a. But marked variations within this pattern, and hence different loops from one occasion to another: e.g. 760 days one time, 775 another, etc.

b. Planetary speeds vary too: e.g. roughly 40 percent variation in apparent longitudinal motion per day of Mars from one extreme to another while away from retrograde

4. Each of the five planets has its own distinct basic pattern of periods of retrograde motion, and its own distinct pattern of variations on this basic pattern

a. Can be seen in examples of Mars and Jupiter, where loops vary

b. An anomaly on top of the anomaly of retrograde motion

c. Well before 300 B.C. the Babylonians had discovered "great cycles" in which the patterns of retrograde loops and timings of stationary points repeat: e.g. 71 years for Jupiter (see Appendix for others)

5. The problem of the planets: give an account ('***logos***') of retrograde motion, including basic pattern, size of loops, and variations for each of the five planets

a. Not to predict longitude and latitude every day

b. Focus instead on salient events – i.e. ***phenomena***: conjunctions, oppositions, stationary points, longitudinal distance between them

c. For the Babylonians, just predict; for the Greeks, to give a geometric representation of the constituent motions giving rise to the patterns

6. Classical designations: "***the first inequality***": variation in mean daily angular speed, as in 40 percent variation for Mars and smaller variation for Sun; "***the second inequality***": retrograde motion, as exhibited by the planets, but not the sun and moon

D. Classical Greek Solutions

1. Various classical solutions to the problem, but with epicylic theory coming to dominate for the second inequality in the 3rd century B.C.

a. No evidence of motion of earth, hence reasonable to conclude that retrograde motion arising from motion on a second circle -- i.e. an epicycle, the center of which moves along a circle called the "deferent"

b. Epicycle consistent with planets being brightest during retrograde motion

c. Aristarchus in 3rd century B.C. the one notable exception, who had the earth and the five planets going around the sun

2. In 4th century B.C. Eudoxus had devised a system of nested homocentric spheres in response to the problem -- see Aristotle, *On the Heavens* and quote from *Metaphysics* (Lambda) in the Appendix

a. Basic idea of solid spheres retained in epicycle theory

b. But with spheres rotating on rotating spheres instead of nested homocentric spheres

3. Most of what we know about Eudoxus's solution comes either from Aristotle (or from modern efforts to recreate it on the basis of what Aristotle says

a. Aristotle himself modifies Eudoxus's system for the planets slightly, seemingly to make it more physically tractable

b. While not altering his homocentric sphere models for the sun and moon

4. In his *On the Heavens* Aristotle provides philosophical and quasi-empirical arguments to support key features of Eudoxus's system

   a. The natural motion of the four elements is toward the center of the earth, while the natural motion of the celestial ether is circular

   b. All motions in the heavens have to be (eternal) uniform circular motions insofar as any speeding up and slowing down would require an external cause

   c. The earth is a sphere at the exact center, because of the natural motions toward its center

   d. The earth does not move, so that the apparent diurnal motion of the stars has to arise from motion of the sphere of the fixed stars

   e. The earth is small compared to the stars

5. These doctrines of Aristotle remained influential over the next fourteen centuries, leading to a number of conflicts with Ptolemaic astronomy

   a. Ptolemy did not have the earth at the exact center of the motions of either the sun or any of the planets, but instead at different distances from the center of their motion along the zodiac

   b. In the case of the moon and the planets, Ptolemy openly violated the requirement of uniform circular motion, replacing it with equiangular motion about a point off-center

E. Classical Greek Solutions After Aristotle

1. The two centuries after Aristotle died (322 B.C.) produced four great figures in classical Greek mathematics who continued to have a dominant influence over the next millennium and a half

   a. Euclid, who thrived in Alexandria around 320-280 B.C.: *Elements*, writings on optics

   b. Archimedes of Syracuse: (287-212 B.C.) writings on science that Galileo took as his model

   c. Apollonius of Perga (ca. 262-190 B.C.): *Conics*, but also writings in astronomy no longer extant

   d. Hipparchus of Nicea (ca. 190-120 B.C.): a fully developed, but inadequate epicyclic system that was the starting point for Ptolemy 300 years later; writings no longer extant, so that what we know of his work is from the *Almagest*

2. Greeks early looked for ways to account for anomalies in the motion of the moon and sun -- i.e. deviations from mean motion, for that reason called ***inequalities***

   a. From proof by Apollonius, recognized epicycle and eccenter as equivalent alternatives, allowing sun and moon to be either merely appearing to be moving at different angular speeds at different times or to be engaged in a compound of two uniform circular motions

   b. Willingness to use the two physically distinct but mathematically equivalent devices interchangeably a sign that their primary interest was calculational -- i.e. calculate locations and timing of salient events among the stars