

Effects of Fundamentals and Subharmonic Pitches on Consonance Perception of  
Complex Chords

An honors thesis for the Department of Psychology

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### Abstract

This study is an attempt to provide empirical evidence for two theories of consonance using musically complex stimuli: harmonicity theory first proposed by Helmholtz (1863) and a more recent theory involving subharmonics proposed by Cariani (2001). 21 musically-trained Tufts students listened to groups of four chords composed of complex tones (a four-note chord, the same chord with a fundamental predicted by harmonicity theory added, the original chord with a lower pitch not predicted by any consonance theory added, and the original chord with a subharmonic tone predicted by Cariani's model added) and ranked them in order of dissonance. The control chord (with a low pitch not predicted by any model) was ranked as significantly more dissonant than the other three chords by participants, and the subharmonic predictions were ranked as significantly less dissonant than the harmonic predictions. These results suggest that both the harmonic series and the subharmonic series play a role in consonance perception, with the subharmonic series possibly playing a greater role. In addition, the results show that musically complex stimuli can be compared in music cognition studies and still lead to significant results, at least within a musically-trained population.

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Despite rapid growth in the field of music psychology over the last several decades, it can still be difficult to find topics of research that lead to results that are of comparable interest to musicians and psychologists. The musical stimuli used in music cognition studies are often very simple, which leads to results that have robust and clear effects that are of use to the field of psychology but are of little interest to music theorists, composers, or performers. Nowhere is this more evident than in the study of consonance and dissonance: although it has been about a hundred years since the relatively widespread adoption of atonality by composers (not to mention extended tonality, polytonality, and non-Western scale types), musical consonance studies still primarily focus on well-tempered, tonal, triadic chords. This severely limits what can be learned about human perception of consonance and dissonance. The present study is an attempt to address the issue of consonance and dissonance perception from a more musically compelling standpoint.

Cazden (1962) outlines several of theories of dissonance. A prominent theory dating back to Hermann von Helmholtz in the 19<sup>th</sup> century is the *beating theory* of dissonance. When two tones are played simultaneously, they interact in our ear canal and the difference between the two frequencies manifests as its own frequency. When the difference between two tones is relatively small (for instance, between 440 cycles per second (Hz) and 445Hz, where the difference in frequency is 5 cycles per second), the resulting frequency is correspondingly small, which we perceive as a rhythm, or a beat.

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As the distance between the two tones increases, the frequency of the beats increases until we cease to hear it as a pulse and rather as a tone itself (this line is somewhere around 40Hz). When we hear this resultant frequency as a tone, it is referred to as a difference tone. As in general tones that are close together (or whose overtones are close together), which produce noticeable beating, are considered more dissonant than tones farther apart, proponents of the beating theory suggest that the beats are causing our perception of dissonance. There doesn't seem to be an explanation of the mechanism by which beats themselves result in dissonance apart from the fact that they may interfere with our processing of the pitches. Beating is also sometimes referred to as roughness, and has been extensively studied as a possible cause of dissonance (McDermott & Oxenham, 2008).

Another theory is *harmonicity theory*. Instead of ascribing dissonance to a particular acoustical phenomenon and describing consonance as the absence of dissonance like the beating theory, proponents of harmonicity theory (including Stumpf in the 19<sup>th</sup> century and McDermott et al., 2011) claim that notes “go together” when they all exist early in a single overtone series. This could explain why the octave, fifth, and major third are so privileged in our Western music idiom, but has trouble when addressing a reasonably stable chord, the minor triad, and would predict that a dominant seventh chord would be quite consonant and presumably stable (it is certainly not a harsh dissonance, but in traditional tonal music it is always expected to resolve).

Finally, a more recent theory proposed by Cariani (2001) posits that the subharmonic series (the inversion of the harmonic series) is the basis for our perception

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of consonance. While the subharmonic series has not been observed acoustically, subharmonic relationships appear when the activity of the auditory nerve is examined. The auditory nerve fires at the peaks of waveform captured by the ear, but due to a refractory period it is unable to fire on every peak. It fires probabilistically, only phase-locking with some of the peaks of the waveform, leading to the presence of the subharmonic series below the fundamental pitch entering the ear. Cariani (2001) developed a neural model of the auditory nerve that predicts salient subharmonic pitches when given acoustical input.

There are other theories, as well, but in essence they are variants or combinations of the first two theories outlined above. The inconclusiveness of the myriad theories of consonance suggest that many factors contribute to how we determine how dissonant a collection of tones sounds. Accordingly, Johnson-Laird et al. (2012) found that dissonance judgments were dependent not only on the acoustical features of the chords themselves but also on the tonal context or lack thereof of any chords surrounding the target chord, and proposed a dual-process bottom-up and top-down theory of dissonance perception. This notion of distinct bottom-up and top-down components is supported by the work of Trainor et al. (2002), which found that infants show a preference for sensory consonances, ruling out cultural habituation as the only cause of consonance. There is also evidence that when two tones have overtone series that are in partial alignment, they “lock in” to some extent in perception and we hear them as being in tune (Cohen, 1984).

While there is not a wealth of empirical studies on overtone relationships and dissonance (most look at dissonance and consonance in tonal contexts), some studies

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have looked into adjacent issues. Johnson-Laird et al. (2012) carried out a study exploring the relationship between roughness and tonal experience of the listener on dissonance judgments. In doing so, however, they incidentally provided some initial examples of one of the effects being sought in the present study.

Using both musicians and non-musicians in Experiment 1, Johnson-Laird and colleagues presented the participants with 55 trichords (in this case meaning chords composed of three pitches, not necessarily triads, or third-based chords). In the 12-note division of the octave, there are only 55 possible three-note combinations (ignoring transpositions). The chords were presented in random order using a piano MIDI sound, and participants were asked to rate their consonance (or “pleasantness”) or dissonance (or “unpleasantness”) on a scale from 1 to 7. The roughness of the chords was calculated using Parncutt’s (1989) method. The mean dissonance scores given by the participants for each chord were then calculated. The authors found that while roughness does predict dissonance ratings to a certain extent (higher roughness leads to higher dissonance scores), it is not always true. For instance, the diminished triad (B-D-F) has a higher roughness score than the augmented triad (C-E-G#), but participants rated the augmented triad as significantly more dissonant than the diminished triad.

In Experiment 2, the authors used four-note chords instead of three-note chords and once again found that roughness predicts dissonance judgments up to a point, but that tonally referential chords (i.e. chords we hear more often) are rated as less dissonant than chords with more obscure or absent tonal references. In Experiment 3, the authors played three- and four-note chords in tonal contexts and in non-tonal contexts.

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Johnson-Laird and colleagues used their findings to support a dual-process theory of musical dissonance, whereby the roughness provides the bottom-up cue for dissonance and lack of tonal reference provides the top-down cue for dissonance. This theory is incidental to the goals of the present study. However, some specific results of the dissonance ratings can be used to support exploring the effect of adding a fundamental to an otherwise dissonant chord. In Johnson-Laird and colleagues' Experiment 1, one of the chords presented to participants is a diminished triad (F3-B4-D5). This aligns closely with the 5<sup>th</sup>, 6<sup>th</sup>, and 7<sup>th</sup> partials of the overtone series of G0, with the B and D two octaves higher than they should be. The participants gave this chord a mean dissonance rating of 3.667, and its roughness rating is 16.83. In Experiment 2, one of the four-note chords presented to participants was a dominant seventh chord (G2-F3-B3-D4). It contains the same pitches as the diminished triad from Experiment 1 (some transposed in octave), plus a G, the implied fundamental of the chord from Experiment 1. Participants rated the chord from Experiment 2, which has a higher roughness rating of 22.10, with a dissonance rating of 2.54, much lower than the chord from Experiment 1. Both chords are quite common in tonal contexts (in fact they serve near-identical harmonic functions in tonal Western music), so the dual-process theory proposed by the authors should predict that roughness would be the primary predictive factor, but we see that that is not the case. This suggests that adding a fundamental to an otherwise dissonant chord may cause it sound more consonant, at least when the upper tones are relatively low partials of the fundamental (Moore, 1989).



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Determining whether the addition of a fundamental frequency to a chord (treating the notes of the chord as upper partials of some lower fundamental pitch) makes a chord or collection of tones sound more consonant, even if the chord itself contains dissonances, would have a number of interesting musical applications as well as establishing a new contributing factor to consonance perception and showing that our ability to hear fundamental-overtone relationships extends farther than is generally reported.

The goal of the present study is to test what effect adding the implied fundamental to a dissonant chord has on the perceived dissonance of the chord. These results will provide some measure of empirical evidence for or against harmonicity theory. In addition, this study is a first attempt to empirically test the subharmonic-based auditory nerve model created by Cariani (2001). Rather than using musically trivial stimuli (tonal triads presented using simple tones), this experiment is also an attempt to explore musically compelling stimuli in a music cognition study by using unusual, dissonant chords presented using complex tones. The use of unusual musical stimuli (in this case including microtones, or divisions smaller than the half-step) also helps limit the complicating role that tonal reference can play in consonance and dissonance studies.

### **Methods**

#### **Participants**

The study consisted of 21 participants, all of whom were Tufts students with musical experience. The mean age was 21.24 ( $SD = 2.32$ ), with a minimum age of 18 and a maximum age of 29. Nine of the participants were male, and the other 12 were female.

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On average, the participants had 13.31 years of musical training ( $SD = 3.60$ ), with a minimum of six years and a maximum of 17 years. The participant pool was recruited by contacting Tufts undergraduate music majors, many of whom directed other music students to the study. Participants were not compensated.

### **Materials**

Participants listened to ten groups of chords, with each group consisting of four chords. The chords, generated using the MIDI piano sound in Sibelius 6, were played for participants over Bose headphones. Each group of chords included four different chords:

1. A four-note chord built from partials 2 through 11 of two different overtone series, microtonally altered to the resolution of a 16<sup>th</sup> tone,
2. The original four-note chord with its implied fundamental (F0), calculated using a mathematical model in the program OpenMusic, added,
3. The original four-note chord with a pitch one semitone lower than its implied fundamental added, and
4. The original four-note chord with a salient pitch from the chord's implied subharmonic series added, chosen using the auditory nerve model developed by Cariani (2001).

Each group of chords was presented on a PowerPoint slide, with icons (labeled "Chord #1" through "Chord #4") to play each of the chords. Two different PowerPoint forms were prepared; 11 participants received Form A and 10 received Form B. In each form, the order of the chord groups and the position of the individual chords within each group was pseudo-randomized. Participants were provided with written instructions and spaces to mark their responses (see Figure 4). Participants were also given a questionnaire which inquired about their age, gender, and years and type of musical training.

## **Procedure**

Testing took place in a quiet room. Prior to testing, participants were provided with an informed consent form to read and sign. After doing so, participants were directed to adjust the volume on the computer to a comfortable level using a sample chord with similar intervallic content to the experimental chords. Once the volume was properly adjusted, the participants began the testing phase. Participants listened to the four chords of each group as many times as they needed to before ranking the chords from most dissonant (or harsh) to least dissonant. Participants were directed to rank the chords not based on their preference but rather their own assessment of which chords they perceived as “objectively” more dissonant. Participants were also made aware that there were no correct answers. After recording their rankings for the chords in all ten groups, the testing phase concluded and participants filled out the demographic questionnaire. Finally, participants were given a debriefing form and the opportunity to ask questions.

For the purposes of data analysis, a chord rated as the most dissonant in a group received a score of 1, the second-most dissonant a score of 2, the third-most dissonant a score of 3, and the least dissonant (or most consonant) a score of 4. These scores were then aggregated for each participant based on which of the four chord categories a chord belonged to, and the mean across all ten chord groups for each participant was calculated. These means were then aggregated into means across all the participants for each of the four chord categories: the original chords, the chords with added fundamental, the control chords, and the chords with added subharmonic tone.

## Results

The mean consonance ranking for the original chords (on a scale of 1 to 4, 1 being the most dissonant, 4 being the most consonant) was 2.81 ( $SD = 0.72$ ,  $SE = 0.16$ , 95% CI [2.48, 3.13]). The mean consonance ranking for the chords with added fundamental was 2.62 ( $SD = 0.36$ ,  $SE = 0.08$ , 95% CI [2.46, 2.78]). The mean consonance ranking for the control chords, with an added tone one semitone below the implied fundamental was 1.72 ( $SD = 0.34$ ,  $SE = 0.08$ , 95% CI [1.56, 1.88]). The mean consonance ranking for the chords with added subharmonic tone was 2.86 ( $SD = 0.43$ ,  $SE = 0.09$ , 95% CI [2.67, 3.05]).

As predicted, there was a significant difference ( $p < 0.05$ ) between the control chords and the other chords, with the controls being ranked as more dissonant than the original chords ( $t(20) = 5.2732$ ,  $p < 0.0001$ ), the chords with the harmonically-predicted tone ( $t(20) = 8.9790$ ,  $p < 0.0001$ ), and the chords with the subharmonically-predicted tone ( $t(20) = 9.3208$ ,  $p < 0.0001$ ). Additionally, the subharmonic predictions were considered significantly more consonant than the harmonic predictions ( $t(20) = 2.2582$ ,  $p = 0.0353$ ). There was no significant difference between the original chords and the harmonic predictions, or between the original chords and the subharmonic predictions.

There were no significant effects of years of musical training, age, or gender on the results. There was also no significant effect of salience of predicted subharmonic tone on participants' rankings of the subharmonic predictions (see Figure 3). However, the form participants received (functionally, the order the chord groups and the position of the chords within each group) did have a significant effect on their rankings of the

original chords and the subharmonic predictions. Participants who received Form B ranked the original chords as more dissonant ( $t(19) = 3.0653, p = 0.0064$ ) and the subharmonic predictions as less dissonant ( $t(19) = 2.1282, p = 0.0466$ ) than participants who received Form A (see Figure 5 for the order of each form).

### **Discussion**

The results of this study provide empirical grounding for both harmonicity theory and the subharmonic-based auditory nerve model proposed by Cariani (2001). Despite making the chords more complex by adding a pitch, the harmonic and subharmonic predictions were considered no more dissonant than the original four-note chords by the participants. On the other hand, the control chords, which involved an added pitch not predicted by any consonance model, were considered significantly more dissonant by participants than the original chords, as well as more dissonant than the consonance model predictions. In terms of a comparison between the two models, the chords with subharmonic tones from the auditory nerve model were considered less dissonant than the chords with the fundamentals predicting by harmonicity theory.

These results are a preliminary empirical indication of a number of things: a) the harmonic series plays a role in perception of consonance, b) the subharmonic series plays a role in perception of consonance, c) subharmonic relationships (i.e. collections aligning with the subharmonic series) may be judged as more consonant than harmonic relationships (i.e. collections aligning with the harmonic series), and d) these effects are observable in musically complex stimuli (i.e. stimuli using complex tones, unfamiliar pitch collections, and microtonal intervallic content). Significant further study is needed

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in order to determine how far these effects extend. Future studies should test non-musically-trained participants, and should use a larger sample size. While it is unlikely that these effects would be observed with complex tones, as they were here, and not simple tones (sine waves), an experiment that duplicates these stimuli using sine waves instead of MIDI piano sounds would also be useful. Additionally, due to the forced-choice paradigm used in this study, future studies should offer participants an opportunity to indicate how sure they were of their dissonance rankings (e.g. whether they were guessing, whether they were extremely confident, etc.).

Finally, the fact that participants who received Form B displayed significantly different ranking tendencies on certain types of chords suggests that a study exploring the role of stimuli ordering effects on dissonance perception would be very useful. While every attempt to balance out any ordering effect was made in this study (by having the two forms present the chord groups in different orders, by ordering the four chords in each group differently across the two forms, and by allowing participants to listen to the chords as many times as they needed to), the fact that any effect was observed begs follow-up research.

On the whole, this study represents an early step in understanding the role the subharmonic series plays in consonance perception, as well as providing further evidence for the role of the harmonic series in consonance perception. This study also provides support for the use of complex musical stimuli in certain music cognition studies, as opposed to the more traditional use of simple (and sometimes near-trivial) musical stimuli.

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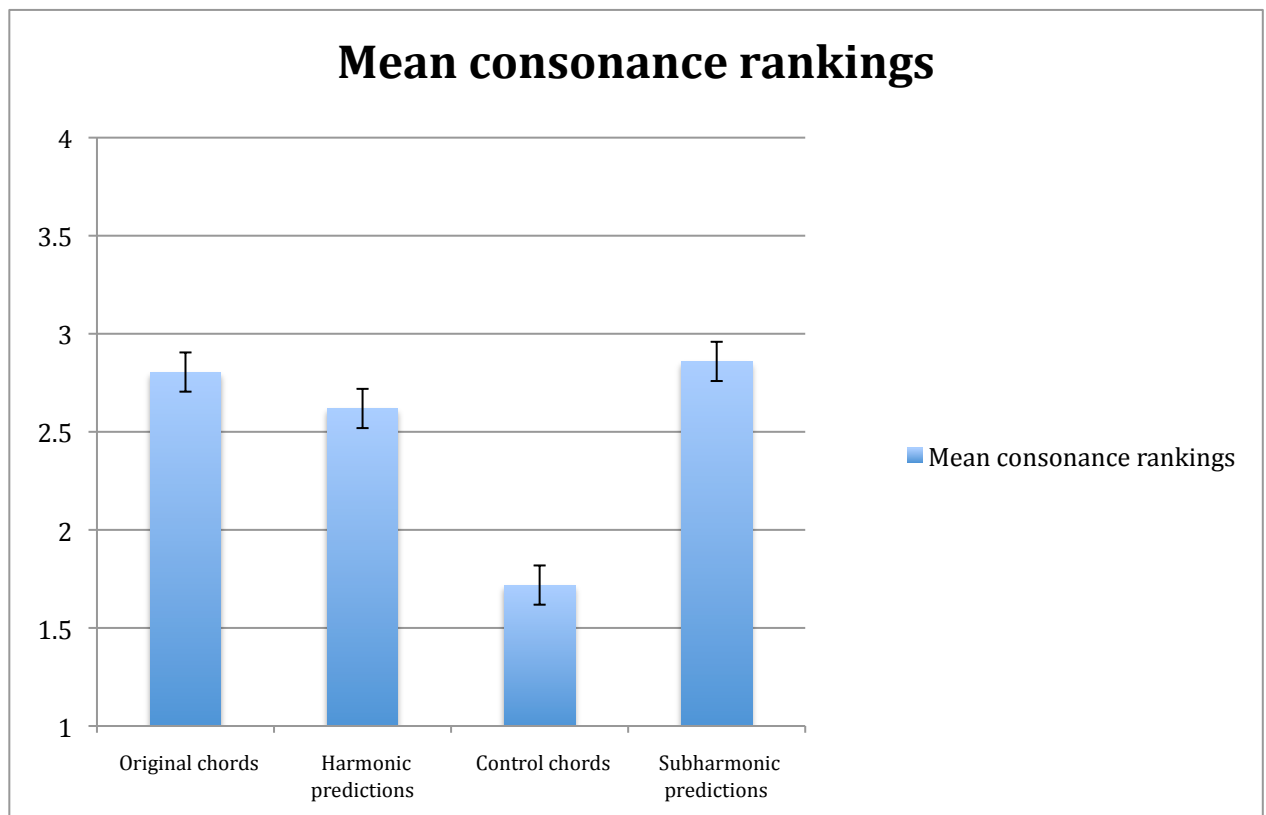
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Figure 1. Mean consonance rankings of four chord categories.



*Figure 1.* Bar graph of mean consonance rankings (minimum = 1, maximum = 4) across all participants. Original chords were four-note chords with no added pitch; harmonic predictions included a lower fundamental predicted by harmonicity theory; control chords included a low tone one semitone lower than the fundamental predicted by harmonicity theory; and subharmonic prediction included the most salient subharmonic tone predicted by the neural model developed by Cariani (2001).

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Figure 2. Salient pitch estimations for ten original chords.

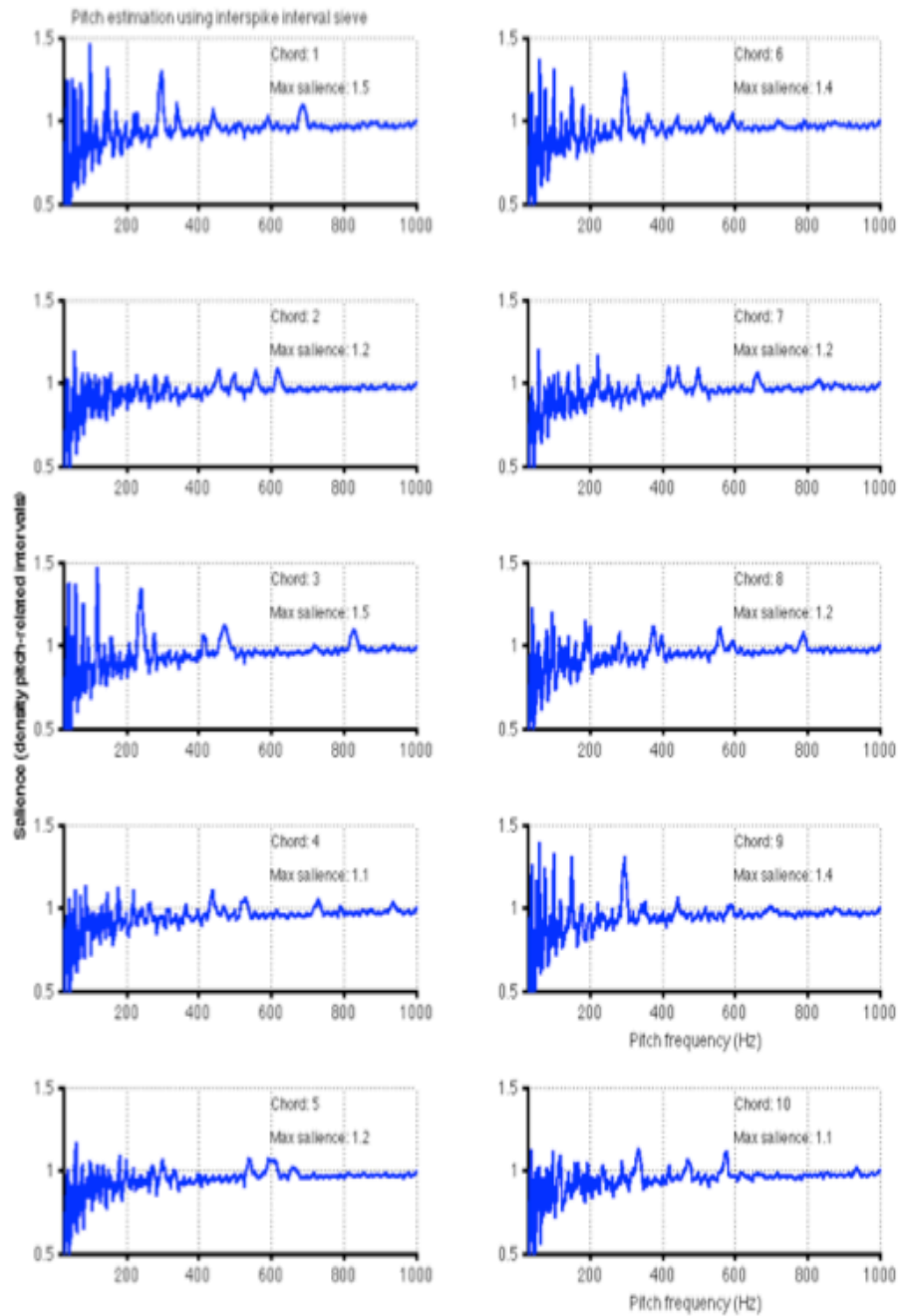


Figure 2. Pitch estimates (in Hz) of most salient subharmonic pitches predicted by the neural model developed by Cariani (2001) for the ten original chords used in the study.

Figure 3. Estimated maximum salience values for ten original chords.

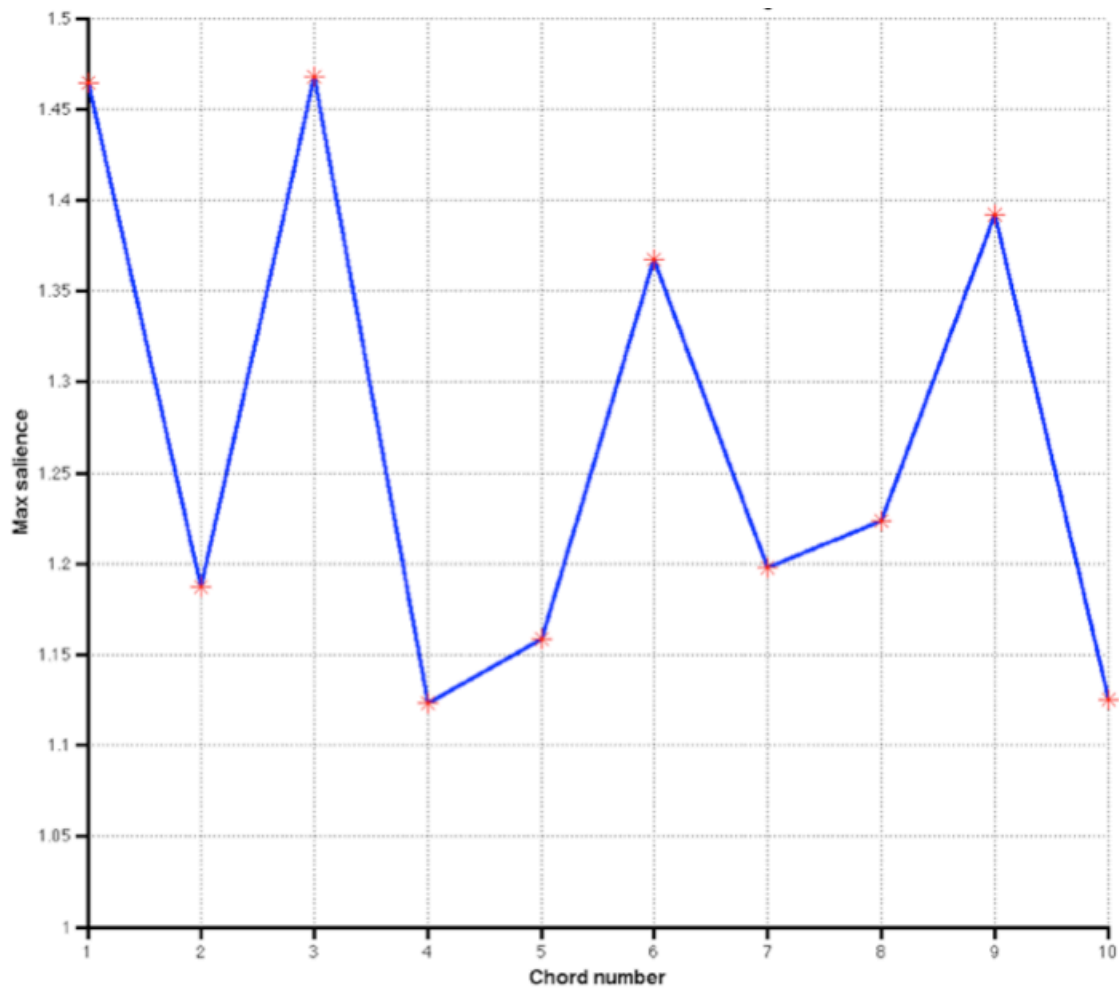


Figure 3. Estimated salience values for most salient subharmonic pitch predicted for each original used in the study.

Figure 4. Instructions given to participants before testing.

**Participant Response Sheet**

In this study, you will hear groups of four chords. Please rank them from **most** dissonant (most harsh) to **least** dissonant (least harsh). Try not to rank them based on your preference for one chord or another, but rather on your perception of how dissonant or harsh the chords are. You may listen to the chords as many times as you want to make your judgments. When you have marked your responses to one group on this sheet, click "next group" to proceed to the next group. There will be 10 chord groups in total. There are no right answers - the judgments are entirely up to you.

Before you begin, adjust the volume to a comfortable level. Click the play icon on the first slide to hear a sample chord, which you can use to help adjust your volume level.

As a reminder, you may cease participation in the study at any time if you feel uncomfortable for any reason.

When you are ready to start, click BEGIN to proceed. When you have listened to the chords in a group enough to rank them, please fill out the corresponding table below. Please write legibly.

**Example:**

In this example, a participant considered chord #3 to be the **most** dissonant, followed by chord #2, chord #4, and finally chord #1, which would be the **least** dissonant.

<b>Most dissonant</b>			<b>Least dissonant</b>
<b>3</b>	<b>2</b>	<b>4</b>	<b>1</b>

Figure 4. Instructions given to participants about how to rank chords.

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Figure 5. Order of groups and chords in each form.

### Form A:

	<u>1 2 3 4</u>
Group #1 (Chord A):	a c b d
Group #2 (Chord B):	c b d a
Group #3 (Chord C):	d b a c
Group #4 (Chord D):	c d a b
Group #5 (Chord E):	d b c a
Group #6 (Chord F):	b c a d
Group #7 (Chord G):	a b c d
Group #8 (Chord H):	b d a c
Group #9 (Chord I):	d c b a
Group #10 (Chord J):	c a d b

### Form B:

	<u>1 2 3 4</u>
Group #1 (Chord J):	b d a c
Group #2 (Chord E):	a b c d
Group #3 (Chord B):	d a b c
Group #4 (Chord H):	c b d a
Group #5 (Chord F):	c a d b
Group #6 (Chord I):	a d c b
Group #7 (Chord A):	b c d a
Group #8 (Chord D):	a c b d
Group #9 (Chord G):	d a c b
Group #10 (Chord C):	c b a d

*Figure 5.* Order of groups and chords within groups for Form A and Form B. “a” refers to original chords, “b” to harmonic predictions, “c” to control chords, and “d” to subharmonic predictions.