   b. Can extend to motion constrained by an inclined plane (ignoring difference between rolling and falling); but note that he is unable to extend theory to pendular motion – curvilinear motion constrained to remain at some specified distance from a point
   c. Only result for pendulums experimental: $T \propto \sqrt{\text{length}}$

C. Some Other Interesting (and Testable) Results
   1. Many of the other mathematical results provide at least a basis for a test of the theory, if not a basis for an experimental program
      a. E.g. can test claim that 45 deg plane fastest for covering a given horizontal distance (though problems in maintaining rolling likely to yield exactly the wrong result)
      b. Many of the claims permit (potential) qualitative tests of the theory, and qualitative tests in general more tractable -- something Galileo appreciated
   2. Probably the most interesting of these is that an inclined plane, though the shortest distance between two points, does not yield the shortest time between the points in natural motion (Prop. XXXVI)
      a. So long as slope of plane no greater than 45 deg, less time via two planes forming chords of same circle (assuming complete transfer of speed at point of intersection)
      b. Since proof depends on a large number of earlier results, this theorem in effect pulls a lot of them together
      c. Main idea: speed gained from D to B shortens time from B to C more than time lost in going from D to B
   3. Scholium that follows extrapolates this result into an even more striking claim: circular arc faster than inclined plane even though distance traveled greater than along any sequence of planes
      a. Argument simply by taking reasoning on planes to limit
      b. Galileo wrongly asserts that circular arc the fastest, but correct in asserting that circular arc faster
      c. Problem of determining the fastest path becomes famous in 1691 with Jacob Bernoulli challenge -- answer, the cycloid
   4. This consequence of the theory is quite unexpected and counterintuitive, and hence provides basis for a challenging test of theory
      a. At 90 deg arc the effects of rolling along plane and falling at beginning of circle will confirm the theory for the wrong reason -- reduced acceleration in rolling
      b. But at 45 deg, or maybe 30, with proper surfaces, maybe can maintain rolling of sphere in both cases, providing a true test
      c. But test very difficult to execute -- more likely to get confounded results than meaningful ones, raising the problem of how to assess actual results
   5. Note logic of evidence here: an anomalous consequence of the theory which, if confirmed, shows that theory yielding new knowledge and hence not just recapitulating what is already known

a. A real chance for falsification -- bold conjecture leads to extraordinary claim that is amenable to being shown false
b. But also notice the danger of confirmation for unrecognized wrong reasons -- rolling along plane vs. falling on part of circle
c. Testing not necessarily so conclusive as one might think!

D. The Theory as a Question-Answering Device

1. Given the effort required just to understand the various propositions -- much less the proofs! -- natural to ask whether any of this display of mathematical facility of scientific interest
   a. A question often asked by scientists when faced with a highly ramified mathematical development of a theory -- is all of this worth the effort of understanding
   b. Brought home even more strikingly in the case of Galileo by the fact that much of his mathematical development can be telescoped into a single algebraic formula which can then be manipulated to yield the various results
2. One purpose served by such a ramified mathematical development has already been noted: generate a bunch of results that can serve as basis for testing the theory -- i.e. its basic claims
   a. Unexpected qualitative contrasts and distinctive phenomenal quantitative patterns a basis for tests to confirm -- or at least to falsify -- the theory
   b. Wider the variety of such results, the greater the supporting evidence for the theory if it "passes" all of the tests
   c. Also, given problems of experimentation in early stages of a science, the wider the variety of such results, the greater the chances of at least one meaningful test
   d. The predictive power of a theory: the range of (confirmed) predictions deducible from it
3. A second purpose served by putting a theory in the form of a small number of basic claims and a highly ramified mathematical development out of them is that the deductions can be turned around to yield explanations
   a. E.g. why do the incremental distances display essentially a 1,3,5,... progression: because the motion is one governed by uniform acceleration
   b. Explain a phenomenon by showing that it is a logical consequence of some basic mechanism -- i.e. by subsuming it under a law (deductive-nomological explanation)
   c. The explanatory power of a theory: a reflection of how much -- range and variety -- follows from how little initially assumed
   d. Explanatory power and predictive power in opposition to some extent, for generally can expand either at sacrifice of other insofar as the way in general to increase predictive power is to include more principles and qualifying complications in order to achieve closer agreement
   e. The fact that Galileo's theory of free-fall follows from only two principles gives it potentially great explanatory power

4. A third purpose served by a ramified mathematical development of a theory is that it provides a general question-answering device
    a. Given a question, theory identifies what combinations of information needed to determine answer and provides a way of then determining the answer (a la Bromberger: vicarious experiments): Galileo's solved Problems
    b. A crucial role of theories in science -- most of what you learn when you study sciences
    c. The form in which an indefinite amount of knowledge becomes contained in a fairly simple theory, fairly simple to learn
5. These three purposes, though somewhat distinct, are not unrelated
    a. Question-answering feature plays a critical role in the design of many experiments (as well as in engineering applications)
    b. Testing critical to be able to rely on question-answering when answering novel questions or ones with answers that cannot be readily checked empirically
    c. Explanatory power and predictive power both tied to question-answering power
6. The question-answering aspect of theories in physics more important than is sometimes noted -- look at e.g. Kepler's theory of planetary motion from this point of view
    a. In fact, a tradition dating back to Ptolemy in astronomy, Archimedes in mechanics
    b. Galileo can be thought of as bringing this tradition into a science of motion within mechanics

III. "The Fourth Day": Conceptual Development of the Theory

A. A Parabolic Path -- The Basic Idea

1. Galileo's proposed conceptualization of "projected" motion: suppose uniform motion along a plane which suddenly ends, so that vertical fall commences (with uniform acceleration)
    a. A basic case of a more general type: motion "compounded from two movements; that is, when it is moved equably and is also naturally accelerated." [268]
    b. Uniform horizontal plus uniformly accelerated vertical, "all impediments being put aside"
    c. Idea, then, is to combine the two theories from the "Third Day"
2. Note that Galileo here again seemingly committing himself to a limited version of what we call the principle of inertia -- for equable horizontal motion is taken to be without impediment
    a. I.e. horizontal motion would remain uniform and hence "perpetual" in nature because nothing in the basic natural process to change its speed (since resistance etc. excluded)
    b. Of course, once horizontal ends, then something in nature to change motion, namely propensity (*propensionem*) to fall to ground
    c. Hence a further assumption, open to challenge: uniform horizontal and uniformly accelerated vertical "propensities" continue unaltered when compounded -- no "cross-talk"!
3. Compounding two independent orthogonal motions was in some ways a radical move in the 17th century, however simple it is to us