Risk-Based Trend Detection for Climate Change Adaptation

Thesis research submitted by
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Abstract

The usual procedure for detecting climate change impacts from historic records chooses statistical criteria (usually α=0.05) to minimize the probability of Type I error, claiming a trend exists where it does not. However, it ignores Type II error, failing to detect an increasing trend in storm surges.

For coastal climate change adaptation, the physical implication of a Type I error is wasted money on unneeded infrastructure. Repercussions of a Type II error, however, are major storm damages and flooding due to inadequate protection. Decision-makers are poorly served by statistical methods that do not carefully consider this type of error.

We propose a new method that combines hypothesis testing, Risk-Based Decision Making, and decision analysis, to evaluate adaptations for a possibly costly but highly uncertain increase in storms. We propose a new metric, Expected Regret, that integrates the statistical certainty and the economic impacts of a trend. This method gives needed attention to the risks of under-preparing; conveys the statistical uncertainty in a physically meaningful way; and addresses the question, "Should we adapt now, despite the uncertainty?"

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1. Introduction and Problem Motivation

1.1. Uncertainty in the Adaptation Decision Process

In planning for climate change and making decisions on how to adapt, coastal communities want to know

Will local storm surges increase in the near future?

What are advantages of adapting to protect against increasing storm surges and damages?

Methods exist to address these questions, but any answers have great uncertainty. So, for decision-makers, the key remaining question is

Can we detect the increase with enough certainty to invest now?

Of the many previous studies which have sought to determine whether a trend exists in storm surges, precipitation, or other hydroclimatic processes, the vast majority define no trend as the null hypothesis, and set a high criteria (usually $1 - \alpha = 0.95$) for considering the trend statistically significant. This minimizes the possibility of claiming a trend exists when it does not. This type of error is known as Type I error, with a probability of occurrence denoted by α.

The usual procedure, however, ignores the other type of error: the possibility that there is an increasing trend in storm surges but that observed records do not provide enough information to detect it. This other type of error is known as Type II error, with a probably of occurrence β. This possibility of a failure to detect a trend is rarely considered in trend detection studies, though for climate change adaptation, this represents the error with the greatest potential societal damage.

The physical implication of a Type I error in adaptation decisions for storm protection is wasted money on unneeded infrastructure. The physical repercussions of a Type II error, on the other hand, are major storm damage and coastal flooding due to inadequate protection. Decision-makers are poorly served by statistical methods that do not carefully consider this type of error.

1.2. Traditional Decision-Making Process with Trend Detection

In the usual decision-making process, a trend is evaluated for statistical significance completely separately from the economic context. First, the trend is detected and the statistical significance of the trend is estimated. If α is below a pre-specified critical value, usually $\alpha_{critical}$=0.05, the analysis continues to evaluate the economic viability of a proposed adaptation plan. If α exceeds that critical value, though, the trend is dismissed and the economic analysis is omitted. Figure 1.1 shows how the trend and economic viability are considered separately in a traditional analysis.

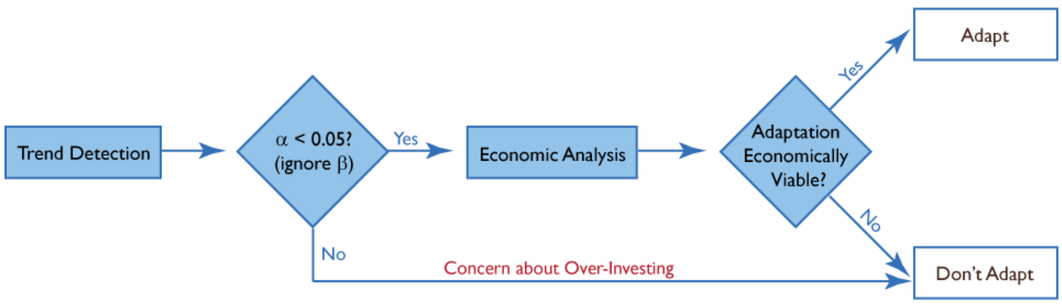

Figure 1.1 Traditional trend-detection decision process

The decision not to adapt if α is high, as is indicated on the diagram, addresses concerns about over-investing when there is no trend. Importantly, concerns about under-preparing when there is a real but undetected trend (concern about Type II error) does not appear anywhere in the traditional analysis. Given the tremendous amount of attention devoted to climate change and other forms of environmental trends, this is quite surprising. In a recent editorial, Kevin Trenberth (one of the lead authors on three IPCC reports) states, "As a whole the community is making too many Type II errors." (Trenberth 2011)

Though a risk-based decision making process would calculate the Net Benefits of adaptation (the estimated damages avoided less the cost of the proposed adaptation), the economic analysis of damages due to increasing storms would only be performed if a trend had been accepted. If the trend is rejected, the consequences of under-preparing are not even calculated and thus are never considered.

1.3. Risk-Based Trend Detection

We propose a new approach which integrates risk-based decision theory and hypothesis testing to yield a Risk-Based Trend Detection method. The steps in the process are outlined in Figure 1.2.

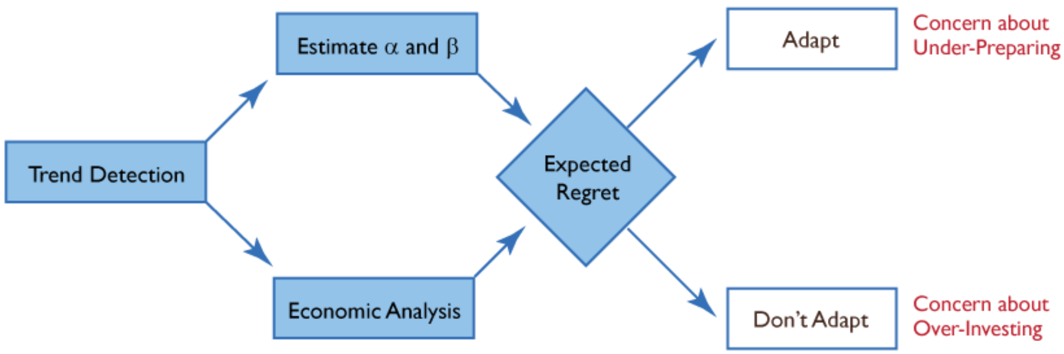

Figure 1.2 Risk-Based Trend Detection decision process

A trend analysis is performed, and the probabilities of both Type I and Type II error (α and β) are estimated. From the trend detection, we also estimate the magnitude of the trend, which is used to conduct the economic analysis. We estimate the damage with and without adaptation, and with and without an increasing trend in the events. These can be used to determine the damages avoided under either scenario. We consider what we term "Cost of Regret", that is the cost of avoidable damages if we under-prepare, and the cost of unneeded infrastructure if we over-invest.

Similar to the calculation of Expected Utility in a traditional decision analysis, the probability and the consequences of each error are multiplied to yield new metrics: the Expected Regret. The Expected Regret encapsulates both the statistical certainty and potential consequences of a trend. The Expected Regret of Over-Investing can then be weighed against the Expected Regret of Under-Preparing. This relative comparison of risk and cost can be presented to decision-makers to assist them in evaluating their various adaptation (investment) options.

1.4. Advantages of Risk-Based Trend Detection

Risk-Based Trend Detection gives needed attention to the risks and damages of under-preparing. It not only addresses the possibility of Type II error, it integrates the probabilities of both Type I and Type II errors with their associated economic repercussions to describe statistical uncertainty in terms of physical risks.

Through the Expected Regret of Under-Preparing and the Expected Regret of Over-Investing, this method delivers a combined analysis of the statistics and the economic consequence in an intuitive, meaningful way that will help stakeholders make well-informed decisions for climate change adaptation.

Instead of presenting the costs and benefits of the adaptation recommended, with a footnote about the uncertainty of the trend, this method integrates them to help address the question, "Should we invest now, despite the uncertainty?

1.5. Outline of this Work

Risk-Based Trend Detection utilizes a few different families of techniques, and brings them together to address the climate change adaptation problem. Therefore, Chapter 2 provides a short introduction and discussion of previous work in the following areas:

-Hypothesis testing, and use of α and β for error estimation

-Risk-based Decision Making, using Extreme Value Theory to characterize rare events

-Type II errors in climate change detection, and their relation to Type I error, minimum detectable trend, and minimum required detection time

-Decision theory for addressing the uncertainty in climate change adaptation

The details of the Risk-Based Trend Detection methodology are explained in Chapters 3 and 4; chapter 3 outlines all the preparatory steps and calculations required, and the chapter 4 describes how these many estimates are put together into a quantification of Expected Regret. Chapter 5 demonstrates this method through a case study application in Mystic and Groton Long Point, CT.

2. Literature Review and Background on Technical Procedures

2.1. Error Estimates and the Significance of Statistical Significance

Our method makes use of hypothesis testing methods for calculating α and β. Frequently in hypothesis testing, only α is calculated. Further, the value of α is often used only so far as to determine whether it is below a critical value ($\alpha_{critical}$), deeming the conclusion statistically significant. Instead of a binary indicator, we use the calculations of α as well as β as estimates probabilities of Type I and Type II error. (See Section 2.4 for calculation of α and β.)

Adaptation planning depends critically on trend detection, hence it is important to understand the limitations of and concerns surrounding tests of statistical significance of trends. Numerous fields, including psychology, economics, social sciences, and medical research, have called into question the value of statistical significance tests (Ziliak and McCloskey 2008; Cohen 1994). One of the main arguments against Null-Hypothesis Significance Testing (NHST) is its adherence to a typical $\alpha_{critical}$ of 0.05, or sometimes even 0.01 or 0.001, all of which are arbitrary and may frequently lead to the dismissal of valuable correlations.

Concerns about NHST are of vital concern to climate sciences and water resources engineering, where the trend analysis could have an impact on major infrastructure decisions. Recent articles in these fields have brought up additional arguments of particular concern to climate applications (Nicholls 2000; Trenberth 2011). One of the main arguments here is the emphasis on Type I error, without regard to such a criteria's effect on the occurrence of Type II error.

Many of these arguments from across the fields are particularly relevant to the usual process of trend detection for decision-making. We highlight three main issues below.

The requirement that α be less than 0.05 is followed almost universally and unquestioningly. Though this value for $\alpha_{critical}$ has some meaning due to its being near universal acceptance, it is nonetheless a physically arbitrary number.

Use of NHST implicitly places disproportionate emphasis on Type I error. As mentioned throughout this paper, the power 1-β is rarely reported, despite the importance of Type II error in climate applications. Without an estimate of β, it is difficult to interpret the value of the trend based on α alone. By placing a disproportionate amount of emphasis on one type of error, the definition of the null hypothesis becomes hugely important. Trenberth argues that the usual null hypothesis of a stationary climate should be reversed. According to the precautionary principle, we believe that it is warranted to either reverse the null hypothesis, or to demand rigorous consideration of the Type II error.

In addition, NHST provides inadequate tools for handling uncertainty. Once a dataset is shown to meet the criteria of $\alpha<0.05$, little more may be done to represent the uncertainty. Further, both α and β are abstract quantities that are difficult to relate to the physical context of the trend in question. Due to their lack of physical meaning, even if α and β are reported, they present the uncertainty in a way that is difficult to incorporate into the decision-making process.

2.2. Risk-based Decision Making

Risk-Based Decision Making (RBDM) is an established methodology (see Tung 2005 for some history on RBDM's development) that determines appropriate levels of infrastructure based on the expected damages avoided vs. the cost of the infrastructure. (Tung 2005, Nat'l Research Council 2000, Queensland Govm't 2002) It can be used in place of the more traditional design storm approach, whereby a certain return period is chosen (usually specified by regulation), and then infrastructure is chosen that will protect against the size of event with that specified return period. Instead, RBDM aims to choose a level of protection investment that will minimize the total expected annual cost, taking into account the annualized cost of the infrastructure (labeled "Annual Installation Cost" in Figure 2.1 and the annual expected damages, as shown in Figure 2.1.

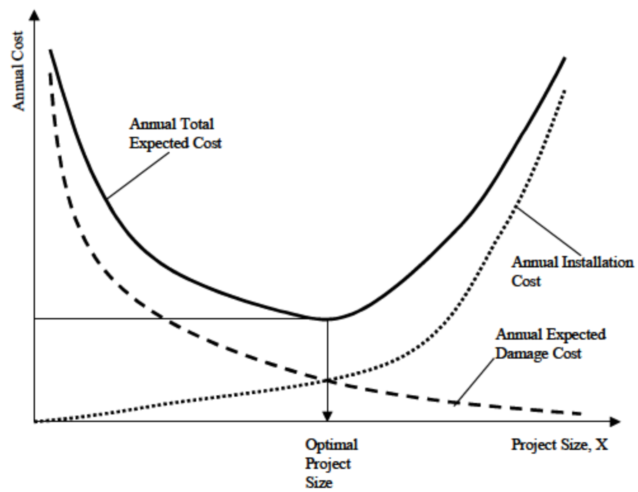

Figure 2.1 Schematic diagram of Risk-Based design (Source: Tung 2005)

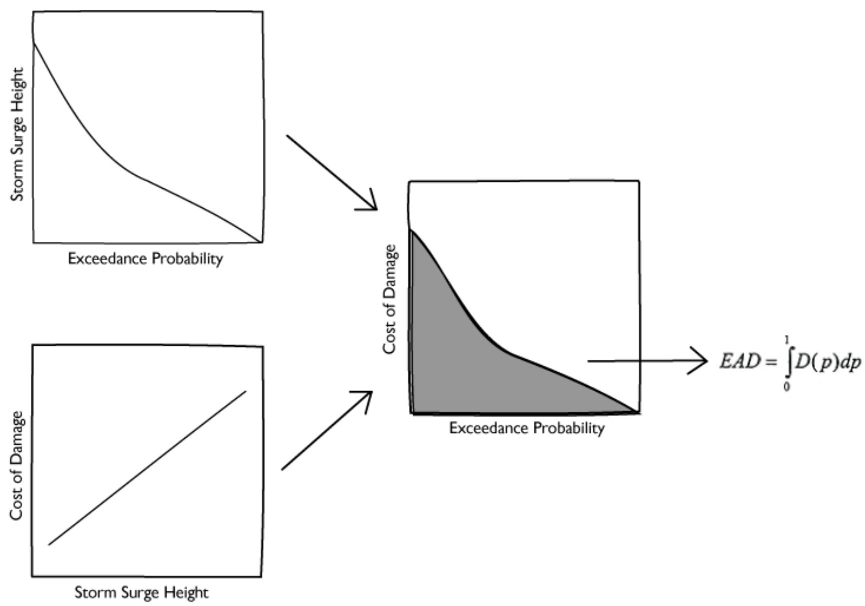

Figure 2.2 Calculation of Expected Annual Damages from height-frequency and damage-height relationships

In order to calculate the Expected Annual Damages, we first use the exceedance probability to determine a relationship between the frequency and the size of an event, illustrated at top left in Figure 2.2. We also use the empirical relationship between the size of the event and the cost of damages (bottom left of Figure 2.2). These two relationships are combined to yield the damage-frequency relationship (Figure 2.2, right), which is defined as a function of exceedance probability.

Since we designate the x-axis the exceedance probability, we can take the area under the curve (or the integral of the damages as a function of elevation), to determine the total expected damages in any given year. The expected annual damages can then be compared with the annual cost of installing and operating the adaptation, or (as is illustrated in Figure 2.1), we can look for an adaptation level that will minimize the sum of the annual adaptation costs and the annual expected damages.

2.3. Power in Climate Change Detection and the Detectability of a Trend

As discussed in chapter 1, very few studies have addressed the issue of Type II error in climate change detection, or considered the impacts of such an error. Two recent papers address Type II error in detection of climate change impacts on precipitation. These two examples examine the relationship between the coefficient of variation of precipitation; the magnitude of a trend in precipitation; "statistical certainty" or 1-α; and the period of time that would be required to detect a trend with such certainty.

Ziegler et all (2005) used GCMs to predict trends in annual precipitation, and then performed simulations to determine the period of record going forward required to detect trends of those magnitudes. Focusing on the Mississippi River basin, they calculated detection time required to predict the magnitude of trends predicted by the GCMs, using assumptions of α=0.05 and β=0.10. For the changes predicted within the basin of 0.6 to 1 cm per decade, they found that between 82 to 143 years would be required to detect the trend with the specified certainty.

Morin (2011) performed simulations to estimate the minimum magnitude of change in annual precipitation that could be detected over a 50-year period. He generates Monte Carlo simulations based on historical from 9,000 stations globally. He then set a criteria of α=0.05 and β=0.50. The choice of these criteria is appropriate, as it demonstrates the detectability of trends under assumptions commonly used; however, as we argue later, based on the physical and economic consequences of Type II error, these assumptions about acceptable levels for α and β need to be reexamined.

He reports minimum detectable trends given α=0.05 and β=0.50 (as given above), and a detection period over the next 50 years. For North America (below the Arctic Circle, the minimum detectible trends range from 5 - 37.5 cm, 10 - 75% changes from historic precipitation. Even trends of those magnitudes would only be detected under 50% of the simulations. Figure 2.3 shows minimum detectable trends globally, as a relative change from historic values. Note that for much of the globe temperate areas, the minimum detectable trend constitutes more than a 10% change from historic values; for arid areas, the minimum detectable change constitutes a much higher percent, 20-75% for large portions of the globe.

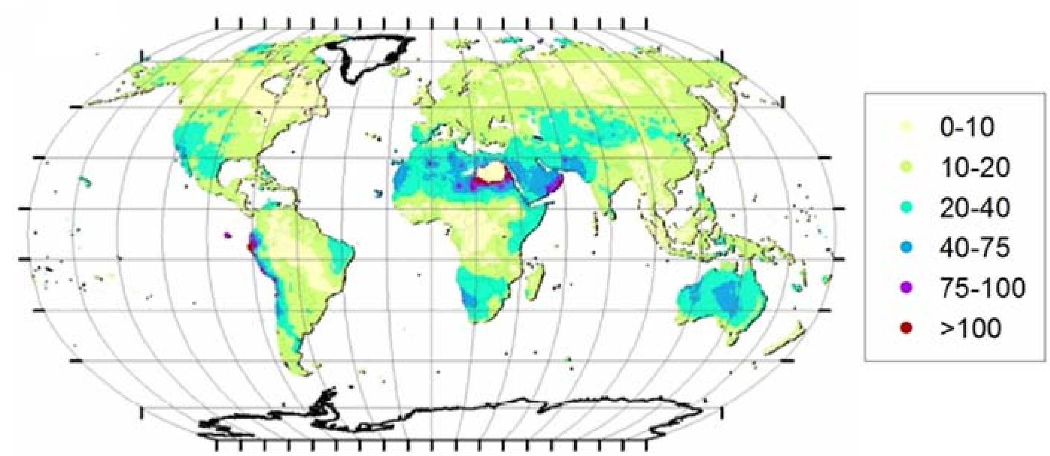

Figure 2.3 Minimal detectable trend in precipitation over the next 50 years, shown as percentage change from historic values. Adapted from Morin 2011. Based on criteria for α=0.05 and β=0.50.

Morin also focuses on one site (with precipitation mean of 600 mm and CV 0.2) to demonstrates the relationship between α and β (or 1-β, as shown in the graph), reproduced in Figure 2.4. In interpreting this figure, Morin states that criteria chosen for α and β are "somewhat arbitrary parameters that obviously affect the minimal detectable absolute trend." Note that in Figure 2.4, the graph region representing both low α and β is n the lower right corner; this region is dominated by high values of minimum detectable trends, such as 35 mm which would constitute a change of 29% of the mean.

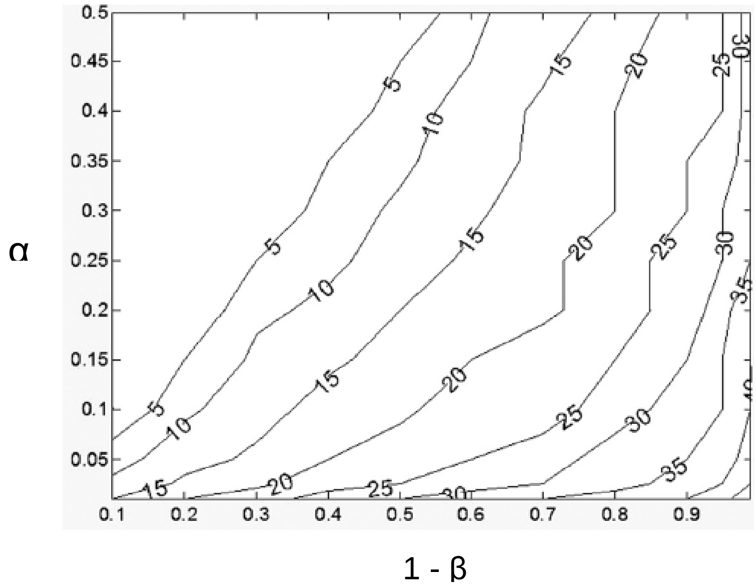

Figure 2.4 Minimum detectable trend (in mm/decade), related to α and 1-β. Adapted from Morin 2011. Minimal detectable trend precipitation in mm/decade, based on Monte Carlo experiment of 50 years of precipitation with mean 600 mm and CV 0.2.

2.4. Analytical Estimates of α and β

The analytical calculation of the p-value, which we use a proxy estimate of α, is widely used. These calculations are derived from the principle that coefficients of a linear regression are themselves random variables that follow a t-distribution. This calculation can be found in various statistics texts (for example, see Haan 2002, Onoz 2003), and is built in to many statistical software packages (such as R).

Dupont and Plummer (1990, 1998) describe an analytical calculation of β (or power, 1-β) for a linear regression. Unlike α, the analytical calculation is not as widely used, though Dupont and Plummer provide easily usable equations, as well as software they developed. The main equations to calculate β are provided in Figure 2.5.

$$\delta = \lambda \cdot \frac{\sigma_x}{s}$$

$$\beta = 1 - \left[T_v\left(\delta\cdot\sqrt{n} - t_{v,\alpha}\right) + T_v\left(\delta\cdot\sqrt{n} + t_{v,\alpha}\right) \right]$$

For a regression model $y_i = \gamma + \lambda \cdot x_i$

Where σ_x is the standard deviation of the independent variable

s is the standard error of the regression

n is the sample size

T_v is the cumulative probability distribution for a random variable having a t distribution with v degrees of freedom

$t_{v,\alpha} = T^{-1}(1-\alpha)$ denotes the critical value that is exceeded by such a t statistic with probability α

Figure 2.5 Equations from Dupont and Plummer (1998)

Using equations for Dupont and Plummer, we created a graph to illustrate the relationship between theoretical values of α vs. β (shown in Figure 2.6). As the requirement for α is lowered, β increases; this increase is steeper for cases with a stronger correlation (indicated by ρ^2). The commonly used value of α=0.05 is indicated (by the dotted vertical line) on the graph. It can be seen that (since the non-linear relationship is fairly steep at the point α=0.05), as small increase allowed in α would correspond to a substantial decrease in β.

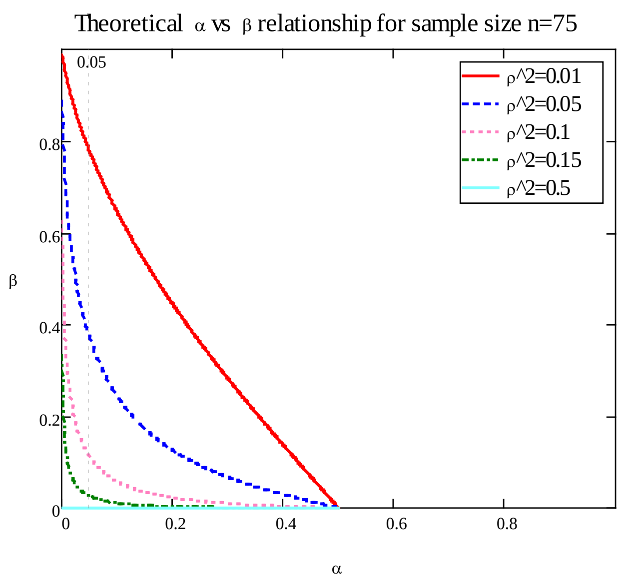

Figure 2.6 Theoretical α vs. β relationship for varying ρ^2 values

2.5. Decision Analysis Applications for Climate Change Adaptation

A decision tree is a structured graph allows us to formalize the decision points, probabilistic or deterministic process points, and outcomes that constitute the process we are trying to describe. Decision analysis helps us make sense of the two main components of estimating and assessing risk: the likelihood of an event, represented by chance nodes, and the consequences of that event, represented by outcome nodes. Decision analysis is ideally suited for the natural variability and uncertainty of hydroclimatic event, and has been used for choosing infrastructure and other adaptations for many years. (Beven 2009).

Thus it is natural for this technique to be applied to a new kind of uncertainty in hydroclimatic processes: climate change, and the possibility of non-stationarity in storms and floods. An early example of decision analysis for climate change applications is provided by Fiering and Matlas in 1990. They described a framework for using Bayesian decision theory to evaluate water resources projects in general, allowing for assessment of the probabilities of climate change based on scientists degree of belief. Chao and Hobbs (1997) give a brief history of decision analysis applications to climate change; they also describe their own method for evaluating breakwater adaptation under possible climate change impacts on Lake Erie, using stochastic dynamic programming.

Hobbs and Chao (1997) provide another example of decision analysis for water resources management under climate change. They demonstrate use of subjective likelihood of climate change impacts, comparing the results of decision analysis that sets this probability either at 1 or at 0.5. They describe a method to update posterior probabilities of climate change impacts in light of future observations, again using subjective probabilities. To our knowledge, no study has formulated a decision analysis in terms of the probability of Type I and Type II errors, and used the results of hypothesis testing as estimates for these probabilities.

3. Risk-Based Trend Detection Methods Part I

3.1. Applications and Required Assumptions

The Risk-Based Trend Detection method has been developed for situations in which there is a suspected but highly-uncertain increasing trend in damaging events; and there is a proposed adaptation that would reduce those damages, but requires a significant investment.

The strength of this method is its ability to deal with situations in which the adaptation is economically beneficial if there is an increasing trend in damaging events; but, in the absence of such a trend, the cost of the adaptation outweighs the damages avoided, and therefore is not economically viable.

Though it is by no means a requirement, this method is ideally suited for situations in which the certainty of the suspected trend is low, but where the repercussions of under-preparing are high. These are precisely the adaptation situations for which few quantitative methods are available to aid decision makers: a combination of high uncertainty, substantial investment, and large risks of damages. The value of this method is in guiding precisely these types of decisions, of bringing together the separate assessments of uncertainty and of damages in a meaningful way.

This chapter describes the data and calculations required to carry out the Risk-Based Trend Detection method and (many of which would also be required for a traditional Risk-Based Decision Making process); the next chapter describes how these calculations are utilized in the Risk-Based Trend Detection decision process.

Though this method could be readily applied to a variety of problems, we will describe its use for a general storm protection adaptation decision. In Chapter 5, we will demonstrate it through a case study in Mystic and Groton Long Point, CT; they are exploring the need for coastal protection against climate change, and are weighing proposed adaptations including hurricane barriers and flood-protection.

3.2. Data Requirements

This method entails very modest data requirements, which combined with the relatively simple computation required for the analysis makes the methodology feasible for a variety of users, with varying resources or technical knowledge.

The analysis considers all relevant, quantifiable damages; this is often limited to monetary property damages, but could also include ecosystem or recreational damages if they can be quantified in a way that they can be compared with the monetary value of the adaptation investment. The method is designed to utilize past, observed records which may or may not exhibit a trend that could be projected into the future. The method could be reformulated to utilize model output, or any other type of data for which the Type I and Type II errors of trend detection could be estimated.

Due to some of the caveats (discussed at the end of this chapter) associated with trend detection methods used and/or extrapolation into the future, this method should be applied to a decision with a reasonably short planning horizon into the future.

The required data are itemized in Figure 3.1:

Required Data

Observations	Observed record of past events (i.e. storm surges)
Damage Estimate	Function to relate level of event (i.e. elevation of storm surge) to amount of damage
Adaptation Costs and Information	Estimation of adaptation's initial capital cost and costs of operation and maintenance. Level of protection provided by adaptation. Lifetime of adaptation.

Additional Required Assumptions

Planning Horizon	Near-future planning horizon for adaptation decision
Interest Rate	Economic assumptions, including interest rate and possibly inflation rate.

Figure 3.1 Required data and assumptions

3.3. Trend Detection Steps

For extreme value analysis, we employ a data set constructed from the maximum event (i.e. storm surge) for each year on record. (See Appendix section 8.2 for more information) We will examine this Annual Maxima Series (AMS) for a trend over time. We make the following assumptions: (1) there may be a trend in the mean of the AMS, but higher order moments (standard deviation and skew) do not vary from the beginning of the period of record through the end of the planning horizon, and (2) the trend in the mean can be projected a short time into the future.

As the trend detection method is not the primary focus of this work, we have sought to utilize a well-known and relatively simple technique, and have thus chosen a linear regression of transformed data for this demonstration. Given the records of storm surges for the coastal Connecticut case study in Chapter 5, and given the short projection period, we believe a trend in the mean of the AMS can be reasonably-well estimated by fitting a power law model. The methodology could as easily be applied using a different non-parametric or non-linear model of the trend, given that a technique was available to estimate the probability of Type I and Type II errors.

In order to meet the requirements of normal and heteroscedastic residuals, we performed a natural log transformation on the observed anomalies. We then performed ordinary least squares (OLS) to fit the linear regression model. From that regression, we can calculate the conditional mean of the AMS of storm surges for every year of the planning horizon. Well-established analytical methods are available to estimate the probability of Type I errors for models fit using OLS linear regression (See discussion in Section 2.4). For this study, the one-tailed α estimate is used.

The probability of Type II error, β, is calculated using the power analysis method outlined by Dupont and Plummer (1998), also described in Section 2.4. These calculations are done in R, using a custom-written function to estimate α and β, based on the built-in function for an OLS linear model. We have chosen an analytical method to estimate β, though a Monte Carlo technique could also be used to calculate β.

It is important to note that, though we have calculated α and β, we do not evaluate them at this point in the process. Instead of comparing α and β against some predetermined critical values of $\alpha_{critical}$ and $\beta_{critical}$, the interpretation is carried out at a later stage, in the context of the economic risk of either erroneously accepting a non-existent trend or rejecting an actual trend. Thus, we continue the analysis even if the regression yields a value of α well over 0.05, which would traditionally cause the trend to be considered statistically insignificant and further analysis precluded.

3.4. Extreme Value Analysis

Most often, in extreme value analysis, a Generalized Extreme Value (GEV) distribution is fit to the series of annual maximum storm surges. (Kirshen et al. 2008, van den Brink et al. 2003, Flather and Williams 2000) This is a three-parameter distribution, consisting of location, size, and shape parameters, which is now widely used for probabilistic assessment of natural hazards. Previous to the GEV distribution, the Gumbel distribution which is a special case of the GEV was widely used in the analysis of natural hazards; the Gumbel is still sometimes used when the GEV shape parameter is close to zero.

For a stationary analysis, the mean, standard deviation, and skew of the AMS are used to estimate the GEV location, size, and shape parameters, respectively. The goodness of fit of the GEV distribution is evaluated by comparing predicted values against ranked observed values in a probability plot (also known as a quantile-quantile plot) and by calculating a probability plot correlation coefficient.

Once the parameters are estimated, the GEV distribution can be used to predict the expected elevation of storm surges at any given probability. Following US Army Corps of Engineers (USACE) guidelines, we calculate the expected storm surge elevations for probabilities between 1/2 through 1/500 (Nat'l Research Council 2000). In this way, the fitted GEV distribution is used to describe a storm surge elevation-frequency relationship.

3.5. Extreme Value Analysis Under a Trend

We begin the non-stationary analysis by combining the trend model with the GEV model. In the stationary analysis we estimated GEV parameters using the moments of the AMS. Here we estimate GEV parameters using the trend regression which provides an estimate of the mean of storm surges conditioned on the year of interest; the stationary standard deviation; and the stationary skew. Thus, we define a non-stationary GEV distribution where resulting quantiles now depend on the year of interest. The fitted non-stationary GEV model can now be evaluated using a variant of the probability plot, by comparing the ranked observed values against estimated maxima values for the year in which the observed value occurred.

Just as we described a storm surge height-frequency relationship for the stationary assumption, we can also use the non-stationary GEV distribution to describe a specific height-frequency relationship for any specified year. We calculate the height at probabilities between 1/2 through 1/500, and for each year in our planning horizon.

3.6. Risk-Based Decision Making

In this section we adapt existing ideas from the field of Risk-Based Decision Making. We begin with an estimate of the damages based on the elevation of the storm surge. This information is often available as a function which describes the storm surge elevation-damage relationship. If such information is provided as a series of damage estimates at varying elevations, we would fit an appropriate function (i.e. linear, quadratic, or spline) to the data points.

We then combine the storm surge elevation-frequency relationships with the damage estimates to yield a damage-frequency relationship. This entails development of a single damage-frequency curve for the stationary analysis, and a different damage-frequency curve for each corresponding year in the non-stationary analysis.

To summarize the damages over all possible outcomes, Risk-Based Decision Making calculates the Expected Damages (ED) on an annual time-step. The ED are calculated as the integral of the damage-frequency curve over all possible outcomes (exceedance probabilities from 0 to1).

The stationary analysis results in a single value for the Expected Damage, which is identical for every year in the planning horizon. For the non-stationary analysis, a different Expected Damage is results in each year, because the mean of the AMS is assumed to increase over the planning horizon. Each value of ED is then adjusted for interest and/or inflation, and summed to yield the present worth of the total Expected Damages for the stationary and non-stationarity scenarios.

In order to account for inflation and interest rate, the costs of damages and the Operations & Maintenance costs of the adaptation are converted to Present Worth (PW) values. Any capital or other costs that do not occur in year 1 of our analysis are also converted to Present Worth using the planning horizon and interest rate assumed in Figure 3.1.

3.7. Summary of Preparatory Calculations

In summary, the steps described in this chapter are outlined in Figure 3.2. A traditional Net Benefits Analysis would only require the stationary analysis steps shown in the left column of Figure 3.2. Our new non-stationary, risk-based method requires consideration of the costs based on both the stationary and non-stationary analyses.

Stationary Analysis: no change in mean of storm surges	**Non-Stationary Analysis:** increasing trend in mean of storm surges
Annual Maxima Series of storm surges	Annual Maxima Series of storm surges
	+
	Increasing trend in AMS (Regression magnitude used here. Statistical significance 1-α and power 1-β of regression considered later in process.)
↓	↓
GEV distribution parameter estimates	**Non-Stationary** GEV distribution parameter estimates
↓	↓
Elevation-Frequency curve	Elevation-Frequency curve **for each year**
+	+
Elevation-Damage estimate	Elevation-Damage estimate
↓	↓
Damage-Frequency curve with and without adaptation	Damage-Frequency curve with and without adaptation **for each year**
↓	↓
Expected Damage (integral) with and without adaptation per year of planning period	Expected Damage (integral) with and without adaptation **for each year** of planning period
↓	↓
Damages converted to Present Worth dollars	Damages converted to Present Worth dollars
↓	↓
Total Expected Damages for planning period with and without adaptation	Total Expected Damages for planning period with and without adaptation with increasing trend

Figure 3.2 Outline of calculation steps for stationary and non-stationary analysis

The analysis summarized in Figure 3.2 results in estimates for the costs summarized in Figure 3.3, all converted to Present Worth dollars:

Expected Damages with	no adaptation	no change in mean of events
	with adaptation	no change in mean of events
	no adaptation	increasing trend in mean of events
	with adaptation	increasing trend in mean of events
Capital Cost of Adaptation		
Operations & Maintenance of Adaptation		

Figure 3.3 Resulting cost and damage estimates

These cost estimates summarized in Figure 3.3 are combined with the probabilities of type I and II errors associated with trend detection (estimated by α and β) in later steps of our Risk-Based Trend Detection method.

3.8. Caveats

There are many caveats and justified warnings associated with detecting a trend observed in the past record and projecting it into the future, as is required by this method. Even if we were certain of a trend during the period of record, there is no assurance that this trend will either continue into the future, or continue as it has in the past. Extrapolation of any statistical model is fraught with uncertainties. With these considerations in mind, we maintain that using past records to predict future expectations is nonetheless useful for planning, given that it is done carefully and our concerns raised here are considered.

We stipulate a few requirements to counterbalance possible errors posed by projecting a trend into the future. Since predictions are expected to be less reliable the further into the future the trend is projected (i.e. the more distant from the last observed record and the longer the projection period relative to the period of record), applications of this method are restricted to only those with relatively short planning horizons. For example, it might be reasonable to use 75-years of observed storm-surge records to consider a 20-year planning horizon for future storm protection. In addition, a conservative model is chosen so that the projection will most likely err on the side of under-estimating future levels. We recommend that confidence intervals be constructed about the trend as well as the trend extrapolation to clarify the degree of uncertainty associated with the trend extrapolation.

We also acknowledge that α and β as actually proxies for probabilities of Type I and Type II error. The use of the p-value to estimate Type I error in particular, is a point of contention in the debate over the use (and misuse) of Null Hypothesis Significance Testing. Hubbard (2011) explains the technical inaccuracy of such a practice, as well as describing in depth the many cases of the practice in books and peer-reviewed journals, many by experts in statistics and applied fields. Hoenig and Heisey (2001) argue that analytical calculations of β from observation data are not meaningful, because they depend on the calculated p-value.

We argue that, while the analytic calculations of α and β are only proxy estimates, they still offer a lot of value in conveying the likelihood of Type I and Type II error. Further, as opposed to the field of psychology, out of which Hubbard's work comes, our data do not constitute a small sample from a much larger population, and from which a second sample could be taken; when attempting to detect a trend that we believe has only been in effect for a relatively short time period, our sample may actually constitute the entire population (for that site). Therefore, using α as a proxy may be appropriate for climate change detection, where it would not for other applications. Finally, we believe that the frequent use of the p-value as a proxy for α in peer reviewed literature reflects its acceptance, and a recognition of the value of an estimate of Type I error.

4. Risk-Based Trend Detection Methods Part II: Decision Tree Framework

4.1. Comparison with Traditional Net Benefits Interpretation

We begin by describing how the cost estimates generated by the procedure described thus far would be used in a traditional approach. A Risk-Based Decision Making process is employed, which uses Net Benefits as a metric to evaluate a proposed adaptation plan.

First we would determine whether to plan for the future based on an increasing trend of storm surges in the future, or to plan based on stationarity. Ordinary least squares (OLS) regression may be used to fit the regression line, first applying a logarithmic transformation to ensure normally distributed residuals. We would calculate α to characterize the statistical significance of the regression.

If α were greater than 0.05, traditionally we would dismiss the trend; complete the analysis based on no change in the mean of storm surges; and likely not consider the possibility of an increase at any further point in the analysis. If α were below 0.05, we would complete the analysis assuming an increasing trend in the mean of the events, and likely would not consider the uncertainty of the trend again beyond reporting the value of α. Given the high natural coefficient of variation and typically small sample size of observations of storm surges, α will frequently be well above 0.05. Thus, the economic analysis will frequently be completed without considering the possibility of an increasing trend.

Calculation of the Net Benefits is a method that allows us to use a single number to assess the relative advantage of an adaptation vs. no action (given a single scenario of the future). Once all the costs are estimated, the Net Benefits are easily calculated as the damages avoided through adaptation, less the adaptation investment required to prevent those damages as shown in Figure 4.1:

$$\boxed{\text{Net Benefits}} = \boxed{\text{Expected Damages without adaptation}} - \boxed{\text{Expected Damages with adaptation}} - \boxed{\text{Total cost of adaptation}}$$

Figure 4.1 Net Benefits calculation

If the Net Benefits are positive, then the damages avoided justify the cost of the investment and adaptation is recommended. If the Net Benefits are negative, adaptation is considered not economically advantageous.

It is important to note that before we can evaluate the Net Benefits of an adaptation, a single climate scenario must be chosen. For example, if we assume that storm surges are stationary (which would likely happen if α>0.05), then Figure 4.2 illustrates the calculation of Net Benefits given that climate scenario. Unlike the decision trees we will introduce later in this chapter, Figure 4.2 does not contain a probabilistic component; in other words, it contains decision nodes, but has no chance nodes.

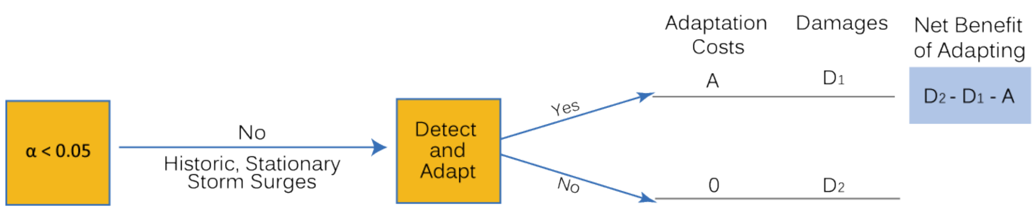

Figure 4.2 Net Benefits following prediction of stationary storm surges

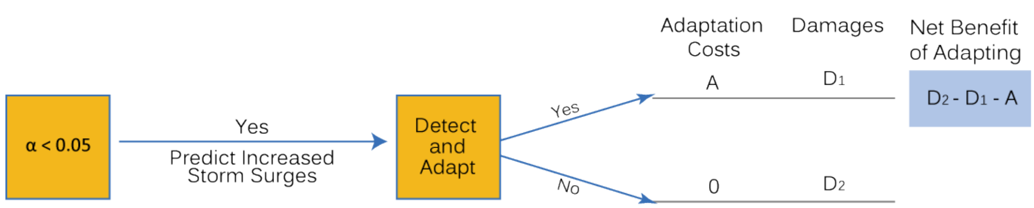

Figure 4.3 Net Benefits following prediction of increasing storm surges

We could do another Net Benefits calculation assuming that there will be an increase in storm surges as indicated by our trend detection, illustrated in Figure 4.3. However, there is no means to consider all four possibilities (with and without an increase in storm surges, with and without adaptation).

The level of uncertainty that caused the trend to either be rejected or accepted may be reported along with the Net Benefits. Using this traditional approach, however, there is no mechanism for interpreting the Net Benefits in light of the uncertainty, or interpreting the uncertainty in light of the economic considerations.

4.2. A Decision Tree Framework for Risk-Based Cost Estimates

Here concepts of risk-based decision analysis (described in Chapter 2), are used to guide the adaptation decision process. A traditional decision tree considering an increasing trend in storm surges and proposed storm protection adaptation is depicted in Figure 4.4.

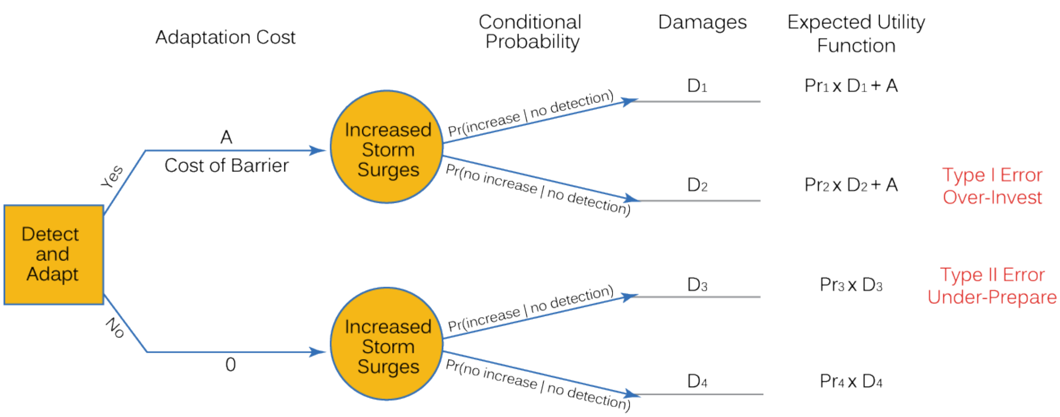

Figure 4.4 Decision tree with conditional probabilities

Our decision to (accept the increasing trend and) adapt is indicated by the decision node at left. Branches from that node represent the decision to adapt along with the associated costs (calculated in the previous chapter), or the decision not to adapt with a zero cost. Unlike the illustrations of Net Benefits consideration in 4.1, all four possibilities considered previously are represented in the decision tree. The probability nodes represent the likelihood of experiencing increased storm surges. To describe these likelihoods, we would need to calculate conditional probabilities: i.e., the probability that there will be an increase given that we accepted a past trend, and the probability that there will be an increase given that we did not accept a past trend.

Next, the damages estimates (calculated in the previous chapter) would be added for each combination of adaptation choice and increasing trend. The conditional probability of each branch would be multiplied by the cost of damage of each branch, plus the cost of the adaptation, to yield the expected utility associated with each outcome. The decision to adapt could be evaluated by comparing the expected utility of the upper branches (proceeding from the decision to adapt) with those of the lower branches (proceeding from the decision to not adapt).

Unfortunately, there is no generally accepted technique to estimate the required conditional probabilities. Experiments have been done to approximate the conditional probabilities using Bayesian analysis, or by aggregating predictions from suites of model predictions, such as Global Circulation Models (GCMs) or Regional Circulation Models (RCMs) (Hobbs 1997, Manning 2009). However, both approaches require a number of assumptions, and neither has attained general acceptance.

Consideration of the decision tree above, however, leads us to consider another approach. We notice that the second branch (predicting an increase in storm surges but experiencing no such increase) is a Type I error. The third branch (experiencing an increase in storm surges after predicting no change, or incorrectly accepting the null hypothesis) is an error of Type II. As described earlier, approaches are available to estimate the probabilities of Type I and Type II errors, α and β, respectively.

4.3. Assigning Probabilities to the Decision Tree Framework for Risk-Based Trend Detection

The 2nd and 3rd branches of the decision tree are the two which represent the Type I and Type II errors, and are the branches for which we have probability estimates α and β. A new metric is introduced to compare the relative cost of these errors. Similar to the way in which Net Benefits summarize cost estimates of two options in a single measure, the Costs of Regret summarizes the costs of all four branches of the decision tree with only two measures.

For each possible chance outcome (increased storm surges or no increased storm surges), the Cost of Regret represents the price of having made the wrong choice. This is also the cost of either a Type I or a Type II error. In this way, decision-makers can evaluate the choice before them not only by considering their certainty in the need for adaptation, but also by considering what is at risk if their conclusion about this need turns out to be incorrect.

Calculations for the two measures, the Cost of Regret of Under-Preparing, and the Cost of Regret of Over-Investing, are summarized in Figure 4.5 and Figure 4.6.

For example, if there turned out not to be an increase in storm surges, how much better off would a community have been if they had not invested in adaptation, than if they had wrongly predicted an increase and invested? This is the Cost of Regret of Over-Investing (adapting when not adapting would have been preferable shown in Figure 4.4:

$$\boxed{\begin{array}{c}\text{Cost of}\\ \text{Over-Investing}\end{array}} = \boxed{\begin{array}{c}\text{Total Cost of}\\ \text{Adaptation}\end{array}} + \boxed{\begin{array}{c}\text{Expected Damages}\\ \text{without increase}\\ \text{with adaptation}\end{array}} - \boxed{\begin{array}{c}\text{Expected Damages}\\ \text{without increase}\\ \text{without adaptation}\end{array}}$$

Figure 4.5 Cost of Regret of Over-Investing

This calculation takes into account the fact that adapting could prevent some damages, even though it does not prevent enough damages to have made the adaptation worth the expense.

If there turned out to be an increase in storm surges, how much better off would a community have been if they had adapted, than if they'd wrongly predicted no increase and hadn't prepared? This is the Cost of Regret of Under-Preparing (failing to adapt when adapting would have been preferable), calculated by the equation shown in Figure 4.6:

$$\boxed{\begin{array}{c}\text{Cost of}\\ \text{Under-Preparing}\end{array}} = \boxed{\begin{array}{c}\text{Expected Damages}\\ \text{with increase}\\ \text{without adaptation}\end{array}} - \boxed{\begin{array}{c}\text{Expected Damages}\\ \text{with increase}\\ \text{with adaptation}\end{array}} - \boxed{\begin{array}{c}\text{Total cost of}\\ \text{adaptation}\end{array}}$$

Figure 4.6 Cost of Regret of Under-Preparing

This calculation takes into account the fact that there would have been some residual damages, even if the adaption plan had been implemented.

In the same way Expected Utility is calculated in a traditional decision tree, estimates for the Cost of Regret and probability of error, can be multiplied to yield Expected Regret.

As there are two values for Costs of Error, there will be two values of Expected Regret: that of Over-Investing and that of Under-Preparing.

A sample decision tree for a coastal adaptation problem is shown in Figure 4.7:

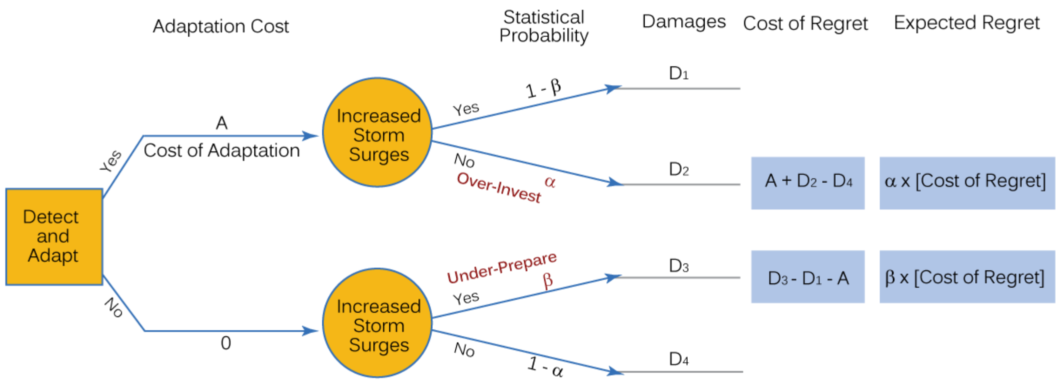

Figure 4.7 Decision tree with probabilities α and β

As with the previous tree, the decision to adapt is represented by the decision node at far left, and the subsequent storm surge levels are represented by the chance nodes to their right. The two branches representing possible errors are labeled in red: Over-Investing (with estimated probability α) and Under-Preparing (with estimated probability β). The costs of all four branches are encapsulated by the Costs of Regret following the two error branches. Finally, the right column shows the two values of Expected Regret: the product of the probability and cost of Over-Investing; and the product of the probability and cost of Under-Preparing.

4.4. Expected Regret Criterion for Adaptation Decisions

A traditional analysis focuses only on whether or not the Net Benefits are positive. The Risk-Based Trend Detection method provides two values of the Expected Regret for comparison. Generally, one would recommend to invest in adaptation when the Expected Regret of Under-Preparing is greater than the Expected Regret of Over-Investing, and to not adapt otherwise.

Alternately, comparison of the Expected Regrets of Over-Investing and Under-Preparing could be used, not as a binary indicator, but as a quantitative assessment of the risk and certainty that would be used in a qualitative deliberation by the decision-makers. The stakeholders are the experts on their community's tolerance for risk. In some cases, the tolerance for Under-Preparing might be much lower. In other cases, the community might be accepting of a certain level of damage in the cases of extreme events, but less tolerant of committing resources directed to unused infrastructure. Thus, how the values of Expected Regret are interpreted, or at what critical ratio of Expected Regret they would decide to adapt, should be left to the decision-makers in each specific application. This analysis empowers them to make a well-informed decision.

This analysis should be updated regularly, as more observational data is available with each passing year. We recommend a plan to reevaluate the trend and the cost-effectiveness of the adaptation alternatives at pre-determined intervals.

4.5. Results of Risk-Based Trend Detection

Risk-Based Trend Detection satisfies the key needs outlined in Chapter 1:

- It gives needed attention to the risks and potential damages of Under-Preparing.
- It integrates statistical uncertainty of an increasing trend, with the economic impacts (of both over and under design) to provide meaningful metrics for decision-makers in terms of Expected Regret.
- It addresses the question, "Should we invest now, given the uncertainty of a future trend?"

Our analysis considers the implications of under-preparing instead of simply dismissing a trend based only on the probability of Type I error (over-investing) as is done in a traditional analysis. These two approaches can lead to dramatically different results. This is demonstrated in the following coastal Connecticut case study, where the traditional Net Benefits analysis recommended not to adapt, but Risk-Based Trend Detection (RBTD) using the same data led to a recommendation to adapt.

Furthermore, RBTD integrates the measures of statistical significance and economic estimates into a combined metric. Instead of considering the physically abstract measure of statistical significance on one hand, and the economic analysis on the other, Expected Regret encapsulates both the uncertainty and the economic impact of both under and over design. By using Expected Regret, we eliminate the need to arbitrarily set a critical value for α (usually 0.05). Instead we combine computed Type I and Type II error estimates (α and β) with their associated physical and economic implications.

Risk-Based Trend Detection fills a gap that exists in the currently available tools for planning for climate change impacts. Many successful studies have attempted to predict the degree of climate change's impact on storms and floods; consider the damages caused by increasing storms and floods; and consider the ability of adaptation measures to reduce these damages. However in the end, decision-makers are left with impacts of plausible scenarios, and warnings of uncertainty of those scenarios; but they are provided little information to support on-the-ground decisions of whether to adapt now, and further to justify such investments to relevant constituents. The decision-makers must rely on an intuitive tolerance for risk, or subjective "expert opinion" about the likelihood of increasing storms or floods to address the fundamental question: "Should we adapt now despite the uncertainty?" This method equips them with a thorough, defensible approach in which the evaluation of uncertainty is quantitative and transparent.

5. Case Study: Mystic, CT

5.1. Case Study Description

The RBTD method is applied to an adaptation decision for coastal protection in the villages of Groton Long Point and Mystic, CT. Both of these areas are (entirely or partly) contained within the municipality of Groton, CT. The Town of Groton, CT has been actively engaged in the task of climate change evaluation and preparedness. In January through June of 2010, three workshops were held engaging local, state, and federal government officials and various stakeholders; the workshops addressed climate change impacts and adaptation in Groton, with a special focus on Groton Long Point and Mystic, CT. These workshops were conducted by ICLEI-Local Governments for Sustainability and Connecticut Department of Environmental Protection, under the U.S. EPA's Climate Ready Estuaries program. (Stults and Pagach 2011)

We have utilized both the selection of adaptation alternatives, and the cost and damage estimates from these workshops, to demonstrate the application of Risk-Based Trend Detection to this adaptation decision process.

Figure 5.1 Map identifying mystic, Groton Long Point, and New London, CT

5.2. Sea Level Data

NOAA operates a New London, CT tidal gauge station. The station is actually located in the New London harbor, directly between Groton and New London, CT, and is less than 7 miles from either Groton Long Point and Mystic. NOAA provides hourly records of predicted (tidal predictions) and observed sea levels through its Tides & Currents website.

In addition, the Joint Archive for Sea Level (JASL) (a collaboration between the University of Hawaii Sea Level Center and the US National Oceanographic Data Center) has produced a data set of verified (observed) tidal records, based on the NOAA New London, CT gauge. These predictions have undergone a thorough quality control process and are considered "research quality". (Caldwell and Merrifield 2011) Hourly values for predicted and observed sea levels at this site were obtained from 1938 (after the September 1938 hurricane) to present.

Following the method used in a 2008 coastal storm analysis by Kirshen, et al, (2008) each hourly observed level is subtracted from the corresponding predicted level; this yields a series of sea level anomalies, where the sea level is higher or lower than the tide-based prediction. Unlike Kirshen, et al, we do not attempt to separate the effects of mean sea level rise and increasing storm surges; thus a trend in anomalies might be due to a combination of these factors. The result of this analysis is a data set of hourly sea level anomaly heights over the period 1938 to present.

In order to determine storm surge heights, we need to add the anomaly to the regular sea level height. Future anomalies could occur during any point in the tidal cycle. As a precautionary measure, we plan for the case when the anomaly occurs at the high tide. We add the sea level anomaly height to the monthly high tide elevation, known as the Mean High High Water (MHHW), yielding the storm surge elevation.

5.3. Damage Estimates and Proposed Adaptation Alternatives

Reports from the Groton, Connecticut Coastal Climate Adaptation Workshop report a series of both "soft" and "hard engineering" adaptations for "moderate" sea level rise; and "hard engineering" adaptations for "significant" sea level rise. We focus on the alternatives under consideration for "significant" sea level rise. (Bosma 2010)

The options listed in Figure 5.2 were outlined for Mystic and Groton Long Point (Kirshen 2010, Merrill 2010). The adaptations listed below would protect both portions of Mystic in the Town of Groton (the west side of the river), and in the portions in the Town of Stonington (east of the river). It was assumed that if the adaptations were implemented, the costs would be split approximately equally by the two towns. As Mystic is largely around and inland of the Mystic Harbor (see Figure 5.3), infrastructure at the mouths of the rivers flowing into the harbor provide the best protection:

Adaptation Alternative	Protection Level (NAVD88)	Adaptation Description	Initial Capital Cost	Annual O&M
A	5.4 ft (1.65 m)	No action		
B	7.4 ft (2.26 m)	Hurricane Barrier at Mystic River entrance.	$18 Million	$75,000
C	8.9 ft (2.71 m)	(in addition to above) Elevating the railroad and increased diking to east.	$27-30 Million	$100,000
D	10.5 ft (3.20 m)	(in addition to above) Further elevating the railroad and increased diking to east.	$35 Million	$120,000

Figure 5.2 Table of Mystic, CT conceptual design costs of adaptation alternatives from 2010 workshops. Costs given are total costs, to be split by Towns of Groton and Stonington. (Adapted from Kirshen 2010)

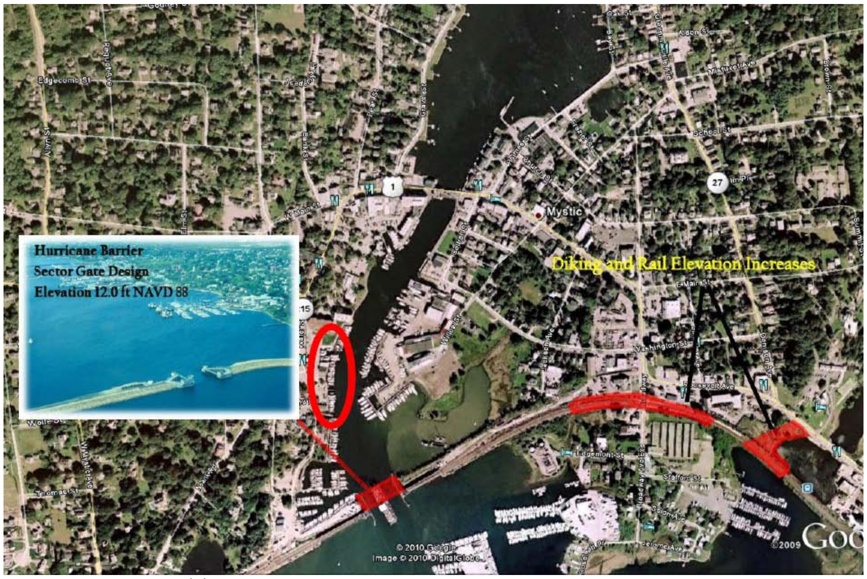

Figure 5.3 Map of Mystic, CT adaptation alternatives from 2010 workshops (Source: Kirshen 2010)

As Groton Long Point is located on a peninsula (see Figure 5.5), a combination of smaller tide gates and flood-proofing measures would be required to reduce storm damages. The adaptation alternatives under consideration are listed in Figure 5.4:

Adaptation Alternative	Protection Level (NAVD88)	Adaptation Description	Initial Capital Cost	Annual O&M
E	3.2 ft (0.98)	No action		
F	4.0 ft (1.22 m)	Barrier (with bottom rising gate) at entrance to lagoon; Modification to culvert and causeway to Island Circle South; Residential flood-proofing or elevation in designated region	$16 Million	$80,000
G	6.5 ft (1.98 m)	(in addition to above) Elevate existing seawall New dike with culvert to Island Circle North Residential flood-proofing or elevation in larger region	$27 Million	$100,000
H	7.4 ft (2.26 m)	(in addition to above) Additional flood-proofing or elevation and evacuation plan	$34 Million	$200,000
I	10.5 ft (3.20 m)	(in addition to above) Additional flood-proofing, elevation, or building upgrades, and evacuation plan	$37 Million	$3 Million

Figure 5.4 Table of Groton Long Point, CT conceptual design costs of adaptation alternatives from 2010 workshops (Adapted from Kirshen 2010)

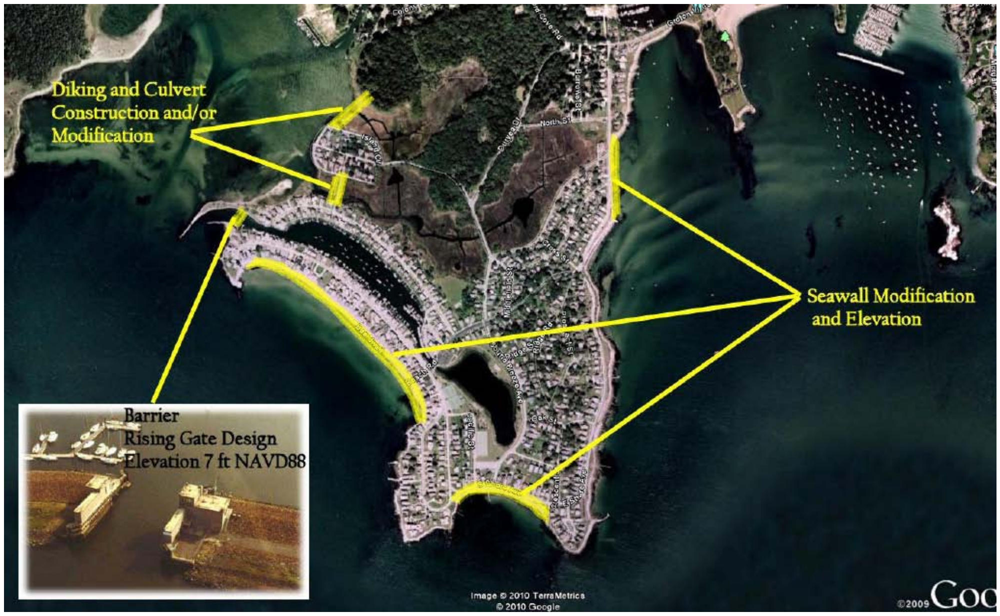

Figure 5.5 Map Groton Long Point, CT adaptation alternatives from 2010 workshops (Source: Kirshen 2010)

The damages at varying elevations for both sites were also estimated and reported at the workshops. (Kirshen 2010) Six point estimates from each site relate storm surge elevation with damage amount, shown in the Figure 5.6.

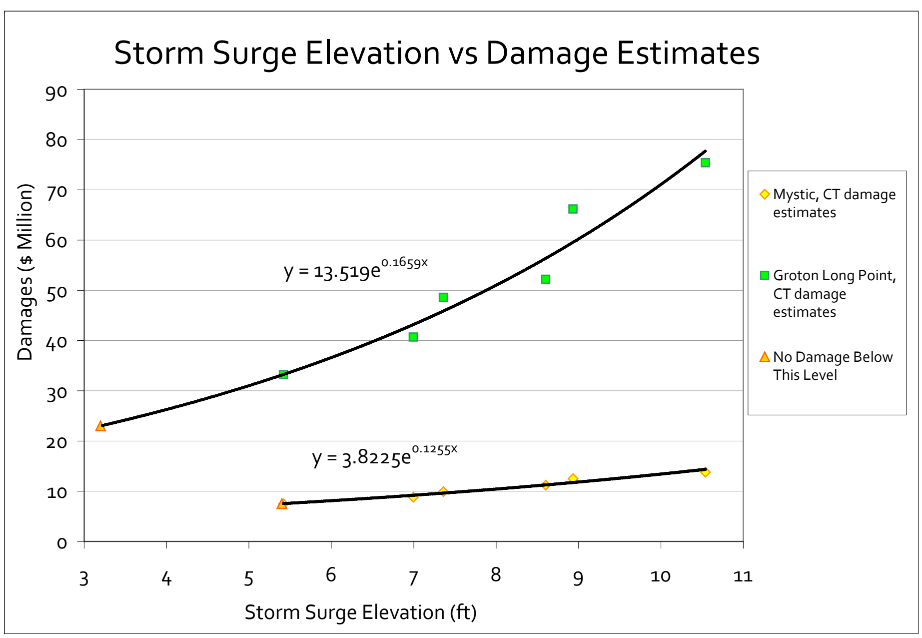

Figure 5.6 Damage estimates for Mystic and Groton Long Point, CT

All estimates include only damages within the Town for Groton; though the Mystic adaptations would also reduce damages in the Stonington portions of Mystic, they are not included in these figures. No damage occurs from storm surges below 5.4 ft in Mystic, or below 3.2 feet in Groton Long Point. Simple exponential functions were fit to the data points to describe the relationships between damage and storm surge elevation for each site, as shown in Figure 5.6. Using these functions, we can estimate the damage at any storm surge elevation.

5.4. Trend Detection in Sea Level Anomalies

Following the extreme value analysis methods (see Appendix 8.2), we analyze the record of sea level anomalies by constructing a series of the annual maxima. This annual time series is then analyzed for a trend. We attempt a number of transformations of the dependent variable, in order to find a transformation that will meet the required assumptions of a linear regression model. The attempted transformations include logarithmic, exponential, inverse, and square root. We performed Ordinary Least Squares (OLS) linear regression on each transformed data set, as well as the original observations, and analyzed the residuals. The linear regression on both the logarithmic- and inverse-transformed data resulted in acceptably normally distributed and heteroscedastic residuals. Of the two, we chose the logarithmic model due to its widespread use. Diagnostic graphs of the residuals of the logarithmic-transformed observations are shown in the Appendix, Figure 8.1.

OLS linear regression also requires that the data be independent. Annual maxima series are used in extreme value analysis precisely to provide this independence. As a further diagnostic, we ran performed an analysis of the autocorrelation function (ACF). No significant autocorrelation was found. Plots of the autocorrelation and partial autocorrelation are also provided in the Appendix, Figure 8.2 and Figure 8.3.

The chosen regression model is plotted in Figure 5.7. The natural log of the anomalies vs. time is shown in the graph at left; the graph at right shows the data and the regression line re-transformed into linear space. The chosen linear regression model provides an estimate of the mean sea level conditioned on the year of occurrence. This can be used to fit a non-stationary probability distribution to the annual maxima data, by using the regression line in place of the stationary mean. (See Section 5.5)

Figure 5.7 Annual maxima of sea level anomalies, and regression of AMS, New London, CT

Next, we compute the Type I and Type II error estimates, α and β associated with the fitted regression. Here a one-tailed α-level is used, since our Type I error focuses on over-prediction of the trend. The power (1-β) is calculated using the method described by Dupont and Plummer (1990, 1998). Estimates of α and β for the New London trend are summarized in Figure 5.8.

α Probability of Type I Error	β Probability of Type II Error
0.14	0.48

Figure 5.8 Statistical properties of regression of log-transformed New London, CT sea level anomalies AMS

Note that for this dataset, α is quite high. Such a high value 0.14 would usually cause the trend to be ignored and further analysis dismissed. However, we see that β is also high. Despite our uncertainty about the trend, the high likelihood associated with missing the trend (β =0.48) justifies further investigation.

5.5. Stationary and Non-stationary GEV Fits

The Generalized Extreme Value (GEV) distribution is utilized, enabling us to describe the annual probability of sea level anomaly heights based on the annual maxima series data.

The stationary GEV distribution is fit to the AMS data, and a non-stationary GEV is fit using the conditional mean regression to estimate the location parameter. The size and shape parameters are estimated using the stationary standard deviation and skew. The stationary and non-stationary GEV parameters for the New London, CT Storm Surge AMS data are given in Appendix Figure 8.4. Since the shape parameter κ is so close to zero, we also fit the Gumbel distribution, a special case of the GEV distribution for which κ=0.

Probability plots (also known as quantile-quantile plots) used to visually evaluate the fits of the stationary and non-stationary GEV distribution and are summarized in Figure 5.9. (See also Gumbel probability plots in Appendix Figure 8.5.)

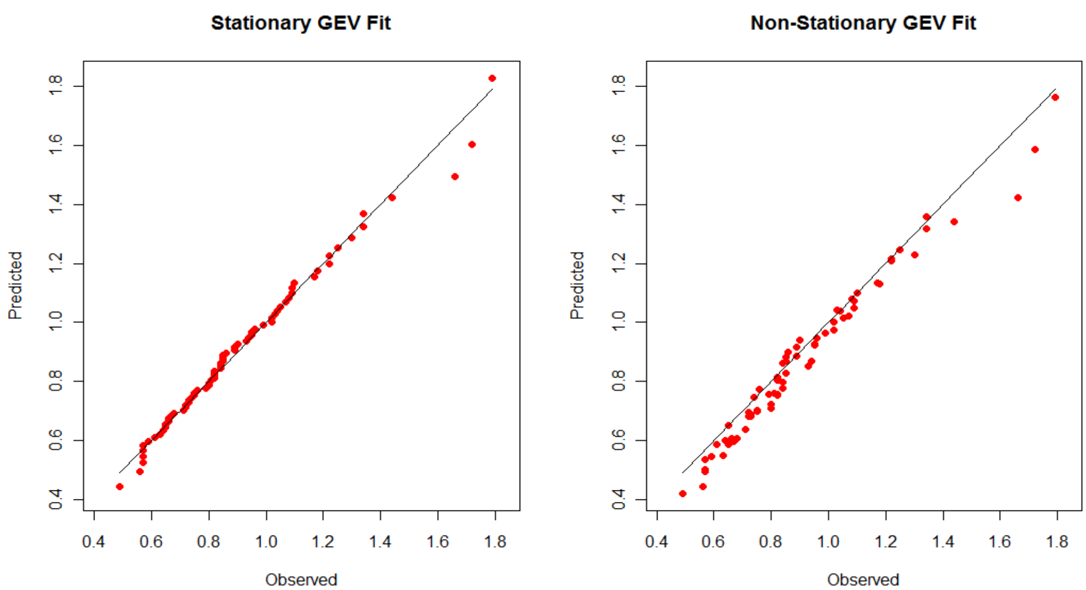

Figure 5.9 Probability plots: stationary and non-stationary GEV fit of New London, CT sea level anomalies AMS

The GEV or Gumbel parameters are then used to calculate elevations of varying expected frequencies; this results in a single height-frequency relationship for the stationary case, and a different height-frequency relationship for each year for the non-stationary case.

Adding the Mean High High Water level sea level anomaly height, we can then desribe the expected frequency of varying storm surge elevations. In order to compare our estimates with those currently used by the Town of Groton, we calculated storm surge elevation estimates for 10, 50, 100, and 500-return periods, and compared them with the values provided in the 2010 workshops. See Appendix Figure 8.6.

5.6. Stationary and Non-stationary Damage-Frequency

U.S. Army Corps of Engineers guidelines recommend consider damages at least between the 1/2 through 1/500 probability events (Nat'l Research Council 2000); we include the 0.001 to .99 probability events. Note that no damage is caused by events below 1.65 m (5.4 ft) in Mystic, or 0.98 m (3.2 ft) in Groton Long Point. The elevation-frequency relationship is illustrated in Figure 5.10.

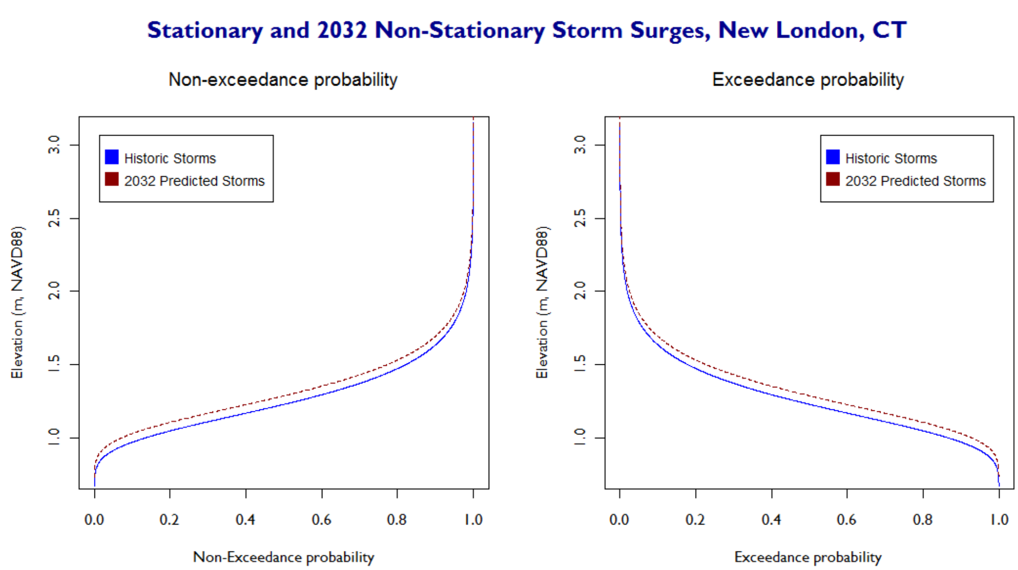

Figure 5.10 Elevation-frequency relationship for New London, CT storm surges

Note that hydrologists may be more familiar with exceedance probability vs. elevation relationships; however, in subsequent steps when the integral is taken, the relationship will need to be presented with the non-exceedance probability on the x-axis. For convenience, both presentations of the elevation-frequency relationship are presented in Figure 5.10.

The equations which describe damages incurred as a function of elevation (developed in section 5.3) are used to estimate the damage-frequency relationship. With no additional adaptation, damages only occur when elevations exceed a specified threshold (5.4 ft in Mystic, or 3.2 ft in Groton Long Point). With each adaptation alternative, and associated protection level, the threshold for damage is raised.

The area under the damage-frequency curve represents the Expected Annual Damage (EAD), which can be calculated by integration under the curve.. A single estimate of the EAD results for the stationary assumption, while for the non-stationary assumption, a different EAD results for each year.

Estimates of the EAD are calculated for each site, and for each proposed adaptation alternative. Figure 5.11 provides example illustrations depicting the damage-frequency relationship: for Mystic and Groton Long Point; under the stationary assumption and the non-stationary assumption for the year 2032; with and without the proposed adaptations B and E (the least expense alternatives beyond “no action”).

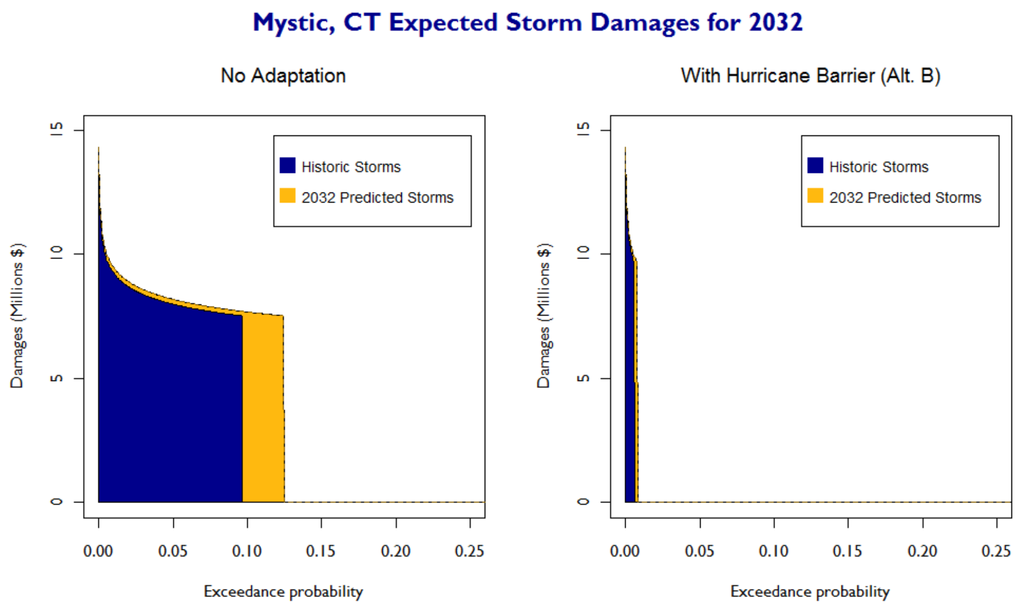

Figure 5.11 Sample damage-frequency curves for Mystic, CT

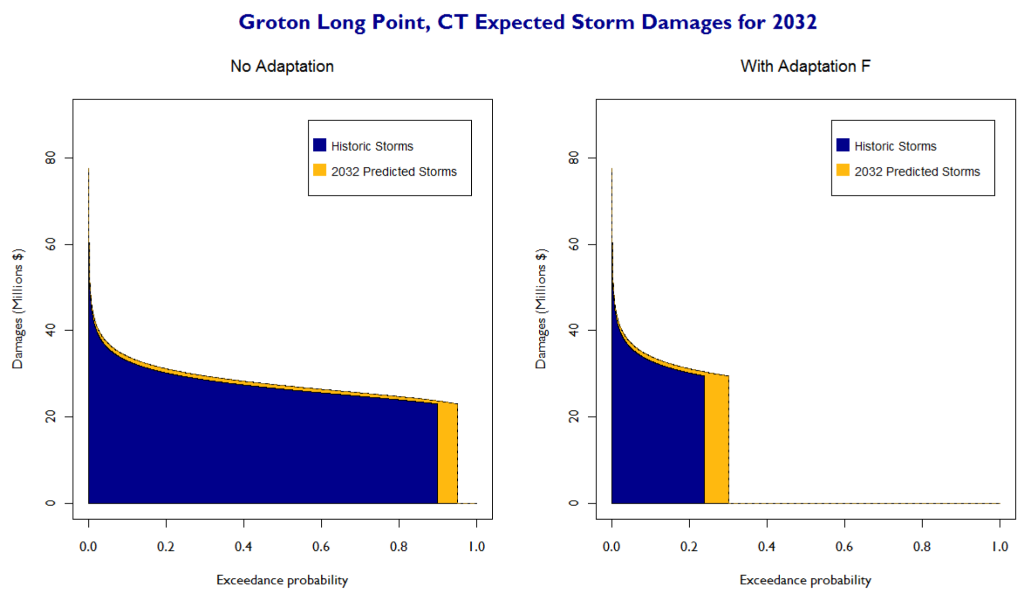

Figure 5.12 Sample damage-frequency curves for Groton Long Point, CT

The expected damages for the entire planning period is calculated by converting the expected damages for each year to Present Worth dollar, and taking the sum. For the conversion, we used the U.S. Army Corps of Engineers Project FY2012 "Evaluation and Formulation Rate" of 4%, as stated in their annual memo. (USACE 2011) Annual operations and maintenance costs of the adaptations were also converted to Present Worth dollars and added to the initial capital cost for the total cost of the adaptation over the planning period. Note that for the Mystic proposed adaptations, the total costs were split in half, as described in section 5.3.

5.7. Results of Traditional Risk-Based Decision Making Net Benefits

Using the traditional Net Benefits approach, we would either decide to accept or reject the increasing trend in storm surge heights, and then consider the damages avoided less the cost of adaptation. For comparison, we calculated the Net Benefits both under stationarity and under a projected increase; results for both sites are listed in Figure 5.13 and Figure 5.14.

	Adaptation Alternative			
	A (no action)	**B (protection to 7.4 ft)**	**C (protection to 8.9 ft)**	**D (protection to 10.5 ft)**
Net Benefits Under Stationarity (millions of dollars)	n/a	-0.68	-5.40	-10.36
Net Benefits given Projected Increase in Storm Surges (millions of dollars)	n/a	1.34	-3.19	-8.15

Figure 5.13 Net Benefits of Mystic, CT adaptation alternatives

	Adaptation Alternative				
	E (no action)	**F (protection to 4.7 ft)**	**G (protection to 6.5 ft)**	**H (protection to 7 ft)**	**I (protection to 8.6 ft)**
Net Benefits Under Stationarity (millions of dollars)	n/a	216.44	302.11	299.77	265.54
Net Benefits given Projected Increase in Storm Surges (millions of dollars)	n/a	216.88	318.28	317.08	284.20

Figure 5.14 Net Benefits of Groton Long Point, CT adaptation alternatives

Regardless of the magnitude or significance of the trend, the Net Benefits for some adaptations are always positive (such as Groton alternative G). Thus adaptation G is always cost-effective even for current climate conditions, and should be recommended. Regardless of the trend, we see that Net Benefits for some adaptations (such as Mystic alternative C) are always negative; therefore, we do not recommend adaptation C for the current planning horizon, given current information about climate change during that period.

However, for some adaptations, such as Mystic alternative B, we find that the Net Benefits would be negative if we dismissed the trend, and positive if we accepted the trend. Therefore, the decision of whether to recommend adaptation B is dependent on our degree of certainty concerning the observed trend. Since the probability of Type I error α for this trend detection is high (0.14), the trend would usually be dismissed and adaptation not recommended. We believe that the trend cannot be dismissed, however, without also considering the probability and consequences of the Type II error. Risk-Based Trend Detection allows us assess the adaptation given the high uncertainty, and probabilities of either Type I or Type II error.

5.8. Results of Risk-Based Trend Detection

A decision tree illustrating the Risk-Based Trend Detection calculations for Mystic, CT adaptation alternative B is shown in Figure 5.15.

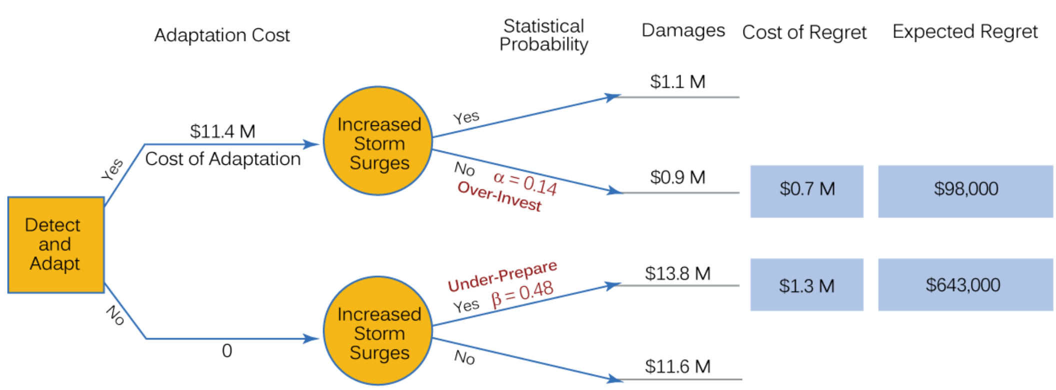

Figure 5.15 Risk-Based Decision Making tree for Mystic, CT, Adaptation B

Recall that the traditional Risk-Based Decision Making analysis for Mystic's hurricane barrier (adaptation alternative B) dismissed the trend that had an $\alpha > 0.05$; yielded Net Benefits under stationarity that were negative; and recommended against building the barrier. From our RBTD analysis, however, we see that the Expected Regret of Over-

Investing is \$98,000, while the Expected Regret of Under-Preparing is more than six times greater at \$643,000. This makes sense, since the Cost of Regret of Under-Preparing is more than twice that of Over-Investing, and in addition the probability of Type II error is more than three times that of Type I error. The result of our RBTD would be a strong recommendation to adapt, despite the uncertainty of an increasing trend.

The decision-makers will need to consider specifics of the situation, as well as their own tolerance for risk. These metrics of Expected Regret, though, empower them to make a well-informed decision.

6. Conclusions and Further Work

6.1. Further Improvements

In the demonstration of the Risk-Based Trend Detection method, a number of techniques were chosen for the intermediate calculation steps, which could be improved in the future. In the above analysis, the Method of Moments were used for estimating GEV parameters. Many studies have showed the benefits of L-moments for estimating GEV parameters, (Kirshen et al. 2008; Stedinger 1993) and we plan to use this technique for future applications.

As discussed in section 3.3, while we chose analytical techniques to estimate α and β, Monte Carlo methods could also be used to estimate the probability of Type I and Type II error. Such an approach could resolve the issues with use of proxy estimates for probability of error, discussed in 3.8. However, it might raise other issues, such as sensitivity to the choice of "stationary" mean and variance for generating a random sample.

As also mentioned in section 3.3, as the trend detection technique itself was not the primary focus of this work, a linear regression was chosen as a relatively straightforward and well-understood method. Further work might use other and non-parametric detection techniques, such as Mann-Kendall. Yue (2002, 2004), Onoz (2003), and Morin (2011) have done work examining the power of Mann-Kendall and other non-parametric techniques, using Monte Carlo analysis. To our knowledge, no published study has provided analytical estimation of the power of the Mann-Kendall test, so Monte Carlo estimates would most likely be necessary. It is interesting to note, however, that Morin (2011) found that for his particular application, the results of the linear regression and the Mann-Kendall were near identical, to the extent that results of only one technique was reported in the paper.

As Yue (2002) points out, however, the non-parametric tests are deemed more suitable for non-normal data; in addition, Onoz demonstrates cases where the non-parametric tests are more robust to mode misspecification than the parametric test.

This leads the conversion us to another source of error to consider, that of model misspecification. Sometimes referred to as Type III error (Kriebel et al. 2001), model misspecification is an important source of error to consider in improvements to this method. During model selection, we performed a simple comparison of the linear regression projections resulting from untransformed, log-transformed, and inverse-transformed dependent variables, and found that the difference was fairly small. However, a more complete analysis would include a comparison of the total estimated damages over the planning horizon based on each of the three models. This would give a sense of the sensitivity of the analysis to model misspecification.

Lastly, this method would benefit from further work to address the question of distinguishing trend from cycles or other "natural" climate behavior. For example, Cohn and Lins (2005) raise the question of Long-Term Persistence, and the difficultly it poses for climate trend detection. Further work could include a reevaluation of techniques utilized by this method with the precautions Cohn and Lins raise.

6.2. Additional Applications

This paper demonstrated Risk-Based Trend Detection for climate-change induced increases in coastal storm surges. Additional, closely-related applications of RBTD include other decisions for climate change adaptation. An application for evaluating investments in urban stormwater management in Somerville, MA is currently under progress.

While a main emphasis of this work was to highlight the need to address consequences of Type II error in climate change adaptation, this need is not limited to water resources decisions. Another possible application is the question of the cost-effectiveness of public health screening tests. In some areas of public health screening, multiple tests are available, some of which allow better early detection, but at a higher cost. Cost-effectiveness of these tests frequently take into account the probability of Type I error, but do not take into account the probability of Type II error. This application provides an interesting comparison, as certain caveats from our climate change adaptation application would not carry over, such as extrapolation of the trend and linearization of a complex relationships. However, we would need to carefully consider the use of the p-value as a proxy for probability of Type I error in applications for which available data comprises a sample from a much larger population.

6.3. Conclusions

We have demonstrated an approach that combines hypothesis testing, Risk-Based Decision Making, and decision analysis to address climate change adaptation decisions made under uncertainty. Whereas traditional Risk-Based Decision Making requires us to choose one climate change scenario, and then calculates the Net Benefits, Risk-Based Trend Detection allows us to consider multiple climate change scenarios by considering the Expected Regret under each.

The strength of this method is its ability to deal with situations in which the adaptation is economically beneficial if there is an increasing trend in damaging events; but, in the absence of such a trend, the cost of the adaptation outweighs the damages avoided, and therefore is not economically viable. These are precisely the adaptation situations for which few quantitative methods are available to aid decision makers: a combination of high uncertainty, substantial investment, and large risks of damages. This method helps guide precisely these types of decisions, by bringing together the separate assessments of uncertainty and of damages in a meaningful way.

Risk-Based Trend Detection gives needed attention to the risks and damages of under-preparing. It not only addresses the possibility of Type II error, it integrates the probabilities of both Type I and Type II errors with their associated economic repercussions to describe statistical uncertainty in terms of physical risks. Instead of presenting the costs and benefits of the adaptation recommended, with a footnote about the uncertainty of the trend, this method integrates them to help address the question, "Should we invest now, despite the uncertainty?"

The Expected Regret encapsulates both the statistical certainty and potential consequences of a trend. The Expected Regret of Over-Investing can be weighed against the Expected Regret of Under-Preparing. This relative comparison of risk and cost can be presented to decision-makers to assist them in evaluating their various adaptation (investment) options.

The considerations that this method allows can lead to dramatically different results than a traditional risk-based analysis. In our Mystic, CT case study, we showed that the traditional Risk-Based Decision Making analysis for the hurricane barrier (adaptation alternative B) dismissed the trend that had an $\alpha > 0.05$; yielded Net Benefits under stationarity that were negative; and recommended against building the barrier. From our RBTD analysis, however, we see that the Expected Regret of Over-Investing is $98,000, while the Expected Regret of Under-Preparing is more than six times greater at $643,000. The result of our RBTD would be a strong recommendation to adapt, despite the uncertainty of an increasing trend.

Generally, one would recommend to invest in adaptation when the Expected Regret of Under-Preparing is greater than the Expected Regret of Over-Investing, and to not adapt otherwise. Alternately, comparison of the Expected Regrets of Over-Investing and Under-Preparing could be used, not as a binary indicator, but as a quantitative assessment of the risk and certainty that would be used in a qualitative deliberation by the decision-makers. Risk-Based Trend Detection helps ensure that this qualitative decision will be well-informed and give appropriate consideration to the possibilities of both Over-Investing and Under-Preparing.

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8. Appendices

8.1. Graphs and Tables

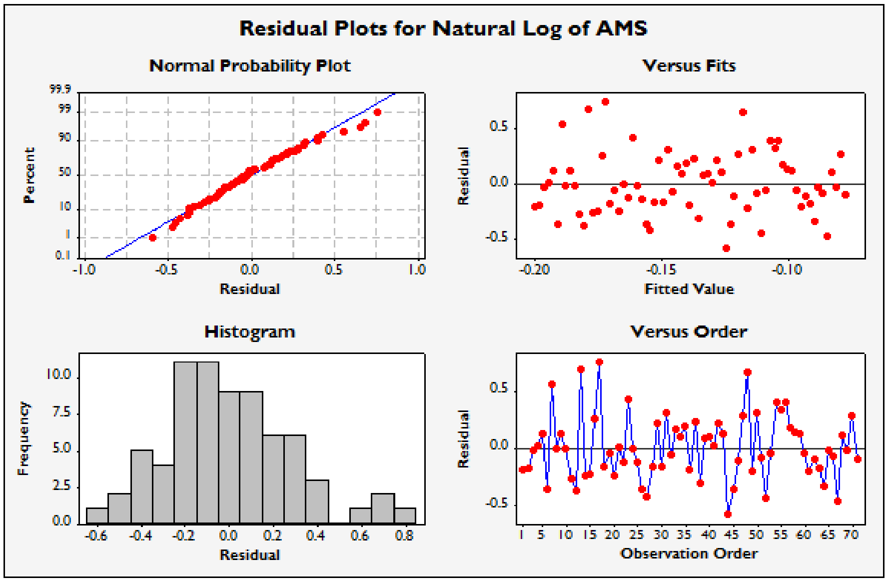

Figure 8.1 Regression residuals diagnostics

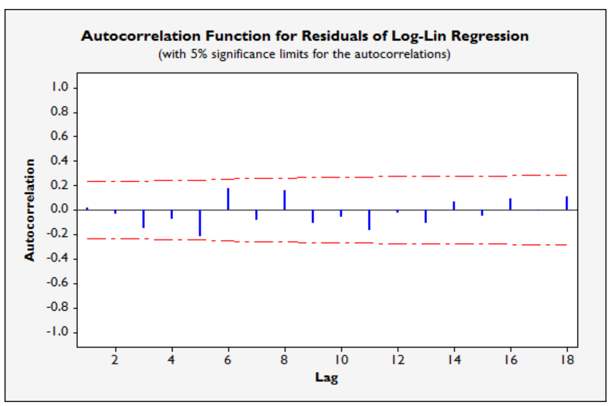

Figure 8.2 Autocorrelation of Regression Residuals

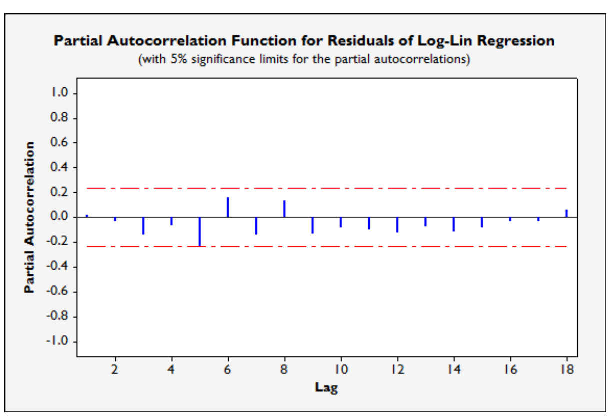

Figure 8.3 Partial Autocorrelation of Regression Residuals

	Stationary location (ξ) parameter	Non-Stationary locations (ξ) parameter	Size (α) parameter	Shape (κ) parameter
GEV	0.90718	0.81859 - 0.92567	0.21502	-0.00209
Gumbel	0.78274	0.69414 - 0.80122	0.21561	n/a

Figure 8.4 GEV and Gumbel parameter estimates (New London sea level anomalies AMS)

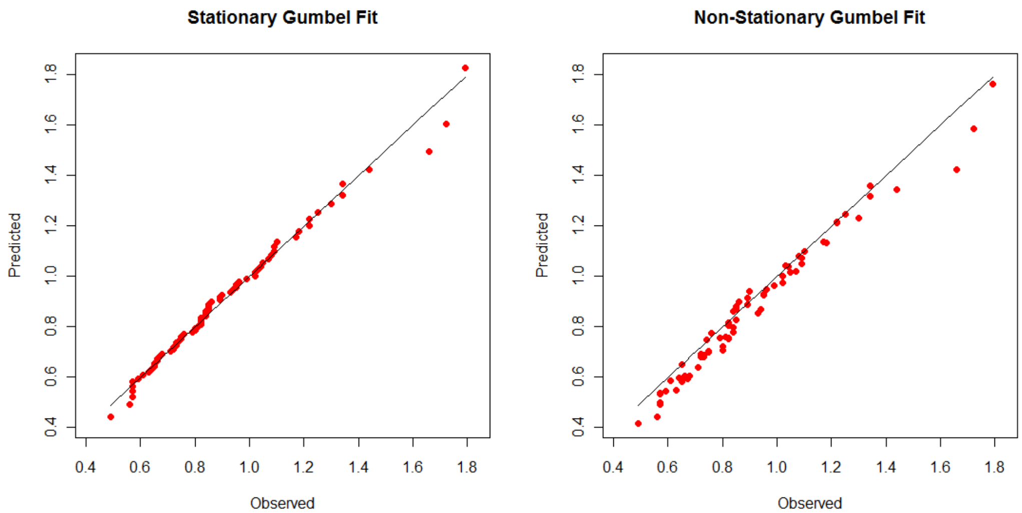

Figure 8.5 Gumbel probability plots

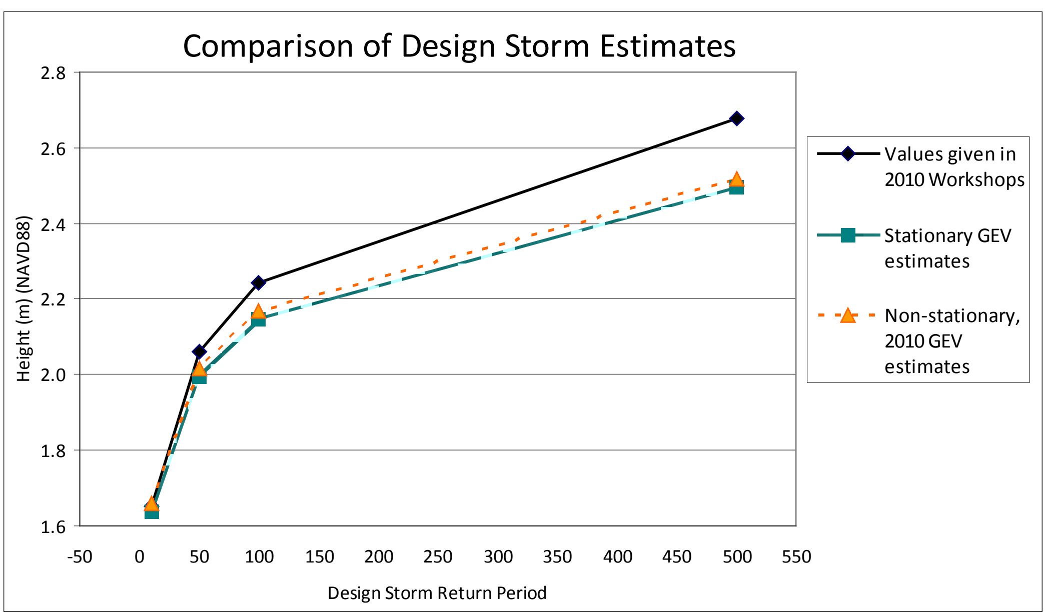

Figure 8.6 Comparison of GEV return periods with estimates used by Town of Groton, CT

8.2. Generalized Extreme Value description and equations

In the established methods of analyzing extreme events, the annual maxima (or minima) of events is often considered, rather than the complete set of observations, in order to ensure a series of independent events. The annual maxima are known to follow a Generalized Extreme Value (GEV) distribution. This distribution is a generalized description, of which the Gumbel, Fréchet and Weibull distributions are special cases. The equations describing the first three moments of the GEV distribution are given in Figure 8.7. The equation describing the Cumulative Distribution Function (CDF) of the GEV, given in Figure 8.8, is used to describe the elevation-frequency relationship, such as those given in Figure 5.10. (Stedinger et al. 1993; Kirshen, et al. 2008)

$$\mu = \xi + \frac{\alpha}{\kappa}(1 - \Gamma(1 + \kappa))$$

$$\sigma^2 = \left(\frac{\alpha}{\kappa}\right)^2 \cdot \left(\Gamma(1 + 2\cdot\kappa) - \Gamma(1 + \kappa)^2\right)$$

$$\gamma = \operatorname{sign}(\kappa)\cdot\frac{-\Gamma(1 + 3\cdot\kappa) + 3\Gamma(1 + \kappa)\cdot\Gamma(1 + 2\cdot\kappa) - 2\Gamma(1 + \kappa)^3}{\left(\Gamma(1 + 2\cdot\kappa) - \Gamma(1 + \kappa)^2\right)^{\frac{3}{2}}}$$

Where μ, σ^2, and γ are the mean, standard deviation, and skew, respectively, and ξ, α, and κ are the location, scale, and shape parameters, respectively.

Figure 8.7 GEV moment equations

$$F_X(x) = \exp\left[-\left[1 - \left[\frac{\kappa\cdot(x - \xi)}{\alpha}\right]\right]^{\frac{1}{\kappa}}\right]$$

Where $F_x(x)$ is the Cumulative Distribution Function (CDF) of x

$$x(p) = \xi + \frac{\alpha}{\kappa}\cdot\left[1 - (-\ln(p))^{\kappa}\right]$$

Where x(p) is the quantile function (inverse CDF) of probability p.

Figure 8.8 GEV CDF and quantile equations