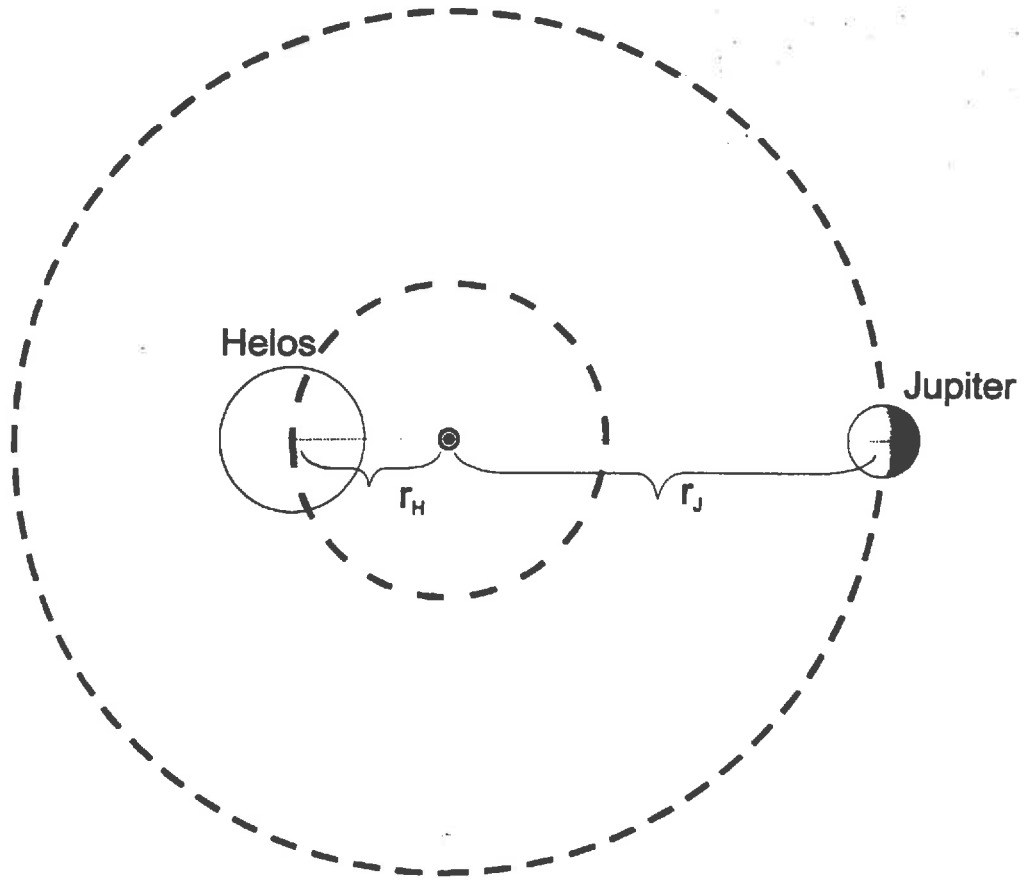


JUPITER INTERACTING WITH THE SUN



$$I_J W_J = I_H W_H$$

$$\frac{r_H}{r_J} = \frac{[a^3/P^2]_J}{[a^3/P^2]_H}$$

$$\frac{[a^3/P^2]_J}{[a^3/P^2]_H} = \frac{W_J}{W_H}$$

TWO PATHS TO THE LAW OF GRAVITY

$$\frac{[a^3/P^2]_J}{[a^3/P^2]_H} = \frac{W_J}{W_H} = \frac{M_J}{M_H}$$

$$\dot{V}_{J_{cent}} \propto \frac{[a^3/P^2]_H}{r_{JH}^2} \propto \frac{M_H}{r_{JH}^2}$$

$$F_{J_{cent}} \propto M_J \frac{M_H}{r_{JH}^2}$$

$$F_{H_{cent}} \propto M_H \frac{M_J}{r_{JH}^2}$$

i.e.

$$F \propto \frac{m M}{r^2}$$

$$F_{J_{cent}} \propto M_J \frac{[a^3/P^2]_H}{r_{JH}^2}$$

$$F_{H_{cent}} \propto M_H \frac{[a^3/P^2]_J}{r_{JH}^2}$$

$$F_{H_{cent}} = F_{J_{cent}}$$

$$\frac{[a^3/P^2]_J}{[a^3/P^2]_H} = \frac{M_J}{M_H}$$

i.e.

$$F \propto \frac{m M}{r^2}$$

Liber Secundus.

Fixas in supremis mundi partibus immotas persisterent, et Planetas
 his inferiores circa Solem revolvi, Terram pariter moveri cunctis annis,
 diurno vero circa axem proprium, et Solem, in ^{cen. focum Universi} omnium centro quiescere,
 antiquissima fuit Philosophantium sententia. Sic enim scripsit
 olim Philolaus, Aristarchus Samius, Plato etate maturiore
 Seleucus Mathematicus, Pythagorae ^{et his antiquior Graecorum} verba, et Romanorum
 Rex ille sapientissimus Numa Pompilius. Et in Symbolum orbis
 rotundi et ignis Solaris in centro, templum erexit Vestae forma
 rotunda, et ignem perpetuum in medio asseruari sancit. Ab Aegyptiis
 autem astrorum antiquissimis observatoribus propagatam esse
 hanc sententiam verisimile est. Et enim ab illis et a gentibus
 contempnitis ad Graecorum gentem magis ~~et~~ philologicam quam philo-
 sophicam, philosophia omnis antiquior, iuxta et sanior mansisse vide-
 tur. Et sacra Vestae ingenium Aegyptiorum sapientiae mysteria captum
 vulgi superantia sacris ritibus et Hieroglyphicis progenitum. Subinde
 docuerunt Anaxagoras Democritus et alij nonnulli Terram in
 centro mundi immotam stare, astra omnia in orbem, aliqua
 aetheris alia tardius moveri, idque in spatijs liberis. Namque orbis
 solidi postea ab Endoxo, Calippo, Aristoteli introducti sunt, deeli-
 nantibus philosophia primitus introducta, et novis Graecorum com-
 mentis paulatim prevalentibus. Cum his orbibus mali consistunt
 Phaeomena Cometarum. Hoc inter corpora caelestia a multis olim
 numeratos Chaldaei rerum astronomicarum peritissimi pro stellis
 errantibus habere, quasi ^{sempel} singulis revolutionibus in orbem velle
 excentricorum partibus infimas descendendo si nobis ^{per viges} conspic-
 endos exhiberent. Eodem ^{postea} in regionibus infra Lunam necessariis
 transit. ista orbium Solis hypoth. ^{et per viges} ~~per viges~~ ^{per viges} ~~per viges~~
 res Astronomorum observationes in caelo Luna superioris ^{et per viges} ~~per viges~~
 hinc contracta ^{supra istos orbis et ex aetheris} ~~et per viges~~ ^{de turbati}
 Quibus vinculis ^{Pythagorae} Chaldaei, Pythagorae, ceteraque Planetas in
 spatijs liberis reineri, idque in saecula saeculorum perpetuo retractos in
 orbem regulariter agi docuisse non constat. In hujus rei ex-
 plicationem orbis solidos excogitatos fuisse opinor. ^{et per viges} ~~et per viges~~
 Philosophi recentiores aut vortices esse voluerunt, aut aliud ali-
 quod hoc impulsus sive attractionis principium, ^{et per viges} ~~et per viges~~

Caelos esse fluidos.

Archimedes in Archeno. distib. Lib. 2 de caelo. Plutarch. lib. 3 de placitis Philos. et in Numa ~~Plutarch~~



Principium motus in spatijs liberis.

From *De Motu Corporum, Liber Secundus* [Add. 3990]

18. *Another agreement [analogia] between forces and bodies. It is proved for heavenly bodies [coelestibus]*

A second agreement between the forces and the attracted bodies is akin to the one just described. Since the action of the centripetal force upon the planets decreases in the duplicate ratio of the distance, and the periodic time is increased in the sesquialteral ratio, it is manifest that if equal planets were equally distant from the Sun, their actions would be equal and their periodic times would be equal, and that if unequal planets were at equal distances, their [^]collective [*collectitiae*] [^]actions would be as the [^]{*pondera*} [^]bodies [^]of the planets. For actions that were not as the [^]{*pondera*} [^]bodies to be moved [^]could not draw those [^]{*pondera*} [^]bodies [^]equally back from the tangents of the Orbits and cause revolutions to be completed in equal times in Orbits that are also equal. But neither could the motions of the satellites of Jupiter be so regular if the circumsolar force were not exerted equally upon Jupiter and all the satellites in proportion to their weights [*ponderum*]. And the same is true of Saturn and its satellite and also of the earth and our Moon, as (from prop. 35, corol. 2 and 3) is manifest ~~and soon will be made more fully clear~~. At equal distances, therefore, there is an equal action of centripetal force upon all the planets in proportion to their [^]{*ponderum*} [^]bodies or quantities of matter in the bodies [^], and thus also upon all the particles of that [^]{*ponderis*} [^]quantity [^]of which the planets are composed. For if the action were greater upon particles of one kind of matter, and less upon those of another, than in proportion to the [^]{*ponderum*} [^]quantity of matter [^], the action upon the planets would also be greater or less not only in proportion to the [^]{*ponderum*} [^]{*corporum*} quantity [^], but also in accordance with the kind of matter, which would be found more abundantly [*copiosius*] in one body and more sparingly in another.

19. *It is proved for terrestrial bodies [terrestribus]*

I have actually tested this proportion with the greatest exactness as possible in different kinds of bodies that exist on our Earth. The action of a circumterrestrial force that is proportional to the bodies to be moved will move them in equal times with equal velocity (by law 2) and will make all bodies that are let fall descend through equal spaces in equal times and will also make all bodies suspended by equal cords oscillate in equal times. If the action is greater, the times will be smaller, and if the action is smaller, the times will be greater. Others have long since observed that all bodies descend in equal times (at least if the very small resistance of air is removed), and it is possible to discern the equality of the times to the highest degree of accuracy in pendulums. I have tested this with gold, silver, lead, glass, sand, common salt, wood, water, and wheat. I got two equal wooden boxes. I filled one with wood and I suspended the same weight [*pondus*] of gold (as exactly as I could) at the center of oscillation of the other. The boxes, hanging by equal eleven-foot cords, made pendulums exactly like one another with respect to their weight [*pondus*], shape, and air-resistance. Then when placed close to each other, they kept swinging back and forth together with equal oscillations for a very long time. Accordingly, the amount [*copia*] of matter in the gold [^](by Prop. ____)[^] was to the amount of matter in the wood as the action of the motive force [*vis motricis*] upon all the gold to this action upon all the wood – that is, as the weight [*pondus*] of one to the weight [*pondus*] of the other. And so for all the others. In these experiments, in bodies of the same weight [*pondus*], a difference of matter that would be even less than a thousandth of the whole could have been clearly noticed. ~~Because of this agreement, I have throughout designated the quantity of matter in each individual body by the word *pondus*, using the name of the measure for the thing measured, as is the common custom.~~

20. *The unanimity of the agreements [Analogiarum consensus]*

And since the action of centripetal force upon the attracted [*attractum*] body, at equal distances, is proportional to the matter in this body, **it is reasonable also to grant [*rationi etiam consentaneum est*]** that it is proportional as well to the matter in the attracting [*trahente*] body. For the action is mutual, and causes the bodies by a mutual endeavor [*conatu mutuo*] (by Law 3) to approach each other, and accordingly the action in one body must necessarily be in conformity with the action in the other. One body can be considered as attracting and the other as attracted, but this distinction is more mathematical than natural. The attraction is really that of either of the two bodies towards the other, [^]and thus is of the same kind in each of the bodies[^].

21. *And their coincidence [Et coincidentia]*

And hence it is that the attractive force is found in both bodies. The Sun attracts [*trahit*] Jupiter and the other Planets, Jupiter attracts its Satellites and similarly the Satellites act on one another and on Jupiter, and all the Planets act on one another. **And although, in a pair of Planets, the action of each on the other can be distinguished and can be considered as paired actions by which each attracts [*trahi*] the other, yet inasmuch as these are actions between two bodies, they are not two but a simple operation between two termini.** Two bodies can be drawn [*trahi*] to each other by the contraction of a single rope between them. The cause of the action is two-fold, namely the disposition of each of the two bodies; the action is likewise two-fold, insofar as it is upon two bodies; but insofar as it is between two bodies it is a simple and single action. **There is not, for example, one operation by which the Sun attracts [*trahit*] Jupiter and another operation by which Jupiter attracts the Sun, but a single operation by which the Sun and Jupiter endeavor to approach each other.** By the action by which the Sun attracts Jupiter, Jupiter and the Sun endeavor to approach each other (by Law 3), and by the action by which Jupiter attracts the Sun, Jupiter and the Sun also endeavor to approach each other. Moreover, the Sun is not attracted [*attrahitur*] by a twofold action towards Jupiter, nor is Jupiter attracted by a twofold action towards the Sun, but there is one action between them by which both approach each other. Iron attracts [*trahit*] a Loadstone [*magnetum*] just as much as a Loadstone attracts iron. For any iron in the vicinity of a Loadstone attracts other iron also. But the action between the Loadstone and the iron is simple, and natural philosophers consider it as simple; the operation of the iron upon the Loadstone is the very operation of the Loadstone between itself and the iron, by which both endeavor to approach each other. This is manifest from the fact that if the loadstone is removed, nearly the whole force of the iron ceases. **In this way conceive that a simple operation, arising from the concurring [*conspirante*] nature of two Planets, is exerted between them; then this operation will be the same with respect to both and thus, being proportional to the matter in one of them, will be proportional to the matter in the other.**

22. The forces of small bodies are insensible

Someone will perhaps say that by this law all bodies must attract [*trahere*] each other, which is contrary to experience in terrestrial bodies. But my answer is that there is no experience at all in terrestrial bodies.... Not even whole mountains would suffice for sensible effects. At the foot of a hemispherical mountain three miles high and six miles wide, a pendulum attracted by the force of the mountain will not deviate two minutes from the perpendicular. It is possible [*licet*] to observe these forces only in the huge bodies of the Planets, but we can discuss lesser bodies as follows.

23. Forces tend toward all terrestrial bodies

—Let ABCD designate the globe of the earth cut by two planes that are parallel and equally distant from the center on both sides. Since the middle part AHEDIB is pressed equally on both sides by the weights [*ponderibus*] of the outermost parts AHEF and BIDC and since, because of the equality of the pressures, the middle part remains in equilibrium, it is manifest that if either part were somewhat raised by some force applied from outside and were slowly withdrawn, the middle part HI would yield to the urging weight of the other part BIDC and would tend toward the withdrawn part. And accordingly, if the part FHG and the sum of the parts HI and IC were forcefully [*violenter*] held back at some certain distance from each other and then were let go simultaneously, both bulks [*moles*] and the part FH and the bulk [*moles*] HC would rush toward each other and thus they have the power of mutual gravitation. The bulk [*moles*] HC gravitates towards the part FH equally as much as the part FH gravitates towards the bulk [*moles*] HC, because both fall towards each other.

{**Inserted by Newton on the facing verso sheet:** [^]It is not legitimate [*non licet*] to imagine that the gravitation of each of the two takes place toward another place distinct from the two bodies. For the imagined space is similar and does not have any [^]specific [^]point towards which gravitation takes place more than any other. If the whole earth were moved out of its place, there is no doubt that its parts would even then gravitate towards its center and would not seek the middle (which is now put out of the center) of its former place. For the properties [*affectiones*] and operations of bodies depend on the bodies and thus will not remain in spaces out of which bodies are moved, but will accompany bodies when they are transferred. Magnetic force follows a magnet, electric force follows amber, and centripetal force a Planet. And similarly the forces with which the bodies FH and HC fall towards each other accompany these bodies when they have been drawn apart....[^]}

... The parts therefore urge each other equally by their weights, that is, are attracted towards each other equally (as the third Law requires) and thus if drawn apart from each other and let go would fall towards each other with velocities that would be reciprocally as the bodies.... It is possible to test and observe all this in a magnet....

fingere non licet gravitationem utriusque fieri in ~~aliquo~~ ^{aliquo} ~~specifico~~ ^{specifico} ~~loco~~ ^{loco} ~~plano~~ ^{plano}
a corporibus distinctum. Nam spatium imaginarij simile est neque
habet punctum, in quo gravitatio magis fiat quam in aliud quovis.
Terra tota de loco suo ~~moveatur non dubium est quin partes eius~~
annuum gravitarentur ~~in yptius clatrum. Nam pendunt affectiones et ope~~
lionis corporum a corporibus neque adeo manebunt in spatijs de quibus cor
moveantur sed comitabuntur corpora translata. Vi magnetica magnetum
electrica electrum, centripeta Planetarum sequitur. Et similiter vires
quibus corpora FH, HC in viciniam cadunt ~~comitantur~~ ^{comitantur} ~~comitantur~~ ^{comitantur} hae corpora
distracta. ~~Quandiu pars media HG contrarijs actionibus exitim~~
FH, IC libratur, cedit ipsa ponderi partis IC parte altera FH ac
etc. At si pars illa FH longius amoveretur et actio ejus in m
HC ob nimiam distantiam cessaret, credendum non esset quod pars H
iam amplius cederet ponderi ^{ipius} IC motusque semper accelerato fugeret
infinitum. Motus in directum ~~perpetuus~~ ^{perpetuus} ~~perpetuus~~ ^{perpetuus} acceleratus haud e
cedendus est viribus naturae. Et, ~~negato, quiescet corpus HC, ad eog~~
dando in corpus vicinum FH ~~demonstrat~~ ^{perpetuum} ~~demonstrat~~ ^{perpetuum} ~~demonstrat~~ ^{perpetuum} ~~demonstrat~~ ^{perpetuum} ~~demonstrat~~ ^{perpetuum}
~~demonstrat~~ ^{perpetuum} ~~demonstrat~~ ^{perpetuum} ~~demonstrat~~ ^{perpetuum} ~~demonstrat~~ ^{perpetuum} ~~demonstrat~~ ^{perpetuum} ~~demonstrat~~ ^{perpetuum}
attractivam ~~quae parum distat~~ ^{quae parum distat} ~~negatam et potest in magna p~~
in aucta distantia ~~diminui~~ ^{diminui}.