# NUMERICAL EXPLORATION OF THE STRING THEORY LANDSCAPE

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*To my parents Vasilis and Alexandra*

## Abstract

String theory is the best candidate to provide a consistent quantum theory of gravity. Its ten dimensional formulation forces us to perform a compactification of the six unobserved dimensions in a very special compact manifold known as Calabi-Yau. The standard way to address this issue is through the flux compactification scenarios. One of the major implications of these scenarios is that the string theory cannot provide a single and unique vacum as a solution. Rather one can find an extremely large set of solutions, each with its own physical properties. This is the string theory Landscape.

In the first part we present the formal description of the flux compactification theory. From the four dimensional point of view this is a supersymmetric theory, fully described only by two functions, the superpotential and the Kahler potential. Their expressions are crucially depend on the geometrical properties of the compact manifold. By writing these functions for the specific Calabi-Yau manifold $P^4_{[1,1,1,6,9]}$ we are looking firstly for supersymmetric and then after breaking the supersymmetry, for non-supersymmetric numerical solutions. These solutions describe the possible vacua and our goal is using statistical analysis to categorize them based on their cosmological properties and to check their stability. Finally we present the existence of stable dS vacua with and without adding an uplifting term on the potential. In the case where there is not an uplifting term the breaking of supersymmetry is done by incorporating $\alpha$' corrections to the Kahler potential.

In the second part we construct a KKLT like inflation model, within string theory flux compactifications and, in particular a model of accidental inflation. We investigate the possibility that the apparent fine-tuning of the low energy parameters of the theory needed to have inflation can be generically obtained by scanning the values of the fluxes over the landscape. Furthermore, we find that the existence

of a landscape of eternal inflation in this model provides us with a natural theory of initial conditions for the inflationary period in our vacuum. We demonstrate how these two effects work in a small corner of the landscape associated with the complex structure of the Calabi-Yau manifold $P^4_{[1,1,1,6,9]}$ by numerically investigating the flux vacua of a reduced moduli space. This allows us to obtain the distribution of observable parameters for inflation in this mini-landscape directly from the fluxes.

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# Chapter 1

# Introduction

Some of the most important quests in human existence involve the sky. People looked at the celestial dome, firstly with awe and then with admiration. They identified planets and observed their orbits and periodicities, but that was not enough. They knew that these planets were not very far away and their spirit could not be restricted in such a small space. So their focus move on to the stars and then to the whole Galaxy. Even though at that time it was impossible to see other galaxies and the large structure of the Universe, questions regarding the creation, evolution and the ultimate fate of the cosmos as a whole, appeared soon. Nowadays, just few moments later in cosmological scales, modern cosmology provides some of the answers for those questions. On the other hand, these theories are continuously challenged in a search for deeper understanding and more universal laws. From the ancient legends of the initial chaos and the god Chronos who created time, to the current leading theories of superstrings and inflation, there is a common theme: the endless need for better understanding of the Universe.

Perhaps the most important success of cosmology is the observational evidence in favor of the standard Big Bang model. This model can describe accurately the expanding universe [1] and its very hot initial stages depicted now in the cosmic microwave background (CMB) [2] . Additionally it can predict with high precision the observed abundances of the light elements produced during primordial nucleosynthesis. However, there are still many questions that this model cannot answer. One of the most important ones is why the universe appears to be homogeneous and isotropic at the largest scales. Based on this standard model, different regions of the primordial universe were very far from each other to be able to reach a common thermal equilibrium. It is therefore hard to see how the CMB could be so perfectly homogeneous. This is the so-called horizon problem.

Another big enigma is the flatness problem. The Universe is observed now to have an energy density very close to the necessary critical one for a flat spatial geometry. However a situation like this at current time requires extreme fine-tuning on that density in the past. Moreover many extensions of the standard model and cosmology predict the existence of magnetic monopoles [3], but there is no evidence so far of such objects in our universe. Finally the standard Big Bang theory does not offer any explanation for the dynamical generation of the small anisotropies observed on the cosmic microwave background as it would be desirable for a theory of the early universe.

A solution to all these issues is provided by an inflationary phase in the very early universe [4]. This inflationary period is characterized by an exponential expansion of the universe driven by a vacuum energy density. The result of this inflationary phase was a tremendous increase in size of a tiny patch of the primordial universe, establishing, in a causal way, the flat, isotropic and homogeneous conditions that we observe in our universe today. Furthermore, the little quantum fluctuations of the density created during this time seem to be consistent with the latest cosmological observations made by the WMAP [5] and PLANCK [6] sattelites, making inflation the leading paradigm for the primordial universe. At the end of this inflationary era, the universe needs to be reheated and filled with radiation setting the stage for its subsequent thermal expansion history accurately described by the Big Bang model.

Many phenomenological models of inflation have been proposed in the literature over the years [7]. But so far the fundamental origin of inflation remains unclear. It is therefore important to try to identify models on inflation within a fundamental theory. One of the leading candidates of such a fundamental model is String Theory. On the other hand, this simple idea of putting together cosmology and String Theory faces several immediate important challenges that have to be addressed before progress can be made.

First of all String Theory seems to be valid only if the space-time has extra dimensions [8]. As a consequence, one needs to introduce a compactification mechanism into this theory that allows us to make a connection to the low energy 4-dimensional dynamics that we observe today. This has been one of the most important challenges for string phenomenology since its early days. On the other hand, new ideas have recently allowed us to find scenarios with stable compactifications of the theory where the curled-up compact internal manifold becomes unobservable at low energies.

From a 4 dimensional perspective one parametrizes the size and shape of the internal manifold as fields that need to be pinned down to some particular point in field space. One does this by introducing some ingredients in the theory that induce a 4 dimensional potential for these fields, the moduli fields.

Unfortunately there is not a unique way to stabilize these fields. It tuns out that one can get an extremely large number of effective potentials with very rich structure of local minima and maxima. The collection of all these minima constitute what has been called the String Theory Landscape [9].

This vast number of vacua clearly presents a challenge to identify the predictions of this theory, which should be now studied in a statistical manner. On the other hand, the extremely large number of different vacua, provides a natural basis to approach great cosmological puzzles, like the cosmological constant problem and the fine-tuning frequently encountered on the parameters of the inflationary models. Using the existence of this vast space of distinct minima, one can argue that there should be many vacua in the narrow band satisfying current observational data on the value of the cosmological constant. On the other hand, one should also be able to find regions of the potential which satisfy the slow roll conditions required for the inflation even if they require a moderate fine tunning of the parameters. Using statistical analysis among the solutions satisfying current observational data one can make predictions about other future observations.

In this thesis we study the form of the potentials induced in a particular corner of the String Theory Landscape. In the first part of the thesis, we generated a large number of these effective potentials and studied the statistical properties of their vacua. We have studied these potentials using their complete field space. In our case, this required the minimization of a 8 dimensional potential function as well as a 10 dimensional case. Finding mimima of these potentials pose some numerical challenges that we were able to overcome guided by some analytical arguments. We could then study the properties of this large set of supersymmetric as well as non-supersymmetric vacua.

In the second part of the thesis and using similar numerical techniques, we were able to generate potentials with the required set of conditions for inflation in the same type of models. We then extract some statistical information about the observational cosmological parameters in these potentials and give some arguments to a natural explanation for the correct initial conditions for inflation.

The outline of this thesis is the following. In the next Chapter we will give some introduction to string theory compactifications with the emphasis on the methods of flux compactifications used in the current thesis. Chapter 3 describes the numerical exploration of a small cornner of the string theory landscape where we were able to find a large number of vacua. We classify and characterize them by their cosmological constants, supersymmetry and stability. In Chapter 4 we give a brief account of the the theory of inflation specially focusing on inflection point inflation. In Chapter 5 we describe how inflection point inflation occurs naturally in this models of flux compactification and how the idea of the landscape can be used to explain the fine tuning needed for a successful model. Finally in Chapter 6 we give some conclusions and discussion of future work.

# Chapter 2

# String Theory Compactification

## 2.1 String Theory Basics

String theory is at the moment the most promising framework in which all elementary particles and all the forces in nature can have a common description at all energy scales. Its key point is that all these particles appear as vibrations of one-dimensional strings. One of these modes corresponds to a massless spin-two state which is identified as the graviton field [10]. This shows that gravity is included in the string theory as well. On the other hand, this is a fully quantum theory and therefore leads to a candidate for a consistent theory of quantum gravity.

The simplest string theory is the bosonic one, which includes only bosonic degrees of freedom. This is phenomenologically not very attractive since it does not describe many particles that seem to exist in Nature. New fermionic degrees of freedom can be introduced by imposing a new symmetry between bosons and fermions, supersymmetry. In general making a realistic string theory requires supersymmetry.

Another important characteristic of String Theory is the number of dimensions of spacetime in which the theory is formulated. While in a classical string theory the number of the dimensions of spacetime is not fixed, in a quantum theory this number is forced to be significantly more than four to be consistent with Lorentz invariance. More specifically in the bosonic case this number is 26. Fortunately supersymmetry reduces that number down to ten. However this crucial observation for string theory is in clear contrast to our everyday experience. We seem to live in an effectively four dimensional spacetime, so a ten dimensional superstring model needs to have its extra six dimensions compactified in a internal manifold.

The only parameter in superstring theory in 10 dimensions is given by the string tension denoted by, $T = 1/(2\pi\alpha')$. Performing the quantization procedure for the string, one can obtain the full spectrum of the theory which is of course characterized by this energy scale. However at energies much below this string scale one should be able to write this effective theory in terms of the massless modes alone. The theory that describes the dynamics of those modes is very constrained once one imposes the dimensionality of the theory as well as supersymmetry. It turns out that the number of such theories is very small. In the current work we will concentrate on Type IIB string theory that leads to Type IIB Supergravity in 10d [12]. The bosonic content of this theory is: a spin-2 particle, $g^{MN}$, a 2-form antisymmetric tensor $B_2$, a real scalar field, the dilaton $\phi$ and the $0-, 2-, 4-$ Ramond-Ramond form fields, $C_0, C_2, C_4$. From these fields we also get the field strengths: $F_{2p+1} = dC_{2p}$ and $H_3 = dB_2$. The action for this theory can be written as:

$$
\begin{aligned}
S_{IIB} &= S_{NS} + S_R + S_{CS} && (2.1)\\
S_{NS} &= \frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\left(R + 4\partial_M\phi\partial^M\phi - \frac{1}{2}|H_3|^2\right) && (2.2)\\
S_R &= -\frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-g}\frac{1}{2}\left(|F_1|^2 + |\tilde{F}_3|^2 + |\tilde{F}_5|^2\right) && (2.3)\\
S_{CS} &= -\frac{1}{2k_{10}^2}\int C_4 \wedge H_3 \wedge F_3 && (2.4)
\end{aligned}
$$

where we have introduced the definitons,

$$
\tilde{F}_3 = F_3 - C_0 \wedge H_3 \qquad (2.5)
$$

$$\tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3 \tag{2.6}$$

One can also include fermionic terms in this lagrangian based on the fact that the theory is supersymmetric. For our purposes here it will be sufficient to only consider the bosonic part of the theory [1]

### 2.1.1 D-branes and fluxes

As we described above, String Theory in 10 dimensions has in its spectrum a set of p-forms that can be considered as a higher dimensional generalization of the 4 dimensional 2-form Maxwell field strength $F_{\mu\nu}$. Similarly to what happens in electromagnetism we therefore expect to find some objects in string theory charged with respect to these forms. On the other hand, we can also see that p-form potentials with higher number of indeces like, $B_2, C_2, C_4$ would not couple to pointlike particles but to other objects of higher number of intrinsic space dimensions. In particular it is easy to see that in the same way as a 1-form potential $A_\mu$ of electromagnetism couples to an electrically charged particle, a 2-form potential like $B_2$ couples "electrically" to a string. This is in fact the fundamental string of string theory.

Following this argument, we see that there should be other objects coupled to the Ramond-Ramond fields. These are called D-branes,[13] where $D$ denotes the number of spatial dimensions of the object, in other words a 1-brane is a string, a 2-brane is a membrane and so on. These objects are not fundamental objects of the theory, like the fundamental string. They are solitonic solutions of the supergravity theory that are charged with respect to these flux forms, much in the same way as a magnetic monopole could be realized in some field theories as smooth solitonic

[1] There is an additional condition that one must impose on the $\tilde{F}_5$ form, the self-duality condition that must be satified by solutions of this theory.

solutions.

Another interesting property of these objects is that they could be considered as boundaries where the open strings can end. These open string degrees of freedom give rise to gauge theories living on the D-branes opening a new avenue for particle physics model building in string theory.

The possibility of confining gauge theories to a lower dimensional hypersurface of the 10 dimensional spacetime, has inspired a large number of models that have been called braneworld scenarios [14]. A particular important point in these type of scenarios is the idea that the volume of the extra dimensional space could be much larger than previously thought since the constraints on this parameter would come solely from gravity which at the moment are not nearly as stringent as the ones coming from particle physics.

## 2.2 Dimensional reduction and moduli fields.

Having disccussed briefly the main ingredients of String Theory, we should now describe how to make contact with a 4 dimensional theory compatible with our current observations. The standard way to do this is to compactify the higher dimensional theory down to a product of a six-dimensional internal manifold times the four-dimensional spacetime. The particular models that we will be focusing on in this thesis are fairly complicated so in order to describe the main issues involved in this procedure in simple terms we will be presenting a couple of toy models in field theory where, hopefully, things will be much more clear.

### 2.2.1 Kaluza-Klein compactification

In this toy model example we will present the simplest scenario for compactification, namely the original Kaluza-Klein model of pure gravity in 5 dimensions compactified to 4d. Our starting point is the five-dimensional Einstein-Hilbert term with an action [15]

$$S = -\frac{1}{16\pi G_5}\int d^5x\sqrt{-g_5}R_5 \tag{2.7}$$

where we will now assume that the extra dimension $x_5$ is in fact compact, meaning $x_5 + 2\pi R = x_5$. We can always choose the following decomposition of the five-dimensional metric

$$g_{MN} = \begin{pmatrix} \left(g_{\mu\nu} + k^2\phi^2 A_\mu A_\nu\right) & k\phi^2 A_\mu \\ k\phi^2 A_\mu & \phi^2 \end{pmatrix} \tag{2.8}$$

where the capital indexes run from 0 to 4 while the small from 0 to 3. Notice that we have assigned different names to different components of the higher dimensional metric. We will see in a second the origin of these names.

Due to the periodicity in the fifth dimension we can now expand these fields in the following way,

$$g_{\mu\nu}(x_5) = \frac{1}{\sqrt{2\pi R}}\sum_n g_{\mu\nu}^{(n)} e^{i\frac{n}{R}x_5} \tag{2.9}$$

$$A_\mu(x_5) = \frac{1}{\sqrt{2\pi R}}\sum_n A_\mu^{(n)} e^{i\frac{n}{R}x_5} \tag{2.10}$$

$$\phi(x_5) = \frac{1}{\sqrt{2\pi R}}\sum_n \phi^{(n)} e^{i\frac{n}{R}x_5} \tag{2.11}$$

Plugging these expressions back into the 5 dimensional action 2.7 and integrating the extra dimension, one arrives at a 4 dimensional theory with an infinite number of fields. This proliferation of fields is not surprising since we are trying to describe a higher dimensional theory from a lower dimensional one, so we need many more fields in the compactified theory to encode the same information. On the other hand, the masses of these fields are given by $m_n \sim n/R$ where $R$ is the size of the extra dimension. We can now imagine that we are only interested in the effective theory below the $1/R$ energy scale and see which fields should be present in this effective theory. The anwers is clearly the massless states of the theory. The action for such an effective theory can be seen to be,

$$S = -\int d^4x\sqrt{-g}\phi\left(\frac{R}{16\pi G} + \frac{1}{4}\phi^2 F_{\mu\nu}F^{\mu\nu} + \frac{2}{3k^2\phi^2}\left(\partial^\mu\phi\right)\left(\partial_\mu\phi\right)\right) \tag{2.12}$$

where as usual $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and where the $k = 4\sqrt{\pi G_4}$ and the four dimensional Newton's constant is related to the five dimensional one by $G_5 = G_4 \cdot 2\pi L$.

We now see the logic behind the names assigned to the fields in the metric decomposition. This 4-dimensional action describes the interaction of an electromagnetic field $A_\mu$ with a real scalar $\phi$ and gravity $g_4$. Of course the interesting point about this theory was pointed out a long time ago by Kaluza and Klein where they describe the idea of using higher dimensions as a means to unify gravity and electromagnetism. The $U(1)$ symmetry of the low energy lagrangian reflects the symmetry of the internal manifold, in this case a circle. More complicated manifold compactifications can give rise to other gauge theory contents.

Here I would like to point out another aspect of this procedure that is generic of other compactifications, namely the presence of a massless scalar field in the lower dimensional theory, the field $\phi$. The value of this field controls the size of the extra dimensional space. On the other hand, these type of massless fields are clearly in contradiction with our observations since they would lead to cosmological problems as well as possible variations of the coupling constants or the presence of fifth forces between objects couple to them. All these are clearly undesirable side effects of compactification and we must avoid them. Therefore, we have to find a way to fix the values of these dangerous massless scalar fields by introducing a mechanism that generates a potential for them and allows them to be fixed at the minimum of that function.

### 2.2.2 String Theory Compactifications. Moduli fields.

There are obviously many different ways in which one can compactify a ten dimensional spacetime into four. In order to restrict the space of possibilities and at the same time obtain some theoretical control over the possible corrections of this theory, we will limit our discussion to the subset of manifolds that keep part of the original supersymmetry unbroken. As we discussed earlier, the higher dimensional theory that we start from is supersymmetric, in fact is $N = 2$ supersymmetric. If we impose that the compactification leave $N = 1$ supersymmetry unbroken in the low dimensional theory one can show that this manifold is in fact a Calabi-Yau manifold (CY) [16] . A CY is a complicated manifold and it would be hard to visualize it except for some simple cases. We therefore relegate to the Appendix some of the more technical aspects of these spaces and concentrate here on the qualitative description of the low energy effective theory that this compactifications generate.

As we explain in the Appendix, these CY manifolds can be deformed in several different ways. Some of these deformations would be such that the new manifold could still keep the CY structure. This is analogous to the situation before in our toy model where variations on the size of the internal circle would keep the structure of the low energy theory. Similarly to what is done in the Kaluza-Klein case, one can then parametrize these deformations as massless 4-dimensional fields. In particular it can be shown that that there are 3 types of fields present in the 4-dimensional theory of a CY compactification that are associated with different variations of the metric as well as other components of the forms in the higher dimensional theory:

- The complex structure fields ($z_a$), that roughly parametrize the possible shapes of the internal manifold.

- The Kahler fields ($T_i$), that control the overall size of the internal space.

- The complexified dilaton ($\tau$) that gives us information about the value of the string coupling constant.

On the other hand, the restriction to a CY compactification tells us that the low energy theory describing the dynamics of these fields should be $N = 1$ supergravity in 4d, where the moduli should be described as chiral fields.

This low energy theory can be specified by two different functions of the moduli fields (collectively called $(\Phi^I)$), the superpotential $W(\Phi^I)$, and a real Kahler potential $K(\Phi^I, \Phi^{\bar{I}})$ [17]. In absence of any D-term we can completely specify the theory by the F-term scalar potential of the form,[2]:

$$V_F = e^K \left( \sum_{IJ} K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right) \tag{2.13}$$

where

$$D_I W = \partial_I W + \partial_I K W \quad ; \quad D_{\bar{J}} \bar{W} = \partial_{\bar{J}} \bar{W} + \partial_{\bar{J}} K \bar{W} \, . \tag{2.14}$$

and $I, J$ are the indeces runnning over all the fields present in the model. The kinetic terms in this Lagrangian are specified by the metric in field space which is, in turn, obtained from the Kahler function, namely

$$\mathcal{L}_{kin} = K_{I\bar{J}} \; \partial_\mu \Phi^I \partial^\mu \Phi^{\bar{J}} \, . \tag{2.15}$$

Performing the compactification on a CY manifold one can identify the effective theory for the massless moduli fields by reading off the Kahler form for the 3 types of deformations we discussed earlier, namely [3]

$$K(\Phi_I) = K_{cs}(z_a) + K_K(T_i) + K_d(\tau) \, , \tag{2.16}$$

[2]We fix $M_P = 1$ and use the standard notation $F_I = \frac{\partial}{\partial \Phi_I}$, $F_{\bar{J}} = \frac{\partial}{\partial \Phi^\dagger_{\bar{J}}}$, $\cdots$, with $F$ being any function of the fields. Also note that indices are lowered and raised with the Kahler metric $K_{I\bar{J}}$ and its inverse $K^{I\bar{J}}$ .

[3]Note that without any other ingredient present in the compactification the superpotential vanishes, $W = 0$ so there is no potential generated for the fields.

where each term of this expression can be found explicitly from the geometry of the CY used in the compactification. In particular, the Kahler potential for the complex structure moduli can be calculated using the expression [16],

$$K_{cs}(z_a) = -\log\left(i\int_M \Omega \wedge \bar{\Omega}\right) \tag{2.17}$$

where $\Omega$ denotes the holomorphic three form and the integral is performed over the Calabi-Yau threefold $M$.

The Kahler potential for the Kahler moduli is given at tree level by,

$$K_K(T_i) = -2\log(\mathcal{V}(T_i)) \ , \tag{2.18}$$

where $\mathcal{V}$ denotes the volume of the Calabi-Yau in string units and can be expressed in terms of the Kahler moduli fields, $T_i$.

Finally, the Kahler potential for the dilaton is given by,

$$K_d(\tau) = -\log(-i(\tau - \bar{\tau})) \ . \tag{2.19}$$

Using this Kahler function one finds the Lagrangian for these scalar fields using Eq. (2.15). The result of this exercise shows that indeed a compactication on a CY leaves many fields massless. We can describe them using a $N = 1$ supergravity Lagrangian given by a complicated kinetic term, but there does not seem to be any potential for them at this level. This is a generic result of string compactifications and constitutes a serious problem that needs to be addressed. In the following we will discussed the idea of flux compactifications and how this cures the problem encountered here.

## 2.3 Moduli Stabilization

### 2.3.1 Flux Compactifications

We would like to present now a simple toy model that gives us an intuitive description of the mechanism of flux compactifications. In order to do this we will consider the following six-dimensional Einstein-Maxwell theory,[18]

$$S = \int d^6x\sqrt{-g_6}\left(\frac{M_6^4}{2}R_6 - \frac{1}{4}F_{MN}F^{MN}\right) \tag{2.20}$$

where the indexes are running from 0 to 5, $M_6$ is the 6d Plank mass. If we were to compactify this theory to 4 dimensions using as the extra-dimensional manifold a 2 sphere, we will find that the size of this sphere is not fixed by the model. In fact the curvature of the internal space would create a runaway behaviour for this scalar, an even worse situation as before[4]. In order to solve this problem we assume that the magnetic flux associated to the Maxwell field is "turned on" along the internal directions. In particular we take the ansatz,

$$A_\phi = -\frac{n}{2e}\left(\cos\theta \pm 1\right) \tag{2.21}$$

This is a magnetic monopole-like configuration on the sphere characterized by the topological number $n$. One can now perform the dimensional reduction of this theory using the following ansatz for the 6 dimensional metric,

$$ds^2 = g^6_{MN}dx^M dx^N = e^{-\psi(x)/M_p}g_{\mu\nu}dx^\mu dx^\nu + e^{\psi(x)/M_p}R^2 d\Omega_2^2 \tag{2.22}$$

where, we see again that the field $\psi$ parametrizes the size of the sphere and $g$ is the metric of the 4d space. Integrating the higher dimensional action over the internal

[4] This is slightly different than in the string theory case where the internal manifold, the CY does not have curvature, is Ricc flat, so it does not induce this runnaway potential. We will use this model even though there are some differences with the real string theory setup.

manifold one arrives to the four-dimensional effective action,

$$S = \int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R_4 - \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - V(\psi)\right) \tag{2.23}$$

where

$$V(\psi) = 4\pi M_6^4\left(\frac{n^2}{8e^2R^2M_6^4}e^{-3\psi/M_p} - e^{-2\psi/M_p}\right). \tag{2.24}$$

where similarly to what we had before we can read off the value of the Planck mass in 4 dimensions to be $M_P^2 = V_{S^2}M_6^4$ taking into account that we have denoted the volume of the sphere by $V_{S^2}$.

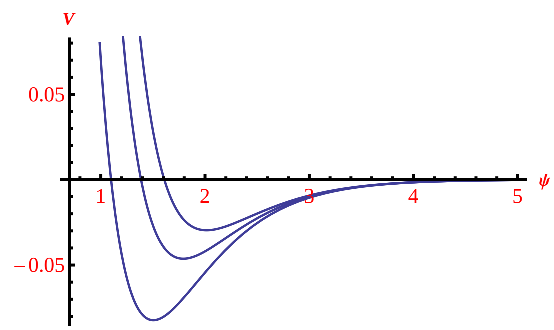


Figure 2.1: Different minima of the potential for different values of the integer n. AdS vacua.

Looking at this potential for the field $\psi$ we see that there is a big improvement over the other cases we had before. The presence of the flux introduced a term in the potential that wants to prevent the collapse of the sphere. On the other hand, the term related to the curvature of the internal space would like to reduce the size of the sphere. These two effects act against one another and one can find an intermediate point where the size of the extra-dimensions is stabilized. This is the beauty of models with fluxes. Once these fluxes are turned on along the internal space they resist their collapse and can help to stabilize the moduli.

This stable minimum for the potential is not unique, since it is parametrized by

$n$ the magnetic field flux through the internal sphere, namely,

$$n = \int_{S_2} F. \tag{2.25}$$

Changing this number one would obtain another solution with a different value of the cosmological constant at the minimum. All the possible values of the cosmological constant are negative in this model. The reason is that there is no appropriate term in the higher dimensional Lagrangian to achieve this. Adding a positive cosmological constant to the higher dimensional theory the 4-dimensional potential receives a new positive contribution, an uplifting term, such that the total potential becomes,

$$V(\psi) = 4\pi M_6^4 \left( \frac{n^2}{8e^2 R^2 M_6^4} e^{-3\psi/M_p} - e^{-2\psi/M_p} + \frac{R^2 \Lambda_6}{M_6^4} e^{-\psi/M_p} \right). \tag{2.26}$$

It is now possible to find, depending on the value of $n$, negative, zero as well as positive values of the cosmological constant at the minima of the potential. This is a very simple model of a flux landscape. We will discuss this idea much more in the following pages.

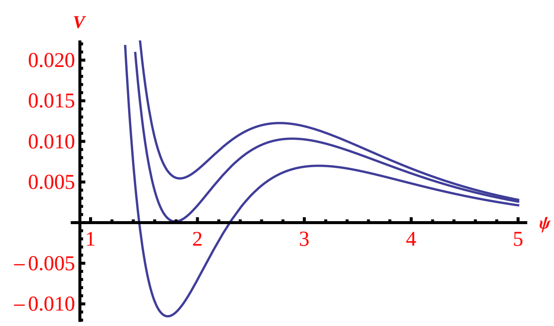


Figure 2.2: Different minima of the potential for different values of the integer n. AdS vacua and uplifted dS vacua.

## 2.4 Moduli Stabilization in String Theory.

### 2.4.1 Flux contribution.

As we describe in the Appendix a compactification of string theory on a CY gives rise to several types of cycles along the internal directions. The size of these cycles are parametrized by the values of the 4d moduli fields described earlier much in the same way as the scalar field $\psi$ parametrizes the size of the internal $S_2$ in the previous example. In particular the complex structure moduli fields ($z_a$) control the size of the 3-cycles. On the other hand, we know that Type IIB string theory has two types of 3-form fluxes that can thread those cycles. In fact one can show that those fluxes are quantized in a similar manner to the monopole case described earlier so we get a list of flux integers given by the integrals,

$$f^a_{A,B} = \frac{1}{(2\pi)^2\alpha'} \int_{A^a,B^a} F_3 \in Z \ , \tag{2.27}$$

$$h^a_{A,B} = \frac{1}{(2\pi)^2\alpha'} \int_{A^a,B^a} H_3 \in Z \ . \tag{2.28}$$

where we denote by $A^a$ and $B^a$ the 3-cycles present in a particular CY.

We can now see following the analogy with the monopole-type example that these fluxes will induce a potential for the complex structure moduli. This potential should then be described in the low energy theory as a contribution to the superpotential of the $N = 1$ lagrangian described above. This superpotential was computed by Gukov et al [19] and can be written as,

$$W_{GVW}(z_a, \tau) = \int_M G_3 \wedge \Omega = \int_M (F_3 - \tau H_3) \wedge \Omega \ , \tag{2.29}$$

where $F_3$ and $H_3$ denote the three form field strengths present in the 10 dimensional theory and $\Omega$ is the holomorphic (3,0)-form of the internal space that depends om the complex structure fields. We also notice that this superpotential includes some dependence on the dilaton field through the relation $\tau = C_0 + ie^{-\phi}$.

The remarkable thing about this is that the $N = 1$ supergravity potential generated from this superpotential and the Kahler function for these fields allows us to fix not only the complex structure moduli but also the dilaton field [17]. This is clearly an important step towards finding a completely stabilized vacuum but we can see that this superpotential is still independent of the Kahler moduli ($T_i$) so it does not fix them to a minimum. We therefore have to incorporate additional contributions to this potential from other sources.

### 2.4.2 Non-perturbative contributions

One of the important properties of supersymmetry is that the superpotential is not renormalized at any order in perturbation theory, however one can consider non-perturbative contributions. In particular, we are interested in contributions to the superpotential that have any dependence of the Kahler moduli. Remember that the Kahler moduli control the sizes of the 4-cycles of the CY, so we should look for terms in the superpotential that depend on these volumes. One such term could be coming from the gaugino condensation contribution on the 4-dimensional $SU(N)$ gauge theory living on a stack of $N$ $D7$ branes that wrap the 4-cycles in question[20]. The gauge coupling of such a theory depends on the volume of the 4-cycle $V_4$ in the following way, $g^2_{YM} \propto \frac{1}{V_4}$. The non-perturbative contribution of the gaugino condensation is then given by a superpotential of the form

$$W_{NP} = Ae^{\frac{-2\pi V_4}{N}} = Ae^{-aT_i} \ . \tag{2.30}$$

In principle there could be several of such terms such that the total contribution to the superpotential would be,

$$W_{NP}(T_i) = \sum_i A_i e^{-a_i T_i} \ . \tag{2.31}$$

### 2.4.3 KKLT model

Putting all these contributions together we arrive to the total superpotential and the Kahler potential of the form,

$$W_{total}(\Phi_I) = W_{GVW}(z_a, \tau) + W_{NP}(T_i) \tag{2.32}$$

$$K(\Phi_I) = -2\log(\mathcal{V}(T_i)) - \log(-i(\tau - \bar{\tau})) - \log\left(i\int_M \Omega \wedge \bar{\Omega}\right) \tag{2.33}$$

this specifies the $N = 1$ effective theory for the moduli completely and one can look for overall minima in this potential.

In [20] a two step process was suggested in order to look for such minima in this model. In the first part, the idea would be to stabilize the complex structure and dilaton fields alone, neglecting the non-perturbative contributions. One can then look for supersymmetric minima of the potential by finding the solutions of the equations,

$$D_{z_a} W_{GVW} = 0 \quad ; \quad D_\tau W_{GVW} = 0. \tag{2.34}$$

Once the values of the moduli were found in this way, one can evaluate the flux superpotential at this point,

$$W_0 = W_{GVW}(z_a, \tau)|_{min} \tag{2.35}$$

and use its value to stabilize the Kahler moduli using the effective superpotential,

$$W_{total}^{KKLT} = W_0 + W_{NP}(T_i) \tag{2.36}$$

Solving the supersymmetric equations for the Kahler moduli given by,

$$D_{T^i} W_{total}^{KKLT} = 0. \tag{2.37}$$

one can find a minima of the potential for the Kahler moduli directions. This method is of course an approximation, based on the idea that there should be a separation of scales between the masses of the complex structure and the dilaton and the ones of the Kahler fields. One should therefore check its validity by looking at the full potential to investigate the persistence of these minima there. We will do this in the following Chapter.

### 2.4.4 Uplifting the potential

Looking at the form of the F-term potential for supergravity one can see that the value of the cosmological constant at supersymmetric minima is given by,

$$V_{|_{D_I W=0}} = e^K \left(-3|W|^2\right) . \tag{2.38}$$

so it is clear that most of these minima would have a negative value of the cosmological constant[5]

On the other hand, our universe seem to located at a minimum dominated by a positive cosmological constant. On the other hand, it is clear that this would require some mechanism of supersymmetry breaking. It is therefore interesting to discuss the presence of other terms in the 4d effective potential and how they help to achieve this goal. One can, for example, introduce an anti-D3-brane in the compactification that explicitly breaks supersymmetry adding a positive contribution to the potential of the form,

$$V_U = \frac{D}{\mathcal{V}^2} , \tag{2.39}$$

where $\mathcal{V}$ denotes as before the volume of the CY, and can be written in terms of the Kahler moduli $T_i$ and $D$ is a constant that depends on the tension of the brane as well as its location in the internal manifold.

One can arrive to similar terms in the effective potential by other mechanisms like D-terms or uplifting by other sectors of the theory by an F-term [21].

[5]One can also obtain a zero value of the cosmological constant by tuning $W = 0$ at the supersymmetric minimum.

On the other hand, the effective theory we described above will in general receive corrections from different sources. A particularly interesting contribution to the effective potential comes from perturbative correction to the Kahler potential originated from $\mathcal{O}(\alpha'^3)R^4$ terms to the 10d type IIB supergravity action. The leading correction has been calculated in some concrete examples [22] and the corrected Kahler function becomes,

$$K_K^{\alpha'}(T_i, \tau) = -2\log\left[\mathcal{V}(T_i) - \frac{\xi}{2i}\left(-i\left(\tau - \bar{\tau}\right)\right)^{3/2}\right] \tag{2.40}$$

where $\xi = -1.3i$. These type of corrections are normally supressed in the large volume supergravity limit but in this case they play a significant role in the structure of the flux vacua. Perhaps the most important realization of this effect is the existence of de Sitter solutions in a theory with this type of corrections included without any other uplifting term [23]. Supersymmetry is spontaneously broken in these vacua and can be therefore studied within the same supergravity Lagrangian without introducing any new external elements. This is however not the only effect induced by these terms. They also lead to the co-existence of multiple non-supersymmetric Anti-deSitter (AdS) vacua, one continuously connected to the de Sitter solutions and another one located at exponentially large volume [24]. In the following we will describe these new vacua in detail in connection to our numerical examples.

# Chapter 3

# A corner of the String Theory Landscape

## 3.1 The origin of the Landscape in String Theory

In the previous Chapter we showed that a combination of fluxes threading the cycles of the internal manifold and non-perturbative effects could stabilize the moduli fields of a CY compactification. On the other hand, different combinations of the fluxes leads to distinct vacua, similarly to what happened in our simple 6d toy model. This provides us with a huge set of configurations stabilized at different values of the parameters and therefore different 4-dimensional physics. The set of all possible vacua constitutes the String Theory Landscape. One can easily see that the number of combinatoric possibilities is extremely large maybe as large as $10^{500}$ [28].

This has inspired people to look at the landscape from an statistical point of view. This type of method has been explored in a number of papers [29, 30, 31, 32, 33, 34] and some general conclusions have been reached. Our work is in some sense complementary to those studies. We explore a particularly simple CY manifold in the most explicit possible way. Our goal is to look for solutions of the full system of equations for all the the moduli fields together using the type of constructions we have described in the previous Chapter.

## 3.2 The moduli space for the $P^4_{[1,1,1,6,9]}$ Calabi-Yau.

In order to investigate the potential for all fields simultaneously we will focus our attention in a particularly simple CY, the orientifold of the $P^4_{[1,1,1,6,9]}$ manifold. This CY threefold has 2 Kahler moduli and 272 complex structure moduli. However we will restrict ourselves to a 2 dimensional slice of the complex structure moduli that can be obtained by imposing a particular symmetry on the manifold. (See [36, 37, 38] for more details on this manifold). This is of course a very small number of complex structure fields and it is by no means representative of a typical CY. However we choose to work with this relatively small number of moduli so we can explicitly perform the numerical calculations in a reasonable amount of time. For other numerical investigations of the complex structure sector see [31].

Taking this particular set of complex structure fields together with its 2 Kahler moduli and the complex dilaton field makes the dimension of the moduli space a 10 dimensional real field space. This is a tractable number of fields to analyse numerically and we have been able to explore a large set of minima of the full 10d supergravity potential without any further approximations.

Several other groups have studied this model in detail using some combination of numerical and analytical approaches[24, 25, 26, 27, 31, 32, 33, 34, 38], here we would like to explore this model in full detail in order to confirm the validity of those kinds of approximate methods. We start our discussion of this model by giving explicitly the form of the Kahler function and superpotential that we will be using in our numerical work.

Following our discussion on Chapter 2 one can completely specify the dynamics of the moduli space by finding the Kahler function and the superpotential as a function of the moduli fields. Focusing our attention to a particular CY, allows us to identify the Kahler function for all the fields. Let's describe this function in our case.

### 3.2.1 Complex Structure

As we described in general earlier, the Kahler potential for the complex structure moduli can be calculated using the Eq. (2.17). This expression can be casted in a different way [16], namely

$$K_{cs} = -\log\ (-i\Pi^\dagger \cdot \Sigma \cdot \Pi) \tag{3.1}$$

by introducing a simplectic matrix $\Sigma$,

$$\Sigma = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} \tag{3.2}$$

and the period vector,

$$\Pi(w) = (w^a, F_a) \tag{3.3}$$

whose components encode the geometrical information of the complex structure and are defined in terms of integrals of $\Omega$ over the tree-cycles, $A^a, B_a$,

$$w^a = \int_{A^a} \Omega \qquad F_a = \int_{B_a} \Omega\ . \tag{3.4}$$

$F_a$ are related to the prepotential $F$ by the relation, $F_a = \partial_a F(w)$. The complex structure moduli fields $z_a$ are then obtained via the normalization condition, $z_a = \frac{w^a}{w^0}$ so, having the expression for the prepotential in terms of the $w^a$ coordinates one can write the Kahler potential as a function of the complex structure fields, $K_{cs}(z_a)$.

In [36, 37, 38], the form of the prepotential was computed for the $P_{11169}$ model.

The result, neglecting instanton corrections is given by,

$$F = (w^0)^2\mathcal{F} = (w^0)^2\left(\frac{1}{6}(9z_1^3+9z_1^2z_2+3z_1z_2^2)-\frac{9}{4}z_1^2-\frac{3}{2}z_1z_2-\frac{17}{4}z_1-\frac{3}{2}z_2+\xi\right) \tag{3.5}$$

where we use the following normalization for the complex structure fields, $z_a = \frac{w^a}{w^0}$, and we have also defined

$$\mathcal{F} = \frac{1}{6}(9z_1^3+9z_1^2z_2+3z_1z_2^2)-\frac{9}{4}z_1^2-\frac{3}{2}z_1z_2-\frac{17}{4}z_1-\frac{3}{2}z_2+\xi\ . \tag{3.6}$$

With this prepotential and using the normalization of $w^0 = 1$, we can now compute the vector periods,

$$\Pi = (1, z_1, z_2,\ 2\mathcal{F}-z_1\mathcal{F}_1-z_2\mathcal{F}_2,\ \mathcal{F}_1,\ \mathcal{F}_2). \tag{3.7}$$

Using Eqs (3.1) and (3.2), the Kahler function for the complex structure moduli in terms of $z_1, z_2$ is given by,

$$K_{cs}(z_1,z_2) = -\log\left[i\left((z_1-\bar{z}_1)(\mathcal{F}_1+\bar{\mathcal{F}}_1)+(z_2-\bar{z}_2)(\mathcal{F}_2+\bar{\mathcal{F}}_2)-2(\mathcal{F}-\bar{\mathcal{F}})\right)\right] \tag{3.8}$$

which in our case becomes, using $z_i = X_i + iY_i$,

$$K_{cs}(z_1,z_2) = -\log\left[4Y_1(3Y_1^2+3Y_1Y_2+Y_2^2)-4i\xi\right] \tag{3.9}$$

following [37] we use $\xi = -1.3i$.

### 3.2.2 The Kahler moduli

The form of the Kahler for the Kahler moduli with $\alpha'$ corrections takes the form described earlier,

$$K_K^{\alpha'}(T_i,\tau) = -2\log\left[\mathcal{V}(T_i)-\frac{\xi}{2i}\left(-i\left(\tau-\bar{\tau}\right)\right)^{3/2}\right] \tag{3.10}$$

where the volume is parametrized in terms of the Kahler fields as,

$$\mathcal{V}(T_i) = \frac{\sqrt{2}}{18}\left(\left(\frac{T_2+\bar{T}_2}{2}\right)^{3/2}-\left(\frac{T_1+\bar{T}_1}{2}\right)^{3/2}\right) \tag{3.11}$$

### 3.2.3 Superpotential

The superpotential generated by the fluxes can be computed using the GVW expression in Eq. (2.29). The result can be written in terms of the dilaton, the complex structure moduli and the integer fluxes through the $A$ and $B$ 3-cycles as,

$$W_F = \sum_{a=0}^{2} [(f_A^a - \tau h_A^a)F_a - (f_B^a - \tau h_B^a)z_a] \tag{3.12}$$

where we have defined

$$f_{A,B}^a = \frac{1}{(2\pi)^2\alpha'} \int_{A^a,B^a} F_3 \in Z \ , \tag{3.13}$$

$$h_{A,B}^a = \frac{1}{(2\pi)^2\alpha'} \int_{A^a,B^a} H_3 \in Z \ . \tag{3.14}$$

Note that this is a complicated function of the complex structure fields through the dependence of $F_a(z_a)$.

Furthermore we will also assume the presence of a non-perturbative superpotential of the form given by Eq. (2.31) whose coefficients $A_i, a_i$ do not vary across the landscape. One could also make this coefficients vary according to some particular statistical prescription but we have not consider this possibility here.

## 3.3 The one Kahler moduli case

We start our numerical exploration by first looking at a slightly simpler model where we will reduce the number of Kahler moduli to one. The Kahler function in this case reduces to,

$$K_K^{\alpha'}(T,\tau) = -2\log\left[g\left(T+\overline{T}\right)^{3/2} - \frac{\xi}{2i}\left(-i\left(\tau-\bar{\tau}\right)\right)^{3/2}\right] \tag{3.15}$$

This simplification will allow us to use some semi-analytic results that will guide us in our search for vacua of the full multifield potential.

### 3.3.1 A KKLT type of construction

The first step in the construction of the KKLT type of vacua involves the identification of supersymmetric points in the complex structure and dilaton sector across the landscape. This has been studied using statistical techniques by [72] and it was argued that one should expect an uniform distribution of values of the superpotential in the supersymmetric vacua sector. In order to check this results we have recently explored in [39] the supersymmetric vacua of the reduced moduli space of the $P_{11169}$ CY explained in the previous section. Here we give a brief summary of the main results disccussed in that paper.

The procedure to find these points is simple, we start with a solution of the equations,

$$D_{z_a} W_{GVW} = 0 \quad ; \quad D_\tau W_{GVW} = 0 \tag{3.16}$$

in a region that respects all the contraints on the fields.(See a description of these constraints below). Having found one such solution we then jump around in the space of flux integers and obtain the new vacua. This procedure allows us to move slowly but potentially very far away from the initial point in the space of fluxes. We can also repeat this procedure several times using different initial points. Finally we calculate the values of the superpotential $W_{GVW}$ at those points in field space and plot the vacua whose superpotential is within a small region around the origin.

The values of the fluxes can not be totally arbitrary, they are constrained by several technical requirements. The first one is due to the tadpole condition, which basically imposes that the total $D3$ brane charge be zero. In practice, this creates a upper limit on a combination of the fluxes over the internal manifold. Here we will follow Douglas et al. [37] and will only consider the sets of fluxes restricted to the condition,

$$N_{flux} = \frac{1}{(2\pi)^4 \alpha'^2} \int_M F_3 \wedge H_3 < 350 \ . \tag{3.17}$$

Also, we will only keep fluxes whose vacua are stabilized at a large value $Im(\tau)$. This will make sure that we will find these vacua in the weak coupling regime. Similarly, in order to disregard instanton corrections in the prepotential in Eq. (3.5) we will only use fluxes that lead to $Im(z_i) \geq 1$ for $i = 1, 2$. Finally we are only considering fluxes that are inequivalent, meaning our set of the fluxes are not related by a $SL(2, Z)$ symmetry transformation. This prevents us from overcounting the number of fluxes in the region of interest for us.[1]

Here we give in Fig. (3.1) a little sample of the vacua generated following this procedure. We focus on the region of the superpotential around the origin on the complex plane where we have checked that the distribution is perfectly uniform. This agrees with the statistical arguments presented in [72]. This result is different from the one found in [33] where the distribution is peaked around the origin of the plane even for a small number of complex structure fields that we have in our case. A similar uniform distribution to the one found here and in [39] has been reported in [40].

In the following we will introduce more fields or other contributions to the potential or solve a different set of equations but we will always use essentially the same procedure to explore the flux landscape.

We now want to move on to find vacua of the full potential where we will treat all the fields in the same way. This is obviously a much harder problem numerically, but we will use our knowledge of the supersymmetric solutions presented above as a guide to lead the numerical search of vacua.

[1] These requirements reduce the number of valid minima from the naive calculation based on the uniform distribution of vacua on the $W_0$ plane since, sometimes, many of these vacua violate some of these restrictions. We have not seen however that this would produce any voids in the $W_0$ distribution, so the argument is still basically valid.

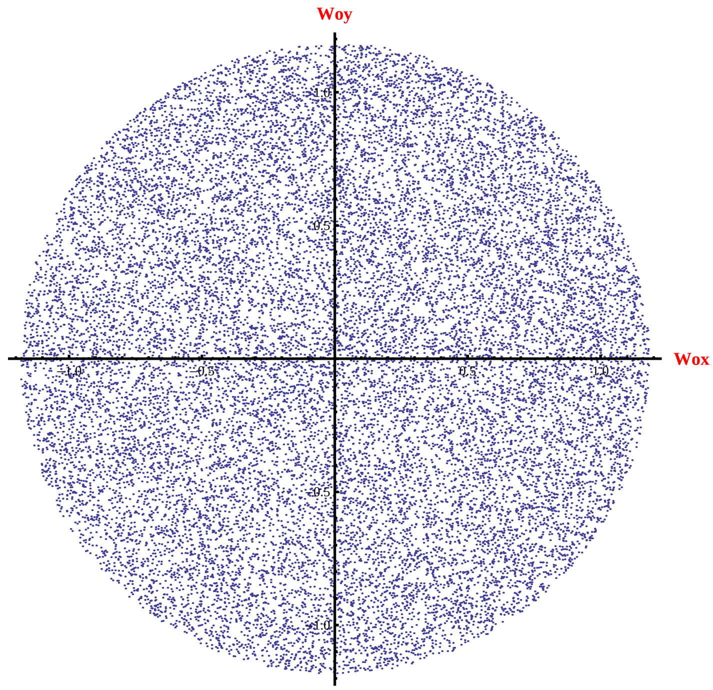


Figure 3.1: Uniform distribution of the superpotential in the complex plane for the supersymmetric flux vacua. Here we use the complex structure and dilaton fields and present the results for large number of sets of flux integers.

#### 3.3.1.1 A fully supersymmetric sector

The first exercise we want to do is to study the supersymmetric sector in this KKLT theory without uplifting. Without the non-perturbative term in Eq. (2.31), the superpotential would be independent of the Kahler moduli and we know that in this case the theory is of the no-scale type, in other words that, $\sum_I K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} = 3|W|^2$, where $i, j$ are the indices associated with the Kahler fields. This means that potential for the complex structure moduli becomes, $\sum_{a,b} K^{a\bar{b}} D_a W D_{\bar{b}} \bar{W}$. Looking at this expression we realize that the structure of this effective theory is such that any supersymmetric point would be found at zero value of the cosmological constant and furthermore it would also be stable due to the fact that any deviation from the supersymmetric point would increase the value of the potential[2]

Adding the non-perturbative term changes the distribution of values of the cosmological constant. Having only one exponential term in the superpotential would force the supersymmetric minima to be AdS. This also affects the stability issue of the supersymmetric point since an AdS vacua in supergravity could have tachyonic directions for the potential [41]. This was indeed found to be the case in simple truncations of our model that includes the dilaton field but freezes the complex structure fields [42]. So this is definetely a potential problem for this model.

Furthermore, it has been recently argued that a typical (random) N=1 supergravity theory with many chiral fields will lead to supersymmetric AdS vacua with unstable directions of the kind described above [43]. The authors found that the number of such vacua increases rapidly with the number of fields, so taking into account that we are studying an 8 dimensional field space we may expect this to happen in our case quite frequently.

We have explored this issue in our 8 dimensional model using the supersymmetric equations for all the fields,

$$D_{z_a}W_{total} = 0 \quad ; \quad D_{\tau}W_{total} = 0, \quad ; \quad D_{T_i}W_{total} = 0 \tag{3.18}$$

where we have taken the parameters of the non-perturbative potential to be, $a = \frac{2\pi}{70}; \quad A = 1.2$[3] Using the same type of procedure as we described above we generate a large number of fluxes and obtain a similar uniform distribution of vacua over the superpotential plane. (See Fig.(3.2)). Increasing the value of the superpotential decreases the volume of the internal manifold in the vacuum. Eventually the volume becomes small enough that one can not trust the supergravity approximation sincer

[2]This is true, except in the Kahler direction, of course, where the potential is flat at those points. This is why one should introduce the non-perturbative terms in the first place.

[3]We expect the values of these parameters to be dependent of the complex structure moduli but in the following we will assume that this is not a strong dependence and keep them as constants.

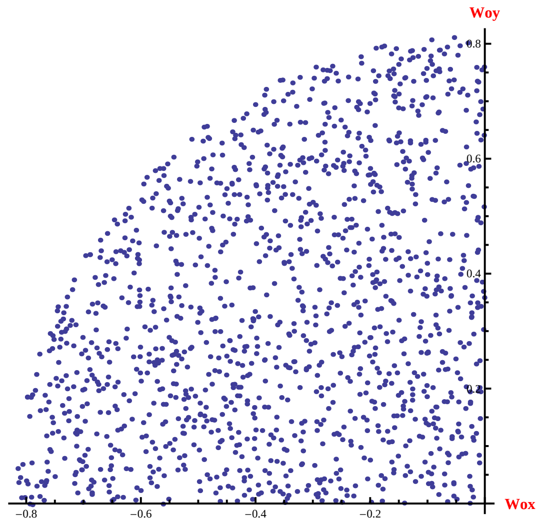


Figure 3.2: Distribution of the flux superpotential for the supersymmetric vacua using eight moduli fields. We show only the solutions found in one quadrant of the complex plane, the other ones are basically the same.

we will have to include other corrections into our action. There is therefore a natural cutoff for the value of the flux superpotential that one can use for this stabilization mechanism. This is roughly the maximum value displayed in Fig. (3.2).

Having found all these supersymmetric points, we can study the distribution of values of the cosmological constant in these vacua. The values of the F-term potential, Eq. (2.13), at a supersymmetric minima are given by the expression,

$$V_{|_{D_I W=0}} = e^K \left(-3|W|^2\right). \tag{3.19}$$

It is clear that the distribution of values of the cosmological constant would be given by the competition between the values of the superpotential and the values of the Kahler function. Note that in this case, it is the total Kahler function and therefore the backreaction on the total volume becomes important for this calculation (See Eq. (2.18). We show in Fig. (3.3) the distribution of cosmological constant values in this model. We notice that it is peaked at zero with a long tail in the negative direction.

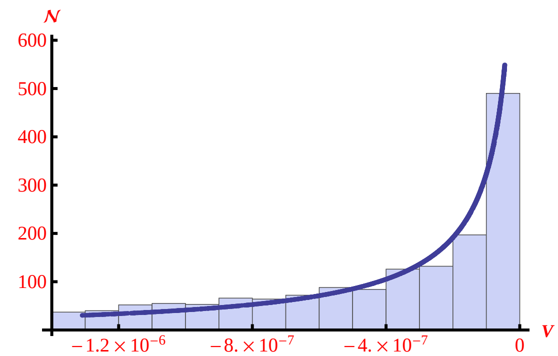


Figure 3.3: Distribution of cosmological constant values for the supersymmetric sector.

We can explain the origin of this distribution by going back to the 2 step stabilization mechanism and calculate using this approximate methods the distribution of the cosmological constants in that case. This was done in [34] with the assumption that the value of the $W_0$ would be uniformly distributed on the plane. This is indeed the case for us, and therefore we can compare our distributions to the one calculated in this way with the Kahler field only.

On the other hand, taking into account that we only allow the superpotential $W_0$ to change as we move around in this landscape (meaning we do not allow $A$ or $a$ to vary from vacuum to vacuum), finding the distribution of cosmological constants knowing the distribution of $W_0$ can be thought of as a somewhat complicated chage of variables. We have performed numerically this change of variables and the resultant distribution shows a good agreement with the data collected in the full problem. This suggests that integrating out the complex structure and dilaton fields and substituting this sector by a uniformly distributed superpotential is indeed a good approximation for this kind of models.

Finally we also compute the masses of the small perturbations around the minimum of the potential for all fields by calculating the eigenvalues of the matrix,

$$N^I_J = g^{IK}\left(\partial_K \partial_J V - \Gamma^L_{KJ} \partial_L V\right) \tag{3.20}$$

where $g_{IJ}$ is the metric in field space and $\Gamma^L_{KJ}$ is the connection computed from this metric. This matrix generalizes the expression for the Hessian to account for non canonical kinetic terms.[4] We have found that all the masses in our vacua are positive which means that there is never an unstable direction in our KKLT construction. This is probably due to the fact that our theory is not a typical representative of a random supergravity but a small deformation of the no-scale type and therefore generic conclusions do not apply in this case. This was also noted recently in a different model in [34].

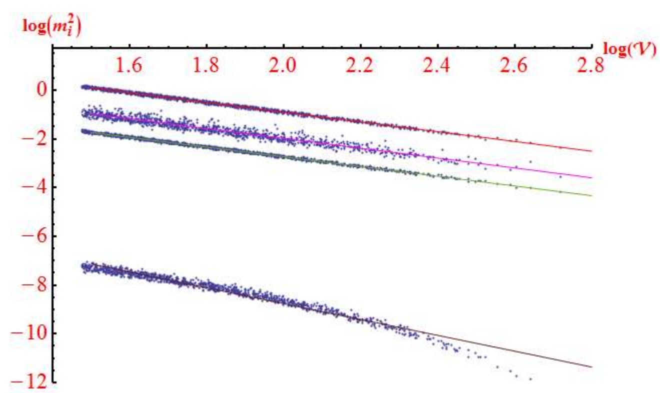


Figure 3.4: Distribution of the masses of the moduli as a function of the internal volume.

We show in Fig. (3.4) the values of the masses as a function of the volume of the internal manifold. One can clearly see the hierarchy of masses between the complex structure and dilaton fields on one side and the Kahler fields on the other. Their dependence with the volume of the CY agrees with the theoretical expectation of a

[4]Note however that this expression gets greatly simplified in our case since we will only be interested in using it at critical points where the second term dissapears.

large volume approximation.

#### 3.3.1.2 Non-supersymmetric sector with explicit uplifting

One of the most important points of the KKLT type of constructions is the fact that one can get de Sitter solutions of the 4d effective potential by using an uplifting term given by Eq. (2.39). On the other hand, it has been argued that this procedure could introduce tachyons in the uplifted full potential [42] ruining the stability of the de Sitter solutions. In order to investigate this idea we introduce an uplifting term in our potential and solve the non-supersymmetric equations for all the fields, in this case we solve the 8 real directions in field space, namely,

$$\partial_I V_F = 0 \ . \tag{3.21}$$

This procedure allowed us to find many de Sitter solutions with a range of values for the cosmological constant. Note that finding these solutions numerically is harder than the supersymmetric ones and in many cases, it requires a special care in choosing the initial guess for the field values. Our strategy to find these solutions was to first use the KKLT two step method and use the solutions found this way as the initial guess to identify the appropiate solution for the non-supersymmetric equations. In most cases, these solutions did not deviate substantially from the KKLT ones.

The distribution of values for the cosmological constant in these vacua is perhaps not so important, since it depends on the particular value of the parameter $D$ which for simplicity we fix in all the cases studied. In a real landscape this parameter should also vary from vacua to vacua.

We have also studied the eigenvalues of the fluctuations around the de Sitter minima in this case and we found them to be all positive, showing explicitly that one can construct stable de Sitter vacua of the full potential in this way. See Fig

(3.5).

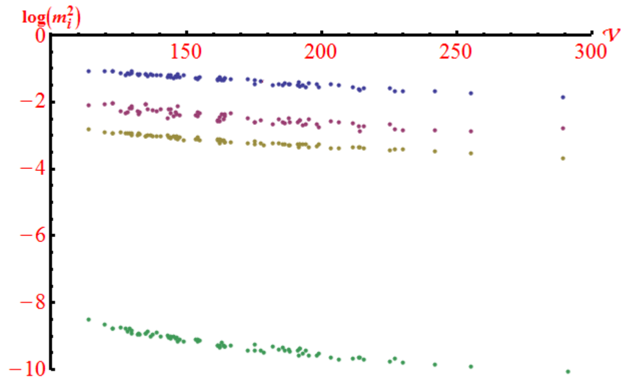


Figure 3.5: Distribution of masses for the uplifted de Sitter solutions.

### 3.3.2 Including $\alpha$' corrections

#### 3.3.2.1 Supersymmetric sector

We would like now to explore the superpersymmetric solutions that one can find by introducing an $\alpha'$-correction to the Kahler potential. Following the same procedure as before we generate a large sample of fluxes that together with the parameters in the non-perturbative terms fix the potential to minimize. In order to find a supersymmetry preserving vacua, we solve the 8 dimensional set of Eqs. (3.18) where we have now included the corrections in the Kahler function.

We plot in Fig (3.6) the values of the superpotential for these supersymmetric vacua. We see that again this distribution is uniform but it is now cut at a smaller value due to the presence of the $\alpha'$ term in the Kahler potential. The reason for this is that, similarly to what happens in the pure KKLT model, increasing the superpotential one decreases the volume of the CY in the solution. Eventually, this volume decreases so much that it is equal to $\xi$ and the supersymmetric solution dissapears since the potential is singular at this point. This behaviour of the supersymmetric solutions was first identified in [23] using analytical arguments in the approximation where one freezes the complex structure moduli and dilaton to their supersymmetric minima.

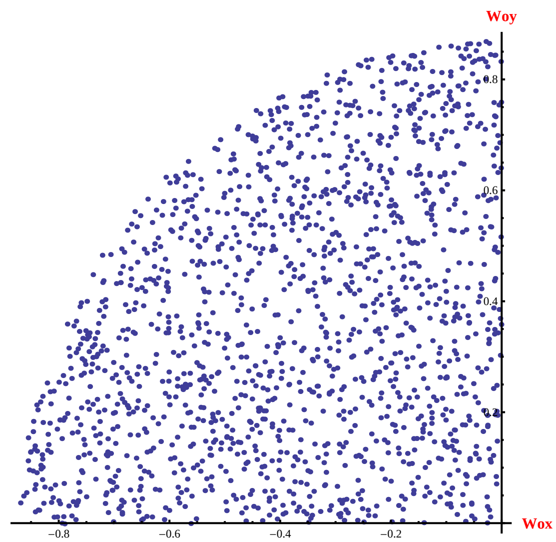


Figure 3.6: Distribution of the flux superpotential in the complex plane

In this case, the F-term potential for the Kahler moduli ($T$), computed with the $\alpha'$ corrections is given by,

$$V_F^{\alpha'} = e^K \left( K^{T\bar{T}}[W_T \bar{W}_T + (W_T \overline{K_T W} + cc.)] + 3\xi \frac{\xi^2 + 7\xi\mathcal{V} + \mathcal{V}^2}{(\xi + 2\mathcal{V})^2(\mathcal{V} - \xi)} |W|^2 \right) \quad (3.22)$$

where we have assumed that $K$ and its matrix elements $K^{T\bar{T}}$ have been computed taking into account all the moduli fields and then evaluated at the point where $D_{z^a}W = 0$ and $D_\tau W = 0$. Considering our non-perturbative superpotential given by Eq. (2.31) one can see the behaviour of this function for different values of the flux superpotential $W_0$. We see in Fig. (3.7) that the potential has a minimum at negative values of the cosmological constant. The value of the volume $\mathcal{V}$ at the minimum decreases as we increase the value of $W_0$. One can show that all these minima correspond to supersymmetric solutions. Eventually, the minimum approaches the barrier $\xi = \mathcal{V}$ and the supersymmetric solutions cease to exist. This determines the critical value of the flux superpotential beyond which we do not get any other supersymmetric solution.

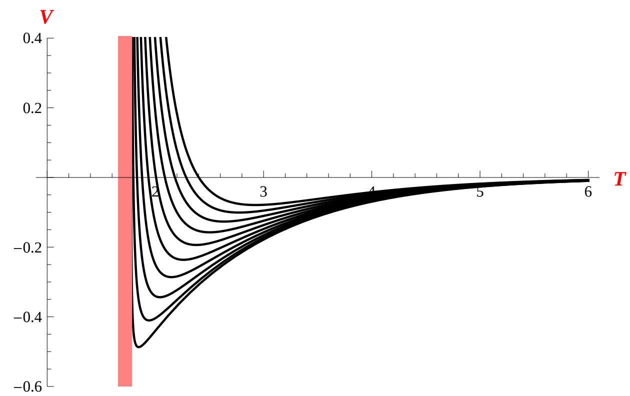


Figure 3.7: We plot in this figure the form of the potential for different values of $W_0$ as a function of the Kahler moduli that controls the volume of the CY. Each potential has a minimum at a negative value of the cosmological constant and their minima aproach the limiting region of $\mathcal{V} = \xi$ (the vertical line) as we increase the value of $W_0$.

Here we have explored the whole set of supersymmetric equations, with the possible backreaction of the variation of the values of the complex structure and dilaton on the equations for the Kahler. However the results are pretty similar to the simple toy model described in the case where one only considers the real Kahler field $T$. We plot in Fig. (3.8) the dependence of the volume and the potential with the absolute value of the superpotential for the set of vacua explored in this case. We see a remarkable agreement with the simple one field model described earlier.

Having the full potential at our disposal we can now obtain the distribution of the masses of the complex structure, dilaton as well as the Kahler moduli for this set of vacua. See Fig (3.9). Note that we do not find any vacua with negative

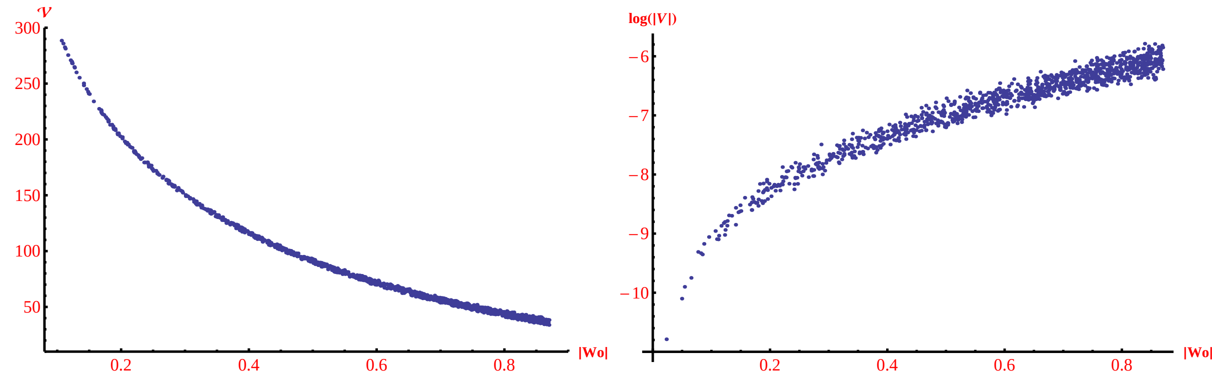


Figure 3.8: Comparison of the supersymmetric sector with the single field approximation. We ilustrate the dependence with the absolute value of the flux superpotential of the cosmological constant (on the right) and of the volume of the CY (on the left).

eigenvalues in this model either and that the masses scale as expected with the volume of the CY.

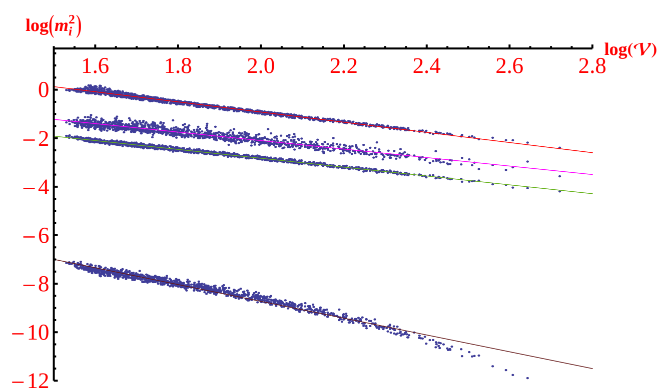


Figure 3.9: Distribution of the masses of the moduli as a function of the internal volume.

Finally we can also obtain the distribution of the values of the cosmological constant for these supersymmetric anti-deSitter vacua. We see that again this probability distribution is peaked at zero. This is easily understood once we realize that the model is described pretty accurately using the description in terms of the Kahler moduli alone. In this case, one can fix the dilaton and the other moduli to obtain the parameters in Eq. (3.22) and find the distributions for cosmological constants assuming an uniform one for the flux superpotential. We show in Fig. (3.10) that this procedure offers a pretty good fit to the real data in the full 8 dimensional space of fields.

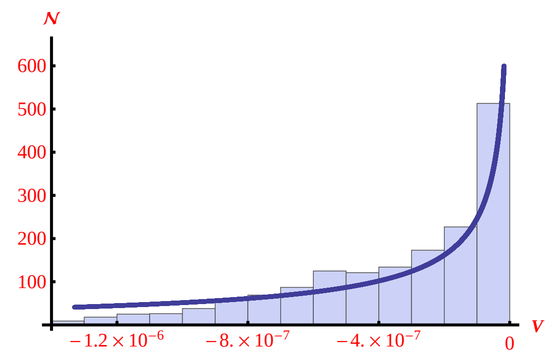


Figure 3.10: Distribution of the cosmological constants for the supersymmetric sector.

#### 3.3.2.2 Non-supersymmetric sector with Kahler uplifting

As we described earlier the supersymmetric equations stop giving solutions when one reaches a critical value for the volume of the internal manifold, namely, $\mathcal{V} = \xi$. As we showed in the previous section this boundary of supersymmetric solutions is located at a particular value for the modulus of the flux superpotential, $W_0$. On the other hand this does not mean that there are no solutions beyond this point in the $W_0$ plane. In fact, it was argued [23] that a non-supersymmetric solution should exist for larger values of $W_0$. We can see that this is indeed the case in our simple single Kahler moduli example. We plot in Fig (3.11) some examples of these non-supersymmetric solutions for $W_0 > W_c$.

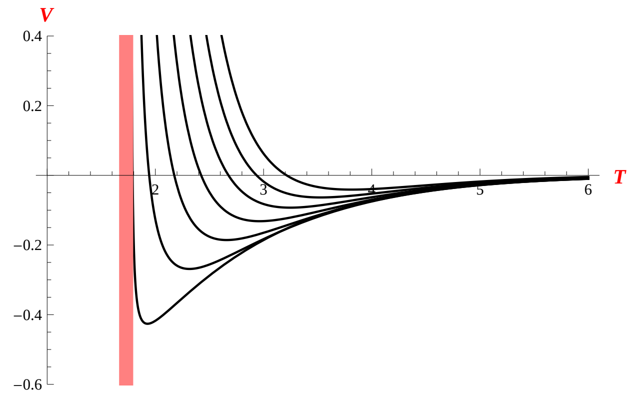


Figure 3.11: We plot the potential for values of $W_0 > W_c$. The minima of the potential ocurrs for larger values of the volume as one increases $W_0$.

Here we explore this region of the landscape in the full 8 dimensional field space. In order to do this we start by searching for flux integers that would give us a solution in this part of the $W_0$ plane. Since we are now dealing with non-supersymmetric equations we need to solve the full equations for the fields, namely,

$$\partial_I V_F = 0 \ . \tag{3.23}$$

These are much harder vacua to find numerically even starting with a pretty good guess of the solution the procedure takes much longer to find the final multifield critical point. We have managed to find a few hundred of those solutions and we can see that they behave similarly to what the single field model described above would predict. There is however considerably more ”noise” in this data due perhaps to the larger variation of the dilaton from vacuum to vacuum. This is of course not captured by the simple single field model. (See the comparison in Fig. (3.12))

We also find that all these points are true minima where all the eigenvalues are positive. It is likely that there are other critical points in this potential but it would be very hard to find them by randomly choosing some initial conditions in this large field space. We only manage to find our solutions because we know where to look for them.

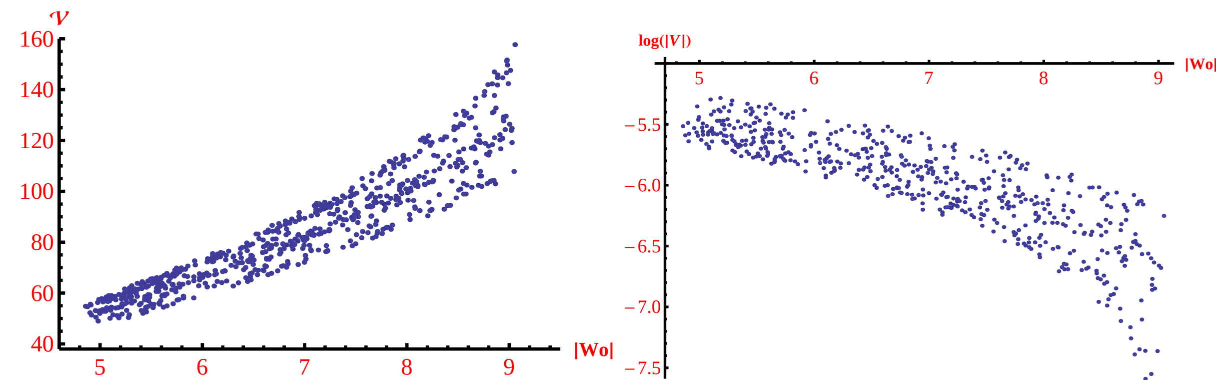


Figure 3.12: Comparison of the supersymmetric sector with the single field approximation. We ilustrate the dependence with the absolute value of the flux superpotential of the cosmological constant (on the right) and of the volume of the CY (on the left).

Finally, it was noted in [23] that the Kahler corrected potential approaches zero in the limit of large volume from above, meaning from the positive side. This means that one should be able to find a de Sitter minimum as one increases the value of $W_0$. This is illustrated in our single field model in Fig. (3.13). Looking at this figure that one can see that there is a fairly small window of values of $W_0$ before the potential develops a runnaway behaviour towards infinite volume. On the other hand, assuming that the uniform distribution of values of the flux superpotential extend all the way to this region, one should be able to find many such vacua.

We have search our 8 dimensional field landscape for these vacua and we have been able to find them by basically selecting flux numbers that could give us values of the $W_0$ in the region of interest. We see in Fig (4.6) that indeed increasing the value of the supepotential one can reach such vacua. Also these are true minima of the full potential, meaning their spectrum of fluctuations are all positive.

We do not have enough data to do a histogram of the cosmological constant values in the non-supersymmetric case for the full theory . We therefore use the

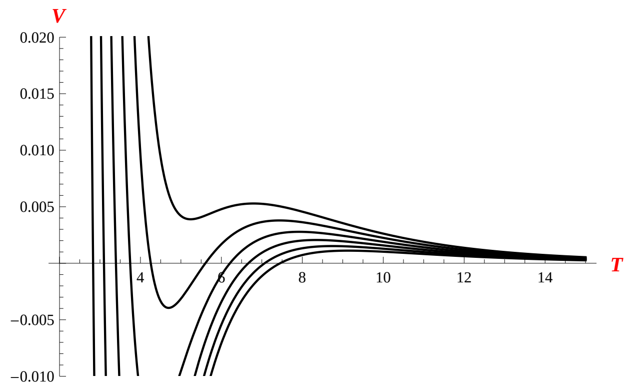


Figure 3.13: Increasing the value of $W_0$ even further one obtains a set of de Sitter solutions for the potential minima.

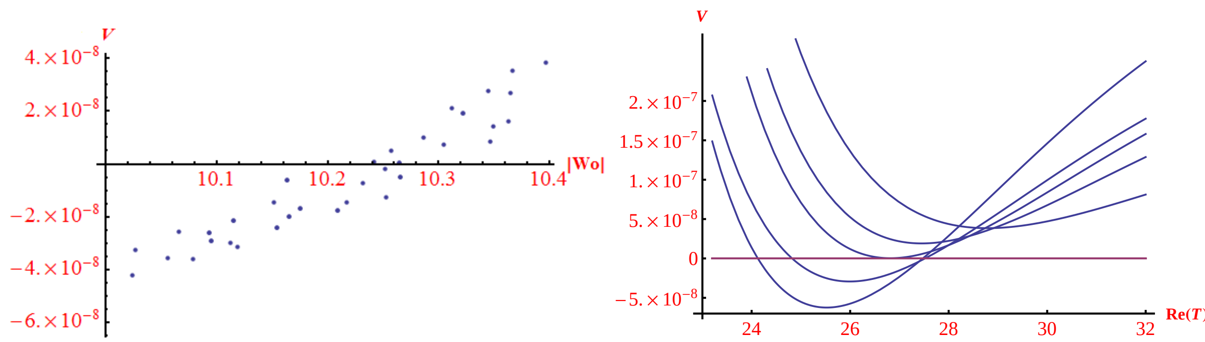


Figure 3.14: Transition from AdS to dS solutions of the non-supersymmetric vacuum solutions of the full potential.

relation to the single field model to simulate what type of histogram one would get assuming an uniform distribution of vacua in this $W_0$ region. We show in Fig. (3.15) the result of such simulated data.

This distribution includes the tiny region of positive cosmological constants buried in the last bin. It is useful to zoom in this part and look at the distribution in the positive cosmological constant side. We show this in Fig. (3.16).

This is a different distribution from the one found in [33]. The reason is due to the fact that we do not assume that the coefficient $A$ in the non-perturbative superpotential is a random variable as they do but keep it constant.

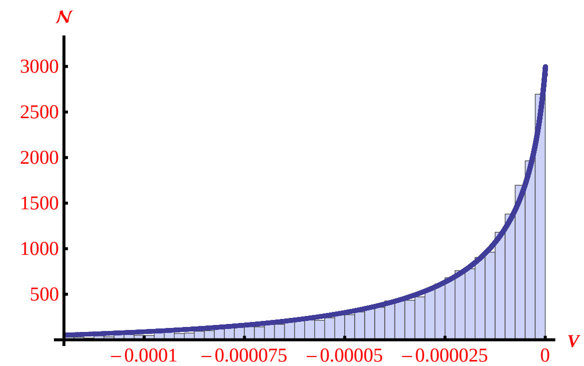


Figure 3.15: Distribution of the cosmological constants for the non-supersymmetric case.

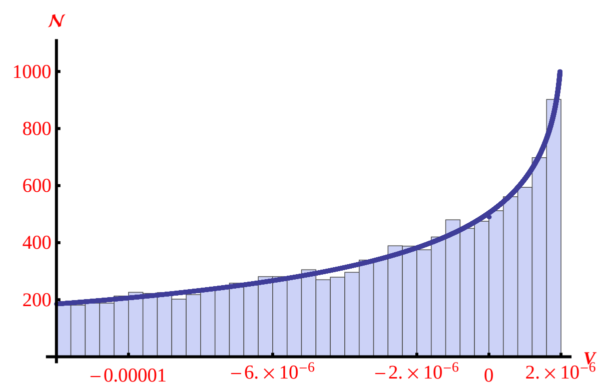


Figure 3.16: Distribution of the cosmological constants for the non-supersymmetric case, with also positive values of the potential.

The results we obtain in this Chapter agree remarkably well with the idea that the landscape of vacua can be described using a 2 step process. In the first step the complex structure and the dilaton are fixed at some value uniformly distributed in the superpotential plane. One can then use this value of $W_0$ to find the supersymmetric vacua of the Kahler moduli fields. The results of this procedure seem to give a very accurate description of the full treatment described here.

# Chapter 4

# Accidental Inflation in the Landscape

## 4.1 Inflation in string theory

As we mentioned earlier, the idea that our universe has undergone a period of inflation seems to fit the current observations of the cosmic microwave background [5]. However, these observations have not taught us, so far, much about its fundamental origin. It is therefore interesting to analyze the different scenarios that can give rise to inflation within a fundamental theory with the hope that one could identify a distinctive signature that allow us to understand the relevant dynamics that were in play in these early stages of cosmic history.

These ideas have led many people to investigate the cosmological consequences of different high energy models and, in particular, string theory (See, for example, some of the reviews on the subject in [45]). On the other hand, it is clear that any model of inflation within string theory should first address the question of compactification. With the arrival of the new ideas in models of flux compactifications like the ones described in previous Chapters and the realization that string theory may lead to a large set of possible vacua, it is obvious that one should rethink the possible scenarios of inflation within this setup.

In particular, the models we discussed in Chapter 3 indicate the existence of perturbatively stable vacua of the theory where we could find ourselves today [20, 47]. Furthermore, this method of compactifying leads to a immense landscape of distinct vacua [28, 48, 49]. Some of these vacua would be similar to our four dimensional universe, but others would have very different properties, for example the value of the cosmological constant [28], the low energy physics [50] or even the number of large dimensions [51]. Transitions between these vacua are allowed by a tunneling event where a bubble of the new vacuum is produced in the background of the parent vacuum. This process can continue due to the presence of metastable vacua with positive cosmological constant leading to the picture of an eternally inflating spacetime where all vacua are explored in what has been collectively called the multiverse.

These ideas in models of flux compactification inspired a flurry of papers on new scenarios of inflation within string theory. Most of these studies concentrate on identifying a particular sector of the effective potential for the $4d$ fields that allows for inflation. We can classify these models depending whether the inflaton field is related to the position of a D-brane along the extra dimensions (D-brane inflation [52]) or whether it parametrizes the shape and size of the internal manifold (Modular Inflation [53]).

Modular inflation is a natural idea in any higher dimensional extension of the standard model since the potential is already present in the construction to fix the moduli fields. On the other hand, it is clear that this could be a very complicated function with many possible forms. In models with fluxes, this effective potential encodes the information about the quantized fluxes that thread the cycles in the internal space. Turning on these fluxes generates a change in energy as a function of the size of the cycles, which in turn is interpreted as potential for these fields. Studying the implications of a particular set of fluxes implies that one is focusing on a single realization of the corresponding potential in the landscape. This has been the approach used in most of the concrete models of inflation in string theory so far. This is a reasonable thing to do since cosmological observations would only allow us to see the last 60 e-folds of inflation[1] which, presumably, would happen within the effective potential with the same set of fluxes.

Furthermore, it is likely that there are many different regions of the landscape that allow for inflation (even within the same model) and if this is the case, we will have to face the question of what is the most likely inflationary scenario on the string theory landscape. The answer to this question will require us to adopt some measure on how to give probabilities to all these inflationary trajectories. This is a hard problem that has been extensively investigated in the last few years and although some progress seems to have emerged from these studies there is not a clear consensus in the literature on how to assign these probabilities (See [57] for some recent discussion on the subject). We will not have anything new to say here about the measure problem and concentrate on another issues where the existence of a landscape can play a significant role, namely, in the fine tuning of the potential as well as the initial conditions required for a successful inflation to occur. This is a first necessary step towards extracting observational predictions in the landscape which would require information about the underlying theory as well as the measure problem [58].

[1]This is not entirely true since it is possible that a tunnelling transition between different vacua could leave its imprint in some of the cosmological observables that we can detect, provided that the total amount of inflation within our bubble is not too large (See [54] or more recently [55, 56]). We will discuss this issue in more detail later on in the paper.

## 4.2 Basics of Inflation

In order to set the stage for the subsequent discussion of inflation in the landscape it would be useful to briefly discuss some of the most basic properties of inflation and its challenges.

The simplest models of inflation are characterized by a single scalar field, $\phi$, called the inflaton whose dynamics are given by the following action[2].

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}\partial_\mu\phi^\mu\phi - V(\phi)\right) \tag{4.1}$$

Using a FRW ansatz for the metric,

$$ds^2 = -dt^2 + a(t)^2 d\Omega_k^2 \ , \tag{4.2}$$

we arrive at the equations of motion for for the scale factor and a homogeneous scalar field [3].

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{k}{a^2} \tag{4.3}$$

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2 + \frac{k}{a^2} \tag{4.4}$$

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V(\phi) = 0 \tag{4.5}$$

where the $H = \frac{\dot{a}}{a}$ is the Hubble parameter and $k$ is the curvature (-1,0,1 for open, flat, closed Universe respectively).

An inflationary period is described as a cosmological phase where the universe is expanding as a quasi-deSitter spacetime, or to be more precise the expansion of the universe is accelerating. This could be achieved in this simple model if the energy-momentum tensor of the field is dominated by the potential energy. We can see this if we recall that in order to have accelerated expansion we need an equation

[2]For simplicity we use in this example a canonically normalized field and also we have set $M_P = 1$.

[3]Note that only two of them are independent.

of state with $w = \frac{p}{\rho} < -1/3$. In our case the energy density and the pressure of this scalar field are such that the equation of state of this fluid is given by,

$$w = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \tag{4.6}$$

so $w < -1/3$ is easy to obtain in this case if the potential energy dominates over the kinetic energy.

### 4.2.1 Slow Roll

The condition for accelerated expansion can be written in terms of conditions of the form of the potential using what is called the slow roll aproximation. Let us show how this works.

The first condition that we want to impose is that indeed the scalar field rolls slowly in its potential. This would clearly be better if one wants to establish an accelerated expansion since at small velocities the kinteic energy would be small. We can see this from the acceleration equation for the scale factor, namely,

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) = H^2(1 - \epsilon) \tag{4.7}$$

where we have defined $\epsilon = \frac{\dot{\phi}^2}{2H^2}$. Clearly, we need $\epsilon < 1$ to have acceleration which means that in this case $\dot{\phi}^2 << V(\phi)$.

On the other hand, in order to have inflation lasting for some period of time we need to make sure that this slow rolling is sustained which in turn means that the acceleration of the field is small, namely

$$|\ddot{\phi}| << |3H\dot{\phi}| \tag{4.8}$$

and

$$|\ddot{\phi}| << |V(\phi)| \tag{4.9}$$

This is satisfied if,

$$|\eta| = \left| \frac{\ddot{\phi}}{H\dot{\phi}} \right| < 1 \tag{4.10}$$

In conclusion, we need to have $\epsilon$ as well as $\eta$ small if we want to have inflation for a long enough period of time to solve the cosmological problems like the flatness problem.

Finally, after a little bit of algebra one can find that these slow roll parameters can also be written solely in terms on the potential and its derivatives as,

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \tag{4.11}$$

$$\eta = \left(\frac{V''}{V}\right) \tag{4.12}$$

One of the challenges of finding an inflationary model is to identify a region of the potential that satisfies this conditions once all the possible corrections have been included.

Assuming these conditions are met, the equations of motion simplify to,

$$3H\dot{\phi} + \partial_\phi V(\phi) \approx 0 \tag{4.13}$$

$$H^2 = \frac{1}{3}V(\phi). \tag{4.14}$$

The solution to these equations is a quasi-deSitter scale factor,

$$a(t) \sim e^{Ht}. \tag{4.15}$$

This inflationary period persists until the slow roll conditions are violated. One can then compute the number of e-folds of the scale factor since the beginning of inflation within the slow roll approximations to be,

$$N(t) = \ln\frac{a_{end}}{a} = \int_t^{t_{end}} \frac{\dot{a}}{a}dt \approx \int_{\phi_{end}}^{\phi} \frac{V}{V'}d\phi = \int_{\phi_{end}}^{\phi} \frac{1}{\sqrt{2\epsilon}}d\phi. \tag{4.16}$$

To solve the horizon and flatness problems one needs to have $N_{total} > 60$. On the other hand, the fluctuations observed in the CMB are created at $N_{CMB} = 50 - 60$ e-folds before the end of inflation.[4]

[4]These numbers have a small logarithmic dependence on the details of reheating. Here will will take them fixed at these values.

Another important aspect of inflation is that it predicts the generation of a spectrum of scalar and tensor perturbations. The scalar perturbations are basically given by the perturbations of the inflaton field such that its amplitude in the slow roll approximation can be computed to be,

$$\Delta^2_{\mathcal{R}} = H^2 \frac{H^2}{\dot{\phi}^2} = \frac{1}{8\pi^2} \frac{H^2}{\epsilon} \approx \frac{1}{24\pi^2} \frac{V}{\epsilon}. \tag{4.17}$$

where these expressions should be evaluated at horizon crossing, in other words when the wavelength of the perturbations crosses the horizon during inflation. Assuming that the spectrum of perturbations is given by a power law in scales one can define the spectral index by,

$$n_s = 1 - \frac{d \ln \Delta^2_{\mathcal{R}}}{d \ln k} \tag{4.18}$$

so in the slow roll approximation we obtain,

$$n_s = 1 - 6\epsilon + 2\eta. \tag{4.19}$$

Both WMAP and Planck have shown that this spectral index is very close but lower than unity having a value $n_s \approx 0.96$, while the value for the amplitude of scalar perturbations is $\Delta^2_{\mathcal{R}} \approx 2.5 \cdot 10^{-9}$.

### 4.2.2 Inflection Point Inflation

As an application of the slow roll approximation we will now discuss a type of inflation that occurs near an inflection point. This is, in fact, an interesting model for the landscape since we expect these type of points to be present in a complicated multidimensional field space such as the one we will discuss in this Chapter.

If the potential possesses an inflection point at $\phi = 0$ it can be expanded as,

$$V \approx V_0 \left(1 - \lambda_1 \phi - \lambda_3 \phi^3\right) \ , \tag{4.20}$$

where $\phi$ denotes the canonically normalized field. From this potential we get the following slow roll parameters,

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \approx \frac{1}{2}\left(\lambda_1 + 3\lambda_3\phi^2\right)^2 \tag{4.21}$$

and

$$\eta = \left(\frac{V''}{V}\right) \approx -6\lambda_3\phi \tag{4.22}$$

as well as the total number of efolds[5]

$$N_{total}(\lambda_1, \lambda_3) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\epsilon}}\,\mathrm{d}\phi = \frac{\pi}{\sqrt{3\lambda_1\lambda_3}}\ . \tag{4.23}$$

We see that different inflection points, characterized by different values of the parameters $\lambda_1$ and $\lambda_3$ lead to different number of e-folds.

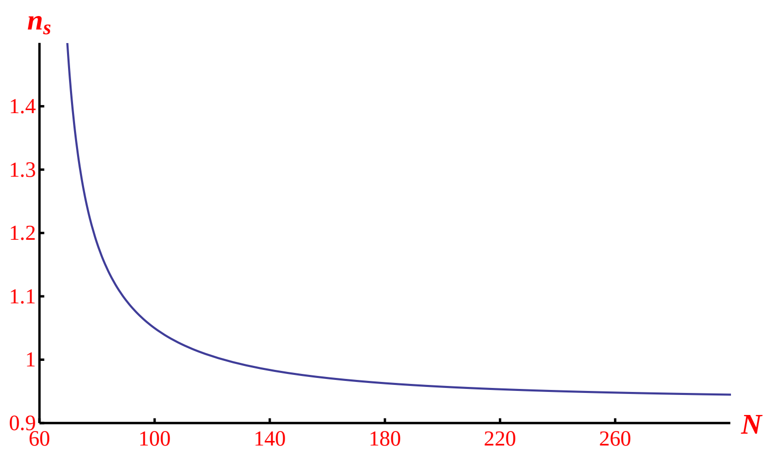


Figure 4.1: $n_s$ versus the number of e-folds, $N$.

From this relation we get the expression for the $\eta$ parameter at the CMB scale, assuming that the end of inflation is given by $\eta\left(\phi_{end}\right) = -1$,

$$\eta_{CMB}(\lambda_1, \lambda_3) = \frac{2\pi}{N_{total}}\left[\tan\left[\frac{\pi N_{CMB}}{N_{total}} - \arctan\left[\frac{N_{total}}{2\pi}\right]\right]\right]\ , \tag{4.24}$$

[5]To perfom this calculation we assume that the scalar field is in a slow roll trajectory as it passes through the inflection point.

which is function of only the first and the third derivative of the potential. The spectral index can then be approximated by

$$n_s(\lambda_1, \lambda_3) = 1 + 2\eta_{CMB}\,(\lambda_1, \lambda_3)\ . \tag{4.25}$$

Using the relations in Eqs. (4.21) and (4.22) we can approximate the position in field space 60 e-folds before the end of inflation as well as $\epsilon$ by the expressions

$$\phi_{CMB} = -\frac{\eta_{CMB}}{6\lambda_3}\ , \tag{4.26}$$

$$\epsilon_{CMB} = \frac{1}{2}\left(\lambda_1 + 3\lambda_3\phi_{CMB}^2\right)^2\ . \tag{4.27}$$

Using this information we can obtain the scalar power spectrum

$$\Delta^2_{\mathcal{R}}(V_0, \lambda_1, \lambda_3) = \frac{1}{24\pi^2}\left.\frac{V}{\epsilon}\right|_{CMB}\ . \tag{4.28}$$

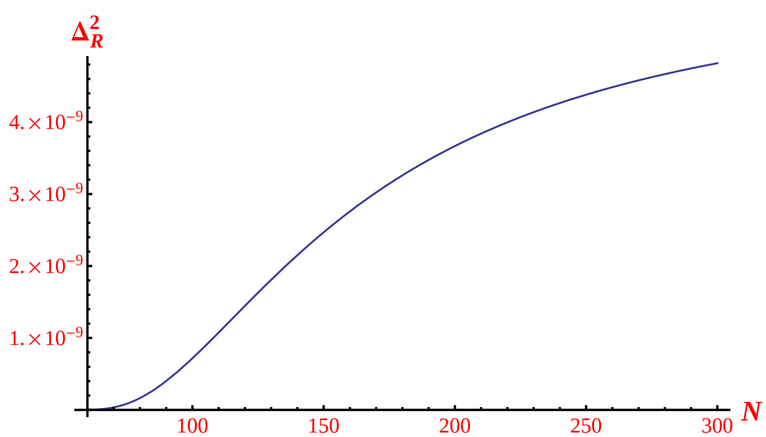


Figure 4.2: Amplitude of perturbations versus the number of e-folds.

These equations will hold for any inflection point model parametrized by the values of $\lambda_{1,3}$. We plot in Fig. (4.1) and (4.2) the observable quantities characterizing the spectrum of perturbations as a function of the total number of e-folds in the model. Later in the Chapter we will compare these analytical expressions to the ones found in a complicated multifield model where we integrate the equations of motion numerically.

## 4.3 Finding an inflationary potential in models of flux vacua.

Most models of inflation so far studied in string theory suffer from some kind of fine tuning to achieve the slow roll conditions. From a purely effective field theory perspective this fine tuning could be considered quite severe, however the idea of the landscape of potentials offers new ideas to explore this issue. Changing the fluxes along the internal manifold as we explored in previous Chapters will in practice allow the parameters in the effective potential to scan over different values so there could be many different sets of fluxes that would lead to a particular value for the coefficient in the potential compatible with observations. The most important example of this phenomenon is, of course, the idea that the cosmological constant problem could be solved by the vast numbers of possible vacua that we have in the landscape, so some of them could land on the very narrow observationally allowed region [28]. In this Chapter we will show that this same idea could help with the fine tuning required to flatten the potential during inflation.

There are several interesting papers in the literature that try to model the complexity of the $4d$ effective potential in the landscape by statistical arguments, see for example [60, 63, 70, 71, 73]. In the following, we will compare our methods to some of these other approaches when appropriate.

### 4.3.1 The KKLT construction for inflationary potentials.

In this section, we want to show that it is possible to find inflationary potentials with an inflection point in models derived from the KKLT type of constructions that we discussed earlier. This is a very economical approach since one does not have to propose the existence of a new potential for inflation but uses the one that has to exist in order to stabilize the moduli. The idea is therefore simple, in the past, the universe had not settled to its overall minima yet and had to roll down this compactification potential to reach the point that we are today. This rolling gave rise to the inflationary period.

We can now use all the machinery previously develop in Chapters 2 and 3 for models of compactification to obtain the potential for the moduli. In particular we will focus here in the $N = 1$ supergravity lagrangian for the Kahler moduli taking the following superpotential

$$W = W_0 + Ae^{-aT} + Be^{-bT} \ . \tag{4.29}$$

where, as before $W_0$ is the value of the superpotential created by the complex structure and dilaton stabilization procedure and the rest is the non-perturbative piece similar to the one suggested in [37].

We will also simplify matters by taking the single Kahler field approximation, whose Kahler potential is given by,

$$K_K = -3 \log \left( T + \bar{T} \right) \ . \tag{4.30}$$

Finally, in order to find an overall minimum of the potential compatible with Minkowski space we choose an uplifiting term of the type:

$$V_U = \frac{D}{\mathcal{V}^2} \ , \tag{4.31}$$

where $\mathcal{V}$ denotes as before the volume of the CY, and can be written in terms of the Kahler moduli $T_i$.

Using the expression of the F-term potential for $N = 1$ one can write the effective potential for this model in the form,

$$\begin{aligned} V(X,Y) \ &= \ \frac{1}{6X^2}[e^{-2(a+b)X}\left(aA^2(aX+3)e^{2bX} + bB^2(bX+3)e^{2aX}\right) \\ &+ \ e^{-(a+b)X}\left(AB(2abX + 3(a+b))\cos(Y(a-b))\right) \\ &+ \ 3aAe^{bX}(W_{0X}\cos(aY) - W_{0Y}\sin(aY)) \end{aligned}$$

$$
\begin{aligned}
&+ \quad 3bBe^{aX}(W_{0X}\cos(bY) - W_{0Y}\sin(bY))] \\
&+ \quad \frac{D}{X^2}
\end{aligned}
\tag{4.32}
$$

where we have separated the real ($ReT = X$) and imaginary parts ($ImT = Y$) of the field as well as the superpotential ($ReW_0 = W_{0X}$) ($ImW_0 = W_{0Y}$). Using this potential one can stabilize the Kahler moduli in a non-supersymmetric de Sitter vacua compatible with our current observations [20].

There are several ways in which one can realize inflatin in these types of models in the following we would like to concentrate on a simple model of inflation, accidental inflation [65]. In order to do this we will need to find the inflationary trajectories in these potentials numerically, so it is neccessary to introduce the cosmological equations of motion for these models.

### 4.3.2 Cosmological evolution of $N = 1$ supergravity fields.

Starting with the $\mathcal{N} = 1$ supergravity action

$$
S = -\int d^4x\sqrt{-g}\left[\frac{1}{2}R + K_{I\bar{J}}\partial_\mu\Phi^I\partial^\mu\Phi^{\bar{J}} + V\left(\Phi^M, \Phi^{\bar{M}}\right)\right] \tag{4.33}
$$

and assuming that the universe is described by a FRW ansatz given by

$$
ds^2 = -dt^2 + a(t)^2 d\Omega_k^2 \ , \tag{4.34}
$$

one can obtain the equations of motion for the moduli fields and the metric

$$
\ddot{\phi}^i + 3\left(\frac{\dot{a}}{a}\right)\dot{\phi}^i + \Gamma^i_{jk}\dot{\phi}^j\dot{\phi}^k + \mathsf{G}^{ij}\frac{\partial V}{\partial\phi^j} = 0 \ , \tag{4.35}
$$

$$
\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{1}{3}\left(\frac{1}{2}\mathsf{G}^{ij}\dot{\phi}^i\dot{\phi}^j + V\right) \ . \tag{4.36}
$$

Note that $k = 0, \pm 1$ parametrizes the spatial curvature of the $3d$ part of the manifold, $\Gamma^i_{jk}$ are the Christoffel symbols for the $\mathsf{G}^{ij}$ metric in field space and $\phi^i$ denote the real components of the chiral fields such that

$$K_{I\bar{J}}\partial_\mu\Phi^I\partial^\mu\Phi^{\bar{J}} = \frac{1}{2}\mathsf{G}^{ij}\partial\phi^i\partial\phi^j \ . \tag{4.37}$$

Taking a single complex scalar field $\Phi = X + iY$ we arrive at the system of equations of the form,

$$\begin{aligned} \ddot{X} &= -3\dot{X}\frac{\dot{a}}{a} + \frac{\dot{X}^2 - \dot{Y}^2}{X} - \frac{2X^2 V_X}{3} \ , \\ \ddot{Y} &= -3\dot{Y}\frac{\dot{a}}{a} + \frac{2\dot{X}\dot{Y}}{X} - \frac{2X^2 V_Y}{3} \ , \\ \left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} &= \frac{\dot{X}^2 + \dot{Y}^2}{4X^2} + \frac{V}{3} \ . \end{aligned} \tag{4.38}$$

Once we have obtained the field trajectories we can calculate the slow roll parameters at any point using the general expressions for a 2 dimensional potential,

$$\epsilon = \frac{1}{2}\left(\frac{\mathsf{G}^{ij}\partial_i V\partial_j V}{V^2}\right) \ , \tag{4.39}$$

while $\eta$ is defined as the most-negative eigenvalue of the matrix:

$$N^i_j = \frac{\mathsf{G}^{ik}\left(\partial_k\partial_j V - \Gamma^l_{kj}\partial_l V\right)}{V} \ . \tag{4.40}$$

## 4.4 Accidental Inflation

This is an inflationary scenario that is easily embedded in a KKLT model of flux compactification. The idea is to look for an approximate inflection point in the Kahler field sector of the potential that allows for sufficient inflation. In [65] the authors found that a region of this type could be generated along the real part of the complex Kahler field in the simplest models with a racetrack type potential.

However, one can argue that the model is quite constrained by what is perhaps more than one type of fine tuning. Firstly, in order for the inflection point to lead to a sufficient number of e-folds the potential has to be fined tuned so that it becomes flat enough around the inflection point. The hope is then that this coincidence can happen at some point of the large parameter space available to string theory, hence the name *accidental inflation*.

On the other hand, these type of models also suffer from an overshoot problem [66]. This is a somewhat generic problem in models where the inflationary region is small because one has to make sure that the field arrives to this point in space with sufficiently low velocity so it can stay in the slow roll inflation region for sufficient amount of time. This problem is related to the question of what are the natural initial conditions for the fields before inflation. This is of course an important question for most of models of inflation but in these kind of inflection point scenarios this is an especially relevant issue.

A particular example of accidental inflation can be obtained by choosing the parameters

$$A = \frac{1}{145}; \quad B = -\frac{1}{145}; \quad a = \frac{2\pi}{580}; \quad b = \frac{2\pi}{600}; \tag{4.41}$$

$$W_0 = 1.01796 \times 10^{-4} + 3.1034287 \times 10^{-5} i; \quad D = 6.0614989 \times 10^{-12} .$$

The idea behind this choice of values for $A, B, a$ and $b$ is that our set of parameters leads to an inflection point whose 3rd derivative is relatively small so one can have a region (not just a point) in field space that satisfies the slow roll condition. This is not an important fine tuning since it can be achieved in a large region of the parameter space but as we will see this becomes quite relevant for our conclusions. We use a $W_0$ superpotential with a real and imaginary part in order to avoid overshooting the overall minima of the potential, in other words to avoid decompactification. The effect of this complex superpotential is to displace the value of the $Y$ component of the field at the inflection point relative to the overall minimum.

One then has a curved trajectory in field space that allows one to reduce the kinetic energy in the $X$ direction and avoid decompactification. We show in Fig. (4.3) the field trajectory around the inflection point and the subsequent evolution in a flat Friedmann-Robertson-Walker (FRW) universe where we have chosen as our initial conditions a point in field space at the beginning of the slow roll region. We note that even though the full trajectory is curved, the relevant part for inflation happens near the inflection point so the predictions of this model are closely related to single field inflection point models [67, 68, 69]. The total number of e-folds for this case is 165 and the amplitude of perturbations as well as the spectral index are compatible with observations.

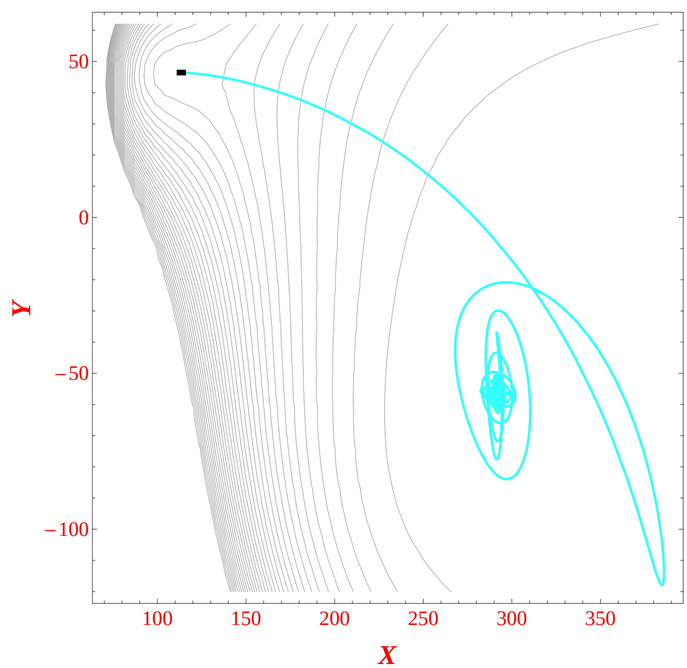


Figure 4.3: Example of Accidental Inflation. We show the inflationary trajectory superimposed over the contour plot of the effective potential in the $X-Y$ plane. We mark in black the small region of the trajectory where the slow roll conditions are satisfied.

One can easily see in this example the problems associated with the fine tuning that we were discussing earlier. Changing the parameters in the potential by a small amount destroys the nice properties of the inflection point and the potential would not give rise to any number of e-folds. On the other hand, choosing the initial conditions far away from the inflection point also has an important effect since the fields pick up too much kinetic energy by the time they arrive at the inflection point to have enough inflation, or even worse, they do not approach that point at all and run directly to the global minimum.

It is clear then that one would like to investigate the possibility of ameliorating some of these fine tunings. In the following we will explore some ideas in the context of this simple model that show how the existence of a landscape could help with both these issues[6].

## 4.5 Accidental Inflation in a corner of the Landscape

As we described in the introduction, string theory provides us with a way to scan different values of the parameters of the low energy effective action. This has important consequences for our understanding of what can be considered fine tuning of the moduli potential as well as the predictions for the observable parameters for inflation.

One could try to investigate this effect by assuming that the parameters of the low energy theory would vary over some range of values in the landscape. In our case this would mean, in practice, to allow the first and third derivatives of the potential around the inflection point to vary with some prescribed distributions,

[6]Note that one can also reduce the severity of this fine tuning problem assuming a particular family of measures [65] that would give a overwhelming weight to any trajectory that inflates compared with the other ones. As described in the introduction, we will not consider these types of measure problems in this paper.

similarly to what was done for D-brane inflation in [70, 71]. On the other hand it is clear that these parameters in the moduli potential are not fundamental themselves but are computed in terms of the fundamental ingredients that vary in a quantized manner over the landscape. In models of flux compactifications based on Type IIB string theory, one specifies the 4d moduli potential once the fluxes along the three cycles of the internal manifold are fixed. This suggests that we should study the mapping between the various sets of fluxes and the values of the parameters in the low energy theory to obtain a more accurate description of the distribution of the parameters of the potential in a real landscape.

Given a set of the fluxes one can obtain, following the prescription given above, the value of the complex structure moduli which, in turn, fixes the value of the superpotential, $W_0$. This means that one can think of this parameter in the potential as being scanned over the landscape. The key point is then to realize that one can control the slope of the potential around the inflection point by choosing $W_0$ appropriately, leaving all the other parameters fixed. This is important for our inflationary models since by decreasing the slope of the potential at the inflection point one increases the number of e-folds in that region. In the following we will use a particular CY to study in detail the distribution of values of $W_0$ in a mini-landscape and to see how this affects the probability distribution of the number of e-folds.

There are, of course, other important fine tunings of this modular potential that one has to address in order to obtain a successful model of inflation in string theory. In particular one should fix the global minimum of the potential at a vanishing value of the cosmological constant which in turn requires us to tune the *uplifting* parameter $D$, so we should also consider this parameter to be scanned over the landscape. This is a much more serious fine tuning than the one required for inflation to happen. We will not try to address these two issues at the same time and in the following we will assume that some other sector of the landscape is responsible for the extreme fine tuning of the parameter $D$ so we can basically consider it a continuous parameter that can be fixed to have an appropriate value of the potential at its global minimum.

### 4.5.1 The $P^4_{[1,1,1,6,9]}$ Calabi-Yau

Similarly to what we did in Chapter 3, we will focus our studies in the well known CY, the $P^4_{[1,1,1,6,9]}$ model with 2 Kahler moduli and 2 complex structure. In Chapter 3 we discuss the relevant information like the form of the Kahler function for the complex structure and the way to compute the flux superpotential in this model. We will not repeat this description in here.

#### 4.5.1.1 Complex Structure

Following the KKLT procedure, we will first find the solutions of the supersymmetric equations for the complex structure moduli, and the dilaton, namely:

$$D_I W = 0 \quad ; \quad D_\tau W = 0 \tag{4.42}$$

where $I = 1 \ldots h^{2,1}$. This is exactly the same of what we did in Chapter 3 for finding the supersymmetric vacua of the complex structure and dilaton fields.

Fixing the flux integers we can now obtain the values of the complex structure fields and the dilaton at a supersymmetric minima by solving Eqs. (4.42). Plugging the solutions back into the expression for the superpotential we can easily compute $W_0$ at that point.

We have followed this procedure for a large number ($\approx 10^9$) of combinations of the fluxes and observe that again the values of $W_0$ seem to be uniformly distributed over a large area (of the order of $10^4$) of the complex plane. Note that this is a much larger area than we previously studied and suggests that this uniform distribution extends quite far away from the origin.

We show in Fig. (4.4) a small sample of $10^4$ randomly selected values over a small region that clearly demonstrates this point.

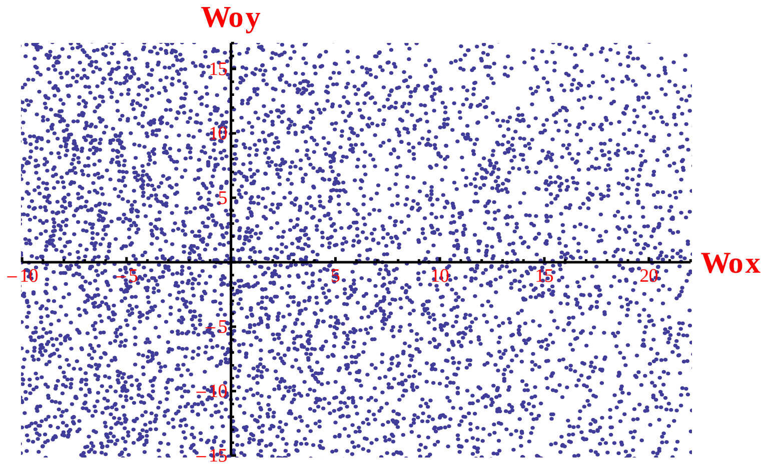


Figure 4.4: Values of $W_0$ for a small sample of supersymmetric solutions in the landscape generated by turning on six fluxes over the $P^4_{[1,1,1,6,9]}$ manifold.

#### 4.5.1.2 The Kahler moduli

For simplicity we will again use the single Kahler field model with Kahler function

$$K_K = -3 \log \left( T + \bar{T} \right) \ . \tag{4.43}$$

We are mainly interested in the effect of the complex structure moduli in our inflationary model so in order to simplify our analysis we will focus on a very simple toy model for the Kahler fields whose Kahler function is given by the previous equation. We will also assume the presence of a non-perturbative potential as well as an uplifting term as described in the KKLT constructions above. Note that one could take the Kahler moduli specific for our CY manifold, but this would make the inflationary model considerably more complicated. We will come back to this issue as part of our future work.

Finally we will also consider the case where the parameters of the non-perturbative superpotential $A, B, a, b$ do not have a strong dependence on the complex structure moduli and take them to be constant for all the values of the fluxes. This is likely to be true for many of our vacua since we have not seen large changes of the values of the complex structure moduli over the scanned vacua. Even if this assumption is violated in some cases, it will be true for many of the vacua and our conclusions are likely to hold.

### 4.5.2 Exploring a mini-Landscape

In the previous section we have shown an example of a successful inflationary model where all the observational constraints of the model were satisfied. In particular one could find a region of the potential that allowed for a large number of e-folds. This was achieved by a quite severe fine tuning of the superpotential around $W_0 = 1.01796 \times 10^{-4} + 3.1034287 \times 10^{-5} i$. This implies that the possible values of $W_0$ to make this model work would be confined to a tiny area in the $W_0$ complex plane of the order of $10^{-17}$. Taking into account that the distribution of vacua on the $W_0$ plane seems to be uniform, we can estimate the fraction of vacua that would be found in this preferred region if one was to generate a large number of flux combinations. Following this calculation we can easily see that we will not be able to explore the landscape finely enough with our mini-landscape in a reasonable amount of time.

On the other hand, the main reason to go to these small values of $W_0$ was to satisfy all the observational constraints, in particular the idea that the scale of inflation should be small to accommodate the amplitude of perturbations. In the following we will sacrifice this requirement in order to demonstrate in a concrete example some of the ideas presented earlier about the fine tuning necessary to achieve large number of e-folds. We therefore explore another region of the parameters where one still needs a considerable degree of fine tuning but can be accessed with our mini-landscape with a sample of generated vacua of a much more manageable size.

#### 4.5.2.1 Specific Example

We start by picking a generic set of values for $A, B, a$ and $b$ in a region of parameter space that leads to a large value of $W_0$. We imagine that these parameters are fixed by the effective theory of a hidden sector. We take the values,

$$A = 5; \quad B = -10; \quad a = \frac{2\pi}{100}; \quad b = \frac{2\pi}{290} \ . \tag{4.44}$$

One can now obtain a range of values for $W_0$ and $D$ such that the potential would have a near inflection point at some positive value of the potential, as well as a global minimum with vanishing cosmological constant. This requires $W_0$ to be within a small area in the complex plane of the order of $\approx 10^{-4}$. This is of course a fine tuned value from the low energy perspective, but the question we would like to address is if one should expect to find several vacua within this narrow strip of values or not, taking into account the existence of the landscape.

In order to address this question we find the values for the superpotential at the supersymmetric minima using the machinery described above. We do this by first generating a large number (of the order of $10^9$) of combinations of fluxes and solving the supersymmetric conditions Eqs. (4.42). Armed with the values of the complex structure and dilaton at their minima, we calculate the superpotential at those values and identify the ones that land within our region of interest. Following this procedure, we were able to find $\sim 50$ combinations of fluxes with the correct values of the superpotential. We show in Fig. (4.5) the region of $W_0$ required to obtain inflation in this model as well as the location of the particular values of the flux vacua the we found following the steps described above.

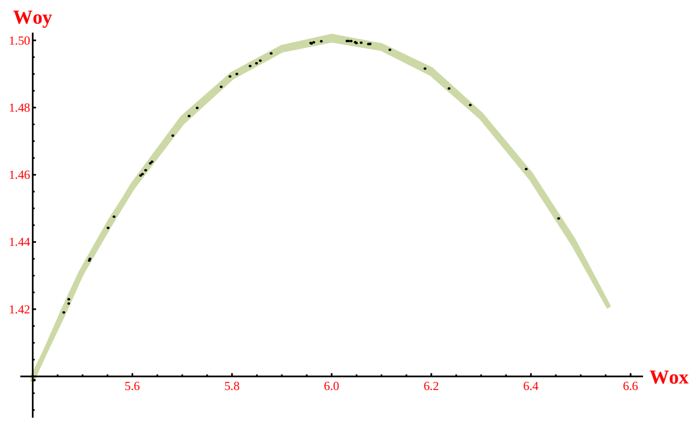


Figure 4.5: We show in this figure the small shaded area in the $W_0$ plane consistent with more than 60 e-folds of inflation for the parameters given in Eq. (4.44). Each of the points in this region represent a particular combination of fluxes that leads to this value of $W_0$.

We note that the number of vacua we found in this region is actually in good agreement with the assumption that the superpotential scans uniformly the values of the complex plane within the area between roughly $W_X = (-100, 100)$ and $W_Y = (-100, 100)$. This suggests that one can follow this calculation for other cases that can not be explicitly done by numerical calculation and therefore one can assume that it is likely that the small fine-tuned region of the $W_0$ is actually achieved by quite a large set of flux vacua.

#### 4.5.2.2 Initial Conditions

In order to make predictions for the observable parameters of this landscape one has to consider some initial conditions. As we argued above choosing these type of values of the parameters relaxes the extreme fine tuning of the initial conditions in this model, since we now have a flat section of the potential where the slow roll conditions can be satisfied, but this does not explain why should the universe start at all close to the inflationary plateau in the vast region of field space.

Here we point out that the idea of the landscape also helps us understanding why this happens. Let us think for a moment on the other flux vacua in the theory, the ones that could be the parent vacua for the one that we find ourselves today. It is clear that there will be many other combinations of fluxes that give a nearby value of $W_0$. The important point is to realize that many of those other values of the superpotential will turn the inflection point in the Kahler moduli potential into a local minima. This makes it possible for the fields to be stuck on a particularly interesting value in the parent vacua, somewhat near the inflationary plateau of the daughter vacuum [7]. One can imagine that the flux changing transition would mainly affect the complex structure fields and would not have a great impact on the values of the Kahler moduli. This is a reasonable assumption since, after all, this transition could happen in a local part of the internal geometry and presumably would not change the overall volume of the internal manifold (parametrized by the field $X$) by a whole lot.

It would interesting to see if this correlation between the end point in field space of the instanton transition and the regions with slow roll is realized in more general models of inflation in the string theory context. If this was the case, this correlation would create a strong bias in favor of these models of slow roll in the landscape [63, 58].

Having this process in mind, one should consider that each of these satisfactory *daughter* vacua can have in principle many predecessors that can give rise to it, (*her parent vacua*). The idea is then that the initial conditions for the field evolution in the daughter vacuum should be set by the conditions in the predecessor. This

[7] Similar ideas were also studied in the context of D-brane inflation in [73].

suggests that we should look for the form of the effective potential outside of the region of the $W_0$ that gives an inflection point inflation and identify the minima of that other vacua. In order to do this in our example, we choose one particular daughter vacum and investigate it in more detail, assuming that we only change the value of $W_0$, in other words we will leave the parameter $D$ constant.

For example, let us consider the following set of flux integers,

$$\begin{aligned} f_A^i &= (17,-2,0) \quad ; \quad f_B^i = (5,-47,-12) \ ; \\ h_A^i &= (-2,-4,4) \quad ; \quad h_B^i = (44,22,3) \ . \end{aligned} \tag{4.45}$$

With these fluxes one can show that the solution of the supersymmetric Eqs. (4.42) for the complex structure moduli and the dilaton take the values,

$$z_1 = -0.749 + 0.991i \ , \tag{4.46}$$

$$z_2 = 2.043 + 0.977i \ , \tag{4.47}$$

$$\tau = -1.28 + 2.87i \ . \tag{4.48}$$

Using these results we obtain the superpotential at this point [8]

$$W_0 = 5.87805764 + 1.49611588i \ , \tag{4.49}$$

while the uplifting parameter in this case should be,

$$D = 0.0642811355 \ . \tag{4.50}$$

Taking all these values into account we use the scalar potential in Eq. (4.32) to find the inflection point in the Kahler moduli fields at,

$$T = 7.8623431 + 14.2923151i \ , \tag{4.51}$$

[8] It is very important to emphasize that all the decimal points on the superpotential are required in order to achieve the large number of e-folds. The superpotential is computed using the moduli fields. These fields are also computed with very high precision but because we are not interested in their actual values here, we present them with just few decimal points.

as well as the global minimum which in this case is situated at

$$T = 66.785459 - 15.343517i \ . \tag{4.52}$$

One can show that this potential qualifies as a successful daughter vacuum leading to roughly 115 e-folds of inflation. Changing slightly the value of the superpotential from this one would make the inflection point region steeper or transform it into a local minimum. Thinking in terms of the complex $W_0$ one can see that there is a relatively large area where one would find a de Sitter local minimum.[9]

On the other hand each set of fluxes gives, at the supersymmetric minimum, a value for the superpotential with a different complex phase and therefore it shifts the inflection point or the minimum in the $X-Y$ field space. We show in Fig. (4.6) the contour plot of the potential for the set of parameters of our *daughter* vacuum together with the location of a few hundred de Sitter minima that we found by varying the combinations of the fluxes. We consider any of these points a good location for the initial conditions for the interior of the *daughter* bubble that forms as a result of the quantum tunneling event. We note that these are not nearby vacua in the sense of a normal metric on the space of fluxes. In fact, some of these vacua may be away from our daughter vacua by changes in several fluxes. It would be interesting to study the distribution of decay rates for this set of vacua along the lines of ([74, 75, 76])[10]. This is important since it enters the final calculation of the probability distribution of any observable in the multiverse [58]. We leave this important issue for future work.

This picture suggests that many of the predecessor vacua for this model would be situated near the inflection point for our daughter vacua. This does not solve all

[9]We will not consider the AdS minima as possible parent vacua since they would likely be collapsing before they have time to tunnel to other flux vacua.

[10]One should also consider other possible ways to induce multiple flux transitions that may be relevant here. See for example [77, 78].

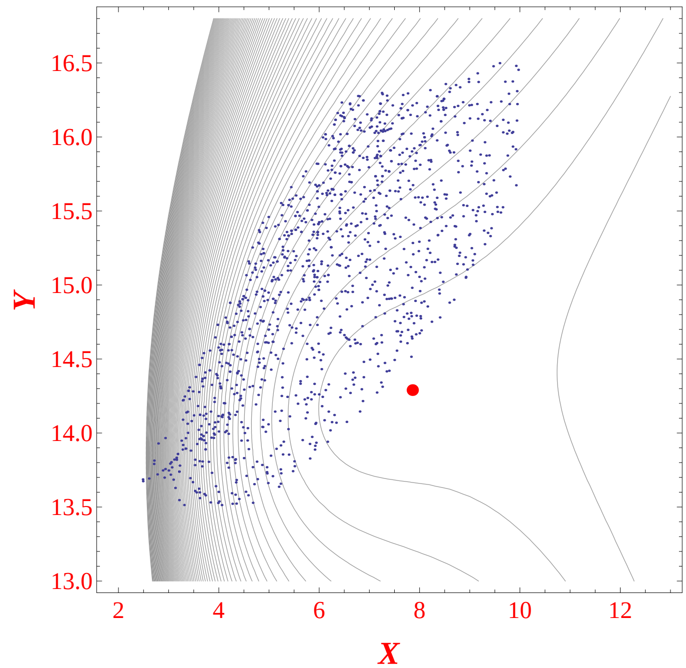


Figure 4.6: Plot of the location of the de Sitter parent minima around the inflection point (big red circle) of the daughter vacuum in the $X-Y$ field space. We show in the background the contour plot of the potential for the daughter vacuum case.

our problems, since even if we start our cosmological evolution from those points, we will still have to face the overshooting problem. Here we argue that the idea that the inflationary regime in our past was initiated by a flux changing transition also helps with this problem.

It was pointed out in [55, 79, 80] that the presence of a curvature dominated regime in the early stages of the interior of a newly created bubble could help solving the overshooting problem by gently depositing the fields over the inflationary plateau. The situation is more complicated in our case, since we have these parent vacua scattered over the $X-Y$ plane which seems to make the problem of dynamically finding the inflection point a little bit harder.

In order to investigate these ideas we take a large number of de Sitter parent vacua for our case and find, using the equations of motion given in the Appendix, the

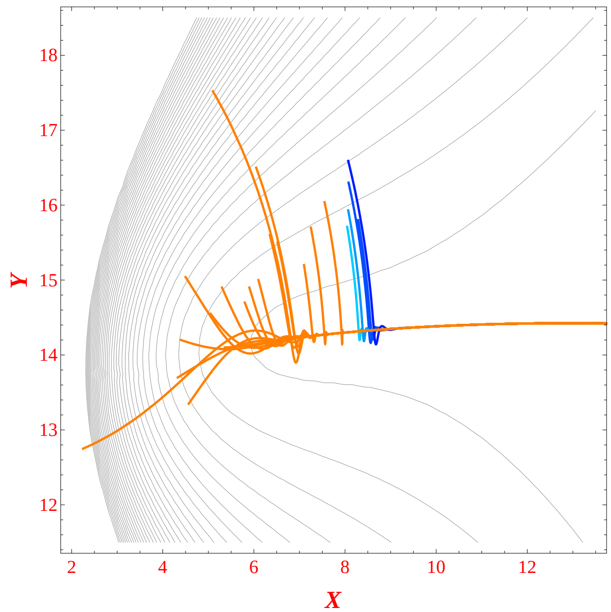


Figure 4.7: Plot of a small number of inflationary trajectories for different initial conditions around the inflection point of the daughter vacuum. All the trajectories converge to the same path at the inflection point demonstrating the attractor-like behaviour in our model.

evolution of the Kahler moduli in the open FRW universe inside of the bubble. We see that even though the initial point is in some cases far away from the inflection point, the fields roll towards it without overshooting it. This is due to a combination of effects. The first one is the one that we described earlier, the help of friction coming from the fact that the universe is open. The second effect is the evolution of the fields along the perpendicular direction, $Y$. This evolution allows for some dissipation of the energy stored in the initial conditions and helps the fields to arrive at the slow roll region without so much kinetic energy. The result is an attractor-like behaviour towards the inflection point that is easily seen in Fig (4.7). This is an important effect since it will increase the range of possible initial conditions that one could take in order to have certain number of e-folds.

#### 4.5.2.3 Distribution of the number of e-folds

One of the interesting questions we can address in this mini-landscape is what is the distribution of the number of e-folds within the inflationary *daughter* vacua. This was studied for a simple model of the landscape in [55] where it was calculated to be a $1/N^4$ distribution. We would like to understand the similar situation in our case taking into account the inflationary daughter vacua found in Fig. (4.5). To calculate this distribution, we want to consider the two effects present here, the fact that the effective potential changes with the value of $W_0$ as well as the possible effect of the distribution of the initial conditions.

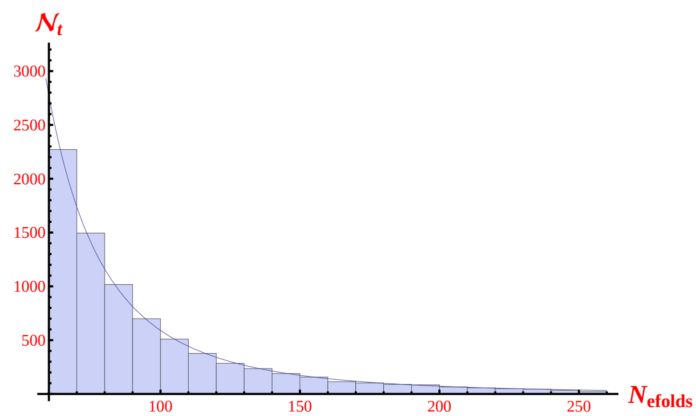


Figure 4.8: Distribution of the number of e-folds. We show the histogram of the number of trajectories ($\mathcal{N}_t$) as a function of the number of e-folds, ($N_{efolds}$).

We investigate this by looking at the evolution of the fields in each of the realizations of $W_0$ by using some random initial conditions near the inflection point [11] as well as the assumption of an open universe. We show in Fig. (4.8) the histogram of the number of e-folds for this set of vacua.

[11] We could, in principle, use the exact location of the de Sitter vacua by calculating the position in each case, but we simplify things a little bit here by taking random initial conditions since the distribution in field space is pretty homogeneous.

The results are well approximated by a $1/N^3$ distribution. One can explain this behavior observing that the distribution of the first derivatives at the inflection point is flat in this ensemble of vacua and assuming that, due to the attractor like behaviour, the initial conditions do not play a significant role in this distribution.

One can then model this landscape by an ensamble of inflection point potentials of the type given by Eq. (4.20) where one only scans the $\lambda_1$ parameter in a uniform way. Taking into account that $N_{total} \sim 1/\sqrt{\lambda_1}$ one arrives at a distribution on the number of e-folds of the form,

$$P(N) \sim \frac{1}{N^3} \; . \tag{4.53}$$

This is a similar result to the one obtained in [70, 71] although in our case we have obtained this distribution directly from the fluxes, and it is not an assumption about the distribution of values of low energy parameters.

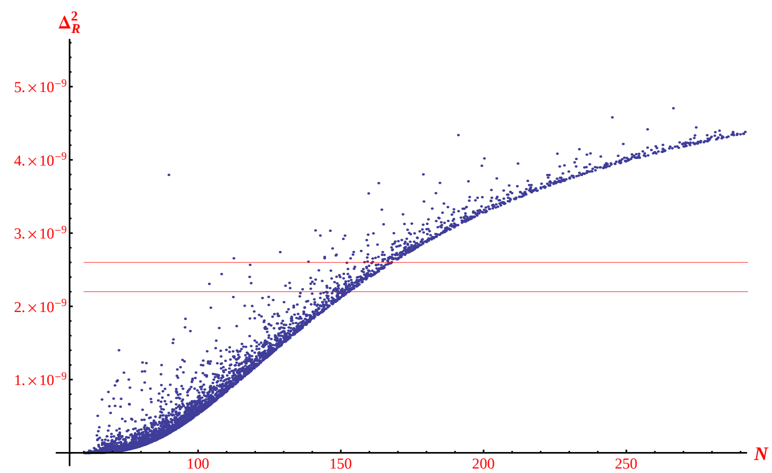


Figure 4.9: Values for the amplitude of perturbations 60 e-folds before the end of inflation as a function of the total number of e-folds for the simulated trajectories in our mini-landscape of accidental inflation.

If this was the only observable prediction of this landscape we would be tempted to argue that it is quite likely to see a small amount of curvature in the universe today, since large numbers of e-folds are hard to achieve inside of our bubble universe. This was first discussed in [55] in a simple toy model for the landscape. (See also the discussion in [81]).

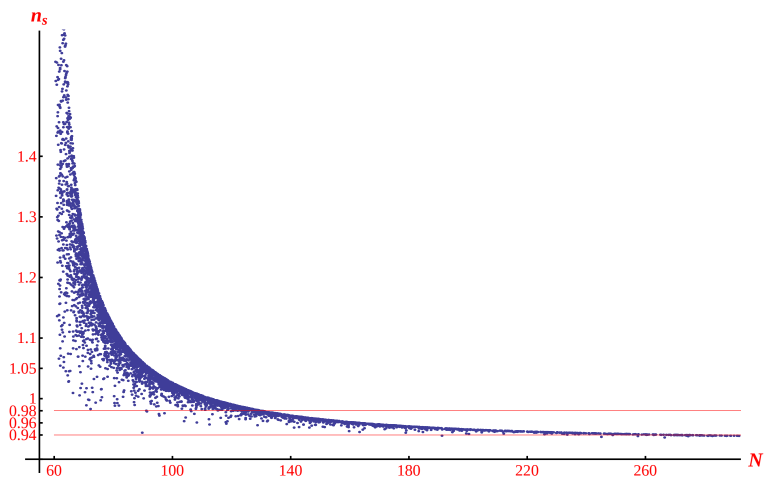


Figure 4.10: Values of the spectral index ($n_s$) versus the number of e-folds for our mini-landscape.

The situation is more complicated in our case if we consider the constraints obtained from the cosmological perturbations associated with the inflaton field. However, we will not discuss the distribution of values of these other observables with the present set of parameters since we are considering a corner of the landscape where the scale of the potential is too high. Remember that this was the prize we had to pay in order to investigate actual vacua of the complex structure minima directly from the fluxes. In the following section, we will return to our original example where we do not have this problem.

## 4.6 Other observable parameters in the Landscape

We can now extrapolate the results of the previous section to other regions of the landscape that we can not directly access numerically since the number of required vacua that we would need to explore would be enormous. In particular, we can investigate the dependence of other observational parameters like the amplitude of perturbations as well as the spectral index in the phenomenologically viable model given by Eq. (4.41). In order to proceed we will assume that the distribution of values of $W_0$ is uniform over the landscape and dense enough in our region of interest and that there are many minima nearby in field space to our inflationary inflection point.

We numerically evolve a large number of inflationary trajectories assuming a random initial condition for the fields near the inflection point in a potential generated by choosing a random value for $W_0$ within the tiny area compatible with more than 60 e-folds.

We plot in Fig. (4.9) the amplitude of scalar perturbations found 60 e-folds before the end of inflation, on a run of 6000 different realizations together with the narrow band of the $2\sigma$ deviation from the observed value [5]. We see that this imposes a pretty strong constraint on the possible trajectories and allows us to discard many of them. We then proceed to calculate the spectral index predicted in this case and we show our results in Fig. (4.10) as well as the $2\sigma$ experimental band observed by WMAP.

We see on these two figures what seems to be a strong dependence of the observables on the number of e-folds together with some scattered points around it. This is again a manifestation of the fact that the most important effect that one introduces by changing the $W_0$ is to modify the slope of the near-inflection point. Most of the trajectories for each individual potential are close to the attractor solutions given by the single field slow roll conditions. Assuming these two effects one can account for the general dependence of these observables with the number of e-folds. In this regard, it is interesting to compare these figures to the ones obtain in the analytic model earlier in Fig. (4.1).

Finally, the distribution on the number of e-folds in this case is again well described by a $1/N^3$ dependence reinforcing the idea that we can think of this landscape as being dominated by a flat scanning of the first derivative of the inflection point inflationary potential.

We conclude that only 4% of all our trajectories are compatible with the current observational constraints. The main reason for this is that most of the trajectories have a small number of e-folds and a blue spectrum, as one would expect for an inflection point [67, 68, 69]. The results for those viable cases are highly peaked around the attractor solution with $N_{efolds} = 160$, $n_s = 0.96$ and $\Delta_{\mathcal{R}} = 2.5 \times 10^{-9}$, but there are a very small number of trajectories that correspond to the edges of the basin of attraction of this solution. An example of this would be a trajectory that started far away from the inflection point and reached the 60 e-folds before the end of inflation mark at the end of the slow roll region having undergone a small number of e-folds. These are interesting solutions where one may be able to observe some curvature. On the other hand, they are highly subdominant.

One could of course imagine a curvaton type scenario where the cosmological perturbations are generated by a different field not related to the inflaton. This is certainly a possibility that one could study in a string theory setup, see for example [82]. Following these ideas one decouples the distribution of the number of e-folds from the other observables related to the perturbations which can have an important effect on the overall predictions on the observable parameters in this landscape.

It is also important to emphasize that these results are obtained assuming the

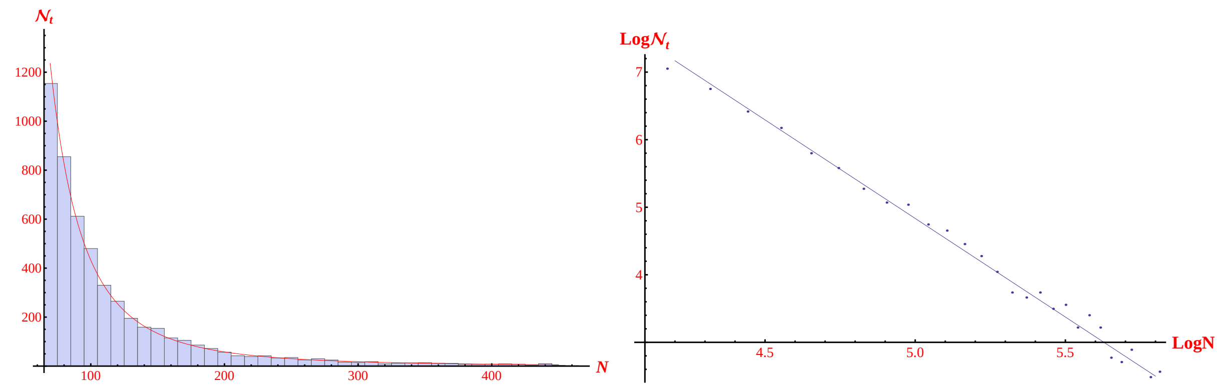


Figure 4.11: Distribution of the number of e-folds in our simulated landscape. We show on the right hand figure the best fit of the data to a curve of the form $P(N) \sim N^{\alpha}$ with $\alpha = 2.92 \pm 0.06$..

same values of the parameters $A, B, a$ and $b$. In practice, this means that we are exploring a particular sub-sector of the landscape with a fixed hidden field theory. One can imagine that these parameters could also be scanned over in different sectors of the landscape. Changing the scale of, for example, $A$ and $B$, would directly affect the scale of inflation so in principle one can rescale the amplitude of the perturbations to include some trajectories and not others.

# Chapter 5

# Conclusions

The subject of this thesis was the numerical exploration of the string theory landscape. The starting point was the $\mathcal{N} = 1$ supergravity theory in four dimensions, which can be constructed by two functions of the moduli fields, the superpotential and the Kahler potential. This theory also can be completely specified by its effective potential, that is written in terms of these two functions. Our main task in the first part was to search for a large set of vacua, coming from the extrema points of this potential in a reduced moduli space of the Calabi-Yau manifold $P^4_{[1,1,1,6,9]}$. In addition to supersymmetric solutions, using appropriate mechanisms which break supersymmetry, we obtained also non supersymmetric vacua, with both negative and positive cosmological constant. Additionally we confirmed that these solutions are stable and a very clear hierarchy exists between the masses of the different moduli. Using statistical analysis we found in the supersymmetric case that the distribution of the cosmological constant peaks as the potential approaches zero from below, while in the non supersymmetric keeps increasing even in positive values of $V$. Finally a very important observation was that by solving the equations in the eight dimensional moduli space, the flux superpotential is uniformly distributed in the complex plane around zero. This allowed us to move in a simpler problem involving only the Kahler moduli. So by choosing random values for the flux superpotential in this region, we performed a much larger number of realizations. The results obtained from this simulated landscape are in agreement with the results from the real fluxes.

In the second part we focused on a particular model of inflation named Accidental Inflation where the potential has to be fined tuned in order to give a substantial number of e-folds. We showed that this apparent fine tuning can be generically obtained by scanning the form of the potentials found in a very modestly small sector of the landscape generated by a family of six fluxes. Furthermore, the existence of a landscape in this model provides us with a theory of initial conditions for the inflationary period. Changing the fluxes from the cosmologically interesting one (the one that we have recently followed in our past history) one sees that the potential develops a local minimum nearby in field space. This is also a generic situation and we can show that there are many other vacua of this kind nearby. This suggests the scenario where the universe evolved from one of these vacua by tunneling out of it by a flux-changing instanton that triggers the transition to the daughter vacuum. This process gives us a natural way to select good initial conditions for our subsequent evolution avoiding overshooting problems that normally occur in these type of models.

# Chapter 6

# Appendix

## 6.1 Topological aspects of String theory

As the string theory is formulated in ten dimensions, its four dimensional description can be obtained by compactifying it in a six dimensional internal manifold. The topology of this manifold has great importance to the final effective theory. For example simple topological spaces like spheres or tori, leave the Supersymmetry of the original theory unbroken, meaning that these are not good candidates for string theory compactification models. In general the requirement for compactifications which leave only some minimum Supersymmetry unbroken in the four-dimensional spacetime, forces us to consider manifolds which admit a covariantly constant spinor. Spaces with this property are the CalabiYau manifolds. A CalabiYau manifold [8] is a Kahler manifold and because of its covariant constant spinor is also Ricci flat. Additionally the CalabiYau manifold admits a nowhere vanishing holomorphic 3-form $\Omega$ which in local coordinates is written as:

$$\Omega(z^1, z^2, z^3) = f(z^1, z^2, z^3) dz^1 \wedge dz^2 \wedge dz^3 \tag{6.1}$$

and it is related to the volume element of the manifold by this equation:[1]

$$\Omega \wedge \bar{\Omega} = -i \left\|\Omega\right\|^2 d\mathcal{V} \tag{6.2}$$

This 3-form $\Omega$ is one of the two existing harmonic forms[2] on the six dimensional Calabi-Yau and plays very important role when its deformations are studied. But before studying these manifold deformations it is useful to provide some mathematical background for the topology of the internal space.

In a real manifold we can define the p-th homology group [10, 11]

$$H_p(M) = \frac{Z_p(M)}{B_p(M)} \tag{6.3}$$

where $Z_p(M)$ is the set of p-cycles and $B_p(M)$ is the set of p-boundaries on it. Actually with the $H_p(M)$ one can find the elements of $Z_p(M)$ which are not boundaries. These non-trivial cycles provide some very important topological properties on the manifold. On the other hand we can also define the p-th de Rham cohomology group, which symbolically is written as [3]

$$H^p(M) = \frac{closed \quad p - forms}{exact \quad p - forms} \tag{6.4}$$

Between $H_p(M)$ and $H^p(M)$ there is a duality. In general the homology group is related to the global properties of the manifold, while cohomology to its local. Having the freedom to work either with the homology or with the cohomology group, one can define the Betti numbers, which are of the most important topological parameters on the manifold. This is because the Betti numbers, which are the dimension of the $H_p(M)$ and $H^p(M)$, count a) the number of the independent closed-forms, b) the number of the independent cycles and c) the number of the harmonic forms defined on the manifold.For example on a simple torus, $b_1 = 2$ .

[1]The volume of the manifold is a (3,3)-form and it will be defined in a little different way.

[2]A form $\omega$ is harmonic if $\Delta\omega = 0$ where $\Delta$ is the Laplacian

[3]A p-form $\omega$ is closed if $d\omega = 0$ and it is exact if there is a (p-1)-form $k$ where $\omega = dk$

This means that on this torus, there are two independent non-trivial one-cycles that cannot be continuously shrunk to a point. In addition with the help of the Betti numbers one can define another very important topological parameter which is the Euler characteristic of the manifold.

$$\chi(M) = \sum_{i=0}^{n} (-1)^i b_i \tag{6.5}$$

For the torus this number is $\chi(T_2) = 0$.

As the Calabi Yau three-fold is a complex manifold, it is necessary also to generalize these definitions for complex dimensions. In a complex manifold a form $\omega$ of bidegree (r,s) is written as

$$\omega = \frac{1}{r!s!}\omega_{\mu_1 \dots \mu_r \nu_1 \dots \nu_s} dz^{\mu_1} \wedge \dots \wedge dz^{\mu_r} \wedge d\bar{z}^{\nu_1} \wedge \dots \wedge d\bar{z}^{\nu_s} \tag{6.6}$$

The exterior derivative of this (r,s)-form is a mixture of one (r+1,s)-form and one (r,s+1)-form. For this reason we can write this as

$$d = \partial + \bar{\partial} \tag{6.7}$$

where the action of $\partial$ in the (r,s) produces the (r+1,s)-form and the this of $\bar{\partial}$ in the (r,s) produces the (r,s+1)-form. Using that we can generalize the p-th de Rham cohomology to the (r,s)th $\bar{\partial}$ - cohomology group

$$H^{r,s}_{\bar{\partial}}(M) = \frac{Z^{r,s}_{\bar{\partial}}(M)}{B^{r,s}_{\bar{\partial}}(M)} = \frac{\bar{\partial}\ closed \quad (r,s) - forms}{\bar{\partial}\ exact \quad (r,s) - forms} \tag{6.8}$$

The complex dimension of the $H^{r,s}_{\bar{\partial}}(M)$ are the Hodge numbers $h^{r,s}$ . These numbers count the number of the harmonic (r,s)-forms on a complex manifold, and give the Betti numbers based on

$$b_k = \sum_{p=0}^{k} h^{p,k-p} \tag{6.9}$$

The Hodge numbers on a Calabi Yau manifold satisfy a series of conditions. Because of the fact that the spaces $H^p(M)$ and $H^{n-p}(M)$ are isomorphic

$$h^{p,0} = h^{n-p,0} \tag{6.10}$$

also from complex conjugation and Poicare duality we correspondingly get

$$h^{p,q} = h^{q,p} \tag{6.11}$$

$$h^{p,q} = h^{n-q,n-p} \tag{6.12}$$

Finally for any compact connected Kahler manifold $h^{0,0}$=1 and because of the vanishing fundamental group of any simply connected manifold

$$h^{1,0} = h^{0,1} = 0 \tag{6.13}$$

So the non-zero elements for a Calabi Yau three-fold are the $h^{1,1}$ and $h^{2,1}$ which give the following Euler characteristic

$$\chi(M) = \sum_{p=0}^{6} (-1)^p b_p = 2(h^{1,1} - h^{2,1}). \tag{6.14}$$

By selecting one Calabi Yau with fixed Betti and Hodge numbers,one can study its size and shape smooth deformations. These deformations are closely related to the massless scalar fields, called moduli which appear in the compactification of string theory from the ten dimensions to four. The formal way to get these moduli is by solving the equation

$$R_{mn}(g + \delta g) = 0 \tag{6.15}$$

which is nothing more than the Ricci flat condition for a small variation of the internal metric. It can be proved that the equations for the mixted components $\delta g_{a\bar{b}}$ and the equations for the pure components $\delta g_{ab}$ decouple. Furthermore the moduli space also has a metric and this metric can be written as a sum of two pieces. Let us see how these two pieces are related to the moduli and to the field deformations in general. As it was mentioned before the real 2-form $B_2$ after compactification gives $h^{1,1}$ zero modes. Additionally the (1,1)-form $J = ig_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}}$ (Kahler form) which defines the volume of the Calabi Yau three-fold

$$\mathcal{V} = \frac{1}{6} \int_{CY} J \wedge J \wedge J \tag{6.16}$$

also after a small variation of $g$ will give $h^{1,1}$ zero modes. As a consequence one can define the complexified Kahler form

$$\mathcal{J} = B + iJ \tag{6.17}$$

from which $h^{1,1}$ complex scalar fields can arise by its deformations. These fields called Kahler moduli leave the Kahler strucure of the metric invariant so are related to size deformations. On the other hand the pure components $\delta g_{ab}$ and $\delta g_{\bar{a}\bar{b}}$ (which are originally zero) can get non-zero values, changing the complex structure of the metric. The responsible variations for this are associated to the complex (2,1)-form $\Omega_{abc} g^{c\bar{d}} \delta g_{\bar{d}\bar{e}} dz^a \wedge dz^b \wedge d\bar{z}^{\bar{e}}$ and after compactification to four dimensions can give $h^{2,1}$ complex structure moduli, which are related to shape deformations of the internal manifold.[4] Taking into consideration all these, we can go back to the moduli space metric and write it as

$$\mathcal{M}(M) = \mathcal{M}^{2,1}(M) \times \mathcal{M}^{1,1}(M) \tag{6.18}$$

.

For a general Calabi Yau three-fold the possible harmonic forms are the nowhere vanishing holomorphic three-form $\Omega$ , $h^{2,1}$ (2,1)-forms $\chi_\alpha$ and their complex conjugates $\bar{\Omega}$ and $\bar{\chi_\alpha}$. The $\chi_\alpha$ is written as

$$\chi_\alpha = \frac{1}{2} \left(\chi_\alpha\right)_{ab\bar{c}} dz^a \wedge dz^b \wedge d\bar{z}^{\bar{c}} \tag{6.19}$$

with $\left(\chi_\alpha\right)_{ab\bar{c}} = \frac{-1}{2} \Omega^{\bar{d}}_{ab} \frac{\partial g_{i\bar{d}}}{\partial t^a}$ and $t^a$ local coordinates for the complex structure moduli space ($a = 1, ..., h^{2,1}$). This expression can be inverted to get the $\delta g_{a\bar{b}}$ and that finally gives the metric of the complex structure deformations, which takes the form

$$G_{\alpha\bar{\beta}} = -\frac{\int_M \chi_\alpha \wedge \bar{\chi_\beta}}{\int_M \Omega \wedge \bar{\Omega}} \tag{6.20}$$

[4]The elements $\Omega_{abc}$ can be constructed using chiral fields, defined on the internal manifold.

. One can prove that under a change in complex structure the (3,0)-form $\Omega$ becomes a linear combination of a (3,0) and (2,1) forms

$$\partial_\alpha \Omega = K_\alpha \Omega + \chi_\alpha \tag{6.21}$$

where $\partial_\alpha = \frac{\partial}{\partial t^\alpha}$. Also any Kahler metric can be expressed as

$$G_{\alpha\bar{\beta}} = \partial_\alpha \partial_{\bar{\beta}} K \tag{6.22}$$

by using the Kahler potential $K$, which at the end takes the compact form

$$K_{cs} = -log\left(i\int \Omega \wedge \bar{\Omega}\right) \tag{6.23}$$

This real Kahler potential $K_{cs}$ carries all the information for the complex structure deformations.

The Kahler potential for the Kahler moduli space, to the leading order is obtained using only the volume of the Calabi Yau. To express this volume firstly we decompose the complexified Kahler form using the moduli fields and a real basis of harmonic (1,1) forms

$$\mathcal{J} = B + iJ = \left(b^a + it^a\right) e_a \tag{6.24}$$

$(a = 1, ..., h^{1,1})$. The volume is

$$\mathcal{V} = \frac{1}{6}\int_M J \wedge J \wedge J = \frac{1}{6} k_{ijk} t^i t^j t^k \tag{6.25}$$

where the intersection numbers are defined as

$$k_{ijk} = k\left(e_a, e_b, e_c\right) = \int_M e_a \wedge e_b \wedge e_c \tag{6.26}$$

. Then by writing the metric of the Kahler deformations as a function of these numbers and the volume one can get the following compact form for this potential

$$K_K = -2log\left(\mathcal{V}\right) \tag{6.27}$$

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