2. The comparison between the circle and the auxiliary ellipse, both of which were within the bounds of uncertainty at apsides and quadrants, indicated that the 100000 circular radius of the orbit at the quadrants needed to be decreased from 858 units to half that value, 429
    a. He says he then stumbled on the fact that the secant of the angle (in the circle), CQS, = 1.00429
    b. That is, get the desired distance CM at the quadrants by setting SM to 100000 there
3. Now generalizes this: the right way to measure the required distances from the Sun to Mars is by taking the projection on the diameter, TP, of the distances to points on the circle
    a. I.e. the SM distances everywhere are given by the rule, $r(1+e\cos x)$, where r is radius of circle and x called the "eccentric anomaly" -- see figure from Wilson in Appendix
    b. Kepler calls these the diametral distances: the "diametral distance" rule
4. Checks this rule against the triangulated distances he had derived from Tycho's observations before
    a. Table (in Appendix) shows that rule gives correct distances within the bounds of uncertainty
    b. Notice that this rule and its confirmation do not presuppose either the area rule or the ellipse; the evidence presupposes only the heliocentric longitude curve-fit of the vicarious theory and Kepler's earth-sun theory with its bisected eccentricity
5. Now asks how the distances in question should be laid off from S, that is, as radius vectors extending from S, under the requirement that the resulting heliocentric longitudes vs. time be correct
    a. First try, consistent with the 429 shortening at the quadrant: intersection between radius vector of appropriate length and radius from center of circle for each value of the angle x
    b. Result: a "puff-cheeked" orbit not symmetric about the diameter at the quadrants, with calculated heliocentric longitudes (using the area rule) outside bounds of vicarious theory uncertainty
    c. Second try, consistent with the 429 shortening at the quadrant: intersection between radius vector of appropriate length and a perpendicular to the line of apsides
    d. Result: an exact ellipse, with the 429 shortening at the quadrant, and one for which the area rule holds exactly; by earlier comparison of discrepancies for circle and auxiliary ellipse, eliminates the discrepancies (vis-à-vis the vicarious theory) in the octants and everywhere else
6. On this construal of Kepler's reasoning, played off triangulated distances against heliocentric longitudes: any two of the three rules -- ellipse, area rule, and diametral distance rule -- if taken to hold exactly, has the vicarious theory entailing that the third holds as exactly as that theory holds!

K. "Phase 7": A Physical Explanation

1. Kepler now had a cinematic model of the Mars orbit accurate to within observational accuracy, give or take a little, but he still had no physical basis supporting the claim that the model gives the true motion, nor for that matter much of any basis for insisting that the true path is exactly an ellipse rather than something closely approximating it
    a. Still no physical reason why the orbit might be an ellipse instead of a circle
    b. Given that the area rule only approximates the inverse distance rule, no physical reason for it

c. And questionable whether to settle on the ellipse as exact if going to infer underlying physics from the trajectory

2. From the outset had singled out the true sun because, as a physical body and not an empty point in space, it can govern speeds: velocity varies inversely with distance from it
   a. That raises a question: why is sun not at center of the trajectory
   b. That question had led Kepler to posit a reciprocating motion of the planet along an epicycle of radius *e* to yield the eccentricity in classic Apollonian style
   c. Had then devised an ad hoc physics to account for that motion, a physics that had later led to his egg-shaped oval (see Chapter 39)
3. With diametral distance rule in hand, replaces the epicycle and its physics with a distance that contracts from aphelion to perihelion and then expands
   a. He thinks such a sinusoidal variation the sort of thing that occurs in nature
   b. Physics proposed now postulates magnetic fibers in each planet, with a north-south polarity typical of magnetic action, as shown in figure in Appendix, as taken from Chapter 57
4. General idea: when at aphelion fibers oriented neutrally, so that their interaction with the Sun results in no net increase or decrease of distance from Sun
   a. As Mars moves away from aphelion, one pole of its magnetic fibers nearer the sun, resulting in a net magnetic attraction toward it
   b. Effect vanishes at perihelion, where again magnetic action becomes neutral
   c. As Mars moves away from perihelion, the other pole of its magnetic fibers nearer the sun, resulting in a net magnetic repulsion away from it
   d. Both the ellipse and its eccentricity physically dictated by the strength of the interaction of its magnetic fibers with the sun
5. Struggles to get the precise diametral distance rule from this physical action
   a. E.g., why should the eccentric anomaly -- an angle defined with respect to the center of the circle -- be the variable governing the distance
   b. And why should the effect bring Mars closer along lines perpendicular to the line of apsides rather than along the radii of the circle
   c. And (in Part V) how are the fibers dictating the angle of inclination of the orbit
6. Still, comes away from this with three conclusions that complete the "war on Mars"
   a. The Mars-sun distances in the elliptical orbit exhibit a variation that is physically reasonable -- this because of the new distance rule
   b. There is a basic relationship between his original velocity rule and the area rule, once the velocity in the original rule is interpreted as the velocity (component) normal to the r-vector
   c. That it was this component that matters was consistent with Kepler's claim of a magnetic vortex from a rotating sun drives the motion, for the vortex should be normal to the radius vectors

d. The discovery that the $1/r$ distance rule can be reconciled with the area rule by having v be the component of velocity perpendicular to the radius vector is nowhere stated clearly in *Astronomia Nova* or quite anywhere else by Kepler, but only subsequently

e. The orbit is an exact ellipse (or, as he adds in a letter to Fabricius, differs insensibly from such an ellipse), for "here the consequences derived from physical principles come into agreement with the results of observation and the vicarious hypothesis." (Wilson, p. 21)

7. Upshot: ellipse and diametral distance rule hold as perturbations of a circular motion produced by a second physical action superposed on the basic action of the Sun, with the second action varying in strength from planet to planet

IV. Kepler's Original Justification for the Two "Laws"

A. The Context of Discovery versus the Context of Justification

1. Now want to turn to the question, what exactly did Kepler's evidence show about the orbit of Mars?
   a. Need to be careful here because the evidence for a scientific claim typically changes over time, sometimes quite rapidly
   b. Want to determine the best evidential argument as of 1609 for the following claim: the orbit of Mars is an ellipse (with minor axis 0.429 percent shorter than major), along which Mars sweeps out equal areas in equal times vis-a-vis the Sun; furthermore, the plane of this ellipse, which passes through the Sun, is tilted 1 deg 50 min relative to the plane of the Sun-Earth orbit
   c. Then want to decide what his best evidential argument in 1609 actually shows about this claim
2. Here doing something a little controversial, for I am implicitly invoking Reichenbach's distinction between the context of discovery and the context of justification
   a. I.e. separating the steps leading to discovery from the steps justifying the conclusion
   b. Former, of course, need not be rational or logical at all
   c. But latter, on the view that the two are distinct, must be rational and logical for justification
   d. To put the point differently, justification will in general require a rational reconstruction of the discovery process, eliminating elements of irrationality and leaving just an evidential argument
3. Kuhn, in particular, has attacked this distinction as one of the primary ways in which philosophers distort science and its history
   a. He argues that acceptance of a new result within the scientific community is often influenced by aspects of the process of discovery that are not amenable to rational reconstruction
   b. These arguments are a central part of his attack on the idea that (rational) evidential considerations drive science
4. Kepler's *Astronomia Nova* would appear to be fertile ground for Kuhn, if only because the book itself argues for its conclusions about Mars by “recounting” the process by which they were arrived at
   a. In truth, Voelkel has shown that Kepler himself engaged in some (presumably) rational reconstruction of the discovery process in *Astronomia Nova*