II. Huygens on Motion Under Perfectly Elastic Impact

A. Hypotheses Underlying the Initial Theory

1. Looking at the Huygens work less from the point of view of the contributions it made to mechanics -- although the assigned readings made major contributions -- than from that of the impact Huygens had on scientific methodology
   a. Huygens one of two key figures (Newton of course is the other) between Galileo and Descartes, on the one hand, and 18th century mechanics -- i.e. J., J., and D. Bernoulli, Euler, Lagrange, etc.
   b. In two weeks will be comparing Huygens's way of dealing with such things as centrifugal force and impact with the way Newton devised in his early, pre-*Principia* efforts
2. Huygens's paper on motion under perfectly elastic impact was written sometime between 1652 and 1668, probably by 1656, but published only posthumously in 1703
   a. Solution plus four key consequences (see Appendix) published (without proofs) in 1669, after he reviewed papers by Wallis and Wren on subject for *Phil. Trans*., and submitted an abridged version of the full paper to the Royal Society, where it remained, unpublished
   b. {All the papers and letters from this episode can be found in a file in Supplementary Material for this class, along with a paper by Murray, Harper, and Wilson}
   c. Experimental checks via ballistic pendulum at Royal Society and (later) by Mariotte
   d. (Huygens held back from full publication because he intended to put out a comprehensive book in mechanics covering all his results, a project undercut by publication of Newton's *Principia*)
   e. (Newton himself came up with the same solution as Huygens had announced, but algebraically and from a different pair of initial principles, in the late 1670s -- see Appendix)
3. Huygens's paper in fact presents two consecutive theories of motion under impact, with the split between them just before Proposition VIII
   a. The initial theory shows that Descartes' rules are incompatible with the principle of relativity that Huygens undoubtedly took him to have espoused
   b. The extended theory then adds a further hypothesis or axiom, beyond any of Descartes', and derives a general solution for perfectly elastic spherical balls impacting head-on
4. The initial theory proceeds from five hypotheses, at least four of which Descartes was surely committed to:
   a. Principle of inertia (applied to moving bodies) -- Descartes' Laws 1 and 2 together
   b. Equal bodies at equal speeds rebound equally -- Descartes' Rule 1, but here in effect a definition of 'hard body'
   c. A (Galilean) principle of relativity (tested by Gassendi): irrelevance of uniform motion of what came to be called the "frame of reference" -- i.e. frame-invariance of laws of impact
   d. Transfer of motion from a large body in motion to a small body at rest -- part of Descartes' third law

e. A much weaker form of Descartes' conservation of motion: if one body retains all of its motion after impact, then nothing will be taken from or added to the motion of the other

5. The main thing to notice about these five announced hypotheses is that most Cartesians accepted all five, and Descartes himself accepted all save perhaps for the principle of relativity
   a. Thus Huygens is here staying (dialectically) within the same general foundational framework as Descartes, though he is avoiding the latter's talk of "forces"
   b. Moreover, by adopting weak versions of some of Descartes' principles and by adding 'uniform' to the principle of relativity, he is staking out a very limited, safe starting point

B. The Initial Theory: Relative Motion Results

1. The "initial" theory, extending to Proposition VII, uses relativity of motion arguments to generate results contrasting with Descartes'
   a. I.e. the "boat-and-shore" arguments, requiring the same basic results to occur for observers in each frame of reference
   b. Arguments simply adjust the speed of the boat so that the conditions of the second, fourth, and fifth hypotheses hold in one frame of reference
2. Propositions I and II pertain to equal bodies, and contrast with Descartes' Rules 6 and 3, respectively
   a. I: If one body at rest, then it will acquire the speed of the other, and the other will come to rest
   b. Argument typical: let boat be moving in opposite direction from moving sphere, but at half the speed, so that from the point of view of the shore the two are moving at equal speeds toward one another, and the result follows from the second hypothesis
   c. II: If at different speeds, then exchange speeds
3. Propositions III, IV, and VI, which drop the equal body requirement, stand in sharp contrast with claims made by Descartes
   a. III: A body however large is moved by impact by a body however small, moving at any speed -- larger body moving toward smaller in one frame of reference has smaller moving toward larger in the other frame
   b. IV: Whenever two bodies collide with one another, the speed of separation is the same, with respect to each other, as the speed of approach -- adjust the speed of the boat so that the conditions of fifth hypothesis hold in one frame of reference
   c. VI: "When two bodies collide with one another, the same quantity of motion in both taken together does not always remain after impulse what it was before, but can either be increased or decreased" -- straightforwardly from Prop. IV, in direct contradiction to Descartes' third law of nature
4. Note that Huygens needs no apologetic explanations for why experience does not accord with these claims, as Descartes does, for experience does accord with them (making allowances for the imperfect hardness of bodies)