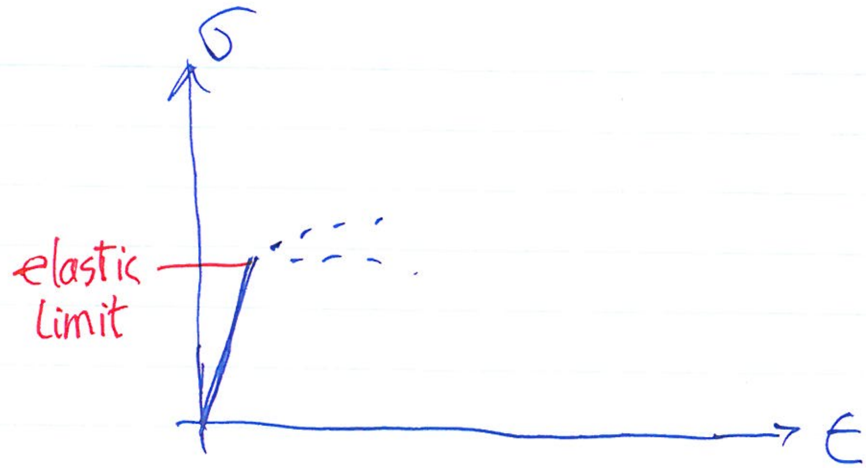
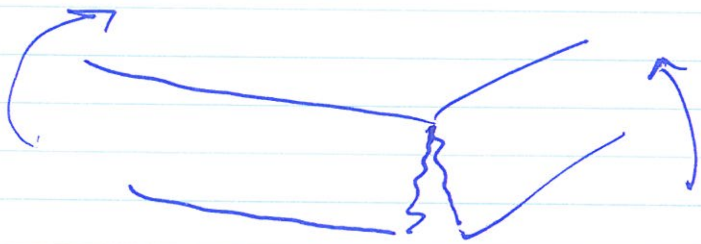
Beyond Elastic Limit



At stresses below elastic limit most materials obey Hooke's law (linear $\sigma - \epsilon$ relation)

Beyond Elastic Limit



- Fracture ...

- plastic deformation

- slow "creep" (high temperature)

Elastic stress analysis : can be done with high accuracy (Many soft. packages)

Yields : stresses, strains

However: in engineering, more interest in :

will the structural element break or not ?

Predictions of fracture : low accuracy (very low)

$\Downarrow$

high accuracy of stress analysis is wasted

Fracture science : not a mature field, yet

Fracture may have different modes for different materials

Highly complex phenomenon , many aspects

Some of the aspects:

- Highly sensitive to non-mechanical factors

  - environment : chemically aggresive environment
    $\Rightarrow$ fracture at relatively low stress

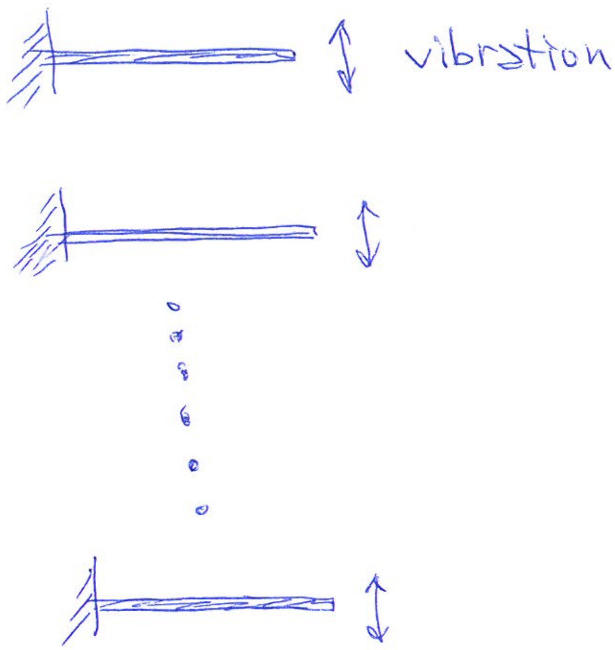
    [water : aggressive for glass, concrete
    salt : ——— for metals]

  - temperature : at high T, creep deformation at low stress
    (turbine blades)

- Highly sensitive to rate of loading

  At high rates (impact; explosion) : mechanics of fragmentation; very different from fracture at low rates

- <u>Statistical aspects</u>



<u>Similar</u> beams, made of the same material will fracture after <u>different</u> number of cycles N

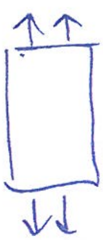
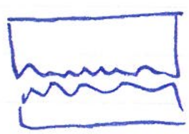
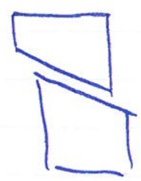
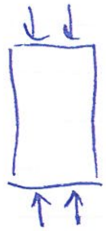
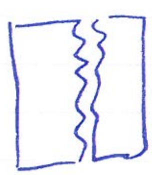
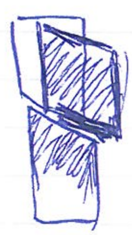
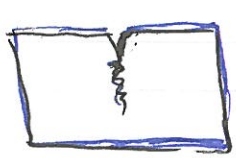
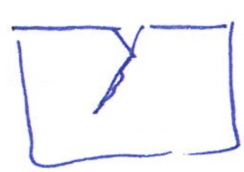
(the differences may be 2–3 times!)

Classification:

Brittle ⟷ Ductile types of fracture :

means : fracture occurs at $\begin{cases} \text{small strains} \\ \text{large strains} \end{cases}$

boundary not clearly defined
usually taken as 2-3% strains

Loading	Type of fracture: Brittle	Type of fracture: Ductile
tension 		
Compression 		
Bending 		

Same material may fracture in brittle or ductile way depending on conditions:

- At high temperatures : materials become more ductile
- At very low temp., most materials become brittle

At room temp..

brittle behavior : concrete
ceramics
glass
rocks

ductile behavior : metals

Simple fracture criteria – in terms of <u>stresses</u>

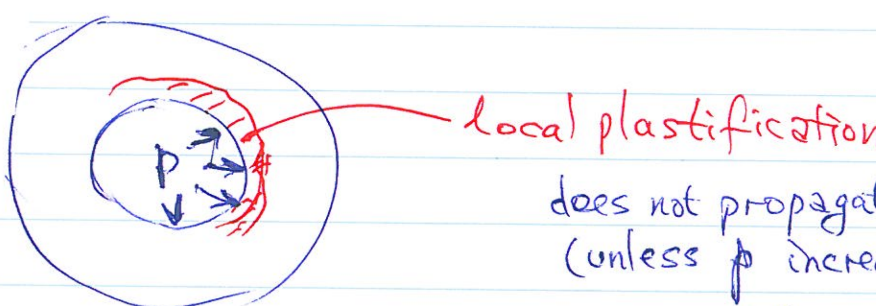
<u>Concept</u>: fracture occurs when certain stress / combination of stresses reaches <u>critical level</u> ← for given material

- <u>Limitation</u>: – effects of {temperature, environment} not accounted for
  – does not apply to cyclic loading

- <u>Limitation</u>: not fully reliable ⇒ "safety factors" (lack of confidence)

- <u>Limitation</u>: They apply to <u>local</u> fracture of the material
  Will fracture <u>propagate</u>? – not answered

Example: pressurized pipe



does not propagate (unless p increases)
⇒ not dangerous

Some simple fracture criteria

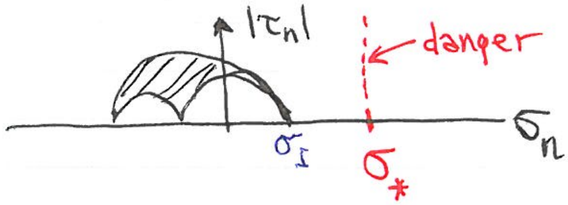
Brittle material: fracture at a given point occurs if, at this point,

Max. tensile stress $(= \sigma_I) = \sigma_*$ ← critical level

With safety factor

$$\sigma_I = \sigma_* / SF$$

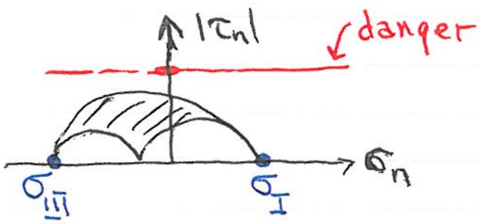
In terms of Mohr' circles:



Ductile material: fracture occurs if

$$\tau_{max} = \tau_* \qquad \left(= \tau_* / SF\right)$$

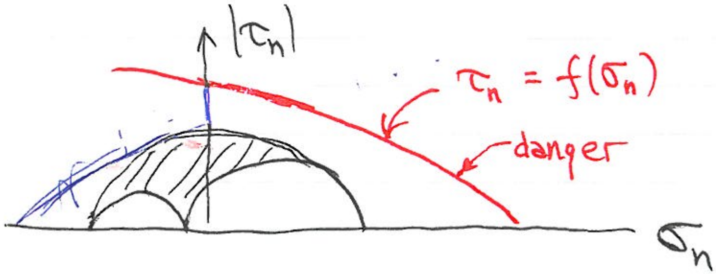
In terms of Mohr' circles:



$$\frac{\sigma_I - \sigma_{III}}{2} = \frac{\tau_*}{SF}$$

for geomaterials (rocks, soils)

. compressive stress increases fracture resistance (internal friction)



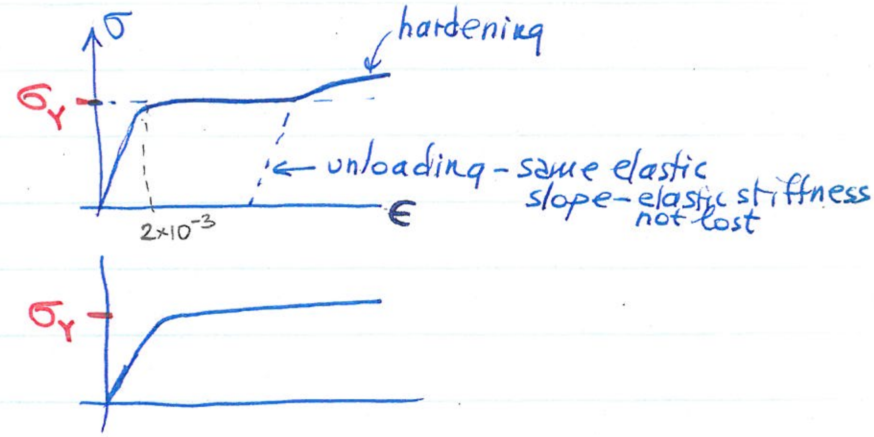
Shear stresses are less dangerous at high compression

Critical curve $\tau_n = f(\sigma_n)$: constructed from (large) number of experim. data

– Difficult to construct –

Plastic Deformation (metals)

Typical uniaxial tests:



σ_Y - yield stress.

(approx. concept ; not always clearly pronounced)

Problem : extend uniaxial test data to complex (non-uniaxial) stress states σ_{ij}

Observation : purely hydrostat. loading $\Rightarrow$ purely elastic volume change
$(\sigma_{ij} = p\,\delta_{ij})$

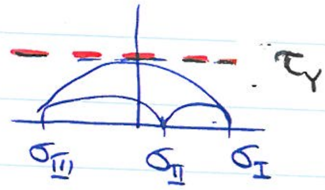
$\Rightarrow$ NO effect of hydrostatic stress on the yield cond.

Two yield conditions (metals)

① **Tresca – St-Venant** $\quad \tau_{max} = \tau_Y$

material constant measured in a simple test

Mohr's circles:



$\tau_{max} = (\sigma_I - \sigma_{III})/2$ – no effect of intermediate stress σ_{II}

② **von Mises**: $\quad T = \frac{1}{\sqrt{6}} \sqrt{(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2} = T_Y$

"intensity of shear stresses"

mat'l const

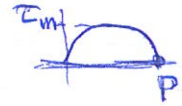
$\frac{1}{\sqrt{6}}$ set to make $T = \tau$ for pure shear ($\sigma_I = \tau$, $\sigma_{II} = 0$, $\sigma_{III} = -\tau$)

Their predictions: relatively close

Say, under uniaxial tension p, yield starts at $p = p_*$ (test)

Predictions for yield at shear τ?

① Tresca: for uniax. tension, $\tau_{max} = \frac{1}{2} p \Rightarrow \tau_* = \frac{1}{2} p_*$



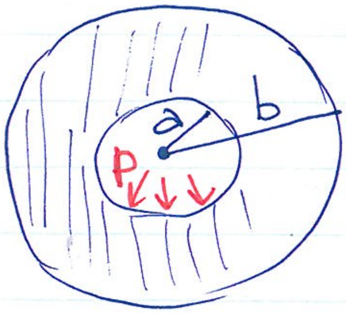
② von Mises: for uniaxial tension, $T_* = \frac{1}{\sqrt{3}} p_*$

Apply to shear: $T = \tau \Rightarrow \tau_* = \frac{1}{\sqrt{3}} p_*$

$\frac{1}{\sqrt{3}}$: 0,58

— relatively close —

Pipe under internal pressure: gradual plastification and final fracture



Elastic sol'n:

$$\begin{cases} \sigma_{rr} = \dfrac{a^2}{b^2-a^2}\left(1-\dfrac{b^2}{r^2}\right)p & < 0 \\ \sigma_{\theta\theta} = \dfrac{a^2}{b^2-a^2}\left(1+\dfrac{b^2}{r^2}\right)p & > 0 \end{cases}$$

$$\tau_{max} = \frac{1}{2}\left(\sigma_{\theta\theta} - \sigma_{rr}\right) = \frac{2a^2b^2}{b^2-a^2}\frac{1}{r^2}p$$

is maximal at $r = a$. Equating it to τ_Y:

pressure at the onset of plasticity:

$$p_0 = \left(1 - \frac{a^2}{b^2}\right)\tau_Y$$

(Thin pipe: $p_0 \downarrow$)

What happens if pressure is raised above p_0?

– plastic zone thickens:

In plasticized zone:

(1) $$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0$$ – equilibrium (should always hold)

(2) Yield condition: $\sigma_{\theta\theta} - \sigma_{rr} = 2\tau_Y$ (Tresca form)

(instead of Hooke's law)

$$\Rightarrow \boxed{\frac{d\sigma_{rr}}{dr} = 2\frac{\tau_Y}{r}}$$ – diff. eq-n for $\sigma_{rr}(r)$

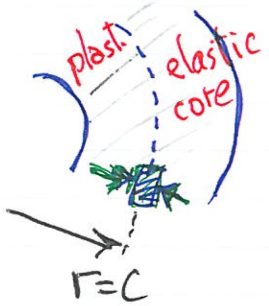
Solution : $\sigma_{rr} = 2\tau_Y \ln r + C$

b.c. $\sigma_{rr} = -p$ at $r = a$ $\Rightarrow$ $\boxed{\sigma_{rr} = -p + 2\tau_Y \ln \frac{r}{a}}$

in plastic zone

Propagation of plastic zone, as pressure increases:

elastic-plastic boundary: from continuity of σ_{rr}



$$\sigma_{rr}^{elastic}\Big|_{r=c} = \sigma_{rr}^{plast}\Big|_{r=c}$$

equilibrium

$$\frac{a^2}{b^2-a^2}\left(1-\frac{b^2}{c^2}\right)p = -p + 2\tau_Y \ln \frac{c}{a}$$

Dimensionless form:

$$2\ln \frac{c}{a} = \left[\frac{1}{b^2/a^2 - 1}\left(1 - \frac{b^2}{a^2}\cdot\frac{a^2}{c^2}\right) + 1\right] p/\tau_Y$$

Eq-n for the elastic-plastic boundary c/a as f-n of p

b/a is a parameter (pipe thickness)

for a given value of b/a solve numerically limit pressure : Max. pressure that the pipe can carry

Entire cross-section plasticizes (elastic core lost)

To find it : recall that $\sigma_{rr}\big|_{r=b} = 0$ (outer boundary)

In the limit state : $\sigma_{rr}\big|_{b} = 0$

from the plasticity solution

$$\Rightarrow \quad \boxed{p_{lim} = 2\tau_Y \ln\frac{b}{a}}$$

Spread between onset of plasticity and limit pressure?

$$\frac{p_{lim}}{p_0} = \frac{2\ln b/a}{1 - a^2/b^2}$$

= 1.10	at $b/a = 1.1$	
1.46		1.5
1.85		2.0

Message:

Onset of plasticity is not dangerous

Pressure can be increased beyond it

Illustrates limitations of simple fracture criteria (in terms of stresses)

They identify the onset of plasticity only