Relational semantics and domain semantics for epistemic modals*

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1. Yalcin's puzzle

Yalcin's data (Yalcin, 2007, 985–986):

(1) # Suppose it is not raining and it might be raining.
Suppose $\neg\phi \wedge \Diamond\phi$.

(2) # If it is not raining and it might be raining, then...
If $\neg\phi \wedge \Diamond\phi$, then...

Contrast with:

(3) Suppose it is not raining and I don't know that it's not raining...

(4) If it is not raining and I don't know that it's not raining...

Standard relational semantics for epistemic possibility modals (e.g. Kratzer, 2012):

- A version of the standard semantics:

$$[\![\Diamond\phi]\!]^{c,w,f} = 1 \text{ iff } \exists w' \in f(w) : [\![\phi]\!]^{c,w',f} = 1$$

- Here f is an epistemic modal base, i.e. a function that maps a world to a set of worlds. So points of evaluation include a modal base parameter (Portner, 2009, 52).
- At a context c, $f_c(w)$ is the set of worlds compatible with what x_c knows in w, where x_c is the individual or group that c determines as relevant for interpreting the modal.

  In a simple version of the theory, x_c might simply be the speaker of c.

The problem for the standard semantics:

- I'm just going to look at the attitude data, but I think what I say about that can be extended to the issue with conditionals.[1]
- Let S_x^w be the set of worlds compatible with what x supposes in w. The standard semantics for attitudes looks like this:

$$[\![x \text{ supposes } \phi]\!]^{c,w,f} = 1 \text{ iff } \forall w' \in S_x^w : [\![\phi]\!]^{c,w',f} = 1$$

- But then x *supposes* $\neg\phi \wedge \Diamond\phi$ is predicted to have a satisfiable truth condition:
  - $[\![x \text{ supposes } \neg\phi \wedge \Diamond\phi]\!]^{c,w,f} = 1$ iff
  - $\forall w' \in S_x^w : [\![\neg\phi \wedge \Diamond\phi]\!]^{c,w',f} = 1$ iff

*These notes are based on a talk I gave at an MIT semantics seminar in November 2012. Thanks to the participants of that seminar (especially Kai von Fintel), and to Seth Yalcin.

[1] I'm also going to take the data at face value, though see Dorr and Hawthorne (2013) for another perspective.

- $\forall w' \in S_x^w : [\![\neg\phi]\!]^{c,w',f} = 1$ and $\exists w'' \in f(w') : [\![\phi]\!]^{c,w'',f} = 1$

- Such a sentence says that all the worlds w' compatible with what x supposes in w are such that not-ϕ is true in w' and there is a ϕ-world that is compatible with the relevant information in w'.
- In other words, the sentences says: x supposes that ϕ is false, but that the relevant information does not exclude ϕ.

  For example: x supposes: that it is not raining, but that for all x knows it is raining.

What has gone wrong?

- Yalcin: the relational semantics for modals is to blame:

  > "The problem... is the idea, practically built into a relational semantics for modals, that the evidential state relevant to the truth of an epistemic modal clause is ultimately determined as a function of the evaluation world..." (Yalcin, 2007, 992)

- So Yalcin gives up the idea that the evidential state relevant for interpreting epistemic modals is determined as a function of the evaluation world; instead, he adds an 'information state' parameter to the points of evaluation.

2. Domain semantics

Domain semantics for modals:

- Instead of a modal base parameter, points of evaluation contain an *information state* parameter i. Formally, this is just a set of worlds.
- Yalcin's *domain semantics* for epistemic modals (Yalcin, 2007, 994):

  $[\![\Diamond\phi]\!]^{c,w,i} = 1$ iff for some $w' \in i$, $[\![\phi]\!]^{c,w',i} = 1$

Domain semantics and the puzzling data:

- Does simply switching to a domain semantics solve the problem *by itself*? No!
  - $[\![x$ supposes $\neg\phi \wedge \Diamond\phi]\!]^{c,w,i} = 1$ iff
  - $\forall w' \in S_x^w : [\![\neg\phi \wedge \Diamond\phi]\!]^{c,w',i} = 1$ iff
  - $\forall w' \in S_x^w : [\![\neg\phi]\!]^{c,w',i} = 1$ and $\exists w'' \in i : [\![\phi]\!]^{c,w'',i} = 1$
- This too is a satisfiable truth condition. The sentence ends up saying two quite distinct things: (i) that, in w, x supposes not-ϕ; and (ii) that ϕ is compatible with the *initial* information state i.
- In other words, the sentence says: (x supposes $\neg\phi$) and (it might be that ϕ).
- Two problems with this:
  (i) there are points of evaluation that satisfy this condition; and
  (ii) the epistemic modal clause doesn't place any constraint on x's state of mind.

- On point (ii), notice that, on this semantics, the following sentences will have (more or less) the same truth conditions:

  (5) John supposed that it might be raining.

  (6) It might be raining.

- The upshot: *merely* moving to a domain semantics for epistemic modals is not sufficient to solve the problem. One must also move to an appropriate semantics for attitude verbs.

Domain semantics for modals with 'shifty' attitude semantics:

- Although he doesn't mention these problems, Yalcinavoids them by altering the semantics of attitude verbs, so that an attitude verb shifts *both* the evaluation world *and* the information state parameter (Yalcin, 2007, 995):

  $$[\![x \text{ supposes } \phi]\!]^{c,w,i} = 1 \text{ iff } \forall w' \in S_x^w : [\![\phi]\!]^{c,w',S_x^w} = 1$$

- Note that the information parameter is now shifted to S_x^w, the set of worlds compatible with what x supposes in w.

- Now we get a better prediction for the problem sentences:

  - $[\![x \text{ supposes } \neg\phi \wedge \Diamond\phi]\!]^{c,w,i} = 1$ iff
  - $\forall w' \in S_x^w : [\![\neg\phi \wedge \Diamond\phi]\!]^{c,w',S_x^w} = 1$ iff
  - $\forall w' \in S_x^w : [\![\neg\phi]\!]^{c,w',S_x^w} = 1$ and $\exists w'' \in S_x^w : [\![\phi]\!]^{c,w'',S_x^w} = 1$

- This truth condition is *not* satisfiable, since it says that every world compatible with what x supposes in w is a not-ϕ-world and that some world compatible with what x supposes in w is a ϕ-world.[2]

3. Relational semantics for modals, shifty semantics for attitudes

Observation:

- Yalcin blames the problem on the standard relational semantics for modals. But the problem really arises from the *combination* of a relational modal semantics *and* the standard attitude semantics.

- Yalcin solves the problem with two innovations: (i) domain semantics for modals, and (ii) a shifty semantics for attitudes.

- This raises a question: Could we solve the problem for the standard semantics by combing a relational semantics for modals with a shifty semantics for attitudes? (And what would 'shifty' mean here?)

- If so, then we would have shown that a relational semantics for modals is compatible with Yalcin's data.

Relational semantics for modals plus shifty semantics for attitudes:

[2] I'm assuming here that ϕ is not itself a modalized sentence. This is reasonable given the datum we are attempting to explain, namely the infelicity of (1).

- Let Sup_x^w be a function from worlds to sets of worlds, such that Sup_x^w maps a world w' to the set of worlds compatible with what x supposes in w.
- So Sup_x^w is a *constant* function.

  If S_x^w is the set of worlds compatible with what x supposes in w, then for any world w', $Sup_x^w(w') = S_x^w$
- Shifty semantics for *supposes*:

  $$[\![x \text{ supposes } \phi]\!]^{c,w,f} = 1 \text{ iff } \forall w' \in Sup_x^w(w) : [\![\phi]\!]^{c,w',Sup_x^w} = 1$$
- Let's look at what happens to our problem sentence:
  - $[\![x \text{ supposes } \neg\phi \wedge \Diamond\phi]\!]^{c,w,f} = 1$ iff
  - $\forall w' \in Sup_x^w(w) : [\![\neg\phi \wedge \Diamond\phi]\!]^{c,w',Sup_x^w} = 1$ iff
  - $\forall w' \in Sup_x^w(w) : [\![\neg\phi]\!]^{c,w',Sup_x^w} = 1$ and $\exists w'' \in Sup_x^w(w') : [\![\phi]\!]^{c,w'',Sup_x^w} = 1$
- The key at this point is to remember that that Sup_x^w is a constant function, mapping any world w' to S_x^w, the set of worlds compatible with what x supposes in w. This means that our final line is equivalent to:
  - $\forall w' \in S_x^w : [\![\neg\phi]\!]^{c,w',Sup_x^w} = 1$ and $\exists w'' \in S_x^w : [\![\phi]\!]^{c,w'',Sup_x^w} = 1$
- This is not a satisfiable truth condition: it says that every world compatible with what x supposes in w is a $\neg\phi$-world and that some world compatible with what x supposes in w is a ϕ-world.[3]

Summary:

- The standard relational semantics for modals is, I think, compatible with Yalcin's data involving how *might* embeds under *supposes*.
- I think a similar point can be made with respect to conditionals, but it would mean adopting a similarly shifty semantics for conditionals.
- Architectural point: we might not need a domain semantics at all, though we may need to allow Kratzerian modal bases to be shifted by various expressions (or bound, if modal bases are represented by covert elements of the syntax).
- Are the two theories notationally equivalent? Is there any data that might favor one over the other?

References

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Kratzer, Angelika. 2012. *Modals and Conditionals*. Oxford: Oxford University Press.

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[3]Again, I'm assuming here that ϕ is not itself a modalized sentence.