5. Perhaps best possibility for strong supporting evidence lies in the intervening body results at the end
    a. In the Galilean tradition, a salient, unanticipated result that can readily be confirmed qualitatively -- e.g. huge amplifications of speed, as indicated in the last paragraph; e.g. 5 spheres with bulks in 1:2 ratio each yield 68% more motion on impact than with 3 middle ones removed
    b. All fundamental principles from which the overall theory is derived implicated in these results
    c. Huygens undoubtedly included them precisely because they permitted such comparatively telling tests of the theory -- even qualitatively for imperfectly elastic spheres
6. All things considered, then, Huygens's theory of impact is best thought of as a Galilean alternative to Descartes' theory
    a. It employs Galilean principles, it rejects certain Cartesian principles, and it conceptualizes the problem without forces, though (implicitly) with Cartesian version of principle of inertia
    b. Its mathematical development is thoroughly Galilean in style
    c. And the opportunities it presents for evidence have a Galilean character

III. Huygens on Circular Motion and Centrifugal Force

A. The Basic Conceptualization of the Problem

1. As remarked earlier, Huygens had begun work on centrifugal force by 1659 as part of an effort to measure the acceleration of gravity
    a. The version we read, which was published posthumously in 1703, does not bring this goal out so clearly as earlier drafts, nor even as the announcement of most of the results without proofs in an Appendix to *Horologium Oscillatorium*
    b. What it loses in that regard, however, it gains in clarity and polish
    c. (Clarity and polish in part because this version pieced together from several other versions by Huygens's posthumous editors -- as shown by Yoder
        (1) Proofs written to support theorems given in Appendix of *Horologium Oscillatorium*
        (2) A couple of proofs the work of the editors, not of Huygens himself)
2. Problem posed: let us see what [sort of] and how great a *conatus* belongs to bodies attached to a string that revolves (or is restrained by a circular barrier)
    a. Tension in the string a measure of this *conatus*, and thus a measure of the accelerative tendency at the first instant the body would leave the string
    b. Just the move Descartes had proposed in Article III, 59 of his *Principia*, but now carried out by solving for the tension in a string required to maintain uniform circular motion
    c. Initially want to see whether this tendency is of same kind as that involved in gravitational fall
3. Consider in Figure 3 the distances that would form between the object on the circle and the straight line it would follow if it were released -- an indication, indeed a proposed measure, of the *conatus* of the body to move away from the end of the string
    a. BK, KL LN = arc length of BE, EF, and FM since v constant

b. But for small angles -- i.e. for the first instants in time -- EK, LF, and MN differ negligibly from EC, DF, and MS

c. EC, DF, and MS are proportional to $r*(1/\cos(\theta) - 1)$

d. For small θ, proportional to θ^2 and hence to t^2

4. Therefore the accelerative tendency of a ball in uniform circular motion is of the same general type as that of a ball in free fall -- i.e. distances traversed are proportional to t^2; in other words the accelerative tendency is uniform and hence akin to Galilean motion

   a. Since the two accelerations are of the same type, the way opens to comparing them and to determining the strength of one via a measure of the strength of the other in suitable circumstances!

   b. Specifically, under circumstances where the two strengths are known to be same in magnitude

5. Finally, just as the tension in a string on which a body is suspended vertically is proportional to the weight or bulk of the body as well as to its accelerative *conatus*, so the tension in a string on which a body is suspended in uniform circular motion is proportional to the weight or bulk of the body as well as to its accelerative *conatus*

6. Problem in the original 1659 effort at this point was first to determine what the accelerative *conatus* of a body revolving uniformly is proportional to, and then find some circumstance in which the accelerative *conatus* of gravity is of the same magnitude

   a. Would then be able to measure acceleration of gravity -- i.e. the distance of fall in the first second -- by determining the centrifugal accelerative *conatus*

   b. In effect reformulating the former measurement problem in a more tractable domain

7. Of course, in this later draft had the pendulum methods of determining g, and so end up with different, though related goals -- i.e. giving an account of centrifugal force in terms of g

B. The Basic Results and their Significance

1. Propositions I through III, which concern "equal bodies", develop the fundamental, classic result: the centrifugal force -- i.e. the tension in the string -- is proportional to v^2/r

   a. I: if same period, and hence same angular velocity, in two different circles, then centrifugal force proportional to r

   b. II: if same radii, but two different velocities, then centrifugal force proportional to v^2

   c. III: if at same v, but in two different radii, then centrifugal force proportional to 1/r -- via combining I and II

2. Once the fundamental result is at hand, a variety of other results can be obtained -- more easily by us using algebraic means than by Huygens using geometrical arguments

   a. E.g. Proposition IV: if equal centrifugal force, then period of revolution proportional to $\sqrt{r}$

   b. A result Huygens did not bother to deduce, but that Newton, among others, did: if the square of the period is proportional to the cube of r, then the centrifugal force is proportional to $1/r^2$

c. I.e. Kepler's third law entails inverse square centrifugal forces for concentric circular orbits!

3. Huygens's interests lie in the direction of Propositions V and VI; V states that the centrifugal force is equal to the weight of the body -- the tension in the revolving string is equal to that in the vertical string -- when the velocity is that acquired through a fall of r/2

   a. In effect, a Galilean "sublimity" result, providing a commensurate basis for comparing centrifugal and gravitational accelerations

   b. Algebraically because $v = \sqrt{(2gh)}$, so that $v^2/r = g$ when $h = r/2$

   c. VI then solves a problem: Given the acceleration of gravity, can determine the radius of the circle for which centrifugal force = weight and period = 1 sec: 9.5 Rhenish inches since $P = 2\pi\sqrt{(r/g)}$ when $v^2/r = g$, and g = 375 in/sec/sec

4. Huygens is here flirting with a notion of mass, as distinct from weight, insofar as he is identifying weight with the tension in a vertical string, which is proportional to $B*g$

   a. He is also flirting with the modern form of Newton's second law: weight = force in string = $B*g$

   b. But to extract a notion of mass, he would need to compare the effect of B on centrifugal tension with a body of unit weight, at the very least

5. Notice, however, that result given in Proposition VI is useless as it stands for determining g, for to invoke the conditions of the proposition and hence to infer g from P given r, must have centrifugal force equal to the weight

   a. I.e. need uniform circular motion in which the centrifugal force is known to equal the weight

   b. Will then be able to infer g from P given r on the basis of Proposition VI

C. A Theory of the Circular Conical Pendulum

1. Proposition VII gives the first result allowing such a determination of g, namely from equilibrium conditions for a ball on a rotating parabolic conoid

   a. At the latus rectum, the slope of the conoid is 45 deg, and hence the equilibrium position will automatically have the centrifugal accelerative tendency = gravitational accelerative tendency via Lemma I (from Stevin)

   b. Via a geometrical argument and Lemma II, the very same rate of rotation will then put a ball anywhere else on the conoid into equilibrium

      (1) An isochronism result: the same period for all equilibrium positions on a parabolic conoid

      (2) For a given parabola, one distinct angular velocity and hence period at which equilibrium occurs everywhere

   c. Finally, an algebraic argument exploiting Proposition V yields the fact that this period = the period of a small arc pendulum of length 1/2 the latus rectum (i.e. r at the latus rectum)

   d. Thus can measure g by determining the equilibrium period for a parabolic conoid: $g = (\text{latus rectum}/2) * (2\pi/P)^2$

2. Huygens now switches to the conical pendulum, which provides in some respects a more tractable experimental device than the conoid
    a. The tension in the string of a conical pendulum is from gravity and the centrifugal *conatus*
    b. Equilibrium when centrifugal accelerative tendency produces a tension in the string, the vertical component of which exactly balances the gravitational accelerative tendency -- i.e. when $v^2/r = g*\tan(\theta)$; see Huygens's static equilibrium sketches for conical pendulums in the Appendix
    c. The relationship between $\ell=r/\sin(\theta)$, $h=r/\tan(\theta)$, v, and r is thus fixed for a conical pendulum
3. Less clear than it might be from the De Vi Centrifuga paper is the extent to which Huygens relied on results from statics, especially from his countryman Stevin, in conceptualizing conical pendulums
    a. The figures and results from statics in the Appendix bring this out to some extent
    b. Of particular note is the figure from Huygens's notebook that did not appear in De Vi Centrifuga showing how the conical pendulum amounts to a problem in static equilibrium
4. Propositions VIII through XI give the basic results for a conical pendulum, which prepare the way for the formula, $P = 2\pi\sqrt{(h/g)}$
    a. VIII: If equal heights, equal periods
    b. IX: Period proportional to $\sqrt{\ell}$ when theta constant
    c. X: Period proportional to $\sqrt{h}$ in all cases
    d. XI: Period proportional to $\sqrt{\cos(\theta)} = \sqrt{\sin(ABE)}$
5. The remaining propositions on the conical pendulum now provide a basis of calculation corresponding to the above formula
    a. XII: Period of a conical pendulum approaches that of a small-arc simple pendulum of same length as θ approaches 0
    b. XIII: Centrifugal force = weight when period in conical pendulum = period of simple pendulum of same length
    c. XIV: Period = time for fall through ℓ when $\arcsin(90-\theta)=0.05062$ -- a sublimity result
    d. XV: Centrifugal force in conical pendulum of given h proportional to ℓ
6. As remarked before, Huygens constructed a conical pendulum clock in late 1659 and used it to measure g, obtaining a (rounded-off) value of 15.6 Rhenish ft of fall in the first second (979 cm/sec/sec), as well as using a later clock based on the paraboloid (see Appendix)

D. Comparing Centrifugal Forces with Gravity

1. The remaining results in the paper compare the centrifugal tension in a string with the gravitational tension under various circumstances
    a. So far have a comparison in circumstances where they are in balance
    b. Now looking for results when not in balance – large-arc pendulum and body revolving non-uniformly in a vertical circle

2. Proposition XVI: force in string at bottom of a 90 deg pendulum is 3 times the weight of the object
    a. For, $v = \sqrt{(2gh)}$ in order to conserve Bv^2
    b. Hence centrifugal tension proportional to 2g at bottom, and addition of gravity gives 3g
    c. Huygens's argument approximates the circle with a parabola to get the precise result at the bottom geometrically, instead of via the algebra above
3. Proposition XVII: a vertical rotating body produces at least 6 times the tension that the weight of the body alone does -- 6 times in order to keep the string taut
    a. To keep the string taut at the top, v^2/r must at least be g
    b. But then velocity at bottom must be large enough to yield v^2/r at top after deceleration while going up
    c. "Sublimity" reasoning, converting velocity to height, then yields the result
4. This last proposition has a striking corollary: the specific height at which to intercept a 90 deg pendulum and complete a circle after the intercept is $(2/5)*\ell$ from the bottom
    a. If intercept any higher, will not complete a circle
    b. A testable result related to Galileo's claims about intercepted pendulums
5. Clearly, Huygens was looking to extend his results on centrifugal forces to the large-arc pendulum in these last propositions, but this was as far as he could get
    a. Probably prompted by the 90 deg pendulum Mersenne used in trying to measure g -- a measurement Huygens had repeated more than once with increasing care, before giving up on it
    b. No theory of the large arc pendulum until math for elliptical integrals in second half of 18th century

E. Empirical Evidence for the Overall Theory

1. Once again Huygens, in the tradition of Galileo, says that the results agree with experiments without giving us any experimental data
    a. Since the mathematical arguments involving infinitesimals raise some doubts, empirical evidence an appropriate concern here
    b. And in fact Huygens had performed a variety of relevant experiments in late 1659 and subsequently
    c. So, the decision not to report results reflects a style
2. Outwardly, the theory is making claims about the tension in a string, which at that time could not be measured to any accuracy at all except in certain cases of static equilibrium
    a. Direct measurements become possible only in late 19th century, with the advent of strain gauges
    b. Measurement via determining breaking points not adequate for strength of string too irregular
3. But Huygens himself had managed to obtain an accurate measure of periods of conical pendulums and hence of an implied value for g