a. In Galileo's case the Tables are in effect being recalibrated for each different velocity and projectile size and shape

b. In Newton's case, by contrast, the only calibration involves a presumable constant

5. The trouble, of course, comes when multiple measurements for this coefficient fail to yield (remotely) consistent values over a range of velocities and projectiles, which is in effect what Huygens had discovered years earlier.

A. Newton is here assuming that resistance effects are proportional to velocity, while the truth is more accurately represented as if by two distinct effects, one roughly proportional to velocity and another, usually dominant, roughly proportional to velocity squared

b. The *Principia* will offer such a further refinement

V. The Significance of *De Motu Corporum in Gyrum*

A. Advances Made by Version 1 of De Motu

1. Even though *De Motu Corporum in Gyrum* was not published, it gained enough circulation through the Royal Society that it is appropriate to ask, what exactly did it, by itself, contribute

a. Or, maybe more appropriately, what exactly would it have contributed if it had not been followed up two and a half years later by the *Principia*

b. A question that is undoubtedly best answered from the perspective of Halley, though also from that of Newton, since he was evidently not satisfied with the contribution as it stood

c. In other words, instead of reading the tract through the lens of the subsequent *Principia*, we should try to read it through the lens of the state of science as of late 1684

2. One thing that this tract did not contribute in its own right is universal gravity, for the tract as such does not even require mutual attraction between celestial bodies, much less between every two particles of matter

a. I see nothing in this version of De Motu that gives any reason to think that Newton had yet even thought of universal gravity among particles

b. No reason to think that Newton had yet even reached a clear concept of mass, as distinct from weight, for the one place where he needs *mass* in the final scholium he uses *weight*

3. In the version sent to the Royal Society, the one key place in which gravity is referred to is at the end of the Scholium to Problem 5, where Newton speaks of the "hypothesis" of gravity being an inverse-square force

a. But the manuscript shows him to have been using "*gravitas*" in Problems 1, 2, 3, and 5, only then to delete it, usually replacing it with "*vis centripeta*"

b. What he had in mind when he first used it and why he then replaced it with a more abstract locution is unclear; but no one looking at the public version of De Motu saw any sign of this

c. The effect is clear, however: the text presents an abstract theory of motion under centripetal forces until after Problem 5, where it turns to motion under hypothetical terrestrial gravity

4. Something the registered version nonetheless did achieve was to tie Kepler's three "laws" together for the first time as manifestations of a specific mathematically describable situation -- inverse-square departures from uniform motion in a straight line toward a single point in space
   a. Keplerian motion no longer a deviant form of uniform circular motion involving a second mechanism; instead uniform circular motion now a special case in which velocity has a peculiar (vector) value for which the eccentricity of the elliptical trajectory becomes 0
   b. Tying the three "laws" together within such a Galilean (or, more properly, Huygensian) mathematical theory also greatly increases the grounds for taking them to be nomological, and provides added reason for thinking that they might be exact
      (1) The former because of the added grounds for holding that the three are manifestations of a single underlying physical mechanism -- indeed, one not peculiar to orbits around the Sun
      (2) The latter because have now identified a comparatively simple circumstance in which the three would hold exactly, viz. if the only force acting on the orbiting bodies is an inverse-square centripetal force
   c. Notice here that a correlative effect De Motu has on the status of the three "laws" is the enormous increase in importance of any evidence of even minor deviations from them
      (1) Major deviations, of course, would tend to undercut the physical applicability of the overall theory
      (2) But minor deviations may provide critical information about celestial physics -- e.g. some forces other than just inverse-square centripetal forces are present
5. Further, the tract puts the area rule on a totally different footing, not only tieing it to centripetal accelerations, but also clarifying Kepler's $r*v_{\perp}$ invariance
   a. Puts burden of proof on alternatives to the area rule, requiring similar simple kinematic underlying principle
   b. Eliminates any remaining worries about whether any possible physical basis for the area rule
6. Tract shows Galileo's "laws" of free fall and parabolic trajectory probably hold only approximately, even in absence of air resistance
   a. That is, under the assumption that acceleration of terrestrial gravity is inverse-square, as supported by the "Moon test," which of course is not mentioned in the tract
   b. Still, both hold to high approximation near surface of Earth, for parabola closely approximates end of high eccentricity ellipse
7. Results on motion under resistance offer promise of being able to test Galileo's "laws," confirming that they would hold to high approximation in absence of resistance
   a. Even if resistance not exactly proportional to *v*, may be near enough over limited distances to allow corrections to Galileo's to be determined

   b. Offering a prospect of solving artillery problem
   c. And also substantiating Galileo's separation of free fall and resistance, with former independent of weight (and shape), but not the latter
8. Results lend added support for Descartes' and Huygens's idea of contraposing principle of inertia, inferring forces from deviations from uniform motion in a straight line
   a. Simple rules of force for two different ellipses promising in just the way a simple rule for force in uniform circular motion was obtained from contraposed inertia
   b. Now have reason to adopt this approach as a general strategy
9. Finally, Problem solutions offer promise of advancing state of art in calculating celestial trajectories
   a. License for Horrocks-Streete use of 3/2 power rule to infer mean distance (i.e. a) from P
   b. An iterative method for determining specific ellipses
   c. Promise for determining comet trajectories

B. A "Galilean-Huygensian" Theory of Orbital Motion

1. From a broader perspective, De Motu presents a mathematical theory, a la Galileo and Huygens, in which uniform acceleration along parallel lines toward the surface of the earth is replaced by acceleration, especially inverse-square acceleration, toward a point in space
   a. In modern sense, a kinematic theory, for talk of force notwithstanding, only accelerations in the manner of Galileo within the theory until resistance is added
   b. Following Galileo, resistance is treated as a separate mechanism, and no cross-talk between components of motion assumed in parallelogram rule
   c. Following Huygens, deviation from uniform straight line motion, taken as proportional to t^2 over first small increment of time, gives measure of acceleration and force
2. Key enabling theorem a direct generalization of Huygens on uniform circular motion and centrifugal force, enabling inference of centripetal force from geometry and speed
   a. Key step for this: geometric representation of time by area for centripetally governed motion
   b. Again, a direct generalization of uniform circular motion, but with centripetality the key factor
3. As solved Problems show, a rich mathematical theory from limited initial assumptions, yielding question-answering power
   a. Key assumption: all deviations from uniform motion in a straight line are directed toward a single point in space!
   b. Different motions -- defined by trajectory and "force center" -- yield different rules of centripetal acceleration
   c. Inverse-square case most richly developed, yielding solution to initial value problem, with different conics from different initial situations
4. In the style of Huygens, takes trouble to tie results for more general curvilinear motions to Galilean free-fall and parabolic trajectory, enhancing opportunities for evidence

a. By providing a contrast between Galilean and inverse-square free fall and projection

b. By proposing solutions to Galilean motion in a resisting medium that have at least some promise of separating resistance effects from motion without them

5. Note too that theory has a Huygensian testable surprising consequence that, if verified, will provide strong evidence of an inverse-square centripetal acceleration "field" around the sun, namely comet trajectories of a sort that were novel to propose at the time

6. Finally, approach to resistance "Galilean" in adopting simplest assumption -- linear in velocity

a. Same approach Huygens took in the 1660s, undoubtedly for the same reason

b. Hoping to yield good enough approximation to allow corrections for confounding effects of resistance in experiments

C. Departures from Galilean-Huygensian Science

1. Given the extent to which De Motu follows the existing tradition and the strength of interest in inverse-square forces after publication of *Horologium Oscillatorium* in 1673, why didn't someone else come up with Newton's main results earlier

a. The mathematics was not out of the reach of Wren or Halley, for no calculus was involved

b. Huygens took himself to task for not having done so when he saw these results in the *Principia*

2. Part of the answer to this question lies in the several respects -- some obvious, some subtle -- in which De Motu reaches beyond Galilean and Huygensian science

a. Obviously, the mathematics of inverse-square accelerations and elliptical trajectories is more complicated, even than that of cycloidal trajectories

b. And not confined to local motion near surface of earth, but extending to celestial motions in a way that both Galileo and Huygens never attempt

3. Still more subtly, even though theory is strictly speaking kinematical, talk of forces is not gratuitous, for dealing with departures from uniform motion in a straight line

a. This talk reaches beyond Galileo in just the way Descartes' treatment of motion reaches beyond geometry in appealing to forces

b. Also, with talk of impressed forces, external to moving body, question of cause cannot help but arise in a way that it doesn't in Galileo's work

4. A further departure: treating forces mathematically, in the abstract, and not tieing them to static forces in string or on wall in the way Huygens did

a. Breaking away from statics, with 'force' being used in a somewhat new way, raising questions about legitimacy

b. No longer a means for confirming measures assigned to forces through appeals to statics

5. This departure is amplified by Newton's step to what we now call fields -- i.e. to idea that every point in space surrounding a "force center" has associated with it a distinctive accelerative tendency toward the center -- $[a^3/P^2]/r^2$

a. In the process, center endowed with a new property, [a^3/P^2] -- at least whenever a body orbits it

b. Save perhaps for glimmers of this idea in work on magnetism, nothing like this in prior science save for the incorrectly proposed universality of the distance of fall in the first second

6. Finally, question of mechanism producing inverse-square centripetal accelerations is left entirely open in De Motu, as if not worth raising

   a. This might or might not have bothered Galileo, but it not only would have, but did bother Huygens (and it appears to have bothered Newton)

   b. What is sneaking into the Galilean-Huygensian tradition here is the English tradition in magnetical philosophy, where appeals to analogy with magnetism were permitted

   c. Would have been more apparent if Newton's had not replaced 'gravitas' in several places

D. Loose Ends in Version 1 of De Motu

1. The absence of mechanism for inverse-square centripetal forces is not the only loose end in version 1 of De Motu

2. Legitimacy of the reasoning in the Scholia is unclear: infer inverse-square from uniform circular and 3/2 power rule, then infer exact Keplerian ellipse from no need for anything beyond inverse-square

   a. Planets are not in uniform circular motion: 40 percent variation in (angular) speed from min to max in case of Mars and 100 percent variation in case of Mercury

   b. Have not explicitly eliminated other bounded trajectories besides ellipse

3. No independent evidence that inferred inverse-square force in case of planets and their satellites is real, *unless comet trajectories turn out to be calculable from inverse-square centripetal accelerations*

   a. A lot seems to be riding on the motions of a handful of bodies, motions that are known at most to approximate Keplerian

   b. Moon provides a clear counterexample to Keplerian motion, and orbits of Jupiter's satellites known from Cassini not to be stationary

4. No evidence for inferred inverse-square variation of terrestrial gravity, save for unstated "Moon test"

   a. A lot seems to be riding on a single number that may be a mere happenstance

   b. Worse, that number predicated on false assumption of uniform circular motion

   c. Would like to have other evidence, such as the claimed variation in surface gravity with altitude observed by Halley at St. Helena and mentioned by Hooke in one of his letters to Newton

5. No evidence that resistance varies linearly with velocity, even to reasonable approximation

   a. Newton provides a way of measuring the constant of proportionality for any one body

   b. But until measurements show that this "constant" is a constant, an open question

6. Orbital theory in De Motu refers all motions to a single point in space, but there are at least four "centers of force," three of which are orbiting the sun

   a. I.e. Saturn, Jupiter, and earth are not stationary centers in space

   b. This raises the question, to what center should orbital motions be referred

c. Because this was the very issue raised by the Tychonic versus Copernican controversy, those at the time would have been quick to notice this

7. If there are inverse-square acceleration "fields" around the earth, Jupiter and Saturn, then not only comets should experience them, but the sun as well
   a. I.e., the results on their face imply an interaction between the sun and e.g. Jupiter
   b. Is this real, and if so, does it nullify the results derived mathematically in Version 1 of De Motu

E. Version 1 of De Motu Versus the *Principia*

1. While the main body of De Motu offers a purely mathematical theory, and not as such a physical theory (save for the way of conceptualizing motion), the theory and the Scholia clearly point to a physical hypothesis
   a. The physical hypothesis: the curvilinear motion of every celestial body, especially bodies orbiting about a principal, is governed by gravity-like, inverse-square centripetal forces
   b. Not universal gravity, but a hypothesis closely akin to one Hooke and Wren were entertaining since mid-1670's, and Newton appears to have been entertaining as well when drafting De Motu
   c. This may have been one reason why Halley was so excited with the tract
2. The standard picture, both then and now, is that Newton's response to the results of Version 1 of De Motu was to leap to a much bolder conjectural hypothesis: every particle of matter gravitates toward every other particle of matter in accord with the inverse-square rule
   a. I.e. a leap of genius to a bold explanatory hypothesis that reaches far beyond any of the results in De Motu and any phenomena then known
   b. An explanatory hypothesis that turned out, over the next century, to explain all sorts of other things
   c. A picture under which De Motu served only to stimulate a hypothesis, and subsequent evidence for this hypothesis was all hypothetico-deductive
3. A consequence of this picture now is the standard answer to the question of this course, how did we ever come to have high quality evidence in any science
   a. Answer: Newton happened upon an extraordinary hypothesis that turned out to be (nearly) true, and the evidence then fell into place
   b. Corresponding picture of science: carry out limited investigations until in a position to leap to general explanatory hypothesis, after which test this hypothesis, replacing it as need be
4. A consequence of this picture then: substantial reluctance in many circles to accept Newton's universal gravitation precisely because it did leap so far beyond De Motu
   a. E.g. Huygens accepted without reservation Version 1 of De Motu plus the "Moon test" and the conclusion from it that the moon is retained in orbit by inverse-square terrestrial gravity, and the planets by inverse-square gravity one in kind with terrestrial gravity, but regarded everything beyond it as unwarranted conjecture -- a view that upset Newton

b. Not just a complaint about lack of mechanism -- and hence the implication of action at a distance -- but also about lack of evidence for interactive celestial gravity, much less universal gravity

c. Universal gravity ultimately came to be accepted as its consequences fell into place between 1740 and 1790, so that its historical acceptance actually did conform largely with a hypothetico-deductive picture

5. This standard picture of what Newton did, once he had De Motu, is in fact not what he did, nor what he said he did

   a. Clearest evidence for this: if he had leapt directly to universal gravity, he would have turned right away to question whether an inverse-square acceleration toward the sun can be composed out of inverse-square accelerations toward each particle of matter comprising it

   b. But did not turn to this question until March or April 1685, after several intervening steps

   c. Further evidence: anger Newton consistently expressed toward those who took him to have leapt to the hypothesis of universal gravity

6. What Newton did instead, once he had De Motu, was to begin focusing on its loose ends, proceeding step by step to a sequence of further conclusions

   a. Initially to a "proof" of Copernicanism

   b. Then to the law of gravity

   c. Then to a new conception of how to do science, which he much later came to call the new "experimental philosophy"

7. Will start this sequence next week, but need to read entire *Principia* to see clearly how the science in it departed from all science before the *Principia*

   a. *De Motu Corporum in Gyrum* derives eleven propositions in nine handwritten pages; *Principia* derives more than 180 propositions in some 500 printed pages

   b. When Newton first discovered the elongation of the image cast by the prism, he conducted a large battery of further experiments to explore what conclusions are properly to be drawn from his initial discovery

   c. One way to think of the transition from the registered version of De Motu to the *Principia* was that he did something analogous to that from November 1684 to the first three months of 1687, adding a large battery of further propositions that allowed him, in his mind, to draw secure conclusions about celestial motions fom his initial findings

<u>Select Sources</u>

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