5. This method of determining comet orbits Newton soon discovers is inadequate, so that within weeks he has switched to another method, and even in the middle of 1686 he is still seeking an iterative method that will converge on a solution
    a. Notice how important he is taking the problem of comets to be and how resourceful he is in concocting approximate, iterative methods when faced with such a problem
    b. Notice also that Newton is openly speaking of the return of comets, via highly eccentric elliptical orbits -- something that may well have been fairly, if not totally, novel at the time
    c. Halley was the first to confirm this when in 1705 he used modified Newtonian methods to calculate the orbit of the comet of 1682, identifying it with the comet of 1531 and 1607 and predicting its return in 1758 (actually returned in 1759, after he died)
6. A further subtle, yet radical step has been taken in Theorem 4 and Problem 4 by tacitly concluding that any body at any point in space about a "force center" must experience an accelerative tendency toward this center of magnitude proportional to $[a^3/P^2]/r^2$
    a. What we now call an inverse-square centripetal acceleration field, with field strength (a^3/P^2)
    b. And no variation in either orthogonal angular direction
    c. De Motu now far removed from Huygens's restricted talk of force in his paper on uniform circular motion, though Huygensian in appealing to comets as a surprising, testable consequence
7. De Motu now also far removed from Newton's view of comets expressed to Flamsteed in 1681!

IV. Some Extended Results on Galilean Motion

A. Problem 5: Vertical Fall Under Inverse-Square Gravity

1. One virtue of the tenuous solution to the problem of determining comet orbits is that it points to a means of addressing another problem -- free-fall under inverse-square acceleration
    a. I.e. instead of uniform acceleration (along parallel lines), treat the case of inverse-square acceleration toward a center, with the goal of determining s versus t
    b. In effect, addressing the problem of fall to the center of the earth, including the special case of direct vertical fall, on the terms Hooke posed
2. Newton ends up providing a fairly simple geometrical solution to a rather nasty nonlinear differential equation: $(d^2r/dt^2 = -k/r^2)$
    a. Suppose body has a small initial velocity perpendicular to the vertical, so that (just as Hooke said), its trajectory will be an ellipse APB with focus at S, the center of the inverse-square centripetal force
    b. Circumscribe the circle ADB about this ellipse; then the time of fall is proportional to the area ASP, and hence to the area ASD
    c. As the initial perpendicular velocity approaches 0, ASD will continue to remain as the time, but in the process the orbit APB will approach AB, B will approach S, and area ABD, now = ASD, will be proportional to the time

   d. Therefore the distance AC for a given time will be defined by taking the area ASD=ABD to be proportional to the time, thus determining C
3. Solution sufficient to allow evaluation of the difference between uniform acceleration and inverse-square acceleration in free-fall, given the distance of fall in the first second at the surface of the Earth
   a. By stipulating a distance of fall -- e.g. 64 feet or 256 feet -- and comparing the ratios of the time between the first 16 feet and the total with uniform acceleration (2 sec and 4 sec) with the ratios of the corresponding areas within the circle ASD
   b. Difference in the ratios of the order of 10^{-6} over 64 feet and 10^{-5} over 256 feet
   c. In other words, the difference over testable distances of free fall lies beyond the precision with which times could be measured, and hence no *experimentum crucis* at the time
4. Scholium indicates that Problems 4 and 5 together provide solutions to projectile motion and vertical fall in the absence of air resistance, but under inverse-square centripetal rather than the uniform acceleration along parallel lines presupposed by Galileo
   a. Projectile in general not a parabola, but in the cases of interest to Galileo, an ellipse that approximates a parabola over the relevant distance when AC small compared to AS
   b. And vertical fall not uniformly accelerated, although when AC small compared to AS, it is approximately so
5. Thus, if we are willing to accept the claim that gravity is an inverse-square centripetal force, Problems 4 and 5 are telling us that Galileo's laws of free-fall and parabolic projection are merely approximations, indeed ones that do not hold even in the mean
   a. I.e. neither holds exactly in the absence of air resistance, nor would they hold exactly in the absence of other secondary effects, but they hold approximately to the extent that the inverse-square change in g remains negligible and the radius of the Earth is large
   b. Furthermore, the solutions to the two problems provide us means, as above, for calculating the magnitude of the error in the approximation for various cases!
6. In other words, Newton here managing to tie the theory into that of "natural" motion near surface of the earth in just the manner Huygens managed to tie his work on centrifugal force, impact, and the pendulum to this theory
   a. Newton is continuing an approach that Huygens has shown is very fruitful in yielding evidence
   b. Notice that he did not have to do so -- he could have stopped with Problem 4 without mentioning projectile motion and vertical fall under terrestrial gravity

B. The Moon Test Repeated in 1684: Confirmation
1. Notice that the Scholium ends with the assertion that "gravity is one species of centripetal force"
   a. The claim that gravity is a centripetal force is unproblematic
   b. But the same cannot be said of the claim that it is an inverse-square centripetal force -- what is the evidence for that

2. Although Newton provides no evidence for this claim about gravity in De Motu, it would not be presumptuous to think he had repeated the "Moon test" of the late 1660s, but now using current values
    a. No unqualified documentation that he had done so for a few more weeks, where the result appears in a manuscript that is an immediate successor to *De Motu Corporum in Gyrum* (i.e. *De Motu Sphaericorum Corporum in fluidis* -- Version 3, called by DTW the "augmented" version)
    b. But every reason to think that he would have done so in the fall of 1684, if he had not done so in 1680 (as Westfall says)
3. By 1684 Newton had various choices for the number of Paris feet in a degree of longitude -- e.g. Picard's value of 342,360, which Newton subsequently used, and Cassini's value of 342,366
    a. With Picard's value, the circumference of the earth, assuming a sphere, is 123,249,600 Paris ft, giving a radius of 19,615,783
    b. The period of the moon 27d 7h 43m, or 2.36058e6 sec, so that its angular velocity is 2.6617e-6 rad/sec
    c. Taking the distance to the moon to be 60 earth radii, then the acceleration of the Moon, $r*\omega^2$, is 8.33833e-3 ft/sec/sec -- i.e. the Moon falls 4.16916e-3 ft in 1 sec
    d. Dividing this number into Huygens's value for the fall in 1 sec at the surface of the earth -- 15.0833 ft, we get 3617.9 -- only 0.5% off a perfect inverse-square, well within known accuracy of the lunar horizontal parallax
4. So, a repeat of the "Moon test" with Picard's values for the radius of the earth rather than Galileo's would have been a great success
    a. From 4375 -- a 21.5% discrepancy -- in the late 1660s to 3617.9 -- a 0.5% discrepancy -- in 1684, clearly within the accuracy of the value for the mean distance of the moon
    b. Whatever Newton was looking for in the 1660s, he had surely found it by the end of 1684, though his doing so in no way depended on any of the new results in De Motu
5. Notice carefully, however, what I am taking the successful result to have shown
    a. Not just that the moon is held in place by terrestrial gravity
    b. But more so that terrestrial gravity varies inversely with the square of the distance from the center of the earth at least to the moon
    c. Since the moon is the sole body orbiting the earth, there was no basis at this juncture to draw the conclusion that the inverse-square forces governed its motion without the "Moon test"
    d. And there was no other basis for concluding that terrestrial gravity diminishes in accord with the inverse-square rule

C. Problems 6 and 7: Resistance and Galilean Motion

1. Resuming with the text of De Motu, Newton next turns to the problems of uniform and orthogonally uniformly accelerated motion, but allowing for air resistance proportional to velocity