C. Motion Versus Rest: Impetus and Momenta

1. Because the new science is one about motion, the distinction between rest and motion does not as such play much of a role here
   a. For Aristotle a distinction of kind, not of degree
   b. Still, a pertinent question that one might want to ask Galileo
2. In a brief digression Sagredo offers a physical picture of the cause of motion under which the distinction would be more one of degree than of kind [201f]
   a. A body hurled upwards progressively loses the "*virtu*" impressed on it by the thrower -- the *virtu* that continues to drive the object upward after it leaves the hand of the thrower
   b. Once the remaining *virtu* diminishes to the point where it is in equilibrium with the *virtu* corresponding to its heaviness, "the moveable stops rising and passes through a state of rest"
   c. Impressed *virtu* then continues to diminish, so that the *virtu* from heaviness progressively out-balances it, causing progressive acceleration
3. Most interesting aspect of this conceptualization of the process of deceleration and acceleration is that Sagredo continues by trying to reconcile static with dynamic *virtu*

   > "Thus, when you support a rock in your hand, what else are you doing but impressing on it just as much of that upward impelling *virtu* as equals the power of its heaviness to draw it downward.... The rock always starts with just as much of the *virtu* contrary to its heaviness as was needed to hold it at rest" [202]

4. Notice how elusive the (Newtonian) concept of force is, under which only acceleration and not motion itself requires force; and how difficult it is to relate the force exerted by a heavy static object -- say on a pulley -- to forces causing motion
   a. We will be seeing the concept of dynamic force emerging as we proceed
   b. Kuhn has claimed (I think wrongly) that not even Newton was always clear about how to reconcile his concept with that of static force -- reconciliation not complete until the middle of the 18th century
5. Also notice how Salviati immediately badmouths Sagredo's physical conjecture, in the process marking a distinction between what we now call kinematics and dynamics [202]
   a. One of several passages cited as evidence for the claim that Galileo wanted science to answer "how" and not "why" questions
   b. This in spite of the fact that he elsewhere is quick to address "why" questions -- e.g. his theory of the tides

D. Alternative Concepts of Uniform Acceleration

1. Galileo now in a position to appeal to simplicity arguments to justify his definition of uniformly accelerated motion
   a. In effect, try simplest "rule" first and see how it conforms with *naturalia experimenta* [197]

   b. Trouble: why is uniform in time simpler than uniform in space

$$\Delta(v) = a*\Delta(t) \text{ versus } \Delta(v) = a*\Delta(s)$$

2. Notice that at the point Sagredo raises the issue in *Two New Sciences* [203] this is a conceptual, not an empirical question
   a. According to Drake, Galileo himself for a long time thought the two were equivalent, but could find no way to obtain the 1,3,5, ... progression he had observed on an inclined plane from it
   b. But here he presents the issue as a conceptual one, eschewing the appeal to the 1,3,5,... progression
   c. Save for pointing out that impact effects seem to be proportional to height of fall, and Galileo had said these effects came from the speed acquired (and not the speed squared)
3. The argument Galileo offers has generally been interpreted as trying to show that $\Delta(v)=a*\Delta(s)$ is incoherent
   a. Given two spaces, one twice the other, then velocities acquired must be proportional to these; but times for traversing proportional to the spaces, and inversely proportional to the velocities; consequently times the same
   b. The trouble is that the alternative acceleration rule is not incoherent: $s=c*\exp(at)$ and hence $v=c*a*\exp(at)$
   c. The only real fault of the alternative rule: it requires v not to be 0 at the beginning of motion
   d. Hence, Galileo's argument generally regarded as fallacious, though what he says does hold if t varies as s/v
4. {The question is whether the argument he offers is at least on the track of an underlying conceptual difficulty
   a. Drake's suggestion: argument envisages a one-to-one mapping between speeds in the half-space and speeds in the whole, from which Proposition II entails the same time for each space, implying instantaneous motion
   b. Fallacy would then be that he is in effect integrating with respect to space and time as if they are linearly correlated}
5. Of course, Galileo can scarcely be faulted for not having the mathematics needed to see what the alternative rule entails, and the reasons he gives in *Two New Sciences* notwithstanding, he did find empirical reasons to adopt $\Delta(v)=a*\Delta(t)$
   a. But it does bring out the way in which concepts that seem so automatic and clear to us in fact required great effort before they became that way
   b. And, while much of that effort was ultimately conceptual -- in the above case, mathematical -- the ultimate guiding factor for Galileo seems to have been empirical

E. A Fundamental Result: The Mean Speed Theorem

1. The discussion from [198] to [205] has removed the conceptual obstacles facing the proposed definition of uniformly accelerated motion, but it has not provided the reader any way to conceptualize such motion
   a. As the preceding discussion of the alternative rule for uniform acceleration makes clear, not in a position to conceptualize it as we do today: $dv=a*dt$ and $v= ds/dt=a*t$, and so $s=1/2*at^2$
   b. For not even Galileo could visualize -- i.e. conceptualize -- the difference between the two rules from their mere statement
2. The "mean speed theorem" -- Proposition I -- bridges this conceptual gap by allowing one to think of uniformly accelerated motion in terms of a corresponding uniform motion
   a. I.e. space traversed in a given time is equal to the space traversed by an object moving uniformly throughout at half the speed the accelerating object has at the end of the time
   b. Licenses inferences about spaces traversed and times required that are parasitic on results for uniform motion, including inferences enabling measured values of varying velocities!
   c. I.e. mathematically reduces uniformly accelerated motion problems to uniform motion problems, thereby avoiding any need for reference to instantaneous speed
3. A trivial result once integrals are used as above, for simply substitute the value for v at the end of the time for a∗t in the formula for s: $s = [1/2*at]*t = [\text{mean } v]*t$
   a. Galileo had no way of "adding up" all the infinity of speeds to get the distance covered -- i.e. had no way of integrating
   b. {Nor did the Mertonians and others – in particular, Oresme, of whom Galileo must have been aware -- who had discovered the mean speed theorem before Galileo}
4. Galileo's "proof" invokes a one-to-one comparison of momenta of speed in the uniformly accelerated case with the momenta of speed in the uniform motion case [208f]
   a. Once one-to-one correspondence granted, the argument goes through
   b. The lacuna: what assures such a one-to-one correspondence? -- this is the non-geometric step
5. Huygens ultimately achieved a purely geometric proof of the mean speed theorem, though one that uses reductio ad absurdum twice
   a. Galileo was averse to using reductio ad absurdum proofs at all in mechanics
   b. So, he might have continued to prefer his proof

III. "The Third Day": The Evidential Difficulties

A. The Postulate: Empirical Motivation and Grounds

1. What immediately follows the definition of uniformly accelerated motion in the Latin text is a postulate: "the degrees of speed acquired by the same moveable over different inclinations of planes are equal whenever the heights of those planes are equal" [205]