarent neque meditullium loci prioris jam factum excentricum peterent in ipsius centrum. Nam pendent affectiones & ope
tiones corporum a corporibus neque adeo manebunt in spatijs de quibus co
moventur sed comitabuntur corpora translata. Vis magnetica magnetem
electrica Electrum, centripeta Planetam sequitur. Et similiter vires
quibus corpora FH, HC in se invicem cadunt ~~[illegible]~~ comitantur haec corpora
distracta. ~~[illegible]~~ Quamdiu pars media HI contrarijs actionibus extim
FH, IC libratur, cedit ipsa ponderi partis IC parte altera FH ac
eta. At si pars ~~[illegible]~~ illa FH longius amoveretur et actio ejus in m
HC ob nimiam distantiam cessaret, credendum non esset quod pars H
jam amplius cederet ponderi ipsius IC motuque semper accelerato fugaret
infinitum. Motus in directum ~~perpetuo~~ perpetuo acceleratus haud c
cedendus est viribus naturae. Eo per Legem primam negato, quiescet corpus HC, adeoque
dendo in corpus vicinum FH ~~demonstrat ipsius gravitatem attractionem~~
~~[illegible]~~ Domi et non cadendo in longinquum, demonstrat ipsius vi
~~distantia magnam et parva in magna proxime~~
attractivam ~~corpora~~ ~~[illegible]~~ diminui.
ex aucta distantia

The Sequence of Increasingly Implausible Claims Comprising Newtonian Universal Gravity

1. Orbiting bodies are retained in orbit, rather than moving forward uniformly in a straight line, by forces directed toward central bodies.
2. These forces, and hence the resulting “centripetal” accelerations, vary inversely with the square of the distance from the central body.
3. These forces act not only on the principal bodies orbiting the central bodies, but on other bodies as well.
4. In the case of the Moon, the force in question is simply terrestrial gravity.
5. In all celestial cases, the force in question is one in kind with terrestrial gravity.
6. There is a force of this same kind on the central body directed toward each body orbiting it, so that the central and orbiting bodies – e.g. the Sun and Jupiter – interact.
7. There are mutual forces of this kind between all celestial bodies – e.g. between Jupiter and Saturn – as well as between each of these and the Sun.
8. The forces in question vary in accord with the law of gravity – i.e. the “motive” force on a body directed toward another body is proportional to the product of the masses of the two bodies and inversely proportional to the square of the distance between them.
9. The gravitational forces between bodies are composed out of inverse-square gravitational forces between each pair of individual particles of matter respectively comprising the two bodies.
10. The force of gravity is universal – i.e. the law of gravity holds not merely between bodies, but between all particles of matter in the universe.
11. The force of gravity is one of the fundamental forces of nature – i.e. it is not composed out of forces of other (known) kinds.

Ten Milestones Toward Newton's *Principia*

1. **Copernicus's** challenge to the Ptolemaic tradition in orbital astronomy, subsequently reinforced by Tycho, underscoring the issue of true motions

2. **Tycho's** more than a decade-and-a-half body of positional data, all to an unprecedented uniform observational standard

3. **Galileo's** many telescopic observations, especially of Jupiter's satellites and the phases of Venus, initiating a new tradition in observational astronomy

4. **Kepler's** several orbital reforms plus the 3/2 power rule, culminating in the Rudolphine Tables and a new standard for mathematical astronomy

5. **Galileo's** development of a fragment of a mathematical theory of motion under uniform (parallel) gravity by idealizing motion without air resistance

6. **Descartes'** singling out the *conatus* to recede from the center in curvilinear motion and noting its relevance to orbital motion

7. **Huygens's** extension of Galileo's mathematical theory of motion to cover evolute-controlled pendular motion, yielding a standard measure of the strength of surface gravity

8. **Huygens's** development of Descartes' *conatus a centro* into a mathematical theory of circular motion, centrifugal forces, and a complementary measure of the strength of surface gravity

9. **Newton's** discovery (prompted by Hooke) that the theory of uniform circular motion is a special case of a more general mathematical theory of motion under centripetal forces -- a theory that links Kepler's orbital reforms to inverse-square centripetal forces

10. **Newton's** finding a measure of the strength of inverse-square force-centers and then allowing those centers to interact

Some Questions at the Forefront of 17th Century Research on Planetary Orbits

Many questions became important at one time or another in the century extending from Tycho Brahe's efforts of the 1580's and 90's to the day in 1684 when Newton began writing what became the *Principia*. A few important questions even received answers. In particular, Galileo's telescopic observations of the phases of Venus and the lack of phases of Mars, Jupiter, and Saturn had pretty well terminated informed support of the Ptolemaic system, if not by 1620, then by 1633. And by 1675 measurements made in conjunction with Huygens's theory of the pendulum had (i) confirmed Galileo's claim that, at least near the surface of the Earth, objects falling vertically would accelerate uniformly to very high approximation in the absence of air resistance; and (ii) determined the precise distance they would fall in the first second to within an accuracy of two parts in a thousand. The questions singled out below, most of which are formulated here with a precision reflecting hindsight, were all still very much open and in the forefront of discussion in the late 1670's.

Questions From Before Kepler's Orbital Theory

1. Granted that the planets all orbit the Sun, do the Sun and the planets together orbit the Earth, in accord with Tycho's system, or does the Earth orbit the Sun, in accord with Copernicus's?

2. Given the failure to detect any annual stellar parallax, can any empirical evidence whatever show whether the Earth has one or more of the three motions the Copernicans attribute to it -- a diurnal rotation, an annual revolution, and a wobble of its polar axis?

3. What are the actual trajectories of the planets around the Sun and the Sun around the Earth (or the Earth around the Sun) -- eccentric circles and compounds thereof of the sorts Ptolemy, Copernicus, and Tycho postulated, or something else?

4. Given that none of the planets describe perfect uniform circular orbits, what physical mechanism or mechanisms are responsible for the departures from perfectly uniform circular motion?

Questions Subsequent to Kepler's Orbital Theory

1. Do the planets and their satellites really describe elliptical orbits, or is the ellipse just a good approximation (vis-a-vis the accuracy of the Tychonic observations) to their true trajectories?

2. Do the radii extending from the planets and their satellites to their respective principals really sweep out equal areas in equal times, or is this rule merely a good approximation (vis-a-vis the accuracy of the Tychonic observations) to their actual motions along their trajectories?

3. Is the 3/2 power rule a mere numerical quirk, perhaps holding only approximately; or a real, but nonetheless parochial feature of our planetary system; or a general feature of celestial orbiting systems?

4. Granted that the trajectories of the planets, though nearly circular, are either ellipses or some similar ovals, what mechanism or mechanisms are responsible for the departures from circularity, and why are the trajectories nevertheless so near to being circular?

Questions Subsequent to Descartes' *Principia*

1. What causes the planets and other orbiting celestial bodies to describe curvilinear trajectories instead of uniform motion in a straight line (as they would in the absence of imposed forces altering their motions)?[1]

2. Are the motions of the planets truly and permanently regular -- e.g. in the way Kepler's rules suggest -- or are the regularities observed to date mere epochal parochialisms that will give way in the future?

3. Given that the celestial realm is changing -- e.g. new stars are being born and others are dying -- is the planetary system we are part of even stable, or will it completely disappear sometime in the future?

4. What physical mechanism or mechanisms are responsible for the phenomena of terrestrial gravity and weight -- this, usually under the constraint that no (non-miraculous) physical mechanism can involve action at a distance?

Some Miscellaneous Further Questions

1. Given the many irregularities in the motion of the Moon -- both defined and, as yet, not defined -- what is its actual motion, what mathematical steps are required to specify its motion to within a reasonable approximation of observational accuracy, and why is it so different?

2. What are comets, where do they come from, and what trajectories do they describe?

3. Are the lines of apsides of the planetary orbits stationary (as Streete claimed), or do they precess in the manner of the line of apsides of the Moon, only more slowly?

4. What is the horizontal solar parallax -- i.e. the mean distance from the Sun to the Earth, measured in Earth radii (or any other terrestrial measure)?

5. Is a science of resistance -- i.e. a science covering the effects resisting media have on the motion of moving bodies -- possible at all, or are Galileo and Descartes right in saying that no such science is possible at all?

6. What trajectory would a projectile, like a cannon ball projected from the surface of the Earth or above it, describe (i) in the absence of air resistance;[2] (ii) in the presence of air resistance; and (iii) if it could continue, without any resistance, below the surface of the Earth?

7. Given that the basic periodicity of the tides correlates with that of the Moon, yet there are two tidal cycles per lunar day, and not one, what physical mechanism or mechanisms are responsible for the tides?

8. Does the strength of surface gravity, and hence the weight of any object, vary from one location to another, and, if so, in accord with what rule of variation?

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[1] What we now call the principle of inertia was first put forward by Descartes in a work that he finished in the early 1630's, but was not published until much later; the general principle first appeared in print in a publication of Gassendi's in the early 1640's, and then in Descartes' *Principia* in 1644; and it was widely assumed in research on the motions of colliding bodies and on circular and other curvilinear motions thereafter.

[2] While Galileo's answer to this question -- a parabola -- was widely acknowledged, he had himself pointed out that it does not take into account the curvature of the surface of the Earth.

Three Revolutions in Evidence

1. A commitment to the principle that the empirical world is the ultimate arbiter in all matters of astronomy and natural science

2. The discovery that extended mathematical theory can open the way to more telling evidence than can be achieved through testing hypotheses in isolation

3. The emergence of a new conception of exact science in which every systematic discrepancy between theory and observation is taken as telling us something that must be pursued