

- a. Perfection of circular motion vs. calculational tractability of uniform circular motion
- b. Unclear which was more important to Ptolemy, or to anyone else at the time
- 4. Ptolemy's *Almagest* vs. his *Planetary Hypotheses*, some decades later
  - a. Former gave a mathematical account of motion of sun, moon, and planets
  - b. Latter offers hypotheses about the physical basis of this motion -- essentially a rotating solid sphere account, with epicycles from smaller spheres centered on the surface of the larger ones
  - c. Determined the minimum size (in units of Earth-Sun distance) required to fit everything in, with the sphere of the stars on the outside and no spheres overlapping any others
- 5. Ptolemy (around 150 A.D.) prolific: The *Handy Tables*, the *Geography*, the *Tetrabiblos*, the *Optics*, the *Harmonics*, treatises on logic, on sundials, on stereographic projection
  - a. A university professor, in a less anachronistic sense than you might think
  - b. In the world center of learning, Alexandria

### III. Ptolemy's *Almagest*

#### A. Overview of the Book

1. *Mathematical syntaxis* (systematic treatise): "a complete exposition of mathematical astronomy as the Greeks understood the term" (Toomer), in 13 Books
  - a. '*Almagest*': the great one, a Latinization of an Arabic adaptation of Greek; the work preserved in Arabic speaking world during Middle Ages and brought from there into Europe in 12th century
  - b. Very much a textbook, instructing how to calculate a wide range of quantities and solve various problems in spherical astronomy
2. Books I and II: Preliminaries (e.g. earth at center of universe, motionless, etc.); followed by spherical trigonometry and its application to a range of calculational problems of interest, with tables
  - a. Using chord function, rather than sine and cosine:  $\text{chord } \theta = 2 \sin(\theta/2)$
  - b. Calculate such things as terrestrial latitudes from length of longest day, transformations between equatorial and ecliptic coordinates etc., including many calculations of astrological interest
  - c. Approach: formulate problems in terms of spherical triangles, then employ Menelaus's theorem to determine the unknown quantity, given five other quantities
3. Books III - VI: Sun, basic lunar, advanced lunar, and eclipses
  - a. Mean sun provides basic unit of time, employing tropical rather than sidereal year, including a slightly erroneous value for the length of the mean tropical year inherited from Hipparchus
  - b. Everything else built off of solar theory
  - c. Note, however, the privileged position of eclipses: emphasis on predicting special events
4. Books VII & VIII: a catalog of the principal fixed stars
  - a. Stars come after moon because moon used as a marker for the sun, and time connected to the sun
  - b. (Note the calculational orientation of the work)
  - c. A catalog of visible stars that remained a primary reference until modern times

5. Books IX - XIII: on the planets
    - a. IX-XI on longitudinal motion
    - b. XII on stationary points, retrogradations, and maximum elongations: the phenomena of primary classical interest, comparable to eclipses
    - c. XIII on latitudes
  6. Throughout the work the mathematics needed to calculate quantities of interest, including calculating parameters of celestial orbits from observations
    - a. In principle, able to determine values of parameters from preferred observations (specified by Ptolemy) -- if need be, correcting those given by him
    - b. Then can calculate position (geocentric longitude and latitude) of Sun, Moon, any planet, or any star at any time whatever
    - c. I.e. a complete calculational system
    - d. With supplementary easy to use tables to aid in the calculations
  7. Time and again the Almagest reminds the reader of Apollonius's theorem and hence of the possibility of representing various non-uniform motions either by an epicycle or an eccentric
    - a. No one who has read the book should ever have thought that the representations Ptolemy in fact adopted were uniquely determined
    - b. Still, the parameters of the orbits remained essentially the same either way, again by virtue of the equivalence Apollonius had established
- B. General Remarks on the System
1. Basic system, ignoring irregularities of sun and moon and variations in patterns of retrograde motion
    - a. Mercury and Venus tied to (mean) Sun; moon and outer planets not
    - b. Location of outer planets on their epicycles tied to (mean) Sun
  2. No planetary distances, save for distance to moon (via parallax) and (inaccurate) estimate of distance to sun via lunar eclipses
    - a. In each case, set radius of each primary circle  $R=60$  (using Babylonian sexagesimal number system with units or degrees, firsts, seconds, thirds etc. not fractions or modern decimals)
    - b. Nothing in the system that allows radii to be determined from observations on any uniform basis, for planetary parallaxes not observable
    - c. Sequence from Moon to Saturn conjectured on basis of time required for each to return to the same place along the zodiac, and not on any more direct basis, using observations
  3. Primary circle for planetary orbits called the deferent, and secondary circle the epicycle
    - a. Ratio of radii of epicycle to deferent,  $r/R$ , determinable for each outer planet, in effect from (average) angular size of retrograde loops (see Swerdlow in Appendix)
    - b. Even more directly for Mercury and Venus, from (average) maximum elongations (via another theorem of Apollonius)