“Identification of Social Interactions”

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Dallas, November 5, 2010

Outline of talk

Importance of social context in economic decisions

- Individuals or firms influenced by the characteristics of others and the decisions of others
- For individuals in residential neighborhoods, schools, workplace, random encounters, serendipity
- For firms: proximity to suppliers, and to competitors; main ingredient of new economic geography
- For individuals: neighborhood effects, peer effects, role models
- Unified treatment is relatively new, since Manski (1993), big boost by Brock and Durlauf (2001a; b); empirical work followed.
- Literature has learned from other social sciences and seems to be having an effect in the other direction
- For firms, many phenomena well studied by urban economics, such as urbanization versus localization economies. Effort to unify by Ioannides (2010); shall see how it is received.

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Decision making in group contexts

- Individual i in group g chooses ω_{ig},

$$\omega_{ig} \in \operatorname{argmax}_{\lambda \in \Omega_{ig}} V(\lambda, x_i, y_g, \mu_i^e(\omega_{-ig}), \varepsilon_i, \alpha_g). \qquad (1)$$

- x_i An R-vector of observable (to the modeler) individual-specific characteristics;

  y_g An S-vector of observable (to the modeler) group-specific characteristics;

  $\mu_i^e(\omega_{-ig})$ A probability measure, unobservable (to the modeler), that describes the beliefs individual i possesses about behaviors of others in the group; For purposes of the elucidation of the basic theory of choice in the presence of social interactions, we focus on the case where beliefs are latent variables.

  ε_i A vector of random individual-specific characteristics describing i, unobservable to the modeler; and

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Equilibrium condition

- The decision problem facing an individual, a function of preferences (embodied in the specification of V); constraints (embodied in the specification of Ω_{ig}); and beliefs (embodied in the specification of $\mu_i^e(\omega_{-ig})$).
  Completely standard microeconomic reasoning.
- Closed by the assumptions under which $\mu_i^e(\omega_{-ig})$ is determined.
- self-consistency between subjective beliefs $\mu_i^e(\omega_{-ig})$ and the objective conditional probabilities of the behaviors of others given i's information set F_i:

$$\mu_i^e(\omega_{-ig}) = \mu(\omega_{-ig}|F_i). \qquad (2)$$

- Demonstrate by applying to the linear case
- Much of the empirical literature on social economics has involved variations of a general linear model, Manski (1993) the linear-in-means model

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Linear-in-means model

- $$\omega_{ig} = k + cx_i + dy_g + Jm^e_{ig} + \varepsilon_i\,, \tag{6}$$
  where m^e_{ig} denotes the average behavior in the group, i.e.
  $$m^e_{ig} = \frac{1}{n_g}\sum_{j\in g} \mathrm{E}(\omega_j|F_i)\,. \tag{7}$$
- Equations (6) and (7) solve for a common value:
  $$m^e_{ig} = m_g \equiv \frac{k + c\bar{x}_g + dy_g}{1-J}. \tag{10}$$
  Individuals' expectations of average behavior in the group equal the average behavior of the group.
- m^e_{ig} depends linearly on x_i, $\bar{x}_g$, and
  the contextual interactions group $g-$specific, y_g.
  $J < 1$: required for equation (10) to make sense.
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A reduced form and standard practice

- $$\omega_{ig} = \frac{k}{1-J} + cx_i + \frac{J}{1-J}c\bar{x}_g + \frac{d}{1-J}y_g + \varepsilon_i. \qquad (11)$$

- $$\omega_{ig} = \pi_0 + \pi_1 x_i + \pi_2 y_g + \varepsilon_i\ , \qquad (12)$$

  where the parameters π_0, π_1, π_2 are estimated empirically.

- How do estimates of π_0, π_1, π_2 characterize social interactions in the sense of (6)?

  $\pi_2 \neq 0$ is neither necessary nor sufficient for **endogenous** *social interactions* to be present, since $J = 0$ is neither necessary nor sufficient for $\pi_2 = 0$.

  Estimates of (12) are not uninformative; should be mapped to structural parameters in the sense of (6) when identification holds;

  if identification does not hold, what does (12) imply about distinguishing types of social interactions?

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Social reflection and identification of the linear-in-means model

- Manski (1993) identification can fail for the linear in means model when one focuses on the mapping from reduced form regression parameters to the structural parameters.
  Manski's original assumption: $y_g = \bar{x}_g$,
  i.e., contextual effects = average of corresponding individual characteristics.
- Equ. (10) becomes:

$$m_g = \frac{k + (c+d)y_g}{1-J}, \tag{13}$$

  m_g in equation (6) linearly dependent on the constant and y_g.
- Reflection problem: ω_{ig} is correlated with the expected average behavior in a neighborhood;
  From (13): Could it be m_g may simply reflect the role of y_g in influencing individuals rather than it itself

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- **Identification in the linear in means model.** *The parameters k, c, J and d are identified if and only if proj $\{\bar{\omega}_g | 1, y_g, \bar{x}_g\}$ − proj $\{\bar{\omega}_g | 1, y_g\} \neq 0$.*
- Partial linear-in-means:

$$\omega_{ig} = k + cx_i + dy_g + J\mu(m_g) + \varepsilon_i. \qquad (15)$$

  Brock and Durlauf identification functional form of $\mu(m_g)$ known.
- Dynamic linear models:

$$\omega_{igt} = k + cx_{it} + dy_{gt} + \beta m_{g\,t-1} + \varepsilon_{it} \qquad (16)$$

- Exogenous group sizes, variance-based methods:

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identification, continued

- Panel data

$$\omega_{igt} - \omega_{ig\,t-1} = c(x_{it} - x_{i\,t-1}) + d(y_{gt} - y_{g\,t-1})$$
$$[3pt] + J(m_{gt} - m_{g\,t-1}) + \varepsilon_{it} - \varepsilon_{i\,t-1}\,. \quad (32)$$

Why does identification matter?

- Datcher (1982)
- Distinguish c, d, J.
- We know students learn from one another; should we mix them or separate ("track") them?
- For many policy contexts, the structural model is of no intrinsic interest. Brock, Durlauf and West (2003) argue that this is the case for a range of macroeconomic contexts. If policies are available to influence y_g , then these interactions can be identified even if the structural parameters are not identified.
- endogenous social interactions of fundamental policy relevance, like when affect the distribution of individuals across groups.
- Then group, i.e. neighborhood choice, self-selection new layer of complexity

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Self-selection

- treat group choice and behavior within a group as a set of joint outcomes, and conduct empirical analysis from the perspective of both behaviors.
  Brock and Durlauf (2001b) first recognized this possibility and studied the case of self-selection between two groups; Brock and Durlauf (2002; 2006) and Ioannides and Zabel (2008) extended this analysis to an arbitrary finite number of groups.
- Heckman (1979) reasoning, individuals choosing among groups $g = 1, \ldots, G$ based on an overall individual-specific quality measure for each group:

$$I^*_{ig} = \gamma_1 x_i + \gamma_2 y_g + \gamma_3 z_{ig} + \nu_{ig}, \quad (39)$$

  where: z_{ig} denotes those observable characteristics that influence i's evaluation of group g but are not direct determinants of ω_i and ν_{ig} denotes an unobservable individual-specific group quality term.

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Self-selection, continued

- Individual i chooses the group with the highest I^*_{ig}. We assume that prior to group formation, for all i and g, $\mathrm{E}(\varepsilon_i|x_i, y_g, z_{ig}) = 0$ and $\mathrm{E}(\nu_{ig}|\xi, y_g, z_{ig}) = 0$.
- Estimate

$$\omega_{ig} = cx_i + dy_g + Jm_g + \mathrm{E}(\varepsilon_i|x_i, \bar{x}_1, y_1, z_{i1}, \ldots, \bar{x}_G, y_G, z_{iG}, i \in g) + \xi_i. \tag{38}$$

where by construction the Heckman error correction term, $\mathrm{E}(\xi_i|x_i, \bar{x}_1, y_1, z_{i1}, \ldots, \bar{x}_G, y_G, z_{iG}, i \in g) = 0$.

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A binary choice model of social interactions

- $$V_i(1) - V_i(-1) = k + cx_i + dy_g + Jm^e_{ig} - \varepsilon_i. \quad (59)$$
- Individual i chooses $+1$ iff $V_i(1) - V_i(-1) \geq 0$.

$$\mu(\omega_i = 1 | x_i, y_g, i \in g) = F_\varepsilon(k + cx_i + dy_g + Jm^e_{ig}).$$

- Close by imposing an equilibrium condition on beliefs: expected value of the average choice level in the population is given by

$$m_g = 2 \int F_\varepsilon(k + cx + dy_g + Jm_g) dF_{x|g} - 1. \quad (62)$$

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A binary choice model of social interactions

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Social Interactions in Social Networks

- A social network is a **graph** (V, E) where V is the set of individuals and the directed edges in E signify social influence: (i, j) is in E if and only if j influences i.
  Can be represented by **adjacency matrix** A, or sociomatrix: $n_V \times n_V$ matrix, one row and one column for each individual in V. For each pair of individuals i and j, $a_{ij} = 1$ if there is an edge from i to j, and 0 otherwise. $a_{ii} = 0$.
- Identification in social networks: key works
  Cohen-Cole (2006): influences from different peer groups
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Identification in social networks with known structure

- $$a_{ij} = \begin{cases} \frac{1}{|P(i)|} & \text{if } j \in P(i), \\ 0 & \text{otherwise.} \end{cases} \quad (55)$$

$$\omega_i = k + cx_i + d \sum_{j \neq i} a_{ij} x_j + J \sum_j a_{ij} \omega_j + \varepsilon_i. \quad (47)$$

The reduced form in vector notation:

$$\omega = k(I - JA)^{-1}\iota + (I - JA)^{-1}(cI + dA)x + (I - JA)^{-1}\varepsilon \quad (49)$$

where I refers to the $n_V \times n_V$ identity matrix and ι is a $n_V \times 1$ vector of 1's.

Identification in social networks with known structure

- **Theorem 2. Identification of social interactions in linear network models**
  For the social interactions model described by (49), assume that $Jc + d \neq 0$ and that for all values of $J \in \mathcal{J}$, $(I - JA)^{-1}$ exists.
  i. *If the matrices I, A, and A^2 are linearly independent, then the parameters k, c, d and J are identified.*
  ii. *If the matrices I, A, and A^2 are linearly dependent, if for all i and j, $\sum_k a_{ik} = \sum_k a_{jk}$, and if A has no row in which all entries are 0, then parameters k, c, d and J are not identified.*

Identification in social networks with known structure, continued

- **Corollary 1. Identification of social interactions in group structures with different-sized groups.** *Suppose that individuals act in groups, and that the a_{ij} are given by either inclusive or exclusive averaging. Assume that $Jc + d \neq 0$. Then the parameters k, c, d and J are identified if and only if there are at least two groups of different sizes. With inclusive averaging (an individual is a member of his own peer group), the parameters are not identified.*
- **Theorem 5. Generic identifiability of the linear social networks model.** *The set of all matrices $A \in S$ such that the powers I, A and A^2 are linearly dependent, is a closed and lower-dimensional (semi-algebraic) subset of S.*
  This theorem is a complement to McManus' (1992) result on the generic identifiability of non-linear parametric models. For

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Identification in social networks with unknown structure

- $$(I - JA)\omega = (cI + dA)x + \varepsilon \qquad (54)$$

  Classical identification in econometrics

- $$\Gamma\omega = Bx + \varepsilon,$$

  where $\Gamma = I - JA$ and $B = cI + dA$ for known A

- Special case: n_V agents on a circle; interactions with closest neighbors.

  $$\Gamma_{ii} = 1,\ \Gamma_{i\,i-1} = \Gamma_{i\,i+1} = \gamma_1, \forall i, \Gamma_{ij} = 0, \text{otherwise};$$

  $$B_{ii} = b_0, B_{i\,i-1} = B_{i\,i+1} = b_1, \forall i, B_{ij} = 0.$$

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- Special case: circle; with closest neighbors up to distance 2.

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$$\omega_i = cx_i + d\sum_{j\neq i} a_{ij}(\gamma)x_j + J\sum_{j\neq i} a_{ij}(\gamma)\omega_j + \varepsilon_i. \tag{56}$$

$$A(\gamma) = \begin{pmatrix} 0 & \gamma & \gamma^2 & \cdots & \gamma^k & \gamma^k & \gamma^{k-1} & \cdots & \gamma^2 & \gamma \\ \gamma & 0 & \gamma & & \cdots & \gamma^k & \gamma^k & \gamma^{k-1} & \cdots & \gamma^2 \\ & & & & \vdots & & & & & \\ \gamma & \gamma^2 & & \cdots & & & & & \gamma & 0 \end{pmatrix}. \tag{57}$$

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Identification in social networks with unknown structure

- **Theorem 7. Identification of the linear social networks model with weights exponentially declining in distance**
  Part **i** says: Each structural parameter vector is observationally equivalent to at most $2n_V - 3$ other structural parameter vectors in the sense that they all generate the same reduced form.
- Part **ii**: if there are no social interactions, this imposes sufficiently strong restrictions on the reduced form parameters to identify both c and also requires that the matrix of reduced form parameters is proportional to an identity matrix.

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Identification

- Create experimental designs such that $\bar{x}_g$ does not lie in the span of the elements of y_g?
- Eliminate unobserved group characteristics by controlling what group members know about each other.
- Group membership can be explicitly controlled, which addresses the self-selection issues.
- Are the actions of interacting agents jointly determined?
- Do statistics other than mean action matter?
- Does topology of interaction matter?
- Virtual vs. actual social interactions?
  Very relevant for understanding relationships in social media.

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- METCO
  Angrist and Lang
- Moving to Opportunity (MTO)
  Housing vouchers, randomly selected families, residents of high-poverty public housing projects.
  Randomly allocated between two subgroups:
  one received unrestricted vouchers;
  and another (the experimental group) vouchers that could only be used in census tracts with poverty rates below 10%
- Social interaction effects derived from calculations of treatment effects associated with the vouchers.
  Kling, Ludwig and Katz: careful to distinguish between measures of the effects of intent to treat (eligibility for a voucher)
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  science and industry
- AddHealth data set
  But torrents of data becoming available from all kinds of devices of contemporary life
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