2. Although Newton provides no evidence for this claim about gravity in De Motu, it would not be presumptuous to think he had repeated the "Moon test" of the late 1660s, but now using current values
    a. No unqualified documentation that he had done so for a few more weeks, where the result appears in a manuscript that is an immediate successor to *De Motu Corporum in Gyrum* (i.e. *De Motu Sphaericorum Corporum in fluidis* -- Version 3, called by DTW the "augmented" version)
    b. But every reason to think that he would have done so in the fall of 1684, if he had not done so in 1680 (as Westfall says)
3. By 1684 Newton had various choices for the number of Paris feet in a degree of longitude -- e.g. Picard's value of 342,360, which Newton subsequently used, and Cassini's value of 342,366
    a. With Picard's value, the circumference of the earth, assuming a sphere, is 123,249,600 Paris ft, giving a radius of 19,615,783
    b. The period of the moon 27d 7h 43m, or 2.36058e6 sec, so that its angular velocity is 2.6617e-6 rad/sec
    c. Taking the distance to the moon to be 60 earth radii, then the acceleration of the Moon, $r*\omega^2$, is 8.33833e-3 ft/sec/sec -- i.e. the Moon falls 4.16916e-3 ft in 1 sec
    d. Dividing this number into Huygens's value for the fall in 1 sec at the surface of the earth -- 15.0833 ft, we get 3617.9 -- only 0.5% off a perfect inverse-square, well within known accuracy of the lunar horizontal parallax
4. So, a repeat of the "Moon test" with Picard's values for the radius of the earth rather than Galileo's would have been a great success
    a. From 4375 -- a 21.5% discrepancy -- in the late 1660s to 3617.9 -- a 0.5% discrepancy -- in 1684, clearly within the accuracy of the value for the mean distance of the moon
    b. Whatever Newton was looking for in the 1660s, he had surely found it by the end of 1684, though his doing so in no way depended on any of the new results in De Motu
5. Notice carefully, however, what I am taking the successful result to have shown
    a. Not just that the moon is held in place by terrestrial gravity
    b. But more so that terrestrial gravity varies inversely with the square of the distance from the center of the earth at least to the moon
    c. Since the moon is the sole body orbiting the earth, there was no basis at this juncture to draw the conclusion that the inverse-square forces governed its motion without the "Moon test"
    d. And there was no other basis for concluding that terrestrial gravity diminishes in accord with the inverse-square rule

C. Problems 6 and 7: Resistance and Galilean Motion

1. Resuming with the text of De Motu, Newton next turns to the problems of uniform and orthogonally uniformly accelerated motion, but allowing for air resistance proportional to velocity

   a. Commentators have had some difficulty explaining why Newton thought these two problems were germane to the rest of De Motu, especially since gravity not treated as centripetal in them
   b. Thus Whiteside, for example, suggests Newton was just putting on record a solution to a problem he had addressed unsuccessfully in 1674
   c. (Huygens had solved the problem including the projectile problem with resistance proportional to velocity in late 1660's, but had withheld the result when experiments disagreed with it, indicating that resistance varies more closely as v^2 than as v)
2. The solutions themselves are at the cutting-edge of mathematics at the time, in effect yielding fully correct geometrical solutions to two differential equations, with g and k constant

$$d^2x/dt^2 = -k*dx/dt \quad \text{and} \quad d^2y/dt^2 = g - k*dy/dt$$

   a. The modern solution to both has an exponential form, employing Euler's constant e (Euler was decades from being born)
   b. The main difficulty Newton faced, once he had formulated the problems, was to find a geometrical relationship that could represent such an exponential relation between the independent variable t and the dependent variable s
   c. I.e., in Newton's words, to find a geometrical relationship that can represent one variable's increasing (or decreasing) in geometrical proportion as the other increases in arithmetic proportion
   d. For, as Newton says, the consecutive decrements in velocity must be proportional to the velocities, so that Lemma 1 applies, and this Lemma entails that $V_0:V_2$ as $[V_0/(V_0-V_1)]^2$, $V_0:V_3$ as $[V_0/(V_0-V_1)]^3$ etc.
3. Newton's approach is to use hyperbolas with rectangular asymptotes -- i.e. curves of the type $u*w$ = constant -- which define areas that increase in an arithmetic progression for a geometric progression along the abscissa, as was known from Napier, Huygens, and Mercator
   a. Appropriate since integral is $u = c*\log w$ if $u = c/w$
   b. The key then to understanding Newton's solution to these two problems is just to understand how the different variables are being represented in the geometrical constructions (see Turnbull, II, p. 460f, in the Appendix)
4. In the solution to Problem 6 time is being represented by the increasing area BADG, distance traveled by the increasing length AD, and velocity by the decreasing length DC
   a. Newton's solution thus gives exactly the modern solution

$$x = (u/k)(1 - e^{-kt}) \qquad dx/dt = ue^{-kt}$$

   b. For his curve represents $w*(a-x)=ab$, where AC=a, AB=b so that, taking A as origin and the initial velocity $u=a*k$, we obtain $t = 1/k*\log(a/a-x)$ and w [= $ab/(dx/dt)$] is proportional to the reciprocal of the velocity

5. In the solution to Problem 7, for the case of descent, the distance fallen in time represented by the area AB^2G^2D is represented by the area B^2E^2G, and in the further time represented by area ${}^2D^2G^2g^2d$, the distance represented by the area ${}^2G^2E^2e^2g$; and the velocity of descent by 0 at the outset, by the areas AB^2ED after the first time and AB^2e^2d after the second, with the area ABCH representing the terminal velocity
   a. Newton's solution thus again gives the modern solution in descent

$$y = (g/k)t - (g/k^2)(1 - e^{-kt}) \qquad dy/dt = (g/k)(1 - e^{-kt})$$

   b. For his curve represents $w*(c-v)=c$, where $c=g/k$ and, taking A as origin, w $[=g/(d^2y/dt^2)]$ is proportional to the reciprocal of the acceleration; but then the two parts of the area under the curve represent y and $k*y$ respectively, and their sum represents $g*t$

D. Scholium: Resistance and Projectile Motion

1. In the Scholium following Problem 7, Newton compounds the solutions for Problems 6 and 7 to give the solution for projectile motion in the case of uniform vertical acceleration and resistance proportional to velocity
   a. Newton's solution has the projectile reaching r along the trajectory DarFK in the time represented by DRTBG, and the speed at r represented by the tangent to the curve at r, rL
   b. For, with c=1/k and a=(u/k)∗cos(α), Newton's solution for the trajectory and time is simply:

$$y = g/k*\{x - c*\log(a/a\text{-}x)\} \text{ and } t = c*\log(a/a\text{-}x)$$

2. The trajectory with resistance proportional to velocity is thus a skewed parabola, with the amount of skewing dependent on k
   a. In qualitative agreement with what almost everyone had been saying from Galileo on
   b. But now a precise quantitative solution, given k, u, and g
3. Newton gives an empirical method for determining $k*u/g$ from observations of the initial angle of ascent and the final angle of descent (for a given spherical surface area and weight)
   a. Deceleration in resistance for a given medium and speed here assumed to be proportional to surface area and inversely proportional to weight
   b. Can then obtain *u* from a measure of range, and finally infer *k*, so that henceforth *k* can be taken as that value multiplied by (1) the ratio of the surface area of the projectile to that of the reference projectile and (2) the ratio of the weight of the reference projectile to the weight of the projectile
   c. So, the solution is as formally complete as Galileo's
   d. Newton's use of *weight* here, and not *mass*, is evidence that he had yet to distinguish the two!
   e. And his use of surface area means that he is viewing resistance as fluid clinging to the body
4. Still a calibrated solution to the projectile problem insofar as a coefficient of resistance has to be measured, but not calibrated in a way to absorb unaccounted for effects

a. In Galileo's case the Tables are in effect being recalibrated for each different velocity and projectile size and shape

b. In Newton's case, by contrast, the only calibration involves a presumable constant

5. The trouble, of course, comes when multiple measurements for this coefficient fail to yield (remotely) consistent values over a range of velocities and projectiles, which is in effect what Huygens had discovered years earlier.

A. Newton is here assuming that resistance effects are proportional to velocity, while the truth is more accurately represented as if by two distinct effects, one roughly proportional to velocity and another, usually dominant, roughly proportional to velocity squared

b. The *Principia* will offer such a further refinement

V. The Significance of *De Motu Corporum in Gyrum*

A. Advances Made by Version 1 of De Motu

1. Even though *De Motu Corporum in Gyrum* was not published, it gained enough circulation through the Royal Society that it is appropriate to ask, what exactly did it, by itself, contribute

a. Or, maybe more appropriately, what exactly would it have contributed if it had not been followed up two and a half years later by the *Principia*

b. A question that is undoubtedly best answered from the perspective of Halley, though also from that of Newton, since he was evidently not satisfied with the contribution as it stood

c. In other words, instead of reading the tract through the lens of the subsequent *Principia*, we should try to read it through the lens of the state of science as of late 1684

2. One thing that this tract did not contribute in its own right is universal gravity, for the tract as such does not even require mutual attraction between celestial bodies, much less between every two particles of matter

a. I see nothing in this version of De Motu that gives any reason to think that Newton had yet even thought of universal gravity among particles

b. No reason to think that Newton had yet even reached a clear concept of mass, as distinct from weight, for the one place where he needs *mass* in the final scholium he uses *weight*

3. In the version sent to the Royal Society, the one key place in which gravity is referred to is at the end of the Scholium to Problem 5, where Newton speaks of the "hypothesis" of gravity being an inverse-square force

a. But the manuscript shows him to have been using "*gravitas*" in Problems 1, 2, 3, and 5, only then to delete it, usually replacing it with "*vis centripeta*"

b. What he had in mind when he first used it and why he then replaced it with a more abstract locution is unclear; but no one looking at the public version of De Motu saw any sign of this

c. The effect is clear, however: the text presents an abstract theory of motion under centripetal forces until after Problem 5, where it turns to motion under hypothetical terrestrial gravity