OPEN QUESTIONS IN ASTRONOMY, 1642

1. Insofar as Copernican and Tychonic systems are both fully consistent with all accessible astronomical observations and several leading astronomers adhere to the latter, can any decisive evidence be adduced to settle the question whether the Earth is in orbit about the Sun or vice versa?

2. Granted that Kepler's claims about planetary orbital motion hold at least to high approximation, should they be taken to hold (1) for bodies beyond those now known to be orbiting the Sun and (2) for the bodies now known, indefinitely far into the past and future; and should they be taken to hold exactly, or only essentially exactly, or merely approximately; and if they do not hold exactly, should they be regarded as idealizations of some sort, and do they at least hold in the mean?

3. Granted that questions about relative distances of the planets, Sun, and Earth from one another have largely been resolved in units of the mean distance of the Earth from the Sun, what do these distances amount to in earthly units – e.g. in units of the radius of the Earth?

4. Is orbital astronomy ***perfectible*** at all – i.e. can the motions be mathematically characterized in a way that assures that conclusions drawn about the remote past and the remote future will hold at least to the same level of precision as conclusions about the present era?

5. Can the apparent motion of the Moon be mathematically characterized to the same level of precision as has been achieved for the planets?

6. What are comets, what trajectories do they describe in their observed motions, and are they governed by the same physical processes, whatever those may be, that govern the motions of the planets?

"Mechanics"

Classical:

- **Archimedes (ca. 287 B.C. – ca. 212 B.C.)**

Medieval:

- **Mertonians (at Oxford, 1320 – 1350)**
- **Buridan, Oresme (at Paris, 1340 – 1380)**

Italian Renaissance: (16th Century)

- **Leonardo da Vinci (1452 – 1519)**
- **Tartaglia (1499/1500 – 1557)**
- **Benedetti (1530 – 1590)**
- **Guido Ubaldo (1545 – 1607)**

Dutch:

- **Stevin (1548/49-1620)**
- **Beeckman (1588-1637)**

The "Science of Machines"

- **(Balance)**
- **Lever**
- **Screw**
- **Pulley**
- **Wedge**
- **Inclined plane**

Archimedes

ON THE EQUILIBRIUM OF PLANES

OR

THE CENTRES OF GRAVITY OF PLANES.

BOOK I.

"I POSTULATE the following:

1. Equal weights at equal distances are in equilibrium, and equal weights at unequal distances are not in equilibrium but incline towards the weight which is at the greater distance.

2. If, when weights at certain distances are in equilibrium, something be added to one of the weights, they are not in equilibrium but incline towards that weight to which the addition was made.

3. Similarly, if anything be taken away from one of the weights, they are not in equilibrium but incline towards the weight from which nothing was taken.

4. When equal and similar plane figures coincide if applied to one another, their centres of gravity similarly coincide.

5. In figures which are unequal but similar the centres of gravity will be similarly situated. By points similarly situated in relation to similar figures I mean points such that, if straight lines be drawn from them to the equal angles, they make equal angles with the corresponding sides.

6. If magnitudes at certain distances be in equilibrium, (other) magnitudes equal to them will also be in equilibrium at the same distances.

7. In any figure whose perimeter is concave in (one and) the same direction the centre of gravity must be within the figure."

Proposition 1.

Weights which balance at equal distances are equal.

For, if they are unequal, take away from the greater the difference between the two. The remainders will then not balance [*Post.* 3]; which is absurd.

Therefore the weights cannot be unequal.

Proposition 2.

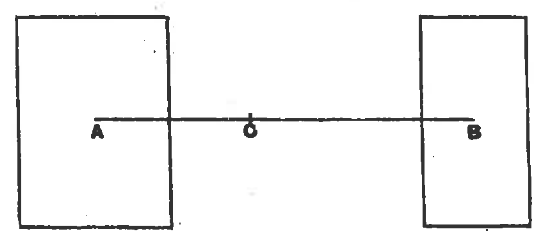
Unequal weights at equal distances will not balance but will incline towards the greater weight.

For take away from the greater the difference between the two. The equal remainders will therefore balance [*Post.* 1]. Hence, if we add the difference again, the weights will not balance but incline towards the greater [*Post.* 2].

Proposition 3.

Unequal weights will balance at unequal distances, the greater weight being at the lesser distance.

Let A, B be two unequal weights (of which A is the greater) balancing about C at distances AC, BC respectively.



Then shall AC be less than BC. For, if not, take away from A the weight $(A-B.)$ The remainders will then incline

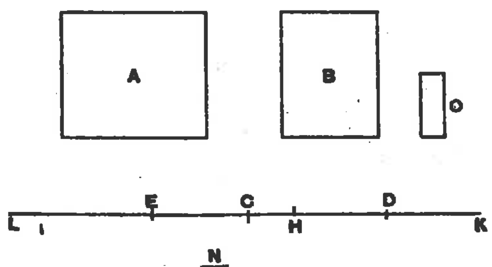
Propositions 6, 7.

Two magnitudes, whether commensurable [Prop. 6] *or incommensurable* [Prop. 7], *balance at distances reciprocally proportional to the magnitudes.*

I. Suppose the magnitudes A, B to be commensurable, and the points A, B to be their centres of gravity. Let DE be a straight line so divided at C that

$$A : B = DC : CE.$$

We have then to prove that, if A be placed at E and B at D, C is the centre of gravity of the two taken together.



Since A, B are commensurable, so are DC, CE. Let N be a common measure of DC, CE. Make DH, DK each equal to CE, and EL (on CE produced) equal to CD. Then $EH = CD$, since $DH = CE$. Therefore LH is bisected at E, as HK is bisected at D.

Thus LH, HK must each contain N an even number of times.

Take a magnitude O such that O is contained as many times in A as N is contained in LH, whence

$$A : O = LH : N.$$

But

$$B : A = CE : DC$$
$$= HK : LH.$$

Hence, *ex aequali*, $B : O = HK : N$, or O is contained in B as many times as N is contained in HK.

Thus O is a common measure of A, B.

Divide LH, HK into parts each equal to N, and A, B into parts each equal to O. The parts of A will therefore be equal in number to those of LH, and the parts of B equal in number to those of HK. Place one of the parts of A at the middle point of each of the parts N of LH, and one of the parts of B at the middle point of each of the parts N of HK.

Then the centre of gravity of the parts of A placed at equal distances on LH will be at E, the middle point of LH [Prop. 5, Cor. 2], and the centre of gravity of the parts of B placed at equal distances along HK will be at D, the middle point of HK.

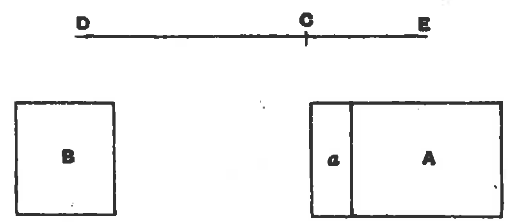
Thus we may suppose A itself applied at E, and B itself applied at D.

But the system formed by the parts O of A and B together is a system of equal magnitudes even in number and placed at equal distances along LK. And, since $LE = CD$, and $EC = DK$, $LC = CK$, so that C is the middle point of LK. Therefore C is the centre of gravity of the system ranged along LK.

Therefore A acting at E and B acting at D balance about the point C.

II. Suppose the magnitudes to be incommensurable, and let them be $(A + a)$ and B respectively. Let DE be a line divided at C so that

$$(A + a) : B = DC : CE.$$



Then, if $(A + a)$ placed at E and B placed at D do not balance about C, $(A + a)$ is either too great to balance B, or not great enough.

Suppose, if possible, that $(A + a)$ is too great to balance B. Take from $(A + a)$ a magnitude a smaller than the deduction which would make the remainder balance B, but such that the remainder A and the magnitude B are commensurable.

Then, since A, B are commensurable, and

$$A : B < DC : CE,$$

A and B will not balance [Prop. 6], but D will be depressed.

But this is impossible, since the deduction a was an insufficient deduction from $(A + a)$ to produce equilibrium, so that E was still depressed.

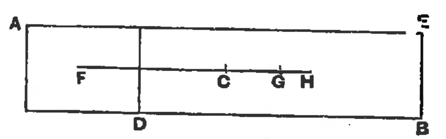
Therefore $(A + a)$ is not too great to balance B; and similarly it may be proved that B is not too great to balance $(A + a)$.

Hence $(A + a)$, B taken together have their centre of gravity at C.

Proposition 8.

If AB be a magnitude whose centre of gravity is C, and AD a part of it whose centre of gravity is F, then the centre of gravity of the remaining part will be a point G on FC produced such that

$$GC : CF = (AD) : (DE).$$



For, if the centre of gravity of the remainder (DE) be not G, let it be a point H. Then an absurdity follows at once from Props. 6, 7.

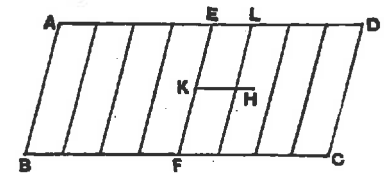
Proposition 9.

The centre of gravity of any parallelogram lies on the straight line joining the middle points of opposite sides.

Let $ABCD$ be a parallelogram, and let EF join the middle points of the opposite sides AD, BC.

If the centre of gravity does not lie on EF, suppose it to be H, and draw HK parallel to AD or BC meeting EF in K.

Then it is possible, by bisecting ED, then bisecting the halves, and so on continually, to arrive at a length EL less



than KH. Divide both AE and ED into parts each equal to EL, and through the points of division draw parallels to AB or CD.

We have then a number of equal and similar parallelograms, and, if any one be applied to any other, their centres of gravity coincide [*Post.* 4]. Thus we have an even number of equal magnitudes whose centres of gravity lie at equal distances along a straight line. Hence the centre of gravity of the whole parallelogram will lie on the line joining the centres of gravity of the two middle parallelograms [Prop. 5, Cor. 2].

But this is impossible, for H is outside the middle parallelograms.

Therefore the centre of gravity cannot but lie on EF.

Proposition 10.

The centre of gravity of a parallelogram is the point of intersection of its diagonals.

For, by the last proposition, the centre of gravity lies on each of the lines which bisect opposite sides. Therefore it is at the point of their intersection; and this is also the point of intersection of the diagonals.

Alternative proof.

Let $ABCD$ be the given parallelogram, and BD a diagonal. Then the triangles ABD, CDB are equal and similar, so that [*Post.* 4], if one be applied to the other, their centres of gravity will fall one upon the other.

Motion, Celestial and Local

***Goal*: a mathematical representation specifying the location of a body versus time along a trajectory, known or unknown**

***Status as of 1638*:**

By adopting Kepler's proposed horizontal parallax of 1′, Horrocks had reduced his eccentricity for the Earth-Sun orbit from 0.0180 to 0.0173, and that in turn had led to an increase in the eccentricity of Venus's orbit from 0.00692 to 0.00750 and in the length of its semi-major axis from 0.74413 to 0.7233 a.u.; the revised value of the semi-major axis eliminated the prior 0.11% discrepancy between its cube and the square of its period, leading him then to take Kepler's 3/2 power rule to be exact and inferring from its period a further revision of the length of the semi-major axis to 0.72333 a.u.; these revisions turned out to reduce discrepancies between observation and the Rudolphine Tables as large as 5′ to less than 2′.

The sole "natural" local motion – that is, near the surface of the Earth – is vertical fall. Galileo had originally concluded that bodies have a characteristic, natural constant speed of descent that depends on their density and the density of the medium. He was now instead proposing that, in the absence of any resisting medium, all bodies are uniformly accelerated as they descend, the rate at which they gain speed is the same for all bodies regardless of their weight and shape, and any observed departure from this results from an effect induced by the motion, and hence a "second-order" consequence of it, namely a resistance to it impressed on the moving bodies by the medium through which they are descending.

DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,

intorno à due nuoue ſcienze

Attenenti alla

MECANICA & I MOVIMENTI LOCALI,

del Signor

GALILEO GALILEI LINCEO,

Filoſofo e Matematico primario del Sereniſſimo
Grand Duca di Toſcana.

Con vna Appendice del centro di grauità d'alcuni Solidi.

IN LEIDA,

Appreſſo gli Elſevirii. M. D. C. XXXVIII.

Table of the Principal Matters That Are Treated in the Present Work[1]

1. This table of contents reversing the essential content of the two first days, was prepared by the Elzevirs.

2. Sometimes called the Sixth Day, this incomplete dialogue was first published in 1718, as part of the second collected edition of Galileo's works. A so-called Fifth Day, first published by Vincenzio Viviani (1622–1703) in 1674, does not belong to this book.

"Natural philosophy" – from Scholastic philosophy

vs.

***"Scientia"* (Lat.), *"Scienze"* (It.) – secure knowledge**

"Theory": ***An inter-connected system of mathematical propositions, linking measurable parameters to one another and to observable phenomena, from which, with appropriate additional empirical information (e.g. values of parameters), one can derive answers to a wide range of questions, including predictive and counterfactual questions.***

Examples: Ptolemaic theories of the Sun, the Moon, Mercury, Venus-Mars-Jupiter-Saturn; Copernican theories of the same; Keplerian theories of the six planets and the Moon

Apollonian Parameters of Orbital Theory

Obliquity of the ecliptic

Longitude and latitude

Sidereal period

Synodic period

Longitude of apogee (or **aphelion)**

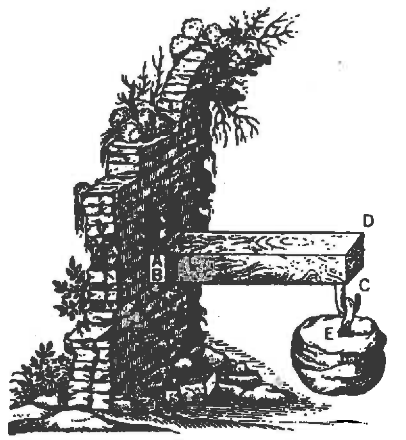
Eccentricity

Radius ratio (of two circles involved in retrograde motion)

Each of these was (1) linked to specific observable phenomena; (2) measurable; (3) assigned a physical significance (that underwent revision from Ptolemy to Copernicus to Kepler, while preserving, at least to high approximation, key propositions involving them)

Galileo's First New Science

Strength of Materials



What parameters govern fracture? Obviously some combination of dimensions of the beam and applied load (i.e. weight), but what combination?

Our answer: *stress* (1820s) and *stress intensity* (1920s)

Galileo's Second New Science

"Local Motion"

I.e. uniform motion, naturally accelerated motion, violent projectile motion

Relevant parameters, in absence of a resisting medium:

distance, height of descent or ascent, time, speed

Irrelevant parameters, in absence of a resisting medium:

weight, shape, density, density of resisting medium

Basis for claim:

(1) "Thought experiments" – e.g. *Discorsi*, [108]
(2) While the heavier of two bodies in vertical fall almost always reaches the ground sooner, the times and hence the speeds are not proportional to the weights of the bodies; and as the heavier the two bodies are, the less the difference in their times of fall
(3) The effects of resistance on a falling body are smaller the less the density of the medium, suggesting they disappear entirely with zero density
(4) Two 5 braccia pendulums, one with a cork bob and the other with a lead bob, remain synchronous, with equal periods, even though resistance affects former more

Galileo on Pendulum Isochronism

… So I fell to thinking how one might many times repeat descents from small heights, and accumulate many of those minimal differences of time that might intervene between the arrival of the heavy body at the terminus and that of the light one, so that added together in this way they would make up a time not only observable, but easily observable.

… Ultimately I took two balls, one of lead and one of cork, the former being at least a hundred times as heavy as the latter, and I attached them to equal thin strings four or five braccia long, tied high above. Removed from the vertical, these were set going at the same moment, and falling along the circumferences of the circles described by the equal strings that were the radii, they passed the vertical and returned by the same path. Repeating their goings and comings a good hundred times by themselves, they sensibly showed that the heavy one kept time with the light one so well that not in a hundred oscillations, nor in a thousand, does it get ahead in time even by a moment, but the two travel with equal pace. The operation of the medium is also perceived; offering some impediment to the motion, it diminishes the oscillations of the cork much more than those of the lead. But it does not make them more frequent, or less so; indeed, when the arcs passed by the cork were not much more than five or six degrees, and those of the lead were fifty or sixty, they were passed over in the same times.

First Day, p. [128f]; see also Fourth Day, p. [277]

NON-ISOCHRONISM OF CIRCULAR-ARC PENDULUMS

$$\frac{P}{P_0} = 1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 k^4 + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 k^6 + \ldots$$

where $k=\sin(\alpha/2)$ and α is the arc in descent.

Arc in descent (deg)	P/P_0	Number of full cycles before a 20% discrepancy
2.5	1.00012	1667
5.	1.00048	417
10.	1.00191	105
15.	1.00430	47
30.	1.01741	12
60.	1.07317	3
90.	1.17996	1+

These precise numbers were not calculable before the late 18th century. (It requires the solution of an elliptical integral.) But both Mersenne and Huygens had observed the qualitative results, the former in the 1630s (published at that time) and the latter in the 1650s (published in 1673). A constant-arc circular pendulum would, of course, have a repeatable period, but the question of the trajectory required to maintain isochronism (that is, same period regardless of arc length) was left for Huygens to discover, in 1659.

Why, when two mechanisms are irremediably involved in an actual process, should one be disregarded, focusing exclusively on the other?

- **A theory of the principal or dominant mechanism is possible, and it is needed in order to make empirical investigation of the other "secondary" one tractable**

- **Experiments yielding results of evidential value are possible provided only that the confounding effects of the other mechanism be largely eliminated or controlled**

- **No theory of the other mechanism is possible at all – for example, because too many variables are involved – so that any science becomes possible only by disregarding it and focusing on the one amenable to theory**

Uniform Motion

In a given time:

$$speed_1 : speed_2 :: distance_1 : distance_2$$

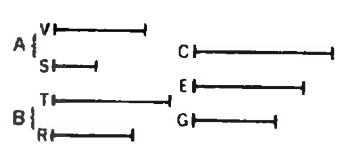
Over a given distance:

$$speed_1 : speed_2 :: elapsed\ time_2 : elapsed\ time_1$$

Therefore, *speed* in uniform motion varies directly with *distance* and inversely with *elapsed time*, the compound to two ratios

Proposition VI. *If two moveables are carried in equable motion, the ratio of their speeds will be compounded from the ratio of spaces run through and from the inverse ratio of the times.*

Proof: Let V and T represent the spaces, and S and R represent the times. Then the ratio of the speeds is represented by the ratio of C to G, compounded from the ratio of C to E (where C:E as V:T) and the ratio of E to G (where E:G as R:S).



Euclid's Elements

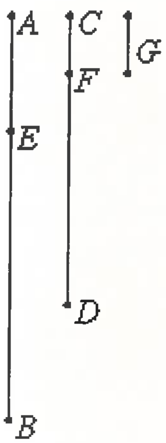
Book VII

Proposition 2

To find the greatest common measure of two given numbers not relatively prime.

Let *AB* and *CD* be the two given numbers not relatively prime.

It is required to find the greatest common measure of *AB* and *CD.*



If now *CD* measures *AB,* since it also measures itself, then *CD* is a common measure of *CD* and *AB.* And it is clear that it is also the greatest, for no greater number than *CD* measures *CD.*

But, if *CD* does not measure *AB,* then, when the less of the numbers *AB* and *CD* being continually subtracted from the greater, some number is left which measures the one before it.

For a unit is not left, otherwise *AB* and *CD* would be relatively prime, which is contrary to the hypothesis. VII.Def.12

VII.1

Therefore some number is left which measures the one before it.

Now let *CD,* measuring *BE,* leave *EA* less than itself, let *EA,* measuring *DF,* leave *FC* less than itself, and let *CF* measure *AE.*

Since then, *CF* measures *AE,* and *AE* measures *DF,* therefore *CF* also measures *DF.* But it measures itself, therefore it also measures the whole *CD.*

But *CD* measures *BE,* therefore *CF* also measures *BE.* And it also measures *EA,* therefore it measures the whole *BA.*

But it also measures *CD,* therefore *CF* measures *AB* and *CD.* Therefore *CF* is a common measure of *AB* and *CD.*

I say next that it is also the greatest.

If *CF* is not the greatest common measure of *AB* and *CD,* then some number *G,* which is greater than *CF,* measures the numbers *AB* and *CD.*

Now, since *G* measures *CD,* and *CD* measures *BE,* therefore *G* also measures *BE.* But it also measures the whole *BA,* therefore it measures the remainder *AE.*

But *AE* measures *DF,* therefore *G* also measures *DF.* And it measures the whole *DC,* therefore it also measures the remainder *CF,* that is, the greater measures the less, which is impossible.

Therefore no number which is greater than *CF* measures the numbers *AB* and *CD.* Therefore *CF* is the greatest common measure of *AB* and *CD.*

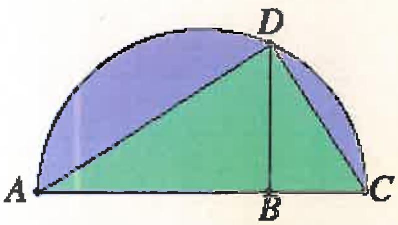
Corollary

Euclid's Elements
Book VI
Proposition 13

To find a mean proportional to two given straight lines.

Let *AB* and *BC* be the two given straight lines.

It is required to find a mean proportional to *AB* and *BC*.



Place them in a straight line, and describe the semicircle *ADC* on *AC*. Draw *BD* from the point *B* at right angles to the straight line *AC*, and join *AD* and *DC*. I.11

Since the angle *ADC* is an angle in a semicircle, it is right. III.31

And, since, in the right-angled triangle *ADC*, *BD* has been drawn from the right angle perpendicular to the base, therefore *BD* is a mean proportional between the segments of the base, *AB* and *BC*. VI.8.Cor

Therefore a mean proportional *BD* has been found to the two given straight lines *AB* and *BC*.

Q.E.F.

Guide

This construction of the mean proportional was used before in II.4 to find a square equal to a given rectangle. By proposition VI.17 coming up, the two constructions are equivalent. That is the mean proportional between two lines is the side of a square equal to the rectangle contained by the two lines. Algebraically, $a : x = x : b$ if and only if $ab = x^2$. Thus, x is the square root of ab.

When b is taken to have unit length, this construction gives the construction for the square root of a.

Use of this proposition

This construction is used in the proofs of propositions VI.25, X.27, and X.28.

Next proposition: VI.14

Previous: VI.12

Book VI introduction

Select from Book VI

Select book

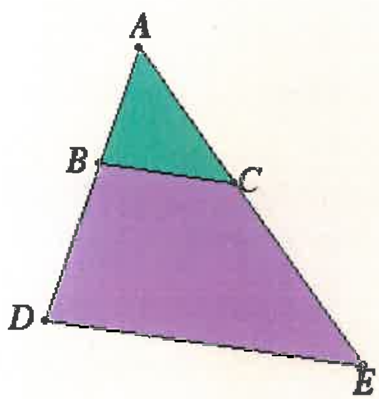
Select topic

Euclid's Elements
Book VI
Proposition 11

To find a third proportional to two given straight lines.

Let *AB* and *AC* be the two given straight lines, and let them be placed so as to contain any angle.

It is required to find a third proportional to *AB* and *AC*.



Produce them to the points *D* and *E*, and make *BD* equal to *AC*. Join *BC*, and draw *DE* through *D* parallel to it. I.3 I.31

Then since *BC* is parallel to a side *DE* of the triangle *ADE*, therefore, proportionally, *AB* is to *BD* as *AC* is to *CE*. VI.2

But *BD* equals *AC*, therefore *AB* is to *AC* as *AC* is to *CE*. V.7

Therefore a third proportional *CE* has been found to two given straight lines *AB* and *AC*.

Q.E.F.

Guide

If *a* and *b* are two magnitudes, then their third proportional is a magnitude *c* such that $a:b = b:c$. The third proportional is needed whenever a duplicate ratio is needed when the ratio itself is known. The duplicate ratio for $a:b$ is $a:c$.

Use of this proposition

This construction is used in propositions VI.19, VI.22, and a few propositions in Book X.

Next proposition: VI.12

Previous: VI.10

Book VI introduction

Select from Book VI

Select book

Select topic

"On Naturally Accelerated Motion"

And first it is appropriate to seek out and clarify the definition that best agrees with that which nature employs. Not that there is anything wrong with inventing at pleasure some kind of motion and theorizing about its consequent properties, in the way that some men have derived spiral and conchoidal lines from certain motions, though nature makes no use of these; and by pretending these, men have laudably demonstrated their essentials *ex suppositione*. But since nature does employ a certain kind of acceleration for descending heavy things, we decided to look into their properties so that we might be sure that the definition of accelerated motion which we are about to adduce agrees with the essence of naturally accelerated motion. And at length, after continual agitation of the mind, we are confident that this has been found, chiefly for the very powerful reason that the essentials successively demonstrated by us correspond to, and are seen to be in agreement with, that which *naturalia experimenta* show forth to the senses. Further, it is as though we have been led by the hand to the investigation of naturally accelerated motion by consideration of the custom and procedure of nature herself in all her other works, in the performance of which she habitually employs the first, simplest, and easiest means. And indeed, no one of judgment believes that swimming or flying can be accomplished in a simpler or easier way than that which fish and birds employ by natural instinct.

Thus when I consider that a stone, falling from rest at some height, successively acquires new increments of speed, why should I not believe that those additions are made by the simplest and most evident rule? For if we look into this attentively, we can discover no simpler addition and increase than that which is added on always in the same way …. [– that is,] whenever, in equal times, equal additions of swiftness are added on.

p. [197f]

Uniformly Accelerated Motion

Equal increments in *speed* in equal increments of *time*

$$v_{acquired} \propto t_{elapsed}$$

$$(v = at;\ s = \tfrac{1}{2} at^2)$$

versus

Equal increments in *speed* over equal increments of *space*

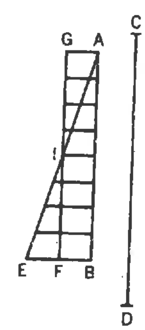
$$v_{acquired} \propto s_{traversed}$$

$$(s = ce^{bt}; v = c \cdot be^{bt})$$

Mean Speed Theorem

Prop. 1. **The time in which a certain space is traversed by a moveable in uniformly accelerated movement from rest is equal to the time in which the same space would be traversed by the same moveable carried in uniform motion whose degree of speed is one-half the maximum and final degree of speed of the previous, uniformly accelerated, motion.**

Let line AB represent the time in which the space CD is traversed by a moveable in uniformly accelerated motion from C. Let EB represent the maximum and final degree of speed increased in the instants of the time AB. All lines reaching AE from single points on the line AB drawn parallel to EB will represent the increasing degrees of speed after the instant A. Next I bisect BE at F and I draw FG and AG parallel to BA and BF; the parallelogram AGFB will [thus] be constructed, equal [in area] to the triangle AEB, its side GF bisecting AE at I.



Upshot: Problems involving uniformly accelerated motion can be reduced to problems involving only uniform motion.

Evidence Problems

In orbital astronomy

1. Determining distances of celestial objects from the Earth (and from the Sun) in a common unit.

2. Distinguishing merely apparent motions and changes of motions from real ones.

In mechanics of local motion

1. Making precise measurements of elapsed time was difficult because the characteristic times of phenomena of motion that could be controlled were short

2. While mean speeds may be measurable, via distance and time measurements, there was no obvious way of measuring speeds that vary with time

3. Theoretical claims that were being set forth usually concerned motions under idealized circumstances like the absence of air resistance – ideals that could not be realized in experimental practice

Fundamental Result

Prop. 2. **If a moveable descends from rest in uniformly accelerated motion, the spaces run through in any times whatever are to each other as the duplicate ratio of their times; that is, are as the squares of those times.**

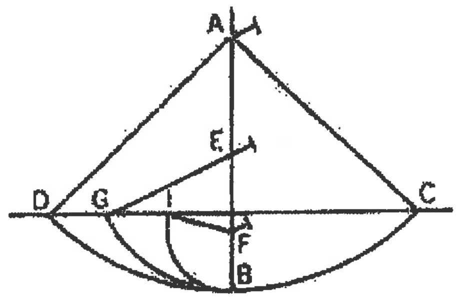
Corol. 1. **From this it is manifest that if there are any number of equal times taken successively from the instant or beginning of motion [during each of which a certain space is run through], ... then these spaces will be to one another as are the odd numbers from unity, that is, as 1, 3, 5, 7, ...**

Corol. 2. **It is deduced, second, that if at the beginning of motion there are taken any two spaces whatever, run through in any [two] times, the times will be to one another as either of these spaces is to the mean proportional space between the two given spaces.**

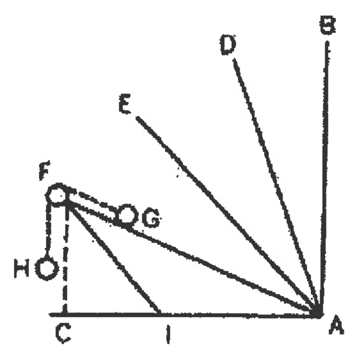
i.e. $\textit{elapsed time}_1 : \textit{elapsed time}_2 ::$
$\textit{space}_1 : \sqrt{(\textit{space}_1 \cdot \textit{space}_2)} = \sqrt{\textit{space}_1} : \sqrt{\textit{space}_2}$

"Galileo's Postulate"

The same speed is acquired from any given height whether in direct fall or along an inclined plane.



First edition



Posthumous Edition

Torricelli: *De Motu Gravium Naturaliter Descendentium Et Projectorum* (1644)

Galileo, when about to discuss naturally accelerated motion, puts forward a principle, that he himself thinks not yet clear, as long as he strives to establish it by the not fully precise experiment of the pendulum, which is: *That the stages of velocity of the same moving object, when amassed over differently inclined planes, are equal when the elevations of the same planes are equal.* From this claim hangs as it were his whole doctrine on both accelerated and projectile motion. If anyone has doubts about the principle, he will not have at all secure knowledge of the things that follow from it. I know that Galileo in the last years of his life tried to demonstrate that supposition, but because his own argument, with his book on motion, has not been published, we have brought forth these few statements on the movements of weights, to be fixed at the beginning of our little book, so that it may appear that Galileo's supposition can be demonstrated and indeed at once by that theorem that he himself selected as demonstrated from Mechanics in the second part of his sixth proposition on accelerated motion, to wit: *The momenta of equal weights over planes unequally inclined are to each other as the perpendiculars of equal parts of the same planes, [that is, as the sines of the angles of inclination of the planes]*.

We set forth

that two weights joined together cannot move of themselves unless their common center of gravity descends.

For whenever two weights are so joined together among themselves that the motion of one follows on the motion of the other, these two weights will be as if one weight made up of two, whether this be a balance, or a pulley, or any other mechanical proportion; however a weight of this sort will not ever move unless its center of gravity descends. And so whenever it is set up so that its common center of gravity cannot at all descend, the weight will remain wholly at rest in its own place; for it will be moved elsewhere without effect; namely, by horizontal motion, which tends downward in vain.

Mersenne: *Harmonie Universelle* (1636)

"I question whether Lord Galileo ever did the experiments of falls along the plane, since he nowhere says so, and the proportion he gives often contradicts experiment." (p. 112)

Mersenne carried out a number of inclined plane experiments before Galileo's *Two New Sciences* was published, from low to high inclinations. In the set of experiments presented immediately preceding the above quotation, he first calculates the expected distance of fall along the inclined plane in a given time, based on a value he had obtained for the distance of fall in 1 second, and then measures the actual distance the sphere travels along the plane. Dominico Bertoloni Meli has reduced the reported data to *the ratio of the observed distance to the expected distance*, using a common denominator of 7 (corresponding to the then unknown fact that a rolling sphere covers 5/7 of the distance of a freely sliding sphere in any given time):

Inclination of the plane	Observed/Expected Distance
15 deg	5.25/7
25	4.97/7
30	5.6/7
40	6.0/7
45	6.0/7
50	5.0/7

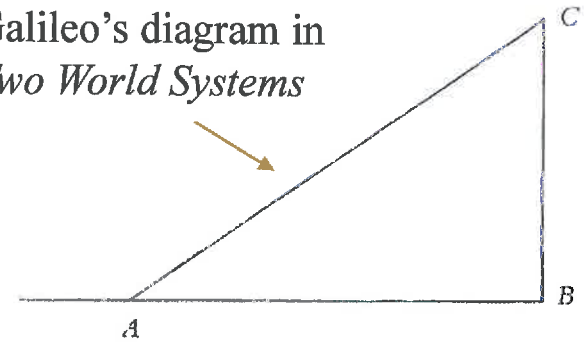
Mersenne remarks on the difficulties of getting well-behaved results at 50 deg and above.

GALILEO'S INCLINED PLANE EXPERIMENTS

"We made the same ball descend only one-quarter the length of this channel, and the time of its descent being measured, this was found always to be precisely one-half the other. Next making the experiment for other lengths, comparing now the time for the whole length with the time of one-half, or with that of two-thirds, or of three-quarters, and finally with any other division, by experiments repeated a full hundred times, the spaces were always found to be to one another as the square of the times. And this for all inclinations of the plane. … We observed also that the times of descent for diverse inclinations maintained among themselves accurately that ratio that we shall find later assigned and demonstrated by our Author."

Two New Sciences (1638) p. 213

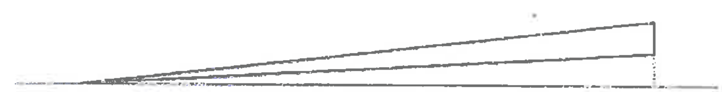
Galileo's diagram in *Two World Systems*



Galileo's announced dimensions in *Two New Sciences:*

Length: 12 braccia

Height: 1 to 2 braccia



Angle: 4.8 to 9.6 deg

Maximum time: 4.9 sec

Minimum time: < 0.9 sec

An Experiment in the History of Science

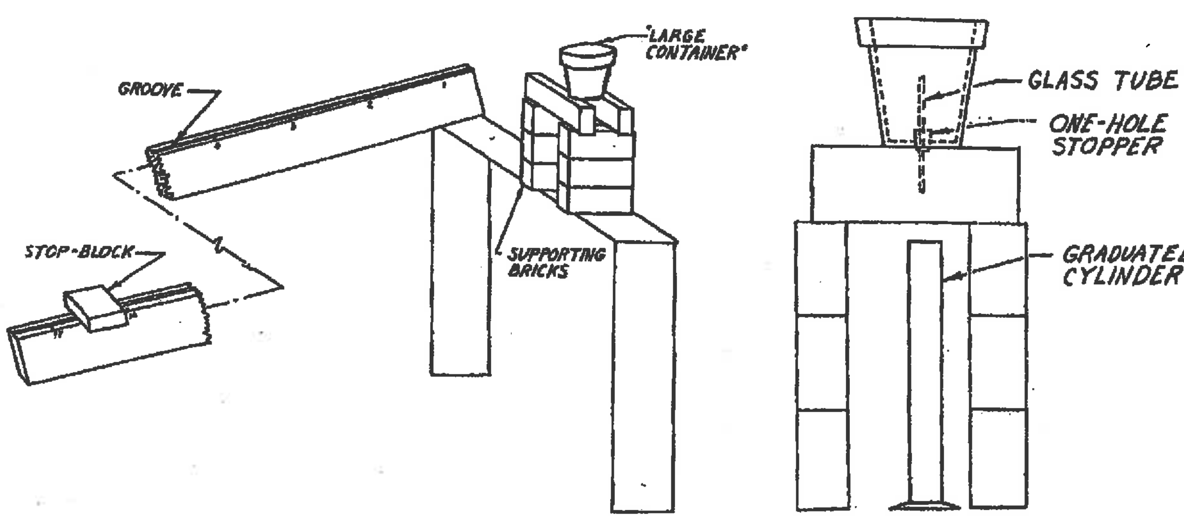
With a simple but ingenious device Galileo could obtain relatively precise time measurements.

Thomas B. Settle

On the "Third Day" of his *Discorsi* (*1*) Galileo described an experiment in which he had timed a ball accelerating along different lengths and slopes of an inclined plane. With it he believed he had established the science of naturally accelerated motion. To get a better appreciation for some of the problems he faced I have tried to reproduce the experiment essentially as Galileo described it. In the process I found that it definitely was technically feasible for him, and I think I gained a good idea of the type of results he probably looked for and of how well they turned out.

He described the experiment because, in his words: "in those sciences where mathematical demonstrations are applied to natural phenomena, as is seen in the case of perspective, astronomy, mechanics, music, and others [,] the principles, once established by well-chosen experiments, become the foundations of the entire superstructure" (*1*, p. 171). In this case his aim was to establish a science based on two principles: (i) a general definition of uniform acceleration, "such as actually occurs in nature" (*1*, p. 154), as that motion in which equal increments of velocity are added in equal times and (ii) an assumption that "the speeds acquired by one and the same body

The author is a graduate student in the history department of Cornell University, Ithaca, N.Y.



(Left) General layout of the experimental apparatus. (Right) The timing apparatus.

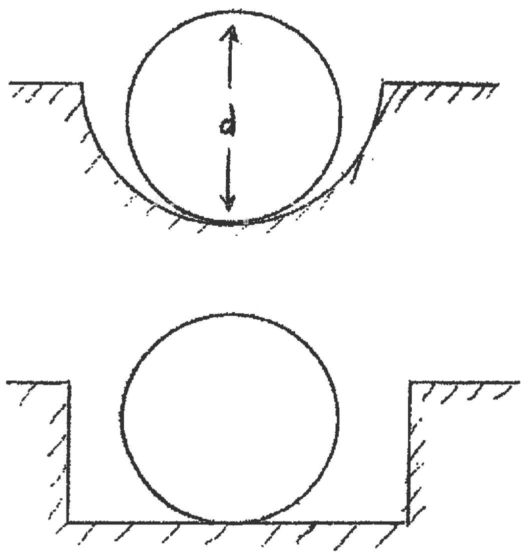
Table 1. Sample of experimental results and calculations which confirm Eq. 2.

Distance	Time (ml of water)		
	(Exp.)	(Av.)	(Cal.)
15	88	90+	90+
	91		
	91		
	90		
	90		
	90		
	90		
	90		
	89		
	90		
13	84	84	84
	84	base	
	84		
	84		
	84		
	84		
	84		
10	72	72+	74−
	73		
	72		
	72		
	72		
	72		
7	62	62−	62−
	61		
	62		
	61		
	62		
	62		
	62		
5	53	52	52+
	53		
	53		
	53		
	53		
	52		
	53		
	51		
	51		
	52		
	53		
	51		
3	40	40	40+
	40		
	40		
	41		
	39		
	41		
	40		
1	26		
	17		
	25		
	24		
	23		
	25		
	23	23.5	23+
	24		
	23		
	23		
	24		
	24		
	24		
	23		
	23		

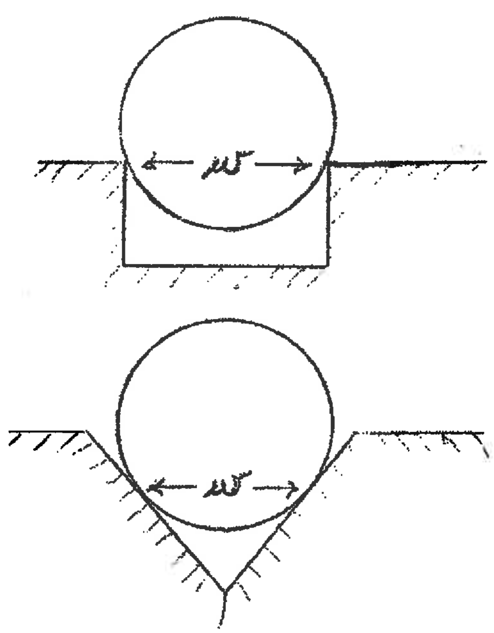
Table 2. Experimental data obtained with the billiard ball for the bases of three slopes, and times computed from one of the other slopes. *L*, slope length; *a*, vertical height; *T*, time.

Slope	Experimental data			Calculated data
	L	a	T	T
a	12	2.92	117	118− (from b)
b	13	6.25	84	85− (from c)
c	9	11.47	52	51+ (from a)

Rolling in a Groove on an Inclined Plane



$$a = \frac{5}{7} g \sin\alpha$$



$$a = \left[\frac{5(d^2 - w^2)}{7d^2 - 5w^2}\right] g \sin\alpha$$

see Hahn (2002)
for details

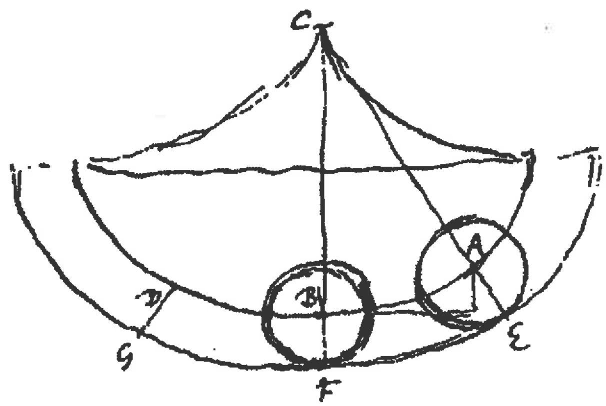
Huygens on Rolling vs. Falling (after the *Principia*)

APPENDICE VI

À LA PARS QUARTA DE L',,HOROLOGIUM OSCILLATORIUM".

[1692 ou 1693][1].

[Fig. 160.]



Pendulum pondus cujus centrum gravitatis A [Fig. 160] per Cycloidem delatum AB, acquirit in puncto B infimo celeritatem qua per arcum BD æqualem BA aſcendat.

Si vero annulus gravitate præditus, et tamen ut peripheria ſimplex conſideratus volvatur in paracycloide [2]) EF, ita ut centrum ejus deſcribat cycloidis portionem AB, is quoque vim collegit, ubi in B pervenit, qua aſcendat revolvendo uſque in D.

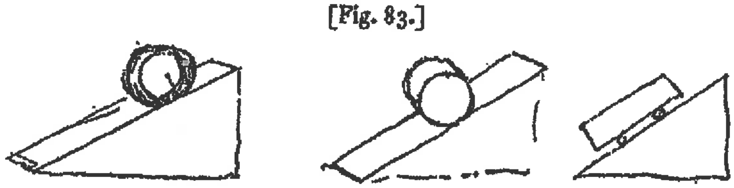
Huygens concludes: "Therefore the total time of revolving in the annulus along the curve EG will be to the time of vibration of the pendulum along the arc AF as $\sqrt{2}$ to 1, or roughly as 7 to 5."

VII.[1]

MOUVEMENT ROULANT SUR UN PLAN INCLINÉ.

Un anneau roule [Fig. 83] moins viſte qu'un cylindre ſur un plan inclinè. le cylindre moins viſte que la ſphere, et la ſphere moins viſte qu'une poutre ſur des rouleaux [2]).

[Fig. 83.]



Riccioli on Measuring Time

***First example*: A pendulum of 3 old Roman feet 4 inches in length with a 1 pound bob**

21706 arcs in 21660 sidereal seconds

***Second example*: A pendulum of 3 old Roman feet 4 inches, with an 8 ounce iron sphere**

87758 arcs in 86400 sidereal seconds

"I set up nine companions (well instructed in this matter, who almost all publicly practiced Philosophy or Theology or Mathematics) so that they succeeded each other in the counting after about every half hour; and in the year 1642 from noon on April 2 to noon on April 3, we maintained a count of simple vibrations, whose number, from the pebbles thrown in the vase every 60 vibrations, was found to be 1466 sixties and in addition 38 vibrations. But a day of the primum mobile contains 1440 of its own minutes. The solar day indeed is 1444 primum-mobile minutes. Therefore such a pendulum in one day of the primum mobile completes sixty times 1462 vibrations and in addition 38 vibrations, when it ought to complete only 1440 if a single simple vibration corresponded to one second; therefore I added one ring to the chain so that the number of vibrations might turn out less, and it might approach more nearly in each of its vibrations to a second of the primum mobile."

***Almagestum Novum*, 1651, p. 86**

***Third example*: A pendulum of 3 old Roman feet 4 +20/100 inches with an 8 ounce iron sphere**

86998 arcs in 86400 sidereal seconds

***Fourth example*: A pendulum of 3 old Roman feet 2 +67/100 inches with a 20½ ounce brass sphere**

3212 arcs in 3192 sidereal seconds

Upshot:

***The one-second pendulum*: 3 old Roman feet 3+27/100 inches – i.e. 3927/100 inches – with a 20½ ounce brass sphere**

***The one-half second pendulum*: 9 +76/100 inches with a little brass sphere**

***The one-sixth second pendulum*: 1 + 15/100 inches with a little brass sphere weighing 4 drachmas**

"Therefore we used a pendulum of this sort for measuruing the natural movement of weights, but, in order to count its vibrations as quickly as possible, it is proper after each set of ten to raise one finger of two clasped hands, and to be extremely attentive. Indeed for greater proof to take two equal pendulums of this sort and have two counters, making their own count separately, so that it is apparent at the end of the operation whether it agrees or not."

***Ibid.*, p. 87**

Proposition IV.

Weights in perpendicular free fall move more and more quickly towards the end, in an increase of speed that is between numbers equally unequal, numbered as wholes; or so as the spaces, traversed in certain times, are among themselves as the squares of the times; or so as the spaces traversed have among themselves a duplicate proportion to that which the times during which those spaces were measured have [*among themselves*].

The whole assertion of Galileo (above) and Baliani has been very often proven by our experiments; now these are the numbers are said to be equally unequal as wholes: 1,3,5,7,9,11,13,15, &c. And so if in the first quarter of an hour some weight has completed 1 stage, in the second quarter it will complete 3, in the third, 5 stages, & thus through the rest of the progression. In order to explore this in truth, Grimaldi & I prepared in advance several clay globes of the same bulk & dropped those of 8 ounces from different towers, or house-windows, or little casements suited for taking measurements, and we first used the towers of Bologna, namely the Asinelli, which is 312 old Roman feet high, and St. Peter, 208 feet high and St. Petronius, 200 feet high, & St. James, 189 feet high, & St. Francis, 150 feet high, though we did not use their whole height, but that which was suited for the aforesaid proportion. Moreover, for discerning more precisely the time in which the dropped globes arrived at the pavement, we used two very small pendulums (see Chap. 20, prop. 13, in the second paragraph), of whom, as is clear, one simple vibration lasts for 10 thirds of the primum mobile. Among many experiments, however, the best two, the most certain of all, written below, I place in the following table, so that so I may not end up too lengthy for my reader.

Experimenta.	Vibrationes simplices Perpendiculi.	Tempus Vibrationibus congruens.		Spatium confectum à globo cretaceo unciarũ 8. in fine tẽporis.	Spatiũ ergo seorsim confectũ singulis tẽporibus æqualibus.	Proportio incrementi velocitatis simplicibus numeris expressa.
				Pedes Romani.	Pedes Romani.	Num.parit.impar.
I	5	0	50	10	10	1
	10	1	40	40	30	3
	15	2	30	90	50	5
	20	3	20	160	70	7
	25	4	10	250	90	9
II	6	1	0	15	15	1
	12	2	0	60	45	3
	18	3	0	135	75	5
	24	4	0	240	105	7
	ferè 26	4	ferè 20	280		

And so in the first experiment, when we observed from a height of 10 feet the aforesaid globe (the operation repeated often) come to the pavement in at least five vibrations of the aforesaid pendulum, we tested the height which a globe equal to that one passed through in 10 vibrations and found it to be 40 feet & thus for the rest. In experiment 2, however, we explored those times with an assumed height, for the height having been found to be 15 feet which the globe passed through in 6 vibrations, we concluded that, if the aforesaid proportion were correct, at the end of 12 vibrations it ought to pass through 60 feet, therefore, a height of 60 feet that would be suitable to action having been sought for, we found this to be correct, & thus for the others; we could not however find the height owed to 30 vibrations which would be useful for the rest of the progression. Now you see in the first experiment, that as the space of 10 was to 40, so the square of 5 vibrations, that is 25, is to the square of 10 vibrations, that is 100; & in the second experiment, as the space of 15 was to the space 60, so the square of 6 vibrations, that is 36, is to the square of 12 vibrations, that is 144, & thus for similar ones.

Riccioli: *New Almagest*, Book 2, Chapter 21, 1651

Constant of Proportionality: A Key Parameter

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velocity $\propto$ ***time*** ***distance*** $\propto$ ***time***^{2}

g : velocity acquired in the first second

d_g : distance of fall in the first second

Galileo (remark in a letter to Peiresc, 15 January 1635)

4 cubits in the first second (197 cm)

Mersenne (in *Harmonie Universelle*, 1636, confirmed in 1640s)

12 Paris feet in the first second (394 cm)

Riccioli (in *Almagestum Novum*, 1651)

15 Roman feet in the first second (467 cm)[1]

{Huygens (in 1659, then in *Horologium Oscillatorium*, 1673)

15 Rh feet 7½ in. in the first second (490.4 cm)}

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[1] Using Riccioli's 312 old Roman ft height for the Tower of Asinelli, which is now said to be 97.2 meters high; if instead one uses 29.57 cm for the old Roman ft (Koyré, Klein), the distance of fall in the first second 444 cm, a nearly 10 percent error that is difficult to explain insofar as the error appears to be uniform across all Riccioli's announced values, and therefore cannot be attributed to either air resistance or to timing errors, but only to a uniform error across all his announced heights.

Galilean Principles of “Local” Motion

In the absence of air resistance, bodies descending from rest

1. **In vertical descent acquire equal increments of speed in equal increments of time.**

2. **Acquire the same speed in descending from the same height regardless of their weight or shape.**

3. **Acquire the same speed in falling from a given height whether falling vertically or along an inclined plane.**

4. **Acquire a speed in descending from any given height which is just sufficient to raise them to that height.**

What experimental evidence did Galileo and those in the decade following him provide in support of each of these principles; and how telling was that evidence in showing whether each holds merely to high approximation or exactly?