2. Proposition XVI: force in string at bottom of a 90 deg pendulum is 3 times the weight of the object
    a. For, $v = \sqrt{(2gh)}$ in order to conserve Bv^2
    b. Hence centrifugal tension proportional to 2g at bottom, and addition of gravity gives 3g
    c. Huygens's argument approximates the circle with a parabola to get the precise result at the bottom geometrically, instead of via the algebra above
3. Proposition XVII: a vertical rotating body produces at least 6 times the tension that the weight of the body alone does -- 6 times in order to keep the string taut
    a. To keep the string taut at the top, v^2/r must at least be g
    b. But then velocity at bottom must be large enough to yield v^2/r at top after deceleration while going up
    c. "Sublimity" reasoning, converting velocity to height, then yields the result
4. This last proposition has a striking corollary: the specific height at which to intercept a 90 deg pendulum and complete a circle after the intercept is $(2/5)*\ell$ from the bottom
    a. If intercept any higher, will not complete a circle
    b. A testable result related to Galileo's claims about intercepted pendulums
5. Clearly, Huygens was looking to extend his results on centrifugal forces to the large-arc pendulum in these last propositions, but this was as far as he could get
    a. Probably prompted by the 90 deg pendulum Mersenne used in trying to measure g -- a measurement Huygens had repeated more than once with increasing care, before giving up on it
    b. No theory of the large arc pendulum until math for elliptical integrals in second half of 18th century

E. Empirical Evidence for the Overall Theory

1. Once again Huygens, in the tradition of Galileo, says that the results agree with experiments without giving us any experimental data
    a. Since the mathematical arguments involving infinitesimals raise some doubts, empirical evidence an appropriate concern here
    b. And in fact Huygens had performed a variety of relevant experiments in late 1659 and subsequently
    c. So, the decision not to report results reflects a style
2. Outwardly, the theory is making claims about the tension in a string, which at that time could not be measured to any accuracy at all except in certain cases of static equilibrium
    a. Direct measurements become possible only in late 19th century, with the advent of strain gauges
    b. Measurement via determining breaking points not adequate for strength of string too irregular
3. But Huygens himself had managed to obtain an accurate measure of periods of conical pendulums and hence of an implied value for g

a. Evidence for overall theory via close comparison with independently measured value (979-981 cm/sec/sec) or by using the independently measured value of g to design e.g. a precise 1 sec conical pendulum

b. Care is needed with experimental approaches using the conical pendulum -- but Huygens had shown that care is enough

4. Other Galilean-type opportunities present themselves for testing the overall theory by verifying some of its salient results

   a. E.g. the equilibrium claims about a rotating parabolic conoid, which will also permit further measurements of g if the period can be measured with sufficient precision

   b. The intercepted pendulum results at the end, which are directly about the variable tension in the string (though must be careful to keep resistance effects to a minimum)

   c. Note that here again the paper ends on a comparatively easy to test salient result

5. Again, therefore, Huygens's theory of centrifugal forces is best thought of as a Galilean response to questions raised by Descartes

   a. Notice the extent to which Huygens has incorporated his theory into the context of Galileo's results on free fall, allowing these results to bring supporting evidence for the theory, and allowing evidence for the theory to support Galileo's results

      (1) I.e. by incorporating this theory into the broader context, allowing diverse, hopefully converging evidence for the whole framework

      (2) Same true more narrowly when he ties results for circle to parabolic conoid, conical pendulum, and then to small arc pendulum -- a unified theory admitting of diverse evidence through cross-comparisons and alternative measures of same quantity

   b. Outwardly, paper displays Galilean approach to evidence too, focusing on verification of striking "quasi-quantitative" predictions

   c. But in fact the theory offers much richer evidential approaches via comparison of alternative measures of g, or use of a preferred measure to make testable predictions about e.g. periods

      (1) Thus, to a far greater extent, this theory provides a basis for looking for small systematic discrepancies that might provide added empirical information

      (2) As remarked above, Huygens seems not to have given much notice to these richer possi-bilities; he seems to have been content with tested striking predictions

6. Finally, notice that here, for the first time, obtain substantial empirical evidence for law of inertia

   a. As Huygens's figure (see Appendix) displays, law presupposed in derivation of v^2/r result and hence in every consequence in paper

   b. All evidence for Huygens's theory thus evidence for law of inertia

   c. This may be the only way to provide significant evidence for the law of inertia: embed it crucially in a theory and develop evidence for that theory

d. The experimentally confirmed solution for centrifugal force in uniform circular motion may help explain why the law of inertia became so quickly accepted in some circles

7. The logic of this evidence is worth noting even though Huygens himself said nothing about it

   a. The principle of inertia, contraposed, says that any deviation from inertial motion requires a "force"

   b. Should be able to characterize the magnitude of the force in any given type of deviation -- e.g. express a measure of the force in terms of the motion

   c. Given such a measure, the question is whether it is correct

   d. Verifying by some other independent way that this measure of the force is correct then legitimates the conclusion that a well-defined force is present, and hence provides evidence for the contraposed form of the principle

8. Huygens's failure to emphasize it notwithstanding, this is an archetypal example of a form of evidence that has been central to physics ever since Newton's *Principia*

IV. Huygens on the Center of Oscillation: Real Pendulums

A. Background: Ideal versus Real Pendulums

1. Huygens's *Horologium Oscillatorium* is divided into five parts, the first of which describes the mechanical features of the clock shown in Appendix and a maritime clock

   a. Part II: theory of free fall, inclined planes, and fall along a curved path, culminating in the isochronism result for the cycloidal pendulum (see Appendix)

   b. Part III: the theory of evolutes, including a justification of the device introduced in the clock to maintain its isochronism (see Appendix)

   c. Part IV: the theory of the real, physical pendulum, in contrast to the ideal pendulum of Part II, plus announcement of cycloidal and small-arc pendulum measures of g

   d. Part V: describes a conical pendulum clock, the bob of which is kept on a paraboloid by a curved plate, and indicates that it too can be used to measure g

   e. Appendix: 13 theorems about centrifugal force, without proofs

2. Theory in Part II employs Galilean principles supplemented by what we now call the principle of inertia throughout -- all the Galilean principles listed in notes for class 7

   a. Three hypotheses at the beginning (see Appendix) license parabolic projection if gravity is taken as uniform and acting along parallel lines

   b. Proposition 1: "derivation" of uniform acceleration of gravity question-begging in assuming same action in each unit of time (Huygens was not always careful about rigor in derivations from listed hypotheses)

   c. Proposition 5: proof of mean speed theorem (via reductio ad absurdum)

   d. Propositions 6-9: generalized pathwise independence principle in ascent as well as descent derived (via reductio ad absurdum) from Huygens's Torricellian principle