Limit loads – 2

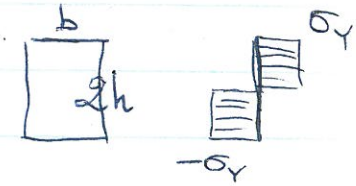
Beams in Bending :

Upper bound (conservative estimate of limit loads)

Plastic collapse: by formation of plastic hinges

At each hinge : - fully plasticized cross-section
holds certain M_u (ultimate) determined by geometry of cross-section

example : rectangular cross-section :

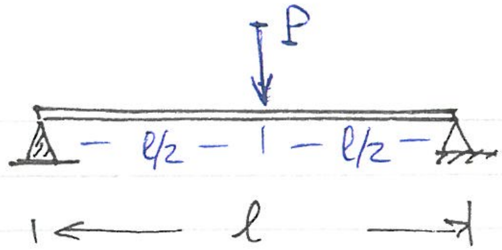


ultimate sustainable → $M_u = \sigma_Y \cdot bh^2$

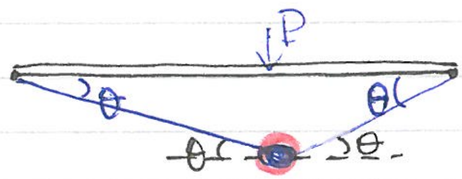
steps : - find M_u for the given cross-sectional geometry
- hypothesize mechanism of collapse (several hinges, with M_u each)
- for this mechanism: equate, in this limit state, (work of external loads) = (work done on rotation of hinges)
- examine several mechanisms (different locations of hinges)
- Choose the one with the lowest load

Points suspected for hinges : - points where point forces applied
- or clamped points

Example



Assumed hinge:



Energy balance during collapse:

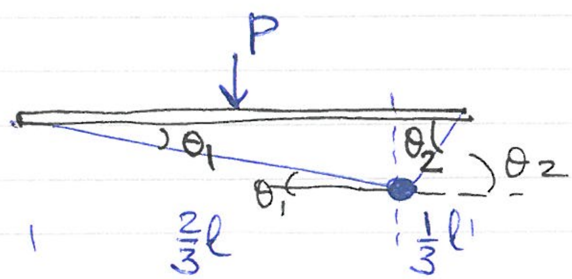
$$\underbrace{P \frac{l}{2} \dot{\theta}}_{\text{rate of work done by } P} = \underbrace{M_u \cdot 2\dot{\theta}}_{\text{rate of work spent in hinge}}$$

$$\Rightarrow \boxed{P_{lim} = 4 \frac{M_u}{l}}$$ – upper bound

(rectangular cross-sect: $M_u = \sigma_y b h^2$)

What if our guess is clearly unreasonable?

Assumed hinge:



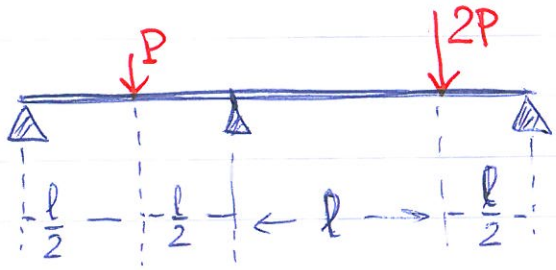
Sin-theorem for triangle: $\frac{\sin \dot{\theta}_2}{\sin \dot{\theta}_1} = \frac{2/3\, l}{1/3\, l} = 2$

Small $\dot{\theta}$ (beginning of collapse): $\dot{\theta}_2 = 2\dot{\theta}_1$

$$P \frac{l}{2} \dot{\theta}_1 = M_u (\dot{\theta}_1 + \dot{\theta}_2) = M_u \cdot 3\dot{\theta}_1$$

$$P_{lim} = 6 \frac{M_u}{l}$$ – estimate got worse

Example:



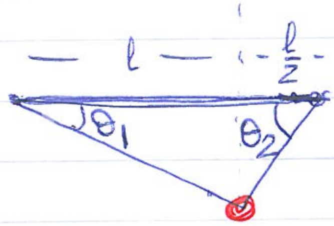
Compare two mechanisms of collapse

(A) In left part:

$$P_l \cdot \frac{l}{2}\dot{\theta} = M_u \cdot 2\dot{\theta}$$

$$\boxed{P_{lim} = \frac{4M_u}{l}}$$

(B) In right part



Geometry:

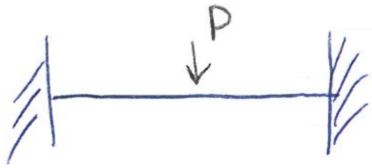
$$l \cdot \theta_1 = \frac{l}{2}\theta_2 \Rightarrow \boxed{\theta_2 = 2\theta_1}$$

$$2P_{lim} \cdot l \cdot \dot{\theta}_1 = M_u(\dot{\theta}_1 + \dot{\theta}_2) = 3M_u \cdot \dot{\theta}_1$$

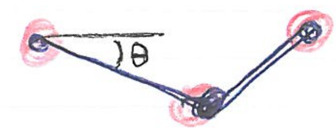
$$\boxed{P_{lim} = \frac{3}{2}\frac{M_u}{l}}$$

— lower — more accurate assessment

what if the beam is built-in?



Add two hinges.



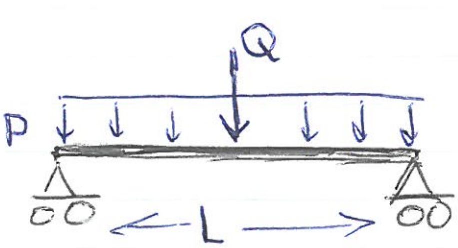
Energy eq-n:

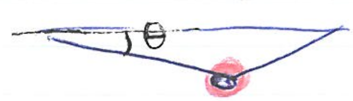
$$P \cdot \frac{\ell}{2}\dot{\theta} = M_u \cdot 2\dot{\theta} + M_u \dot{\theta} + M_u \dot{\theta}$$

$$P_{lim} = 8\frac{M_u}{\ell}$$ – twice as high (compared to simply supported)

[Note: no M-diagram needed!]

What if we have distributed load? (in addition to point loads)

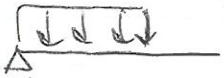




hinge at mid-point

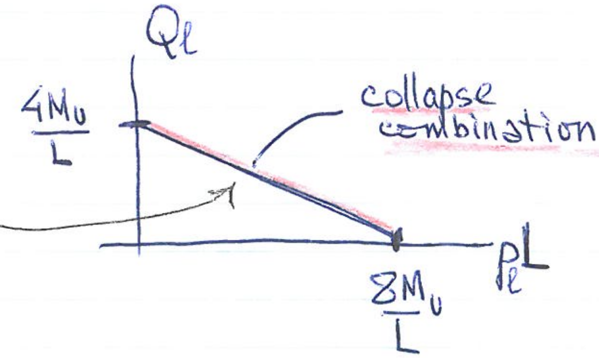
Energy eq: $$p_\ell \dot{\theta}\frac{L^2}{4} + \dot{Q}_\ell \cdot \frac{L}{2}\dot{\theta} = M_u \cdot 2\dot{\theta}$$

from

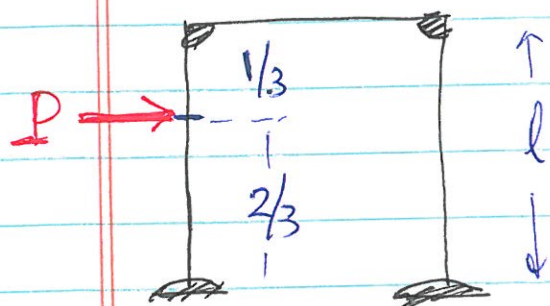


$$2 \cdot \int_0^{L/2} p\, x \dot{\theta}\, dx$$

$$p_\ell L + 2Q_\ell = 8\frac{M_u}{L}$$

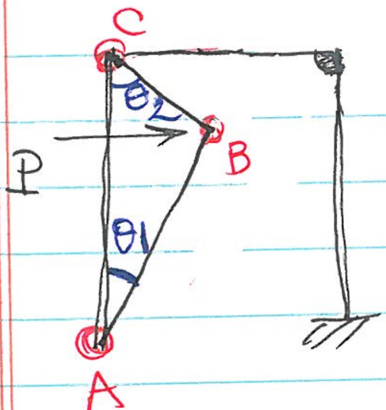


Frame collapse:



Consider two mechanisms:

Ⓐ



$\theta_2 = 2\theta_1$ for small angles (beginning of collapse)

Energy balance at collapse:

$$P \cdot \frac{2}{3} l \dot{\theta}_1 = \underbrace{M_u \cdot (\dot{\theta}_1 + \dot{\theta}_2)}_{B} + \underbrace{M_u \dot{\theta}_2}_{C} + \underbrace{M_u \cdot \dot{\theta}_1}_{A}$$

$$= 6 M_u \dot{\theta}_1$$

$$\Rightarrow P_{lim} = 9 \frac{M_u}{l}$$

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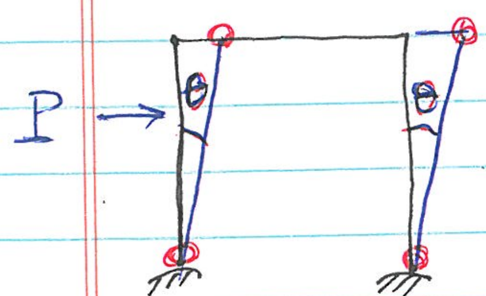

$$P \cdot \frac{2}{3} l \dot{\theta} = 4 \cdot M_u \cdot \dot{\theta}$$

$$\Rightarrow \boxed{P_{lim} = 6 \frac{M_u}{l}}$$ — lower!

This is the actual collapse mechanism!