a. Aristotle himself modifies Eudoxus's system for the planets slightly, seemingly to make it more physically tractable

b. While not altering his homocentric sphere models for the sun and moon

4. In his *On the Heavens* Aristotle provides philosophical and quasi-empirical arguments to support key features of Eudoxus's system

    a. The natural motion of the four elements is toward the center of the earth, while the natural motion of the celestial ether is circular

    b. All motions in the heavens have to be (eternal) uniform circular motions insofar as any speeding up and slowing down would require an external cause

    c. The earth is a sphere at the exact center, because of the natural motions toward its center

    d. The earth does not move, so that the apparent diurnal motion of the stars has to arise from motion of the sphere of the fixed stars

    e. The earth is small compared to the stars

5. These doctrines of Aristotle remained influential over the next fourteen centuries, leading to a number of conflicts with Ptolemaic astronomy

    a. Ptolemy did not have the earth at the exact center of the motions of either the sun or any of the planets, but instead at different distances from the center of their motion along the zodiac

    b. In the case of the moon and the planets, Ptolemy openly violated the requirement of uniform circular motion, replacing it with equiangular motion about a point off-center

E. Classical Greek Solutions After Aristotle

1. The two centuries after Aristotle died (322 B.C.) produced four great figures in classical Greek mathematics who continued to have a dominant influence over the next millennium and a half

    a. Euclid, who thrived in Alexandria around 320-280 B.C.: *Elements*, writings on optics

    b. Archimedes of Syracuse: (287-212 B.C.) writings on science that Galileo took as his model

    c. Apollonius of Perga (ca. 262-190 B.C.): *Conics*, but also writings in astronomy no longer extant

    d. Hipparchus of Nicea (ca. 190-120 B.C.): a fully developed, but inadequate epicyclic system that was the starting point for Ptolemy 300 years later; writings no longer extant, so that what we know of his work is from the *Almagest*

2. Greeks early looked for ways to account for anomalies in the motion of the moon and sun -- i.e. deviations from mean motion, for that reason called ***inequalities***

    a. From proof by Apollonius, recognized epicycle and eccenter as equivalent alternatives, allowing sun and moon to be either merely appearing to be moving at different angular speeds at different times or to be engaged in a compound of two uniform circular motions

    b. Willingness to use the two physically distinct but mathematically equivalent devices interchangeably a sign that their primary interest was calculational -- i.e. calculate locations and timing of salient events among the stars

c. (An instance of the second fundamental evidence problem in astronomy recognized as early as mid third century B.C.: which observed motions with respect to the fixed stars are real, as represented by the epicycle, and which merely apparent, as represented by the eccenter?

3. The two fundamental evidence problems of astronomy are distinct:

a. The problem of distances from the earth: if the distance of every celestial object from the earth were known (on some common measure) at all times, then their observed angular positions would suffice to determine their locations with respect to one another at all times, and hence too trajectories relative to one another -- e.g. is the sun or the earth near the center of Mars's orbit?

b. But still would not be able to say which apparent motions and changes of motion are real; only what the motions and changes of motion are relative to some point taken as at rest; i.e. the problem of the proper point to which all apparent motions should be referred would remain

c. Fully appreciated early in ancient Greek astronomy: stars vs. earth's rotation; sun vs. earth)

5. Apollonius's cinematic models around 225 B.C. used (major) planetary epicycles and (minor) epicycles and eccentricity for sun and moon: the forerunner to Hipparchus

a. Epicycles for retrograde, yielding uniform patterns, with similarly uniform patterns of speeding up and slowing down for sun and moon

b. Thus a first-order approximation to the anomalous motion

6. Hipparchus, unlike Apollonius, had access to Babylonian records stretching back centuries, which, coupled with his own careful observations and refinements (ca. 150 B.C.), showed that this first-order Apollonian approximation is definitely inadequate

a. Need more complexity in models, such as combining eccentricity and epicyclic motion

b. At this juncture, reached beyond mathematical methods available at the time

c. Hipparchus's refined cinematic models essentially the point from which Ptolemaic astronomy takes off, including his models for the Sun and Moon

F. The Problem Inherited by Ptolemy

1. Requisite mathematics developed over the next roughly 300 years

a. Menelaus and spherical trigonometry the most important development, but also classical trigonometry

b. Most of the developments lost because the *Almagest* covered them so well that interest in work leading up to it disappeared, just as in the case of Euclid's *Elements*

2. Problem: give an account (***logos***) of non-uniform patterns of retrograde motion of Mercury, Venus, Mars, Jupiter, and Saturn and non-uniform motion of sun and moon that handles main observable "inequalities" -- systematic departures from uniform (apparent) motion

3. Ptolemy's formulation (inherited from Hipparchus etc.): to develop an account of planets on the basis of the philosophic postulate that the observed irregularities of planetary motion are the result of the combination of uniform circular – or at least equiangular -- motions

a. Perfection of circular motion vs. calculational tractability of uniform circular motion
b. Unclear which was more important to Ptolemy, or to anyone else at the time

4. Ptolemy's *Almagest* vs. his *Planetary Hypotheses*, some decades later
   a. Former gave a mathematical account of motion of sun, moon, and planets
   b. Latter offers hypotheses about the physical basis of this motion -- essentially a rotating solid sphere account, with epicycles from smaller spheres centered on the surface of the larger ones
   c. Determined the minimum size (in units of Earth-Sun distance) required to fit everything in, with the sphere of the stars on the outside and no spheres overlapping any others
5. Ptolemy (around 150 A.D.) prolific: The *Handy Tables*, the *Geography*, the *Tetrabiblos*, the *Optics*, the *Harmonics*, treatises on logic, on sundials, on stereographic projection
   a. A university professor, in a less anachronistic sense than you might think
   b. In the world center of learning, Alexandria

III. Ptolemy's *Almagest*

A. Overview of the Book

1. *Mathematical syntaxis* (systematic treatise): "a complete exposition of mathematical astronomy as the Greeks understood the term" (Toomer), in 13 Books
   a. '*Almagest*': the great one, a Latinization of an Arabic adaptation of Greek; the work preserved in Arabic speaking world during Middle Ages and brought from there into Europe in 12th century
   b. Very much a textbook, instructing how to calculate a wide range of quantities and solve various problems in spherical astronomy
2. Books I and II: Preliminaries (e.g. earth at center of universe, motionless, etc.); followed by spherical trigonometry and its application to a range of calculational problems of interest, with tables
   a. Using chord function, rather than sine and cosine: chord $\theta = 2 \sin(\theta/2)$
   b. Calculate such things as terrestrial latitudes from length of longest day, transformations between equatorial and ecliptic coordinates etc., including many calculations of astrological interest
   c. Approach: formulate problems in terms of spherical triangles, then employ Menelaus's theorem to determine the unknown quantity, given five other quantities
3. Books III - VI: Sun, basic lunar, advanced lunar, and eclipses
   a. Mean sun provides basic unit of time, employing tropical rather than sidereal year, including a slightly erroneous value for the length of the mean tropical year inherited from Hipparchus
   b. Everything else built off of solar theory
   c. Note, however, the privileged position of eclipses: emphasis on predicting special events
4. Books VII & VIII: a catalog of the principal fixed stars
   a. Stars come after moon because moon used as a marker for the sun, and time connected to the sun
   b. (Note the calculational orientation of the work)
   c. A catalog of visible stars that remained a primary reference until modern times