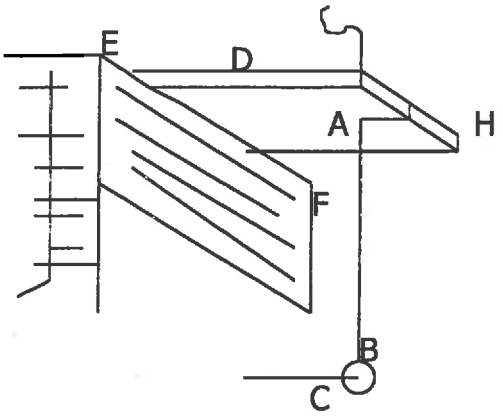
Christiaan Huygens to Thomas Helder.[1]
1686.

Observation concerning the Length of a simple Pendulum.



XXXVI. While ashore at the Cape of Good Hope as well as especially in Batavia[2] if the voyage goes so far, or while the ship is lying very still, one will observe by using the clock how long a single pendulum must be to do each beat in a second, that is [the length] from the top end of the thread until the center of the sphere; for, here I call a single pendulum a copper or leaden little sphere of about a thumb[3] in diameter that is hanging on a thin thread.[4] With regard to the motion of the clock much depends on this experience. For, a certain Frenchman claims to have found at a location about 5 degrees north of the Equator that such a pendulum was a bit shorter there than in Paris, England, and Holland. In order then to observe this perfectly one should hang the pendulum as in the figure, in which EF is the side of a high table or windowsill; DH [is] a flat piece of wood nailed down on it and which overhangs by only 1/2 thumb; AB [is] the thread, wedged in an incision in that wood, and having a length down to the sphere C of about 3 Rhenish feet 1 1/2 thumb.

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[1] Item 2520 in *OCCH*, Vol. 9, pp. 292-293. Even though this document was found separately in the Huygens Archives, in the University Library of Leiden, the paragraph number at the beginning of it (XXXVI), makes it almost certain that it was to supplement the instructions Huygens had given Helder on the use of the clocks during the voyage (Item 2423 in *OCCH*, Vol. 9, pp. 55-76).

[2] Now Jakarta in Indonesia.

[3] A thumb is a Rhenish inch, i.e. 1/12 of a Rhenish foot, roughly 0.965 of a Paris inch, or 2.6 cm.

[4] Although we have been unable to determine what it was, the material used for the thread is of some interest. A 40 deg. F (22 deg. C) change in temperature would cause a copper or bronze thread of the length of a seconds-pendulum to increase in length by roughly 0.16 Paris lines, or 0.037 percent. The thermal expansion of a non-metallic thread could well have been as little as 1/4 of this, depending on what material was used. Huygens's drawing suggests a non-metallic thread. Cat-gut (in truth, sheep's gut), which was used for stringing musical instruments, is a good candidate. There is some evidence for silk. In his instructions to Helder, Huygens mentions that the triangular pendulum is hanging from silk threads. See item 2488, *OCCH*, Vol. 9, pp. 222-223.

One will make this pendulum[5] move very slowly, roughly just 2 or 3 thumb-widths· being very careful that the sphere no longer rotates, as always occurs from the start, for, through this the thread unwinds itself and becomes longer. One can impregnate it with wax, except at the top near A. Furthermore, one will observe the movements of this pendulum against one of the clocks, ensuring that one movement accords with two movements of the pendulum of the clock, and that for about a half an hour.[7] One can thus shorten or lengthen the pendulum AB, until the beats, as was said, accord perfectly. Once then this has been done, one shall measure off neatly with a straight stick, having been shortened to this measure, the true length AB from the top end of the thread until the top of the sphere, so that it just fits between the piece of wood, DH, and the sphere C. After that one can take the length of this little stick as a correct foot-measure. Adding to it half the diameter of the sphere, this yields the total length of a pendulum beating seconds, if the clock has been properly calibrated to the proper mean of the days.[8]

But as the clock usually goes several seconds too quickly or too slowly in 24 hours, so the movement of this single-pendulum will be a bit shorter or a bit longer than a second; let us suppose that a clock goes 1 minute too slow in 24 hours; then turn 24 hours into minutes, resulting in 1440, from which the previous mentioned 1 minute [should be] subtracted, resulting in 1439. Now just as the square of 1439 [is] to the square of 1440, [is] the length found for the pendulum to the correct length of a pendulum that beats in a second. For example, if the length of this little stick, including half the diameter of the sphere, is found to be 37 thumbs, 11 2/3 lines, then saying the square of 1439 [is] to the square of 1440, just as 37 thumbs 11 2/3 [lines] is to another length, the latter being very near 38 thumbs and 1/3 of a line. This is the length of a pendulum for seconds here in

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[5] In Dutch the word here is *slinger*, not *pendulum*.

[6] Huygens knew that a circular arc pendulum is isochronous over only infinitesimal arcs, where it truly approximates a cycloidal pendulum. He had no way, however, of calculating the departure from isochronism as a function of arc length, for the solution for the (large arc) circular pendulum had to await Euler's elliptical integrals three-quarters of a century later. The only way Huygens therefore had for determining a limit on the arc length of a simple seconds-pendulum was through trial-and-error comparison with a cycloidal pendulum. In his ***Horologium Oscillatorium*** he says, "...the pendulum should be set in motion with a small push because small oscillations, for example 5 or 6 degrees, are sufficient to give equal times, but not a large number of degrees," (OCCH, Vol. 18, p. 351; in the translations by Blackwell, p. 168). The instruction here calls for a slightly smaller arc: 2 thumbs corresponds to an arc of about 3 degrees, and 3 thumbs to one of 4.5 degrees.

The incremental error in the period of a 4.5 degree arc circular pendulum is 0.009 percent. This would require the length of a seconds-pendulum of this arc to be 0.085 Paris lines shorter than a one-second cycloidal pendulum. By contrast, the incremental error in the period of a 6 degree arc circular pendulum is 0.017 percent, which would amount to a difference in length between a circular-arc and a cycloidal seconds-pendulum of 0.151 lines. The smaller arc length that Huygens is here recommending to Helder is accordingly of some merit. (A still smaller 2 degree arc length would reduce the difference in length from 0.085 to 0.038 lines.)

[7] Over the course of 30 minutes, a 0.00555 percent discrepancy between the two periods will produce a 10 percent asynchrony between their motions, which would have been quite apparent to the naked eye.

[8] Huygens here ignores the correction for the center of oscillation that he so carefully takes into account in Part Four of his ***Horologium Oscillatorium***. He can safely ignore it here insofar as he is only comparing lengths, and not determining the absolute magnitude of the effective acceleration of gravity. With a 1 Paris inch diameter sphere forming the seconds-pendulum, the center of oscillation is only 0.032 lines beyond the center of the sphere.

Holland as well as in France and England. But the mentioned French observer says to have found this length 5/4 of a line less in Cayenne. When there is great calmness, it will be good to observe this on board the ship, not only at the Equator but also at several other latitudes, and to note the measures found.[9, 10]

[9] Suppose the accuracy of the pendulum clock could be determined sidereally to within at least 4 sec. per day (i.e. 1 part in 21,600); and the seconds-pendulum could be determined to be no more than 1/10 of arc out of synchrony with the pendulum clock after 30 minutes (1 part in 18,00000); and the length of the seconds-pendulum could be determined to within 0.1 Paris lines (1 part in 4405). Following the instructions Huygens gives Helder, the error in the absolute magnitude of the acceleration of gravity as determined by a seconds-pendulum would then have been less then 1 part in 2320, or 0.42 cm/sec^2. Doubling the error of the length in the pendulum to 0.2 Paris lines, to allow for thermal effects, would increase the upper bound on the error to 1 part in 1520 -- still only 0.65 cm/sec^2. Richer was claiming a difference between Paris and Cayenne amounting to 1 part in 352, or 2.78 cm/sec^2.

[10] Translation by Eric Schliesser; notes by Eric Schliesser and George E. Smith

HUYGENS'S PENDULUM MEASUREMENT OF DISTANCE OF FALL IN 1ST SECOND

Distance of fall in 1st second
$= \pi^2 \cdot$length of pendulum$/2T^2$

Length of 1 sec pendulum:

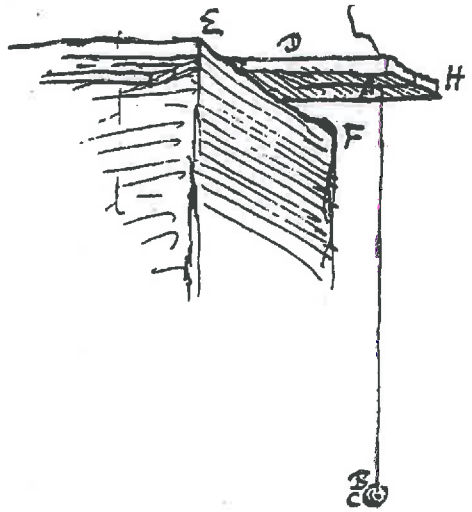
3 Paris feet 8.5 lines

i.e. 440.5 lines

Distance of fall in 1st sec:

15 Paris feet 1.1 inches

(i.e. 980.7 cm/sec/sec)



1. Adjust length until in synchrony with pendulum clock for 30 min.
2. Measure length to bob center and correct for center of oscillation
3. Correct length for any inaccuracy in clock: (86156/no. sec in day)^2
4. Infer distance of fall in 1st sec

MEASURING GRAVITY WITH A SECONDS-PENDULUM

A MODERN ASSESSMENT OF ACCURACY

Suppose the accuracy of the pendulum clock could be determined to within 4 seconds per day : **1 part in 21600**

Suppose seconds-pendulum could be determined still to be within 1/10 of an arc in synchrony after 30 min : **1 part in 18000**

Suppose the length of the seconds-pendulum could be determined to within 0.2 Paris lines : **2 parts in 4405**

Then error in strength of gravity : **≤ 1 part in 1520**

If instead, the length of the seconds-pendulum could be determined within 0.1 Paris lines : **1 part in 4405**

Then error in strength of gravity : **≤ 1 part in 2320**

For a 2 inch diameter bob, the correction for the length of a seconds-pendulum is slightly more than 0.1 lines.