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Key Points:

- Hazard function analysis (HFA) can link return periods and flood magnitudes under nonstationarity
- HFA is an alternative to risk analysis for characterizing properties of nonstationary flood series
- Nonstationary average return periods and reliabilities can be computed using HFA for flood series

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Hazard function analysis for flood planning under nonstationarity

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Abstract The field of hazard function analysis (HFA) involves a probabilistic assessment of the "time to failure" or "return period," T, of an event of interest. HFA is used in epidemiology, manufacturing, medicine, actuarial statistics, reliability engineering, economics, and elsewhere. For a stationary process, the probability distribution function (pdf) of the return period always follows an exponential distribution, the same is not true for nonstationary processes. When the process of interest, X, exhibits nonstationary behavior, HFA can provide a complementary approach to risk analysis with analytical tools particularly useful for hydrological applications. After a general introduction to HFA, we describe a new mathematical linkage between the magnitude of the flood event, X, and its return period, T, for nonstationary processes. We derive the probabilistic properties of T for a nonstationary one-parameter exponential model of X, and then use both Monte-Carlo simulation and HFA to generalize the behavior of T when X arises from a nonstationary two-parameter lognormal distribution. For this case, our findings suggest that a two-parameter Weibull distribution provides a reasonable approximation for the pdf of T. We document how HFA can provide an alternative approach to characterize the probabilistic properties of both nonstationary flood series and the resulting pdf of T.

1. Introduction

Many disciplines are concerned with the time to arrival of a certain magnitude event in excess of a design threshold, which results in a system failure. Depending on the field of application, the time to arrival is often termed the return period, time to failure, or survival time, just to name a few. The analysis of this variable is termed "hazard function analysis" (HFA), and has roots in many fields. HFA is used to determine the onset or relapse of a disease in the bio-statistics, the time until a person becomes employed in economics, the time until a device fails in reliability engineering, and the time to death in actuarial science, among many other fields and uses [*Klembaum*, 1996; *Klein and Moeschberger*, 1997; *Tung et al*., 2006; *Kottegoda and Rosso*, 2008; *Cleves*, 2008; *Finkelstein*, 2008; *Lawless*, 2011]. HFA comprises a well-known set of tools for characterizing the probability distribution function (pdf) of the return period, T, associated with a specific event or process over the course of a time period of interest. Importantly, in nonstationary cases (i.e., where the event likelihood changes through time), HFA can represent T and its distribution. Using HFA to improve our understanding of the probability distribution of T is important because design standards in reliability engineering and manufacturing, as well as policies for clinical trials in public health, are based on the expected time until the failure of a piece of equipment, or even the end of a person's life after diagnosis of a disease.

In hydrology, we are concerned with the return period T of a flood event which exceeds the capacity of current flood prevention systems, and the corresponding reliability that such an event will not exceed the design capacity over a future planning horizon. Flood series are known to exhibit nonstationary behavior due to changes in land use, climate, water infrastructure, and other factors [*Milly et al*., 2008, 2015]. We apply HFA to the return period of flood events, where a failure is defined as a flood event X which exceeds a design flood magnitude x_o. Our goals are (1) to show that HFA can provide a mathematical linkage between the probabilistic properties of X and T for nonstationary processes and (2) to illustrate how HFA can be used to generalize and characterize our understanding of the probabilistic properties of the return period and reliability under nonstationary conditions. The paper is organized as follows: section 2 provides a brief summary of stationary and nonstationary flood frequency analysis; section 3 presents an overview of HFA theory and applications; section 4 details the derivation of stationary and nonstationary Exponential models of flood peaks using HFA; similarly, section 5 evaluates a nonstationary lognormal model of flood peaks using HFA and documents how the Weibull HFA model can represent the distribution of T for this case; and section 6 provides concluding remarks.

2. Summary of Stationary and Nonstationary Flood Frequency Analysis

Stationarity implies that flood observations (e.g., annual maxima), X, are identically distributed with cumulative distribution function (cdf), $F_X(x;\ \theta)$, where θ is a vector of parameters that does not change in time. Under stationary conditions, the design event x_o determines the capacity of the flood control project corresponding to a fixed exceedance probability associated with x_o, equal to $p = 1 - F_X(x_o;\ \theta)$ so that $x_o = F_x^{-1}[1-p, \theta]$. For the stationary case, p is constant over time and the return period, T, follows an exponential (continuous) or geometric (discrete) distribution with average return period equal to $1/p$ (for example, see *Douglas et al*. [2002] for derivation). This is an important result, because under stationary conditions, the distribution of T is always exponential, regardless of the pdf of the process X, of interest.

Under nonstationary conditions, the parameters θ of the distribution of X are not constant, and the nonstationary cdf is given as $F_X(x;\ \theta(t))$. Now the exceedance probability associated with the design event x_o is a function of time and is given by $p_t = 1 - F_X(x_o;\ \theta(t))$. Thus, under nonstationary conditions, the risk of failure and associated reliability change accordingly (see *Olsen et al*. [1998], *Salas and Obeysekera* [2014], and *Cooley* [2013] for mathematical details). In a review of developments in nonstationary flood frequency analysis, *Bayazit* [2015] notes a recent increase in the number of articles on this subject. Despite this increasing attention, fundamental questions still remain concerning whether or not nonstationary methods are needed [see e.g., *Cohn and Lins*, 2005; *Lins and Cohn*, 2011; *Matalas*, 2012; *Montanari and Koutsoyiannis*, 2014; *Serinaldi and Kilsby*, 2015]; and further, the question of how to select an appropriate design event given evidence of nonstationarity and future uncertainty [*Obeysekera and Salas*, 2013; *Salas and Obeysekera*, 2014; *Read and Vogel*, 2015; *Rootzén and Katz*, 2013; *Stedinger and Griffis*, 2011]. Since the work of *Cohn and Lins* [2005], one must always question whether or not a deterministic trend can be distinguished from stochastic persistence.

Interestingly, in spite of the tremendous attention given to methods for characterizing nonstationary flood frequency distributions associated with flood discharges, X, relatively little attention has been given to understand the properties associated with the return period, T, or reliability indices associated with nonstationary hydrologic variables X. We do not attempt to summarize the myriad of recent papers which introduce nonstationary probabilistic models of X (see *Bayazit* [2015] for a recent review). Instead, we focus our attention here on the few studies which have considered the probabilistic properties of T under nonstationary conditions.

Obeysekera and Salas [2013] and *Salas and Obeysekera* [2014] linked the probabilistic properties of X and T using the theory introduced by *Olsen et al*. [1998], *Cooley*, [2009], *Wigley* [2009, reprinted], and others for a few particular cases. Similarly, *Read and Vogel* [2015] used the theory introduced by *Olsen et al*. [1998] and others, along with Monte-Carlo simulations to link the probabilistic properties of X and T for a wide class of systems characterized by a nonstationary lognormal (LN2) model which was found to be representative of actual flood series across the continents of the U.S. [*Vogel et al*., 2011] and the U.K. [*Prosdocimi et al*., 2014]. By citing those studies, we are not promoting the application of nonstationary methods, rather, we are only providing support and evidence for the goodness of fit of that particular nonstationary LN2 model evaluated at thousands of rivers on two separate continents. It may be important to consider the consequences of nonstationarity as illustrated by *Read and Vogel* [2015] who showed that even a small degree of nonstationarity associated with X can lead to extremely complex shapes in the corresponding pdf of T. This study extends the work of *Read and Vogel* [2015] by exploiting HFA instead of the approach of *Olsen et al*. [1998], to generalize our understanding of the impact of nonstationary behavior in X on the probabilistic properties of T. As we discuss in the next section, HFA has been applied to water resources problems to describe the probabilistic properties of T [see e.g., *Tung and Mays*, 1980; *Tung and Mays*, 1981; *Lee and Mays*, 1983; *Plate and Duckstein*, 1987; *Mays and Tung*, 1992]. Nevertheless, this is the first study we are aware of that uses HFA to link the probabilistic properties of X and T.

Table 1. Cross-Disciplinary Examples of HFA: Hazard Function Definitions and Applications

Field	Definition	Example
Manufacturing	Conditional failure rate	Parts wearing out in a machine
Epidemiology	Age-specific failure rate	Number of people in specific age group contracting a disease
Actuary statistics	Force of mortality	Likelihood of dying at a particular age
Reliability engineering	Failure rate	Failure of electronic devices

3. Hazard Function Definitions, Theory, and Applications

Time to failure analysis is a branch of HFA that deals with the length of time T that a system remains operational until experiencing a failure (i.e., exceedance) event. In reliability engineering, such an analysis provides an approach for combining the various factors which tend to cause failures (e.g., environmental and operational deterioration) into a single random variable, T [e.g., *Tung and Mays*, 1980; *Mays*, 1996; *Tung et al.*, 2006; *Kottegoda and Rosso*, 2008]. Table 1 summarizes definitions and applications of the hazard rate function (defined below) in fields that commonly apply HFA, also widely known as survival analysis. Extensive descriptions of survival analysis can be found in monographs on the subject [*Klembaum*, 1996; *Klein and Moeschberger*, 1997; *Cleves*, 2008; *Lawless*, 2011; *Finkelstein*, 2008] and textbook chapters discussing HFA in broader contexts such as probability [*Bean*, 2001], applied statistics [*Kottegoda and Rosso*, 2008], and reliability engineering [*Mays and Tung*, 1992; *Tung et al.*, 2006; *Mays*, 1996]. Nonparametric and semiparametric methodologies in survival analysis, not explored here, are also commonly applied [see *Kaplan and Meier*, 1958; *Cox*, 1972; *Klembaum*, 1996].

The hazard function, or failure rate function $h(t)$, is central to HFA and is defined as the probability that a failure event occurs in a given time interval $(t, t+\Delta t)$ [e.g., *Tung et al.*, 2006]:

$$h(t)=\frac{f_T(t)}{1-F_T(t)}=-\frac{\frac{dS_T(t)}{dt}}{S_T(t)} \tag{1}$$

where $h(t)$ is in units of failures/time, f_T is the pdf of the return period, or time to failure T, F_T is the cdf of T, and $S_T = 1 - F_T$ is the survival function of T, also known as the reliability function, which represents the probability of no failure in the time interval $(0, t]$, given that failure has not yet occurred prior to t.

From the definition in (1), it follows that the cumulative hazard function $H(t)$, which represents the total number of failures over a specified time interval [*Cleves*, 2008], is given as

$$H(t)=\int_0^t h(s)ds=\int_0^t \frac{dS_T(s)}{S_T(s)}ds=-\ln\left(S_T(t)\right) \tag{2}$$

The survival function of the return period T can be rewritten from (2):

$$S_T(t)=\exp\left(-\int_0^t h(s)ds\right)=\exp\left(-H(t)\right) \tag{3}$$

For stationary independent and identically distributed processes, the hazard function is constant as is shown below, and the return period T associated with the design event x_o follows an exponential (geometric) pdf for the continuous (discrete) case, regardless of the form of $F_X(x;\theta)$ [see *Gumbel*, 1941; *Thomas*, 1948; *Douglas et al.*, 2002; *Volpi et al.* 2015]. See *Douglas et al.* [2002] for a discussion of how the assumption of independence impacts the probabilistic properties of T.

The hazard function for the one-parameter exponential (EXP1) case with rate parameter λ can be derived from (1) as:

$$h(t)=\frac{f_T(t)}{1-F_T(t)}=-\frac{\lambda\exp\left(-\lambda t\right)}{\exp\left(-\lambda t\right)}=\lambda \tag{4}$$

The constant hazard rate in (4) reflects the constant exceedance probability p_o, associated with the design event x_o, so that $h(t)=\lambda=1-F_X(x_o,\theta)=p_o$.

Importantly, the hazard function defined in (1) is not necessarily a probability distribution function and is usually only constrained to be nonnegative, $h(t) \geq 0$; it may be increasing or decreasing, nonmonotonic, or discontinuous [*Klein and Moeschberger*, 1997]; however, we can interpret $h(t)$ as the conditional probability of failure in an infinitesimally small time period between t and $(t+\Delta t)$ given that the system has survived until time t. In this sense, the hazard function is a measure of risk: the greater the hazard between t and $(t+\Delta t)$, the greater the risk of failure in this time interval. This probabilistic interpretation of the hazard function sets our work apart from all previous HFA research as discussed below.

To date, all previous applications of HFA that we are aware of begin with an assumed form of the hazard function $h(t)$ and do not derive its form from first principles as we do. Normally, $h(t)$ is introduced as a function that attempts to characterize such nonstationary processes as the deterioration of infrastructure [*Mays and Tung*, 1992], urban transportation [*Hensher and Mannering*, 1994], the remaining "survival" life of a device or human being [*Finkelstein*, 2008; *Lawless*, 2011], time of relapse to a disease [*Klein and Moeschberger*, 1997], or duration of economic events [*Kiefer*, 1988]. In such parametric applications of HFA, the user assumes that $h(t)$ takes a certain shape (e.g., "bathtub," increasing, and decreasing), which reflects ones' intuition and/or experience about the nature of changes in the exceedance probability of a certain hazard over time [*Wienke*, 2010]. We are unaware of any previous work using HFA, other than our companion paper [*Read and Vogel*, 2016] which derives the hazard function from assumptions regarding the probabilistic properties of the original variable of interest X, as is our goal. Consider a time varying model for floods, $F_X(x; \theta(t))$, in which floods are independent but not necessarily identically distributed (i/nid). Let us also restrict ourselves to an increasing trend in the mean of the annual maximum series (AMS) [*Vogel et al*., 2011; *Prosdocimi et al*., 2014]. Now the exceedance probability associated with the design event changes as a function of time, p_t, and thus the expected waiting time until a flood occurs is no longer simply $1/p$. *Tung and Mays* [1980, 1981] first considered this case and introduced the idea of dynamic (or time varying) reliability models to address the issue of modeling risk and reliability under such nonstationary conditions, i.e., with loading as a random independent variable that changes over time and represents a composite risk of failure that may be a combination of multiple stresses on the system. The objective of *Tung and Mays* [1980, 1981] and *Lee and Mays* [1983] was to derive a model for the dynamic reliability of a system over a planning period where the distribution of loads that cause system failure change over time [*Lee and Mays*, 1983]. Our work extends their work by connecting the concept of dynamic reliability modeling with HFA for the purposes of flood planning.

We distinguish our work from others by defining $h(t)$ as the exceedance probability of a failure, defined as the exceedance of an event X in excess of the design capacity x_o, in the time interval $(t, t+\Delta t)$. Thus, we estimate $h(t)$ from the probabilistic properties of the random variable of interest, which here is the annual maximum flood discharge X, and the design capacity x_o of the system. We relate $h(t)$ with the cdf associated with x_o, given by $F_X(x_o; \theta(t))$, so that:

$$h(t)=1-F_x(x_o, \theta(t))=p_t \quad (5)$$

where x_o reflects the chosen design capacity of the system. Our primary goal is to use the hazard function $h(t)$ to deduce the survival function from the probability distribution of the waiting times for the next exceedance flood event $X > x_o$ for i/nid systems given the assumption that $h(t) = p_t = 1 - F_x(x_o, \theta(t))$ analogous to the stationary EXP1 case shown in (4).

To ensure validity of the assumption in (5), we compare our HFA approach for defining the behavior of return periods for nonstationary flood series, with existing approaches introduced by *Olsen et al*. [1998] and others. The approach of *Olsen et al*. [1998] relies on knowledge of p_t, defined as the annual probability of exceeding a design flood. In cases for which p_t is assumed to be continuously increasing every year, the average return period is:

$$T_1=E(T)=1+\sum_{t=1}^{t_{\max}}\prod_{i=1}^{t}(1-p_i) \quad (6)$$

and t_{max} represents the year in which the exceedance probability is equal to one [see *Olsen et al*., 1998; *Cooley*, 2013; *Salas and Obeysekera*, 2014; *Read and Vogel*, 2015]. In the following sections, we compare results of HFA for a few simple cases with the results given in (6).

3.1. Survival and Hazard Function Analysis in the Water Resources Literature

Tung et al. [2006] and *Mays* [1996] provide detailed descriptions of possible hazard functions in the context of reliability of hydraulic infrastructure. Despite the applicability of HFA to hydrologic challenges associated with infrastructure design and planning under nonstationarity, few hydrologic studies even mention HFA in this context [*Katz and Brown*, 1992; *Wang et al*., 2010; *Zhong and Hunt*, 2010]. *Lee et al*. [1986] applied HFA to the problem of multiyear drought durations to identify the logistic model for describing the shape of the hazard rate function.

Much of the literature on hydrologic applications of HFA employs a set of proportional hazard (PH) regression models first introduced by *Cox* [1972]. Such models differ significantly from our work because they require survival data, information often not available in the context of water resource applications. Such PH models are useful when survival data are available because they enable one to employ multivariate regression to relate covariates to failure processes. Cox PH models differ from the parametric models used here because they focus on the probabilistic properties of T alone and do not assume underlying knowledge about the functional form of the hazard function or the distributional properties of the hazard variable X. PH models have been widely applied in the bio-statistics literature and are useful in determining whether covariates (stationary or time varying) influence the probability of occurrence of events (i.e., does climate influence the occurrence of floods, droughts, etc.) [*Klein and Moeschberger*, 1997]. Applications of PH models in the water resources literature include characterizing flood risk [*Futter and Mawdsley*, 1991], climate variation [*Maia and Meinke*, 2010], and changes in flood behavior by the peaks over threshold method [*Smith and Karr*, 1986; *Villarini et al*., 2012], based on covariates other than time. Literature on "trend attribution" in hydrology has applied PH models to identify mechanisms for changes in peak flood regimes [*Cunderlik and Burn*, 2004; *Villarini et al*., 2012], though more research is needed in this area [*Merz et al*., 2012]. A critical limitation of the use of PH models in hydrology is the lack of a linkage between the probabilistic properties of the flood process of interest X and the properties of the survival distribution of T, which is the focus here.

In the following sections, we employ HFA for nonstationary flood series arising from two representative cases for flood systems: (1) a nonstationary one-parameter exponential (EXP1) distribution and (2) a more realistic case considering a nonstationary lognormal (LN2) model. For each case, we provide a derivation of $h(t)$, $S_T(t)$, and $H(t)$ and use these tools to compute design metrics of interest such as the average return period and reliability. Our critical assumption in equation (5) is verified in two ways: by comparing results from the HFA approach with that of *Olsen et al*. [1998] and others in equation (6) for both cases, and through Monte-Carlo experiments in which we generate nonstationary flood series, and then compare sequences of exceedance probabilities associated with a particular design event, for a wide range of flood cases with $h(t)$ derived using the HFA equations. Since those Monte-Carlo evaluations resulted in exact agreement with the analytical results, for all cases considered, we do not graphically illustrate those results here.

4. Hazard Function Analysis for Stationary and Nonstationary Exponential Flood Peaks

The exponential distribution (EXP1) is widely used in the peaks-over-threshold (also called partial-duration-series or PDS) method for characterizing the magnitudes of flood exceedances above some set level [*Stedinger et al*., 1993]. For example, *Stedinger et al*. [1993] document that if the number of PDS flood arrivals follow a Poisson process and their magnitudes follow an exponential distribution, then the series of AMS flood magnitudes will follow a Gumbel distribution. Thus, this analysis corresponds to situations in which the Gumbel pdf provides a good approximation to AMS. Note that until recently, the Gumbel pdf was used as a standard in many countries [see *Vogel and Wilson*, 1996, Table 3].

In this section, we derive general results corresponding to HFA for the case when PDS flood magnitudes follow an EXP1 distribution. This initial analysis enables us to demonstrate how the probabilistic properties of PDS flood magnitudes X, and an associated design event x_o, relate to the probabilistic properties of their failure times, T, for a relatively realistic underlying model of flood magnitudes. The following compares the application of HFA to both stationary and nonstationary EXP1 series. The pdf and cdf of an EXP1 random variable, X, representing the PDS flood magnitudes above some threshold are given by

$$f_x(x)=\lambda \exp(-\lambda x) \quad (7)$$

$$F_x(x)=1-\exp(-\lambda x) \quad (8)$$

where λ is the rate parameter, with E[X] = μ_x = 1/λ. Consider $p_o = 1 - F_x(x)$, the fixed exceedance probability associated with design event x_o given by the quantile function obtained from (8):

$$X_o=-\frac{1}{\lambda}\ln(p_o) \quad (9)$$

Recall that since $h(t) = p$ from equation (5), the survival function is given by substitution of $h(t) = p$ into equation (3), resulting in the survival distribution

$$S_T(t)=1-F_T(t)=\exp\left(-\int_0^t p ds\right)=\exp(-p\cdot t) \quad (10)$$

Similarly, since $f_T(t) = dF_T(t)/dt$, we obtain the pdf of the time to failure T, as an exponential distribution with parameter p, so that the average time to failure is E[T] = 1/p for this simple stationary case and we have verified $h(t) = p$ for a hazard following a EXP1 under stationary conditions (as in equation (4)).

Now consider the nonstationary case in which the random variable X increases with time t due to a trend in the mean, represented by the following exponential trend model:

$$\mu_x(t)=\frac{1}{\lambda}\exp(\beta\cdot t) \quad (11)$$

Note that for no trend, $\beta = 0$ and the nonstationary mean reduces to the stationary mean $\mu_x=1/\lambda$. *Vogel et al*. [2011] and *Prosdocimi et al*. [2014] found that the exponential trend model in (11) provides an excellent representation to thousands of actual flood series in both the U.S. and the UK.

Although the trend parameter β in (11) denotes the magnitude of the flood trend, this parameter is difficult to physically interpret. Instead, *Vogel et al*. [2011] and *Prosdocimi et al*. [2014] define the more easily understood flood magnification factor M as the ratio of the flood magnitude in year ($t + d$) to the flood magnitude in year t (where d is a specified number of years, e.g., $d = 10$ years for a decadal M). Combining the quantile function in (9) under stationary conditions with the model for the nonstationary mean in (11) leads to an expression for M for a nonstationary EXP1 variate:

$$M=\frac{x_p(t+d)}{x_p(t)}=\frac{\frac{1}{\lambda}\exp[\beta(t+d)\ln(1-p_t)]}{\frac{1}{\lambda}\exp[\beta\cdot t\ln(1-p_t)]}=\exp[\beta\cdot d] \quad (12)$$

which is identical to the magnification factor derived by *Vogel et al*. [2011] for a nonstationary LN2 variable. The cdf of a nonstationary EXP1 variable is obtained by inserting (11) into (8) and replacing β with M given in (12) leading to:

$$F_x(x,t)=1-\exp(-\lambda x\cdot M^{-t/d}) \quad (13)$$

If the design event x_o is based on conditions at $t = 0$, then $p = p_o$ and the design event is fixed so that combining (9) and (13) leads to an expression for the hazard rate function $h(t) = p_t$ as:

$$h(t)=1-F_x(x_o,t)={p_o}^{M^{-t/d}} \quad (14)$$

After fixing the design event, the random variable of interest is now T, the time to failure, with t as its realization. The cumulative hazard function $H(t)$ and survival function $S_T(t)$ are obtained by inserting (14) into the relationships in (2) and (3), respectively:

$$H(t)=\int_0^t {p_o}^{M^{-s/d}} ds \quad (15)$$

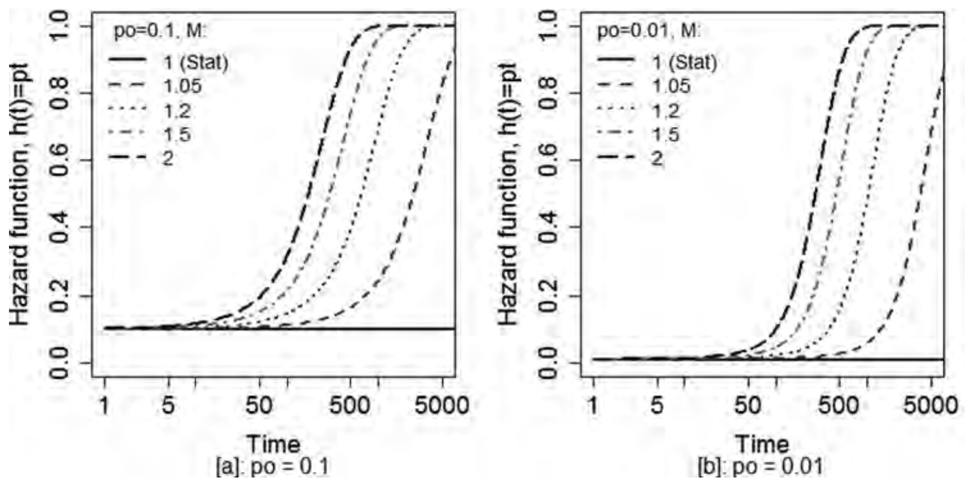

Figure 1. Hazard rate function $h(t) = p_t$ corresponding to flood series which arise from a nonstationary EXP1 model; lines represent trends parameterized by a range of decadal magnification factors ($M = 1$, 1.05, 1.2, 1.5, and 2) for two possible event sizes: (a) $p_o = 0.1$ and (b) $p_o = 0.01$. Note the log scale for the x axis (time).

$$S_T(t) = \exp\left[-\int_0^t p_o^{M^{-s/d}} ds\right] \tag{16}$$

which can both be solved using numerical integration. The pdf of T is computed from the cdf by $f_T(t) = \frac{d}{dt} F_T(t)$ which leads to the following expression, which may be solved numerically:

$$f_T(t) = \frac{d}{dt}\left[\exp\left(\int_0^t p_o^{M^{-s/d}} ds\right)\right] \tag{17}$$

Above we have shown how HFA can be used to relate the probabilistic properties of an EXP1 flood series X combined with a fixed design event x_o, to the properties of the distribution of the time to failure T, associated with the resulting design event. Figure 1 illustrates the hazard function computed from (14) for a set of trends parameterized by decadal (d =10 years in equation (12)) magnification factors ($M = 1$, 1.05, 1.2, 1.5, and 2) assuming (a) $p_o = 0.1$ and (b) $p_o = 0.01$. We note from Figure 1 that the hazard rate function for the nonstationary EXP1 model is no longer constant through time as it was under stationary conditions; and, that greater trends are associated with more accelerated hazard rates. When $h(t)$ approaches unity, the likelihood of no failure approaches zero. While $h(t)$ is an important mathematical function in hazard analysis, and serves as the linkage between the probabilistic properties of the time to failure, T, and the properties of the flood series X, for the purposes of flood planning and risk communication, the survival function, cdf, and pdf of T are more useful tools in practice.

The survival function $S_T(t)$ given in equation (16) and shown in Figure 2 assumes $p_o = 0.01$ for a range of trends (M). The time to failure distribution is clearly impacted by the presence of an increasing trend, evidenced by the departure from the classic exponential survival curve corresponding to a stationary EXP1 model in Figure 2. Realizations of $S_T(t)$ yield important information about the distribution of the time to an exceedance event, or the reliability of a system, and how trends impact this experience. For example, in Figure 2, the reliability (probability of no failure) over a 50 year project life is $S_T(t) = 0.61$ assuming stationarity ($M = 1$) and decreases to $S_T(t) = 0.33$ for $M = 2$.

The cumulative hazard function $H(t)$ given in equation (15) and plotted in Figure 3 provides a way to interpret the total hazard through time. Note that as expected for the stationary case, $H(t) = 1$ at $t = 100$ indicating that only one event is expected within this time period. As the magnitude of the trend increases, the amount of time it takes to experience an event (or magnitude above some threshold) decreases; for example, Figure 3 illustrates that with $M = 1.5$, the $p_o = 0.01$ event may now occur twice in 100 time periods (or once in 60 periods).

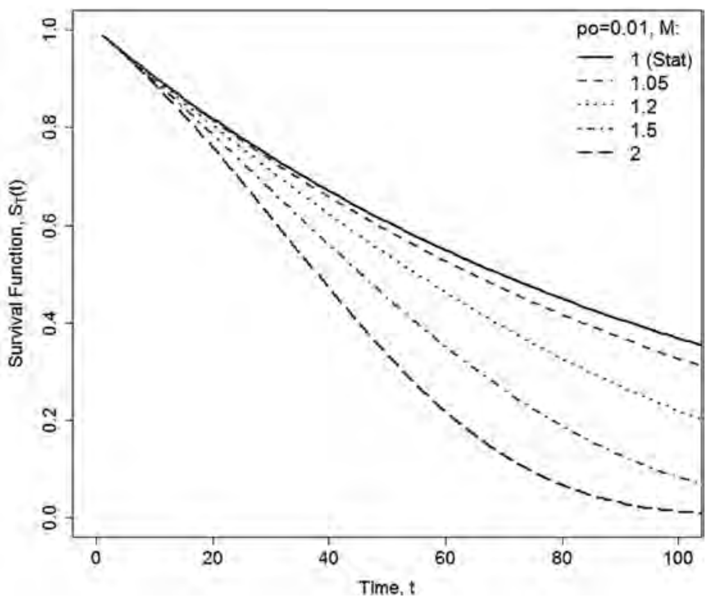

Figure 2. Survival function, $S_T(t)$ (reliability) corresponding to flood series which arise from a nonstationary (EXP1) model with $p_o = 0.01$; lines represent trends parameterized by a range of decadal magnification factors ($M = 1$, 1.05, 1.2, 1.5, and 2).

Figures 4a–4c use equation (17) to illustrate the impact of trends on the shape of the distribution of the return period. Figure 4 documents that the distribution of the failure times change in very complex ways, e.g., the T distribution for a smaller event ($p_o = 0.1$) is less impacted than a larger event ($p_o = 0.001$) for the same magnitude trend.

Overall, we have shown how HFA corresponding to a nonstationary EXP1 model of flood magnitudes can be applied using the functions $h(t)$, $S_T(t)$, and $H(t)$, and how this analysis can provide a window into how the shape of the distribution of T and other probabilistic properties change due to trends. Our results for nonstationary EXP1 flood series show analogous patterns as in the findings of *Read and Vogel* [2015] who studied the change in shape of the pdf of the return period for a nonstationary LN2 model using completely different methods of analysis. Most importantly, analogous to the recent findings of *Read and Vogel* [2015], Figure 4 illustrates that the distribution of the return period under modest nonstationarity no longer takes on a simple exponential shape, so that the average return period may no longer be a "sufficient" summary statistic. We expect that further research of other realistic nonstationary frequency models using HFA may produce similar findings.

5. Hazard Function Analysis for Nonstationary Two-Parameter Lognormal (LN2) Floods

5.1. Preliminary Remarks

In the previous section, we derived general results for a nonstationary exponential model of flood series which provides a good representation of the behavior of PDS of floods. Here we consider the AMS of floods, and thus a more complex pdf of the flood series is needed. We employ the nonstationary LN2 model introduced by *Vogel et al.* [2011] and *Prosdocimi et al.* [2014] based on evidence that the LN2 distribution is a suitable approximation for representing the pdf of AMS flows [*Vogel and Wilson*, 1996; *IACWD*, 1982; *Villarini et al.*, 2009] and that a log linear (exponential) trend model is simple and effective for approximating a change in the mean of the logarithms of the flows through time for thousands of rivers in the U.S. and the UK, particularly, in urbanizing areas. The goal of the following experiments is to use Monte-Carlo simulation to generate equally likely traces of AMS arising from a nonstationary LN2 model to: (1) examine the probabilistic properties of the return period, (2) confirm our fundamental assumption in equation (5), and (3) apply goodness-of-fit measures to select a suitable probability distribution for approximation of the survival function associated with the return period. This approach differs from that of others who have sought a distribution for representing $h(t)$ in that here we use (5) to derive $h(t)$ from properties of the nonstationary LN2 model and the associated design event, which in turn enables us to use (1) to find a parametric distribution to represent the survival function $S_T(t)$.

Read and Vogel [2015] provide details on steps to derive an expression for p_t for a LN2 random variable assuming the log linear trend model $y_t = \ln(x_t) = \alpha + \beta \cdot t + \varepsilon_t$, where an ordinary least squares (OLS) regression yields estimates of the model parameters α and β for the model of the conditional mean of y given by $\mu_{y|t} = \alpha + \beta \cdot t$. Note that this nonstationary trend model implies a proportional reduction in σ_y^2 compared

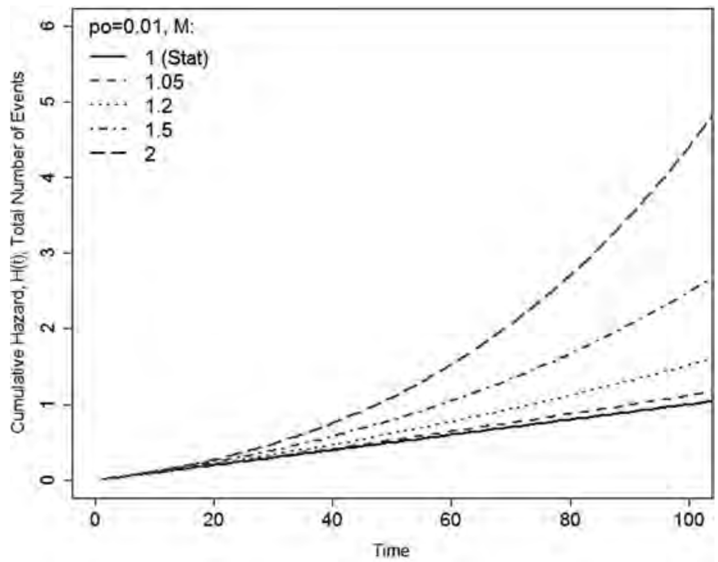

Figure 3. Cumulative hazard function, $H(t)$ corresponding to PDS flood series which arise from a nonstationary EXP1 distribution with $p_o = 0.01$; lines represent trends parameterized for a range of magnification factors ($M = 1$, 1.05, 1.2, 1.5, and 2).

with stationary conditions that depend on the magnitude of the trend. *Read and Vogel* [2015] derive a formula for computing the conditional coefficient of variation $C_{x|t}$ of the nonstationary series X given by,

$$C_{x|t}=\sqrt{(C_x{}^2+1)^{(1-\rho^2)}-1} \quad (18)$$

where C_x is the unconditional or historical coefficient of variation of x, ρ is the Pearson correlation coefficient defined as $\rho=\beta\sigma_t/\sigma_y$ which measures the strength of the linear relationship between the flood series, $Y = ln(X)$, and the covariate time, t. As we stated earlier, we do not recommend the use of the covariate time for predicting trends in floods, instead in practice, a suitable covariate which has been shown to influence physical flood processes should be used. Note the two extreme cases of no trend, in which case (18) reduces to $C_{x|t} = C_x$, and a perfect trend model with $\rho = 1$, which leads to $C_{x|t} = 0$.

Again, we employ the decadal magnification factor introduced by *Vogel et al*. [2011] to reparameterize the slope term β into a value with physical meaning. Substitution of the log linear trend model $\mu_{y|t}$ into the cdf for a LN2 distribution yields an expression for the exceedance probability in year t, p_t, associated with the fixed design discharge, x_o, selected at the beginning of the planning period:

$$p_t=1-\Phi\left[\frac{\ln(x_o)-\mu_{y|t}}{\sigma_{y|t}}\right] \quad (19)$$

where $\sigma_{y|t}=\sqrt{\ln(1+C_{x|t}^2)}$ by definition and Φ denotes the CDF for a standard normal variable.

5.2. Characterization of Nonstationary LN2 Model Using HFA Principles

In the previous case considered for a nonstationary EXP1 PDS flood series, we were able to write theoretical expressions for the pdf, cdf, and survival functions of T, whereas in this case of nonstationary LN2 AMS series of floods, that is not possible. Instead, we resort to an alternative approach based on Monte-Carlo

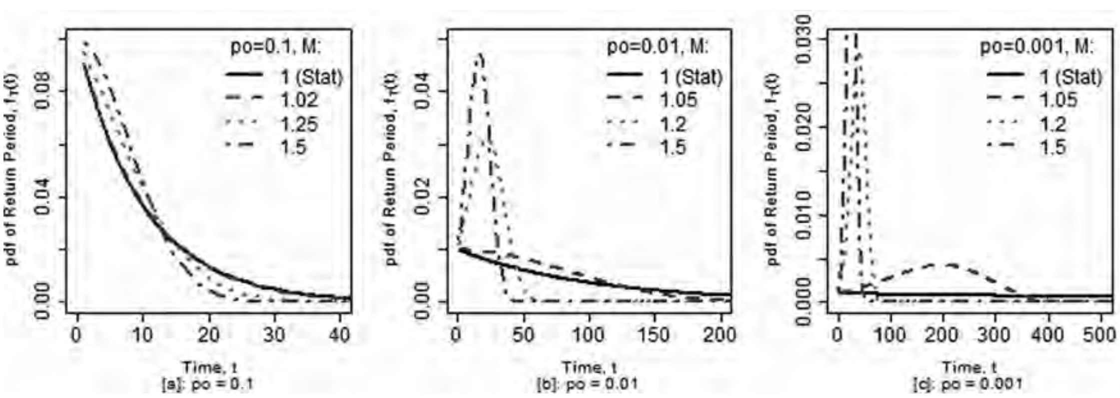

Figure 4. The pdf of the distribution of the return period T corresponding to a nonstationary EXP1 model; each figure shows a range of possible trend values considering a range of magnification factors ($M = 1$, 1.02, 1.25, and 1.5) and plots are per event size: (a) $p_o = 0.1$, (b) $p_o = 0.01$, and (c) $p_o = 0.001$.

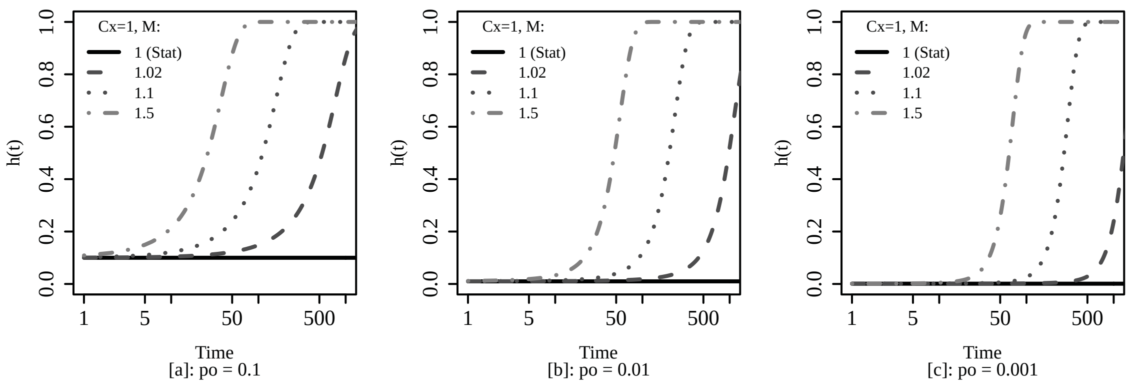

Figure 5. Hazard function $h(t) = p_t$ corresponding to nonstationary LN2 flood series with $C_x = 1$; dotted and dashed lines represent increasing trends ($M = 1.01$, 1.05, 1.1, and 1.5) from stationary ($M = 1$ solid black) illustrating evolution in each plot for different sized events: (a) $p_o = 0.1$, (b) $p_o = 0.01$, and (c) $p_o = 0.001$.

experiments and goodness-of-fit evaluations of various alternative models of the pdf of *T*. Monte-Carlo simulation is used to generate a large number of failure times, along with their associated average return periods, corresponding to a wide range of nonstationary LN2 AMS. The goodness of fit of several alternative distributions are then evaluated in an effort to choose a reasonable probability distribution to represent both the survival function $S_T(t)$ as well as the mean survival times (return periods).

Our experiments proceed as follows: by varying the exceedance probability (p_o) and associated design event at time zero (x_o), the magnification factor (*M*), and the coefficient of variation (C_x) independently, we generated 100,000 sets of 1000 year traces of floods events using the nonstationary LN2 quantile function:

$$x_{p|t} = \exp\left[\mu_y + \beta(t-\mu_t) + z_{p_t}\sigma_y\sqrt{1-\rho^2}\right] \tag{20}$$

where μ_y and μ_t denote the mean values of $y = \ln(X)$ and *T*, and z_{p_t} is the standard normal variate randomly generated by sampling the exceedance probability p_o from a uniform distribution U(0,1); β is derived from the decadal magnification factor, $M = \exp(10\beta)$, and the standard deviation of $y = ln(X)$ is equal to $\sigma_y = \sqrt{\ln(1+C_x^2)}$ if $\rho = 0$ and $\sigma_{y|t}$ otherwise. For each trace, the time to failure, *T*, or the time at which the flow exceeded the design event, was recorded, producing a simulated realization of the return period *T*. With 100,000 realizations of *T*, various pdfs could be considered for approximating the distribution of the return period as described below.

Figure 5 shows that, as expected and similar to the EXP1 model, the hazard rate for the LN2 model is constant under stationary conditions (equal to p_o when $M = 1$), and increases rapidly toward unity as the trend in the mean increases. The plots in Figure 5 illustrate $h(t)$ for three events ($p_o = 0.1$, 0.01, and 0.001), with $C_x = 1$ and a range of trends. The shape of $h(t)$ informs our search for a probability distribution to approximate the distribution of the survival times corresponding to a nonstationary LN2 model, pointing toward one that can accommodate increasing hazards.

We employed the widely used probability plot correlation coefficient (PPCC) to assess the goodness of fit of several two-parameter and three-parameter probability distributions to approximate the probability distribution of failure times corresponding to our simulations based on nonstationary LN2 flood series [see *Heo et al.*, 2008; *Vogel and Kroll*, 1989; *Stedinger et al.*, 1993; *Vogel et al.*, 2011]. With the full range of realizations of *M*, C_x, and p_o described in the experimental design, Figure 6 illustrates the distribution of PPCCs associated with several candidate distributions of the failure times. PPCC values closer to unity indicate greater confidence that the data arise from the hypothesized distribution. As evidenced by smaller values of PPCCs, the distribution of the failure times under nonstationary conditions is no longer well approximated by an exponential pdf, as it is under stationary conditions. Among all the distributions considered, the two-parameter Weibull-2 distribution provides the best overall approximation of the distribution of return periods with PPCC values which ranged from 0.9970 to 0.9999, and a median value of 0.9979. For further information concerning probability plots and PPCC hypothesis tests for the two-parameter Weibull distribution, see *Vogel and Kroll* [1989] who were the first to develop such a test for this pdf.

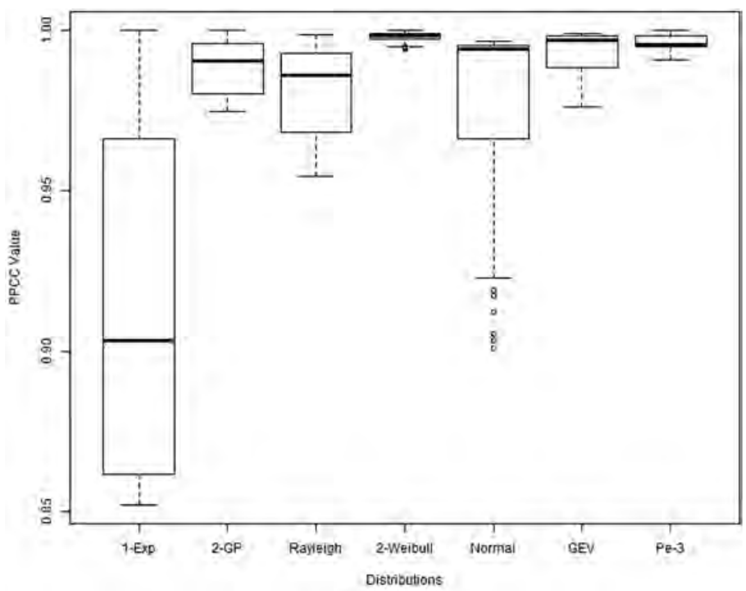

Figure 6. Boxplots of PPCC values that illustrate goodness of fit of simulated failure time data corresponding to nonstationary LN2 flood series for a range of parameters (M: 1–2; $C_x = 0.25$–1.5; $p_o = 0.001$, 0.01, and 0.1) to a range of models including the Exponential (1-EXP), Generalized Pareto (2-GP), Rayleigh, 2-Weibull, Normal, Generalized Extreme Value (GEV), and Pearson Type III (Pe-3) distributions.

Based on our results in Figure 6, we further explore the Weibull-2 model as a distribution for describing the distribution of time to failure corresponding to the nonstationary LN2 model over a feasible range of trends (M), event sizes of interest (p_o), and hydrologic variability (C_x).

5.3. Weibull Behavior of Return Period Distribution for Nonstationary LN2 Models

Consistent with our findings, the HFA literature recognizes the two-parameter Weibull model as one of the most common distributions of survival times across applications. This is due to its flexibility in modeling both increasing and decreasing hazards [*Mudholkar et al*., 1996; *Klein and Moeschberger*, 1997; *Wienke*, 2010]. It is recognized that Weibull-2 survival models can properly describe a number of hazard types, including software reliability (i.e., time between failures of software) [*Pham and Pham*, 2000], bank failure rates [*Evrensel*, 2008], and occurrences of earthquakes from crustal strain [*Hagiwara*, 1974]. In addition, modified Weibull survival models, e.g., the Beta-Weibull distribution, have been suggested to characterize breast cancer occurrence rates [*Wahed et al*., 2009].

Using an empirical analysis, the cdf of the time to failure associated with a design based on AMS from the nonstationary LN2 model can be linked with a Weibull-2 survival model. Consider a two-parameter Weibull cdf of the random variable time to failure, T:

$$F_T(t)=1-\exp\left(\left[-\frac{t}{\sigma}\right]^{\kappa}\right) \tag{21}$$

where the scale and shape parameter are given by σ and κ, respectively. The survival function is written as $S_T(t) = 1 - F_T(t)$ and interestingly, in this case, the corresponding hazard function is also given by a Weibull distribution as shown by *Mudholkar et al*. [1996] as:

$$h(t)=\frac{\kappa}{\sigma}\left[\frac{t}{\sigma}\right]^{(\kappa-1)} \tag{22}$$

Given the relationship in (2), $S_T(t)$ and $h(t)$ can be combined to produce the cumulative hazard function; in the case of the Weibull-2 survival distribution, the result is

$$H(t)=\left[\frac{t}{\sigma}\right]^{\kappa} \tag{23}$$

To fit a Weibull model to the distribution of simulated return periods, the shape (κ) and scale (σ) of the Weibull-2 distribution must be estimated from the LN2-distributed failure times and related to physical parameters of the hydrologic system. Using the *survival* package in R [*Therneau*, 2015], maximum likelihood estimates for κ and σ were obtained from the simulated return periods corresponding to the nonstationary LN2 flood series (see *Vogel and Kroll* [1989] for a comparison of a variety of alternative estimation methods corresponding to the Weibull distribution). We then used multiple regression to relate estimates of κ and σ

to the physical and design parameters of the hydrologic system, i.e., the decadal magnification factor, M, the coefficient of variation, C_x, and the exceedance probability, p_o. The resulting models were

$$\sigma=\frac{1}{-0.0249+0.0489\cdot M-0.0194\cdot C_x+1.0882\cdot p_o} \tag{24}$$

$$\kappa=e^{-0.3252}M^{0.6583}{C_x}^{-0.1196}{p_o}^{-0.1858} \tag{25}$$

with adjusted R^2 values of 0.961 and 0.987, respectively. To determine the predictive goodness of fit of this approach, a leave-one-out regression analysis of both (24) and (25) was conducted. The resulting Nash-Sutcliffe Efficiencies (NSE) for each of the regressions in (24) and (25) were 0.796 and 0.968, respectively. Based on the high NSE and adjusted R^2 of (24) and (25), we are reasonably confident in using these equations to approximate values of κ and σ based on a given set of hydrologic parameters (i.e., M, C_x, and p_o) and using these estimates for hydrologic planning and design.

Given a set of hydrologic parameters (M, C_x, p_o), (24) and (25) can be used to compute $S_T(t)$ and $H(t)$ for any arbitrary system. Figures 7 and 8 show the resulting survival function, $S_T(t)$, and cumulative hazard function, $H(t)$, over a range of trends and flood variability based on the regression results in (24) and (25). Because $h(t)$ is an increasing function, $H(t)$ increases through time, especially for higher trends. The cumulative hazard $H(t)$ function is yet another way to interpret the expected number of events within a certain period of time, and an easy way to visually compare expected exceedances under stationary conditions to those under nonstationary conditions.

The survival function, $S_T(t)$, and the cumulative hazard function, $H(t)$, are impacted by the flood variability (C_x) such that a higher C_x corresponds with greater reliability ($S_T(t)$) and fewer total failures over a given time period. Our explanation for this result is that the mean return period is more influential than its standard deviation in terms of describing the overall time to failure, or in other words changes in the mean return period dominate the system failure response. This is especially true under conditions of nonstationarity, as was shown earlier, and by *Read and Vogel* [2015] where the pdf of the return period becomes more symmetric as the degree of nonstationarity increases.

5.4. Comparison of Weibull-2 Survival Regression Model With Theoretical Values

In this section, we provide a comparative assessment of the ability of (24) and (25) to characterize the PDFs of return periods. Expressions for the average return period, E[T], and system reliability can now be estimated from the Weibull-2 distribution combined with (24) and (25). The expected value of a Weibull-2 distribution is:

$$E[T]=\sigma\cdot\Gamma\left(\frac{1}{\kappa}+1\right) \tag{26}$$

where $\Gamma()$ is the Gamma function. A regression estimate of E[T] is obtained by substituting the regression estimators in (24) and (25) into (26). In addition to this approximate regression approach, the average return period of T was computed by two alternative approaches: (1) calculating the mean of the simulated failure times corresponding to the simulated nonstationary LN2 flood series for a given set of parameters, and (2) determining an exact result for the average return period T_1 in equation (6) where p_t in equation (19) is derived from the nonstationary LN2 model. The comparison of the three approaches provides a validation of the consistency among

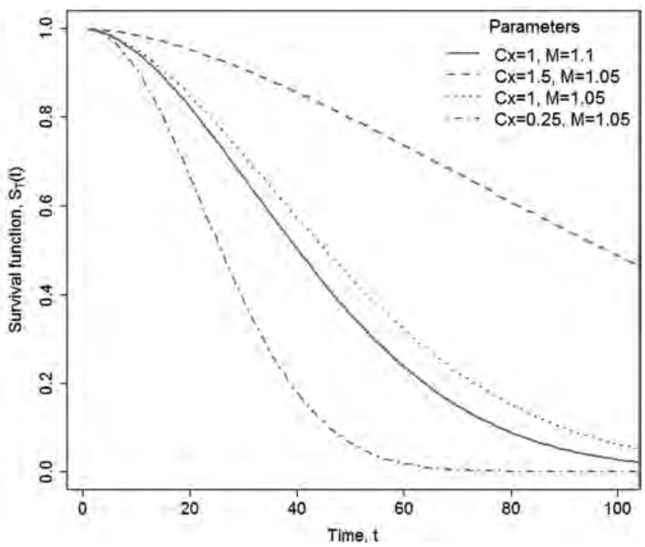

Figure 7. Survival function of simulated time to failure data corresponding to a nonstationary LN2 flood series; traces illustrate parameter subsets for the $p_o = 0.01$ event corresponding to a range of values of coefficient of variation C_x and Magnification factor, M.

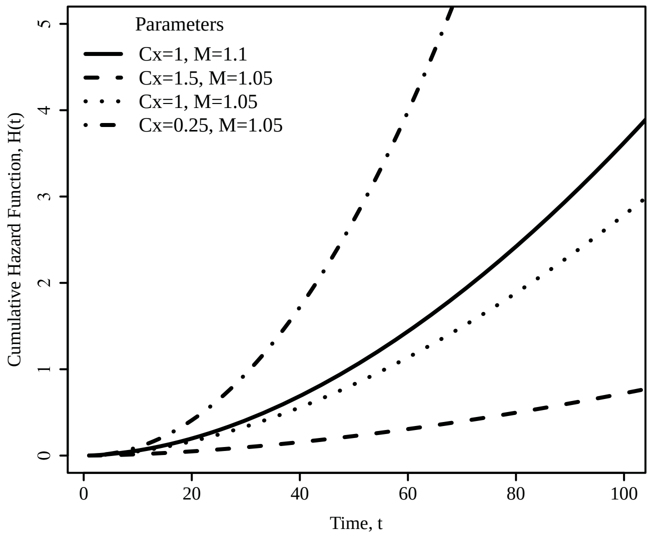

Figure 8. Cumulative hazard function $H(t)$ for simulated time to failure data corresponding to flood series arising from a nonstationary LN2 model; traces illustrate parameter subsets for the $p_o = 0.01$ event corresponding to a range of values of coefficient of variation C_x and Magnification factor, *M*.

methods. For the entire range of *M*, C_x, and p_o values considered here, Figure 9 compares the exact E[T] result (*y* axis) with the regression estimates of the mean failure times based on (24), (25), and (26) against a 1:1 line.

Figure 9 illustrates that the average return periods, T_1, are extremely well approximated by our regression approach in (24)–(26). In practice, it is much easier to estimate the Weibull parameters using the fitted regression and then to use $S_T(t)$ to describe the likelihood of experiencing an event rather than the more cumbersome and time-consuming approaches based on Monte-Carlo simulations or the analytical approach in equation (6). Our regression results, although approximate, may therefore be quite practical and useful in flood planning. This regression approach may be particularly useful if similar approximations hold for a wider range of models of nonstationary flood series including Gumbel, GEV, and Log Pearson Type III distributions. These distributions have been recommended for nonstationary flood frequency analysis [*Villarini and Smith*, 2010; *Salas and Obeysekera*, 2014; *Serinaldi and Kilsby*, 2015], and thus are natural cases to consider for the application of HFA to the field of hydrology. Another promising avenue of research would be an extension to the initial application of HFA to a nonstationary Generalized Pareto model for PDS floods by *Read and Vogel* [2016]. Further extensions to our application of HFA may also benefit from consideration of the PH models recommended by *Smith and Karr* [1986], *Maia and Meinke* [2010], and *Villarini et al*. [2012], which may better enable incorporation of physically meaningful covariates into an HFA analysis rather than the regression approach employed here.

As previously discussed, *Read and Vogel* [2015] and others [see *Bayazit*, 2015; *Serinaldi*, 2015; *Sivapalan and Samuel*, 2009] document concerns with use of the average return period in practice and, instead, recommend use of system reliability over a planning horizon. In fact, *Read and Vogel* [2015] argue this point even under stationary conditions. We compare estimates of reliability (survival function) based on regression estimates of the Weibull-2 model parameters (combining the inverse of (21) with estimates of σ and κ obtained from (24) and (25)) to exact values of reliability computed from the expression given in *Read and Vogel* [2015] and elsewhere: $\mathrm{Re}\,l_n = \prod_{i=1}^{n}(1-p_i)$. Figure 10 compares reliability estimates over a wide range of planning horizons for the exact and Weibull-2 regression

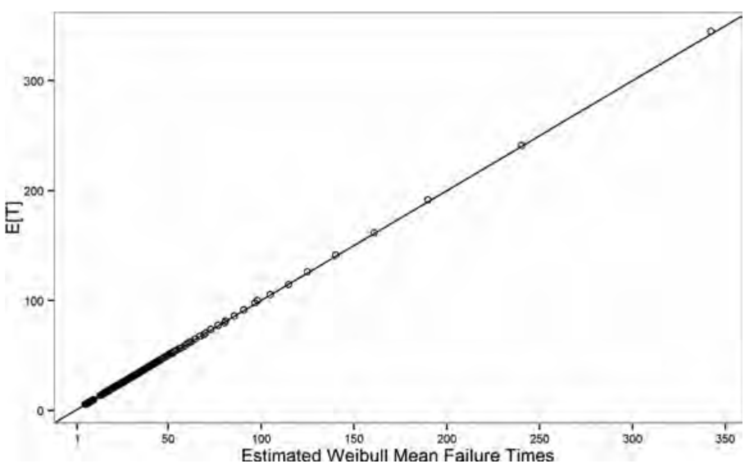

Figure 9. Comparison of exact values of average return period E[T] from (equation (6)) with regression estimates from a fitted Weibull-2 model in (24), (25), and (26). The data range represents a reasonable range of parameters from LN2 distributed flood flows ($M = 1 - 2$; $C_x = 0.25$–1.5; $p_o = 0.001$, 0.01, and 0.1).

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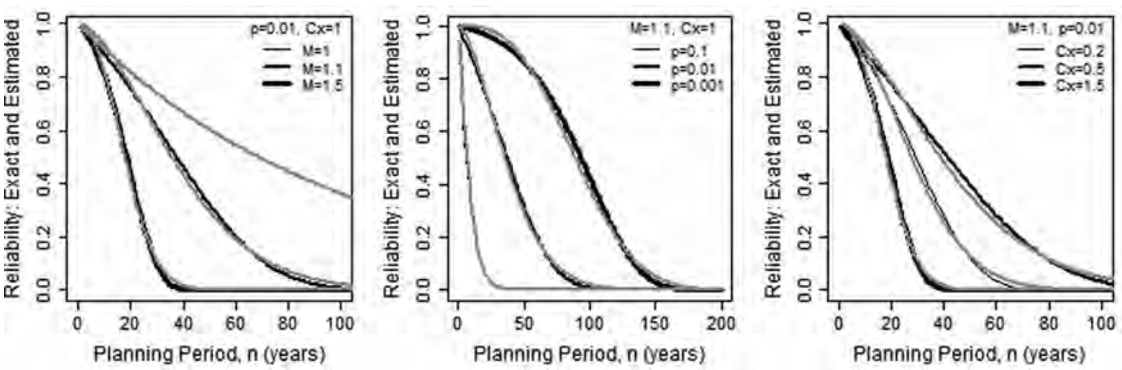

Figure 10. Comparison of reliability values corresponding to nonstationary LN2 flood series using exact simulation results (black lines) with Weibull-2 regression model estimates (grey lines) for a range of experimental values: (a) trends, $M = 1$ (stationary), 1.1, and 1.5; (b) $p_o = 0.1$, 0.01, and 0.001; (c) $C_x = 0.2$, 0.5, and 1.5.

estimates for a range of hydrologic systems (as defined by trends, M, variability, C_x, and initial design probability, p_o). The regression-based Weibull-2 estimates reproduce the exact reliabilities reasonably well for a range of increasing trends and event sizes (Figures 10a and 10b). The greatest difference between exact and approximate regression-based reliability values is only ± 0.04 years, occurring for higher values of C_x.

Both of our previous analyses of the probabilistic properties of the return period corresponding to flood series which arise from a nonstationary LN2 AMS model, and for a nonstationary EXP1 PDS flood series, show that the reliability $S_T(t)$ over a planning horizon is reduced when increasing trends in a flood series exist. This implies that a design engineer may want to reconsider the design event $x_o(p_o)$ and adjust it to maintain a certain level of acceptable risk, if nonstationary conditions are either known to have occurred in the past, and/or expected to persist into the future. The challenge of design event selection is not new, and even under stationary conditions, uncertainty plays a central role in estimating the risk of failure in hydraulic structures [*Tung and Mays*, 1981; *Tung et al*., 2006]. Coupling HFA with a risk-based decision framework under nonstationarity, analogous to the work of *Rosner et al*. [2014], may lead to general guidance on a methodology for selecting an "optimal" design event given uncertainty about past and future trends. Research in risk-based design of hydrologic systems by *Bao et al*. [1987] and others, along with the nonstationary risk-based framework presented in *Rosner et al*. [2014], may benefit from integration with the HFA framework introduced here. In addition to all the existing sources of uncertainty associated with a stationary flood frequency analysis, there is always increased uncertainty associated with additional model parameters needed for a nonstationary model. Such increased uncertainty associated with the application of nonstationary methods will impact design quantile estimation as shown by *Serinaldi and Kilsby* [2015]. Thus, nonstationary methods should be used with great caution and always compared and contrasted with traditional stationary methods of flood frequency analysis.

6. Conclusions

The purpose of this paper is to understand the benefits and limitations in applying hazard function analysis (HFA) to nonstationary flood planning. We provide two examples integrating HFA with standard hydrologic frequency analysis using standard metrics employed by operational hydrologists. Our overall goal was to use HFA to link the probabilistic properties of the hydrologic flood series X with the probabilistic properties of the return period T, associated with future floods which exceed some design flood threshold. We began with a simple yet realistic analytical example based on a nonstationary one-parameter exponential (EXP1) model of partial duration flood series, to demonstrate the application of HFA concepts including the hazard function, survival function, and the cumulative hazard function. Next, we used Monte-Carlo simulation to consider a more realistic model of annual maximum flood series (AMS) which arise from a nonstationary LN2 model. In agreement with results from numerous other fields which have applied HFA, the Weibull-2 model was identified as a suitable model of the survival function for nonstationary LN2 AMS. We developed regression models to estimate the Weibull shape and scale parameters based on known hydrologic system parameters and design requirements, and provided equations for calculating the average return period and reliability, achieving reasonably precise estimates without complex computations required using alternative approaches. We provide useful equations for relating the properties of actual flood control systems to the probabilistic properties of the return period and reliability associated with a particular design event when AMS of floods arise from a nonstationary LN2 model. These findings suggest significant utility and potential associated with the application of HFA to other reasonable nonstationary models of AMS of floods, including the Gumbel, GEV, and Log Pearson Type III distributions. We anticipate that this initial study, introducing an approach for linking the probabilistic properties of X and T for flood applications, should prove useful for improving our understanding of the impact of nonstationarity on water resources design, planning and management. Finally, we remind all future researchers who consider employing nonstationary hydrologic frequency analysis in practice, to heed carefully the warnings and guidance provided by *Serinaldi and Kilsby* [2015] concerning the introduction of additional sources of uncertainty into such an analysis. Thus, in spite of the attention and advances relating to nonstationarity, we are still at a stage in the evolution of nonstationary methods for flood frequency analysis to raise serious questions about their use compared with traditional stationary methods.

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