   a. (Note the tacit assumption, which amounts to counting on nature being regular in a certain way; also the assumption that Mars is in orbit around the sun, for on Ptolemaic theory it is not in the same place every 687 days, only the center of its epicycle is)
   b. Could then use triangulation to infer ratios of three earth-sun distances in each case, from which could calculate center of presumed circle
   c. Calculated distances subject to errors from observational inaccuracies
   d. Not surprising, then, that he obtained varying results, ranging from 0.025 to 0.01653 using his vicarious theory of Mars's heliocentric longitudes, and from 0.01837 to 0.01530 using the preliminary Copernican-type theory Tycho had developed for Mars about the mean sun in the 1590s
   e. But all showing near middle between Sun and equant, and definitely not coincident with latter
   f. Also compared apogeal and perigeal apparent diameters of sun (30/31), giving a value of 0.0164
5. Transporting this same reasoning to a Ptolemaic system showed that Mars's epicycle requires a point around which equiangular motion occurs that is removed correspondingly from the point at which it is attached to the deferent
   a. That is, the vicarious hypothesis and observations of geocentric longitudes of Mar again defined three distinct triangles every 687 days between the earth, the point of attachment of the epicycle on the deferent, and Mars on the epicycle (see figure in Appendix)
   b. These three defined a circular trajectory for Mars on its epicycle showing that its point of attachment and point around which motion is equiangular straddle the center of the circle, at least to reasonable approximation
   c. This, of course, is a conclusion about Mars's epicycle, not about the earth-sun orbit, but that did not stop Kepler:

      That these characteristics belong to the Ptolemaic epicycle, is properly demonstrated. But that they are carried over from the epicycle to the theory of the sun is shown by a probable argument only, pieced together from Ptolemaic opinions (Chapter 26, p. 145)

   d. Arguing that whatever is true of Mars's epicycle should be true of those of Jupiter and Saturn as well, Kepler offers a diatribe against Ptolemaic systems (see Appendix), while nevertheless granting that his bisected eccentricity reform can be incorporated within them
   e. Having shown that the reasoning from observations (and the vicarious hypothesis) can be transported from one system to another, Kepler from here on reasons in terms of the Copernican
6. On this basis hypothesized exact bisection for a circular earth (or sun) orbit, contrary to a tradition dating back to Hipparchus: 0.018, half of Tycho's value, 0.03584, rounded
   a. (Note: did not use average of calculated values; hence an idealization)
   b. Checked by comparing heliocentric longitudes of earth calculated with new model against Tycho's original at the quadrants and octants of anomaly, finding good agreement (9 sec, but correcting for calculation error, 1 min 7 sec)

c. Then carried through a mathematical analysis to explain why the longitudes would be so much more sensitive to the distance between the sun and equant than to the location of the center of the circle, thus explaining Tycho's success with the sun (Chapter 31)

7. The combination of the actual sun and the bisection of the eccentricity of the earth-sun orbit reduces the discrepancies in Mars's orbit from greater than 4 degrees to a few minutes of arc -- a reform that can be introduced just as well to the Ptolemaic system (see Voelkel and Gingerich)
   a. In effect, an order of magnitude-plus reduction in discrepancies between observation and Ptolemaic theory when latter revised for actual sun and epicycles have bisected eccentricity
   b. Had these two reforms been carried out earlier, a very high standard of observational accuracy would have been required to give reason to pursue the further refinements of the area rule and the ellipse

F. "Phase 3" Continued: the Area Rule

1. Now adopts new hypothesis in place of the equant -- arc length velocity everywhere varies inversely with distance from the sun
   a. Kepler had long believed that the sun provides the motive power driving the planets, and thought it must diminish with distance in keeping with the longer periods of outer planets
   b. In Ptolemaic type orbit as in vicarious theory, with equant but with bisected eccentricity, velocity exactly inversely proportional to distance from sun at perigee and apogee
   c. So, now replace the equant with this feature of it, but generalizing across entire orbit: a minimal move beyond the vicarious theory
2. But now finds this new rule for locating Mars versus time calculationally taxing (needed calculus)
   a. Substitutes an approximate method, dividing circular orbit into equal 1 deg segments, computing distance of each arc from sun and adding up these distances, so that the time in each arc determined by the ratio of its distance to the sum of the distances
   b. Verifies this simplification against Tycho's solar theory (within 9 arcseconds)
3. Then happens upon a still simpler approximation to the inverse distance from the Sun rule: equal areas in equal times (Chapter 40: "An Imperfect Method ...")
   a. I.e. area of sector as a measure of all the distances within the sector (which were being summed on above method): assume areas proportional to times for the equal arcs
   b. Again verifies against Tycho's solar theory (within 34 arcseconds)
   c. Note the reasoning here: so long as within observational error bands, both methods okay
4. Proceeds through remainder with two separate motion rules to replace the equant hypothesis, the inverse distance rule and the area rule, though he never states the latter in a fully perspicuous manner until after *Astronomia Nova*

G. "Phase 4": The Orbit is Definitely Oval

1. Given rule (in fact two rules) for motion, now ready to address question of trajectory