Monte Carlo Study of Sensitivities of Data-Driven Correction Factors Used to Model Invisible Z^0+ jets Events

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Abstract

In this thesis, the sensitivities of the ratio of $Z^0 \to \nu\nu$+jets to $Z^0 \to l^+l^-$ to both physics effects and phase space distortions caused by experiment are explored using Monte Carlo simulations, in order to evaluate the usefulness of such a ratio in modeling SM backgrounds for monojet events and develop a scheme for doing so. Monte Carlo simulations of $Z^0 \to \nu\nu$, $Z^0 \to \mu^+\mu^-$, and $Z^0 \to e^+e^-$ channels are generated using Pythia, and ratio histograms of the $p_T^{Z^0}$ distributions are created using ROOT. Effects that cause the ratio of invisible to visible Z^0 decays to deviate from flatness (namely QED radiation, interference from a $\gamma^* \to l^+l^-$ amplitude, overlap removal and phase space cuts) are suppressed to produce a flat ratio, and then systematically added back in to determine their effects. Finally, information gained on the sensitivities of these ratios to different physics effects is used to create a correction factor that accounts for the minimum possible number of necessary correction terms (in this case QED radiation, fiducial cuts on the lepton, and the γ^* interference) to produce an accurate model of the invisible Z^0 decay. Additionally, an analysis scheme for maximizing the deviation of this ratio from flatness with minimal theoretical uncertainties is developed, to be compared with data as an experimental test of this model.

Contents

1 Background

As searches for dark matter and other new physics at the LHC continue, events with one hard jet and a large amount of missing energy (indicated by the missing transverse momentum in the event, E_T^{miss}), known as monojet events, are frequently featured as final states that such new physics might produce. In particular, the large E_T^{miss} of these events lends itself to being the final state of a dark matter candidate, as these particles are frequently theorized to be WIMPs (Weakly Interacting Massive Particles), and hence unlikely to appear on a detector at an accelerator. In order to properly probe these areas for new physics, however, it is necessary to estimate the Standard Model background as precisely as possible for this final state. In the Standard Model, the irreducible background for this final state is the invisible decay of processes the Z^0 boson when produced with jets, which produces a collection of jets and two neutrinos. Conventionally, the method to estimate this final state's contribution to a monojet measurement is to simply use a Monte Carlo simulation. However, because of the presence of strong interactions in this process, the results of the simulation are subject to very large theoretical uncertainties due to QCD parameters. Instead of employing this method, however, one can note that the Z^0 has other, far more easily measurable, decay pathways, into a muon-antimuon or electron-positron pair. While Monte Carlo simulations of this background will be subject to the same large uncertainties, physics governing the jets in both final states will be identical. As a result, any change in a QCD parameter in a Monte Carlo estimation of one channel will identically affect the other channel (to within statistical fluctuations). As a result, if a ratio histogram of the distributions in the invisible decay channel to the distributions in the visible channel is created, this ratio will, in theory, be utterly insensitive to the large uncertainties plaguing each individual channel's Monte Carlo prediction.[1] This ratio can then be multiplied by an experimentally measured distribution of the $Z^0 \to l^+l^-$+jets channels, producing a prediction for the invisible Z^0 contribution that is not subject to any theoretical uncertainties in QCD parameters, and as a result, is a much more precise prediction of this background.

While the physics governing the jets in the visible and invisible channels of the Z^0+jets final state is the same, there are some notable differences between channels with charged, visible leptons in the final state to those with their neutral, virtually undetectable neutrino counterparts. A priori, there are two physical sources of discrepancy between the electron and muon channels and invisible ones: First, the muon and the electron, unlike neutrinos, can radiate photons after being produced in the Z^0 decay, affecting both the reconstructed Z^0 and jet clustering as the photons carry energy away from the leptons in the final state and deposit it where it may be clustered into a jet (depending on jet definition). Second, unlike neutrinos, muons and electrons couple to photons as well as the Z^0 boson. As a result, the production of a photon in association with jets can lead to the same final state as the production of a Z^0 with jets, (namely, l^+l^- + jets), so in the cross-sections for the muon and electron decay pathways of the Z^0, there will be an interference effect from a virtual γ^* that is absent in the invisible Z^0 channel. In addition to these two physical sources of discrepancy, experimental conditions restrict the phase space of the phase space of the leptonic channels: The kinematic cuts on the p_T, η, and ΔR of the visible decay products and the necessity to restrict the phase space of jets to prevent them from being

[1] In addition, large experimental uncertainties, such as those found in jet energy corrections from calorimeter readings, ought to be suppressed in this ratio, although this phenomenon goes beyond the scope of this truth-level study.

collinear with either hard visible lepton (in order to adequately distinguish them from the leptons appearing in the detector) in the visible channels are both unnecessary and impossible to implement for the undetectable neutrino decay products. As a result of these differences, careful Monte Carlo modeling of the ratio must be performed, and the sensitivity of the ratio to each conceivable factor must be accounted for in order to produce an accurate estimate of the correction factor and get a good idea of its uncertainties.

2 Objectives

In this thesis, Monte Carlo modeling is performed to produce a truth-level study of these effects. Because this study focuses on the fundamental sensitivities of the ratios of these Z^0 decay channels to fundamental physical effects and phase space distortions, this study is performed at the truth level, in the absence of any detector effects or reducible backgrounds (such as $Z^0 \to \tau^+\tau^-$ events). Once the sensitivity of these ratios to all the physical effects mentioned in section 1 has been established, an analysis scheme to maximize the sensitivity of the ratios to these effects can be developed. This maximized sensitivity presents an excellent test for the validity of these models: Because the ratios will be highly sensitive to the physical effects (namely QED effects and phase space distortions from fiducial cuts and jet overlap removal), predicted in the model, a measurement of these ratios will in turn be a method for testing the accuracy of the Monte Carlo modeling. Once a test is developed (and assuming that the Monte Carlo simulations pass these tests), a correction factor to account for all effects that cause the ratio to deviate from flatness can be generated by comparing a ratio with these effects suppressed to one where all effects are included.

3 Procedure

3.1 Ntuples

As one would expect of a Monte Carlo study, the first step in this analysis is to use a Monte Carlo generator to obtain data (in this case, in the form of ntuples). To generate the ntuples employed in this study, the Pythia 8 set of libraries, version 8.185 is used, with beam characteristics to approximate the most recent completed run of the LHC: A *pp* beam with a center of mass energy of 8 TeV. In each ntuple, two hard processes are enabled: $q\bar{q} \to \gamma^*/Z^0\ g$ and $q\bar{q} \to \gamma^*/Z^0\ q$; these two hard processes correspond to the production of Z^0 plus jets. Ntuples for three possible decay channels of the Z^0 are generated: One for each of the two leptonic pathways being analyzed, $Z^0 \to e^+e^-$ and $Z^0 \to \mu^+\mu^-$, and one for the invisible decay pathways, $Z^0 \to \nu\nu$. For each decay pathway, three different ntuples are generated: One where the full effects of QED radiation and interference by the γ^* are allowed, one where QED radiation is enabled but the γ^* interference term is disabled, and one where both the γ^* interference and QED radiation are disabled. Finally, each ntuple is generated using two different multiparton interaction tunes (see section below, "Tuning"). In total, then, 18 ntuples are created in this analysis. In each ntuple, the four-momenta of the decay products of the Z^0 are stored. In addition, four jet collections (see section below, "Jet Clustering"), as well as a complete record of all final-state leptons and all photons emitted by Z^0 decay products are stored.

3.1.1 Lepton Definitions

For electrons and muons, three different lepton definitions are employed, and so three different sets of lepton four-momentum data are stored in the event ntuples. First, the leptons emitted immediately from the Z^0 are stored, prior to any final-state radiation. This lepton definition is referred to in the analysis as "born-level". While this lepton definition suppresses the effect of QED radiation on the Z^0 reconstruction, its use in experimental data requires models to predict QED radiation, since the born-level leptons cannot be directly measured. Next, the parton-level leptons after final-state radiation takes place are stored, called the "bare-level" leptons. Because muons are minimally ionizing particles, this lepton definition corresponds most closely to detector-level data for muons. Finally, the bare lepton four-momenta are summed with the four-momenta of any radiated photons within a cone of $R < 0.2$ (where $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$, where η is the pseudorapidity and ϕ is the azimuthal angle of a given particle's four-momentum) around them, and these four-momenta are stored. This lepton definition is referred to as the "dress-level" lepton. Because electrons register on the same calorimeter as their emitted photons, this lepton definition most closely corresponds to detector-level data for electrons.

3.1.2 Jet Clustering

When generating ntuples, the jet clustering is handled by the Fastjet 3.0.3 software package. Jet clustering is performed on final-state particles using the anti-k_T algorithm with an R parameter of 0.4 and a minimum jet p_T of 20 GeV/c. Four different jet collections are formed: The first of these represents the "experimental" collection of jets, and includes all final-state particles except for neutrinos that come out of the hard process (since, in an experiment, these would be invisible). The second of these excludes the born-level leptons (namely, it excludes the bare leptons and all final-state photons which the hard-process leptons radiated); I call this collection "born jets". The third of these excludes bare-level leptons, so it includes all photons radiated in the hard process. This collection is known as "bare jets". The final collection excludes the bare leptons and all photons within an $R < 0.2$ cone around them; in short, this collection excludes the dress-level leptons, and is as such called "dress jets".

3.1.3 Tuning

The particular appeal of using ratios to model the invisible Z^0 decay lies in the fact that the sensitivity to QCD parameters are the same for both channels, and therefore uncertainties in these parameters ought to cancel when the ratio is taken. However, the differing phase spaces between the visible and invisible Z^0 channels (discussed above) may introduce some QCD sensitivity back into these ratios: The phase space of jets considered in the $\mu^+\mu^-$ and e^+e^- channels may be affected differently by changes in tune than the phase space of jets considered in the $\nu\nu$ channel, rendering the ratio sensitive to tune. In order to explore this QCD sensitivity, ntuples with two different QCD tunes are generated. The first of these tunes is Pythia 8's 4C tune. This tune is the default employed by the software, and is tuned to give close to accurate data for pp and $p\bar{p}$ collisions based on early LHC data, but isn't tuned to any specific process. The second tune, AZ, is tuned to ATLAS γ^*/Z^0 data, and as a result, has excellent agreement with experiment for these channels. The parameters that differ between the two tunes are listed in the table below:

Tune Name	4C	AZ
Primordial k_T [GeV]	2.0	1.71
ISR $\alpha_S^{ISR}(m_Z)$	0.137	0.1237
ISR cut-off [GeV]	2.0	0.59

Both of these tunes are parton shower, FSR, and ISR tunes, and so different tuning values will result in different initial and final-state QCD radiation, which in turn results in different hadronic recoil. Because any sensitivity of the ratios to QCD effects must ultimately arise from the differing phase spaces of the Z^0 decay channels' hadronic recoils being differently sensitive to changes in QCD parameters, an estimate of the sensitivity of these ratios to QCD effects can be estimated by observing the effect of varying the parton shower tune for these ratios.

In the interest of thoroughness, in addition to considering two different parton shower tunes, two different underlying-event tunes are considered,both the 4C tune and an ATLAS underlying-event tune AU1 (selected because this tune uses 4C as a basis). Because intuition would suggest that the dominant contributing factor to QCD sensitivity in this scenario would be QCD radiation parameters, this tune is only considered when calculating the correction factor in the final analysis. Below, the tuned parameters for the two tunes are shown:

Tune Name	4C	AU1
MultipleInteractions:pT0Ref	2.085	2.48
MultipleInteractions:ecmPow	0.19	0.19
MultipleInteractions:bProfile	3	2
MultipleInteractions:coreFraction[2]	-	0.23
MultipleInteractions:coreRadius[3]	-	0.64
BeamRemnants:reconnectRange	1.5	1.35

3.1.4 Mitigating Statistical Uncertainties

In order to minimize the effects of statistical fluctuations in this analysis, extremely large samples of 20 million events are generated. However, generating samples of this size in a single run of Pythia is impractical. Instead, 80 smaller samples of 250,000 events each are generated in parallel, each with a different random number seed generated using Python's built-in random number generator. In addition to generating samples of large numbers of events, a generator-level phase space cut of 80 GeV/c on the minimum invariant p_T for the hard process is applied in order to increase the number of events that pass cuts on the jet p_T at the analysis level. Together, these measures allow for uncertainties in ratios (and ratios of ratios) on the level of fractions of 1% within areas of kinematic interest. (namely, high missing E_T).[4]

3.2 Generating Histograms

Once the ntuples are generated, ROOT is used to produce histograms of $p_T^{Z^0}$ using the data contained therein for each ntuple, with statistical error calculated using the sum

[4] In the data analysis, a high cut on E_T^{miss} will be achieved by the trigger. In 2012, this was effectively a cut of about 110 GeV/c

of the squares of the weights in each bin. With the exception of a generator-level cut on the minimum invariant p_T of the event (which is applied uniformly to all events in this analysis, as discussed in section 3.1.4), all analysis cuts are performed during the ROOT analysis. This allows for the same ntuples to be assessed using different cutting schemes, which in turn allows for the effects of each new analysis cut to be quantified with greatly diminished statistical uncertainties (see section 3.2.4). In order to determine the sensitivities of the $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ to different physical effects, a wide variety of possible analysis schemes are applied, involving jet cuts, lepton cuts, and overlap removal schemes. After these histograms are generated, ratios of the electron and muon Z^0 channels to the invisible Z^0 channels are generated using ROOT's methods for dividing histograms, and finally, ratios of ratios are produced, using ROOT's division methods again.

3.2.1 Jet Cuts

In this analysis, two jet cutting schemes are considered: One inclusive, one exclusive. The inclusive cutting scheme simply involves a cut on the leading jet p_T; in order to pass, an event must have a leading jet with a $p_T > 110$ GeV/c and a pseudorapidity between -2.5 and 2.5. Meanwhile, the exclusive jet cutting scheme applies the same requirement on the leading jet, but in order to pass, a jet must also have no secondary jets with $p_T > 30$ GeV/c and pseudorapidity between -4 and 4. Because the parton shower tunes considered here primarily affect soft QCD radiation, it would be expected that the secondary jets would be more sensitive to tuning than the lead jet, so removing events with significant secondary jets would likely reduce events where the parton shower tunes had a significant impact on the hadronic recoil, resulting in a different sensitivity of the ratios to these tunes. In addition to approximating the hadronic recoil part of the final-state definitions for both monojet and more general jets+E_T^{miss} events, by comparing these cut cases to the uncut case, a close inspection of the sensitivity of hadronic recoil to different other physical effects can be achieved.

3.2.2 Fiducial Cuts

Apart from cuts on the jets, experimental conditions require that fiducial cuts restrict the phase space of the Z^0 decay products in the visible channels. When fiducial cuts are applied, any electron or muon events must pass three requirements: First, the pseudorapidity η must range between -2.5 and 2.5, second, these leptons must both have $p_T > 20$ GeV/c, and third, the leptons must be separated by at least a ΔR of 0.2. Since these are only applied to the visible channels in the analysis, and not the invisible channel (since neutrinos will not appear on any detectors); it is expected that the ratio of invisible to visible Z^0 channels will be highly sensitive to these cuts.

3.2.3 Overlap Removal and Overlap Subtraction

In addition to fiducial cuts, experiment necessitates another phase space restriction on the visible channels to avoid clustering Z^0 decay products with the jets. To explore this effect, three different schemes are considered: First, an ideal "truth-level" case is defined by requiring that the lepton decay products are simply omitted from clustering. In this case, the born-level, bare-level, and dress-level jet collections (see 3.1.2 for more detail) are used in the analysis. While this case is identical to the jet clustering used in experiment for the invisible Z^0 decay, it is highly artificial, so two more realistic overlap schemes are considered: The first of these is simple overlap removal, which uses the experimental-like collection of jets (see 3.1.2), but removes the jets closest to each visible decay product in $\phi - \eta$ space (as long as these jets are within a $\Delta R < 0.4$ cone around the lepton). This eliminates jets possibly seeded by the visible Z^0 decay products. While this scheme is certainly experimentally simple to implement, it lends itself to biases compared to the $Z^0 \to \nu\nu+$ jets events when a Z^0 decay product is collinear with a jet, so in addition to this scheme, an overlap subtraction scheme is implemented: In this scheme, the closest jet (within a $\Delta R < 0.4$ cone) to each Z^0 decay product has that decay product's momentum four-vector subtracted from its own. In this case, if a real jet accidentally overlaps a Z^0 decay product, the energy of the decay product will be removed but the original jet will still survive.

3.2.4 Ratios And Statistical Uncertainty

For the current ratio studies, no normalization factor is applied to either decay channel. As a result, this ratio is depicted as though each channel's data was collected at a luminosity that would result in 2×10^7 events for the uncut data sample. In the actual experiment, where the luminosity is the same for both channels, the value of the ratios must be scaled by the ratio of branching ratios of the two channels, approximately 5.94. Since this analysis wishes to analyze deviation from flatness and sensitivity of these correction factors to different effects, it is convenient to have a flat ratio correspond to a value of 1. The only exception to this lack of normalization is in the case of ratios that have the virtual γ^* contribution enabled. The histograms with this effect included must be comparable to the same histograms with the effect not included, which is irrelevant in the $\nu\nu$ channel, but in the l^+l^- channels, the cross section is changed by both the inclusion of a new amplitude contribution at the generator level and the (experimentally necessary to reduce the γ^* contribution) requirement that the mass M_{Z^0} of the Z^0 in the visible channels be within 25 GeV/c^2 of the invariant mass of an on-shell Z^0 (approximately 91 GeV/c^2). As a result, the visible channels when the γ^* effect is enabled must be normalized to the same luminosity as the visible channels when γ^* is disabled. The statistical errors in this histogram are scaled by the same factor as the histogram, to preserve accurate error calculation.

Once the ratios are created, physics sensitivities can be further explored by taking ratios of ratios. When the ratios are taken over two different data sets, the method for creating them is essentially identical to the method for creating single ratios: ROOT's histogram division tools create the new ratios of ratios and propagate statistical uncertainty accordingly. However, as mentioned at the beginning of section 3.2, when ratios are taken over the same data set with different analysis schemes, statistical uncertainty is greatly diminished: The only statistical fluctuations that would still influence a ratio of ratios of the same ntuples would be fluctuations in elements of the event that appear in one cutting scheme, but not the other. For example, a ratio of ratios with dressed leptons over those with born leptons would be subject to the statistical fluctuations in the angular distribution of photons radiated from the Z^0 decay products, but not subject to fluctuations in the momenta or angular distribution of these products, nor any fluctuations in the jets. As a result, with these ratios, ROOT's calculated uncertainty is not depicted, since it grossly overestimates the actual uncertainty in any given bin. Instead, the uncertainty in these ratios is considered to be small enough that for the purposes of this exploratory analysis, it can be ignored.

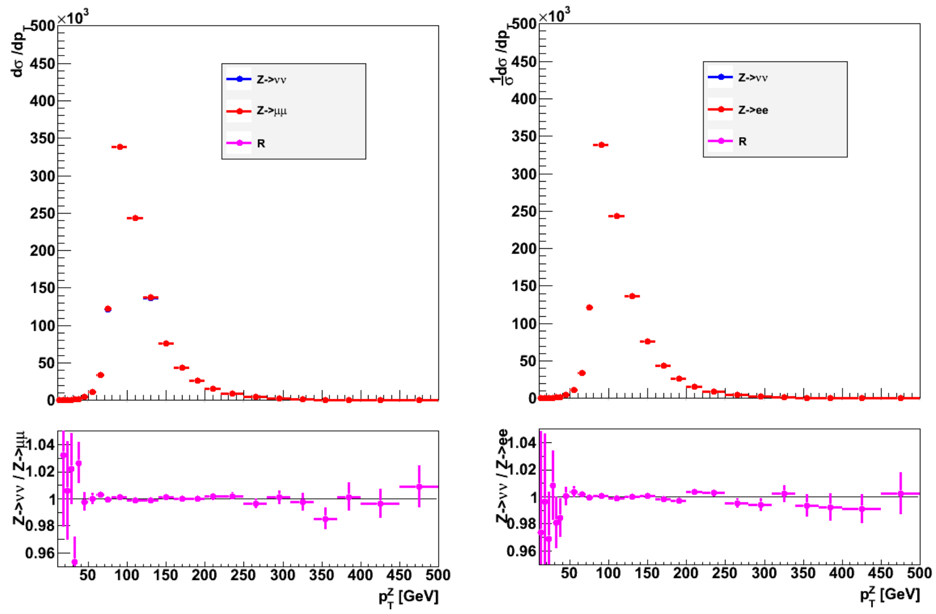
4 Results

4.1 Producing a Flat Ratio

In order to isolate the effects which produce discrepancies between the leptonic and invisible Z decay channels, it is useful to remove the effects that we would expect to produce these discrepancies until we reach ratios of the invisible channel to the leptonic ones that are flat (to within statistical error). Once this flat ratio is achieved, a correction factor to model the invisible Z^0 decay using data from the visible decays can be constructed by correcting for each source of discrepancy between the channels, and this correction factor may be used to model the invisible Z^0 decay channel. As noted in section **??**, the fact that electrons and muons radiate photons, have a first-order contribution to their amplitudes from a virtual γ^* which interferes with the Z^0 contribution, and are subject to phase space cuts and overlap removal schemes causes the phase space of these channels to differ from that of the neutrino Z^0 decay channel. Therefore, it is a priori expected that one needs to suppress QED radiation and the γ^* interference, as well as avoid placing any phase space cuts or jet overlap removal, in the visible decay pathways. Producing a ratio without any cuts using the ntuples that had both QED radiation and the γ^* contribution turned off for both channels produce exceptionally good agreement with this hypothesis: As shown below, for both the $\mu^+\mu^-$ and e^+e^- the discrepancies between the visible and invisible channels are on the order of fractions of a percent. Additionally, the ratio in any given bin is never much more than 1 standard deviation from 1, and deviations from 1 do not appear systematic, but rather random in nature (as we would expect from statistical fluctuations). It is safe to say, then, that any systematic discrepancy between the visible and invisible decays of the Z^0, as well as any sensitivity of this ratio to lepton species, that may be observed later in the analysis is the result of physical effects from QED radiation and γ^* interference, or phase space restrictions imposed on the visible channels.

Figure 1: $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$, made flat with suppression of QED, γ^*, and phase space cuts

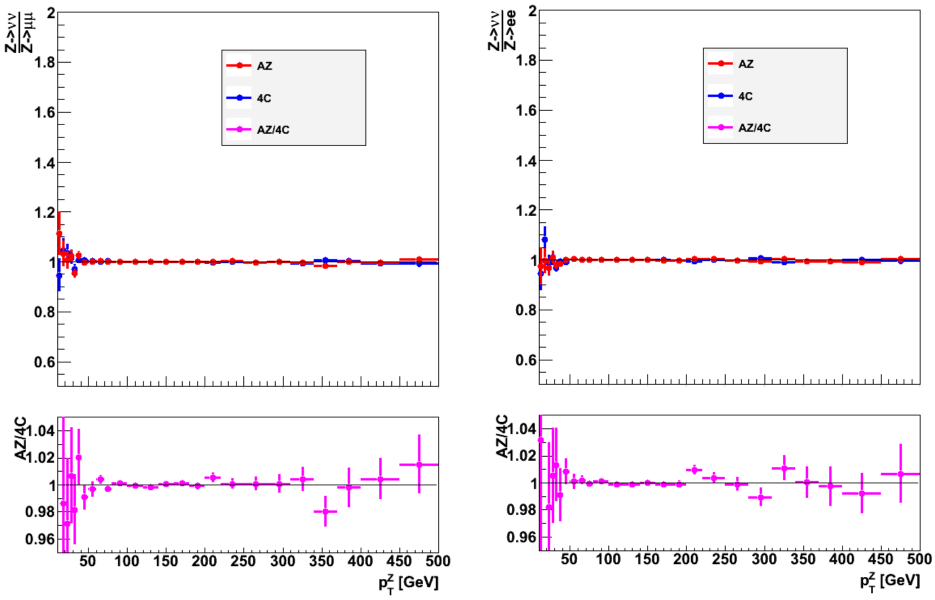
(a) Ratio $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ (b) Ratio $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$



It is also notable that these plots are the same regardless of which parton shower tune is used; both the 4C and AZ tunes offer virtually identical results here, indicating that this ratio is conveniently insensitive to QCD parameters here. To demonstrate this, below are pictured the ratios of ratios for the AZ tune to those of the 4C tune with QED radiation, γ^*, and phase space cuts suppressed, for both lepton species:

Figure 2: Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios for the AZ tune to the 4C tune, with suppression of QED, γ^*, and phase space cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios $(\frac{AZ}{4C})$ (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios $(\frac{AZ}{4C})$

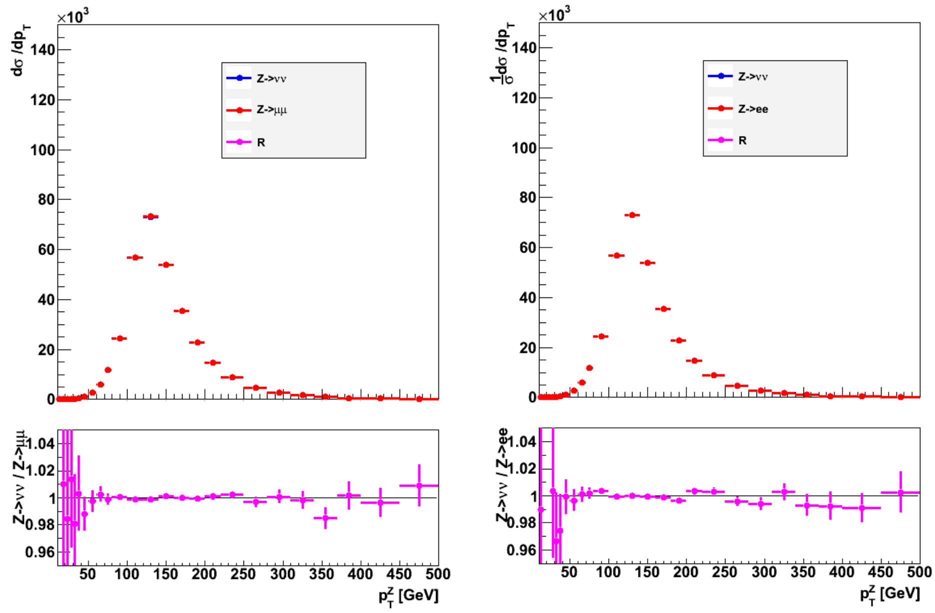


As anticipated, the results for these ratios of ratios are consistent with complete insensitivity to tune: Using a chi-squared fit to a degree-0 polynomial, the muon ratio has a best-fit flat value of 0.99999 +/- 0.000447249 (with a χ^2 value of 23.8075 and a probability value of 85.13%), and the electron ratio has a best-fit flat value of 0.999988 +/- 0.000447249 (with a χ^2 value of 25.3398 and a probability value of 79.20%).

In order to confirm the fact that this ratio is, in fact, completely insensitive to QCD effects, it is also necessary to determine if the ratios stay flat when jet cuts are applied; it is possible that the ratios are insensitive to tune when all jets are included in the analysis, but when the same jet requirement is applied to different tunes, a previously invisible sensitivity could be revealed. Below are the same ratio plots depicted previously, but with a p_T cut of 110 GeV/c applied to the leading jet for both the visible and invisible decay channels. Here again, however, the ratios remain flat to within statistics; past the jet cut threshold, the deviation from a value of 1 remains on the order of 0.5%, consistent with the plot uncertainties, and each bin remains not more than just above one standard deviation from 1. In more quantitative terms, a χ^2 fit of these ratios to degree-0 polynomials yield results incredibly consistent with a flat ratio at 1: For the muon channel, the best-fit value for the ratio is 0.999586 +/- 0.000545276, with a χ^2 value of 15.8091 and a probability of 99.26%. For the electron channel, this fit yields a ratio value of 0.9997 +/- 0.000545276, with a χ^2 value of 24.1331 and a probability of 83.96%. It is apparent, then, that the ratio is insensitive to jet cuts when QED radiation, the γ^* contribution, and fiducial cuts on the visible Z^0 decay products are ignored. It can then be anticipated that the ratios stay flat for various QCD parton shower tunes.

Figure 3: $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$, made flat with no QED, γ^*, or fiducial cuts, but with inclusive jet cuts

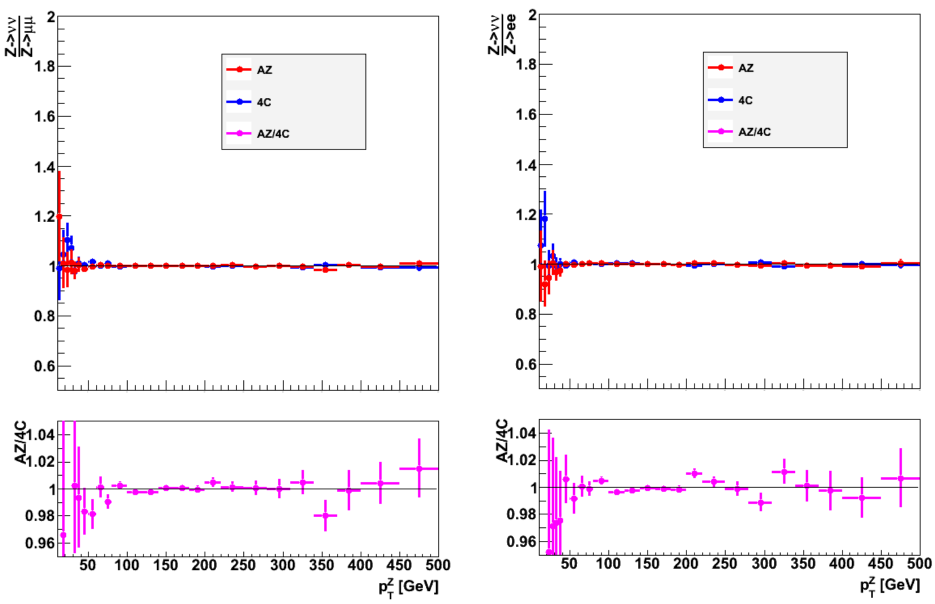
(a) Ratio $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ (b) Ratio $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$



Below are pictured the ratios of ratios comparing the AZ tune to the 4C tune again, but this time with the same cut on the jet p_T that was implemented in the above histograms:

Figure 4: Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios for the AZ tune to the 4C tune, with suppression of QED and γ^*, and inclusive jet cuts implemented

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios $\left(\frac{AZ}{4C}\right)$ (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios $\left(\frac{AZ}{4C}\right)$

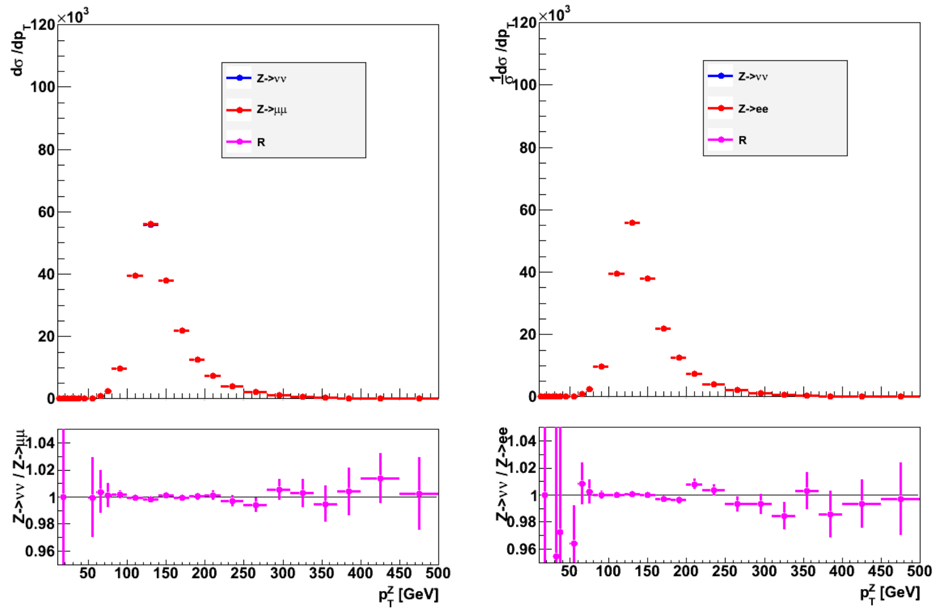


As is readily apparent, the implementation of inclusive jet cuts does not increase the tune sensitivity of this ratio; thus far, the complete insensitivity of this ratio to hadronic recoil is maintained.

After checking the insensitivity of the ratio to jet p_T cuts, it is also useful to check that a requirement of jet exclusivity (that is, apart from the leading jet, an event must have no other jets with $p_T > 30$ GeV/c) also maintains a flat ratio. Requiring an exclusive jet cut suppresses events with soft jets, for which parton shower tuning parameters are most important, so inspecting this ratio under both an inclusive and exclusive cutting scheme is useful in observing the possible effect of differing parton shower parameters on the ratios. Histograms for both the e^+e^- and $\mu^+\mu^-$ ratios are shown, for both the AZ and 4C tunes. Again, the ratios remain flat to within statistics:

Figure 5: $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$, made flat with no QED, γ^*, or fiducial cuts, but with exclusive jet cuts

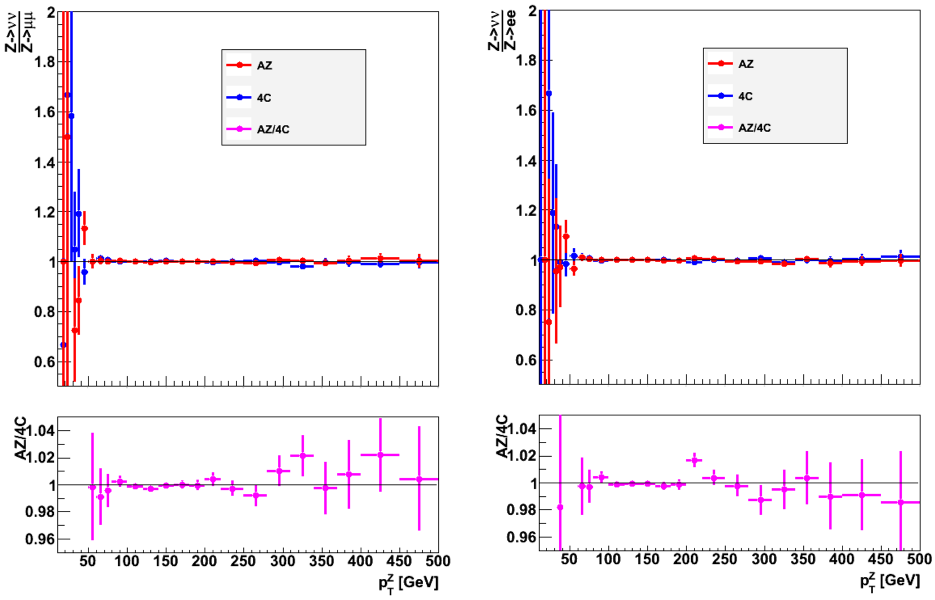
(a) Ratio $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ (No Cuts) (b) Ratio $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ (No Cuts)



To finally confirm the insensitivity of this ratio to any QCD effects, it is necessary to check that tune insensitivity remains intact even when an exclusive jet cut is applied. Below, the ratio of ratios comparing the AZ tune to the 4C tune is depicted, this time with exclusive jet cuts:

Figure 6: Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios for the AZ tune to the 4C tune, with suppression of QED and γ^*, and with exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios ($\frac{AZ}{4C}$)   (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios ($\frac{AZ}{4C}$)



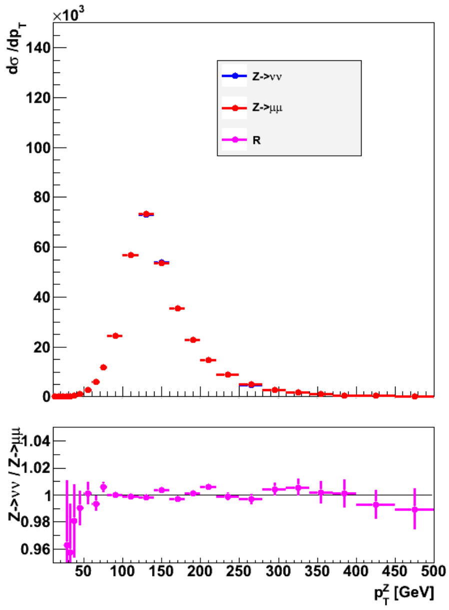
Within statistics, these ratios remain flat. For the muon channel, the ratio has a fit value of 0.999053 +/- 0.000990208, with a χ^2 value of 23.8109 and a probability of 81.82%. For the electron channel, the fit value is 0.999689 +/- 0.000990795 with a χ^2 of 25.019 and a probability of 72.41%. To summarize, then, it can be safely assessed that, in the absence of the phase space cuts, QED radiation, and a virtual γ^* to the amplitude, the ratio of the visible leptonic Z^0 channels to the invisible Z^0 is highly insensitive to any jet selection cuts, exclusive or inclusive, that might be implemented, as well as changes in parton shower tune; as a result, it can be safely determined that this ratio is highly insensitive to any QCD effects that might appear.

4.2 QED Radiation

Having confirmed that the ratios of visible to invisible decays are flat in the absence of QED radiation, virtual γ^* interference, and phase space cuts on the leptons, it is now necessary to introduce the factors which cause the ratio to deviate from flatness, starting with the re-introduction of QED radiation. In principle, electromagnetic radiation affects the results for the e^+e^- and $\mu^+\mu^-$ channels in two different ways: First, when the leptons radiate, part of their momentum is radiated away, so a bare-level reconstruction of the Z^0 will tend to have lower p_T than it would when QED radiation is suppressed. Second, the radiated photons may be clustered into the jets, which may increase the p_T of a leading jet which would not have passed the jet p_T cut without radiation or contribute to or even seed a secondary jet, causing an event to fail exclusivity requirements. In order to confirm the initial hypothesis that these effects are the only areas where QED radiation alters the ratio, it is useful to first suppress these effects in the analysis of the ntuples with QED radiation enabled by taking born-level leptons (using the Pythia event record, locating the born-level hard products is trivial), and omitting photons radiated from the hard products from jet clustering (using the "born-level" jet collection). Below, histograms for the AZ tune using born leptons and born jets are pictured, with both inclusive and exclusive jet cuts:

Figure 7: $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$, made flat with no γ^* and born-level Z^0 reconstruction and jet collections (inclusive jet cuts)

(a) Ratio $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$



(b) Ratio $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$

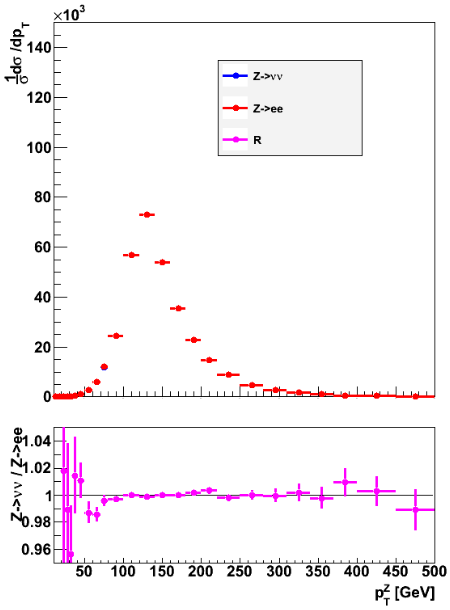
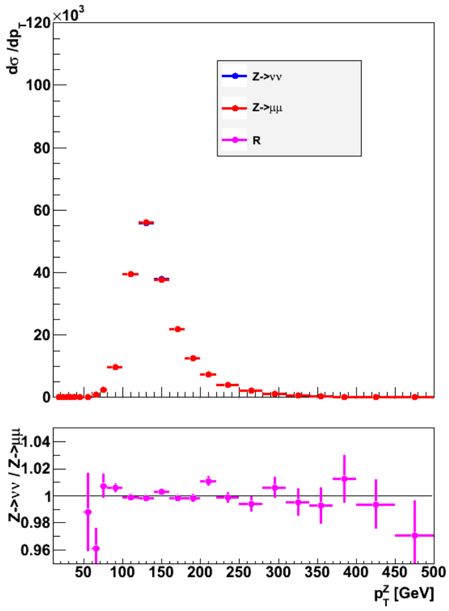
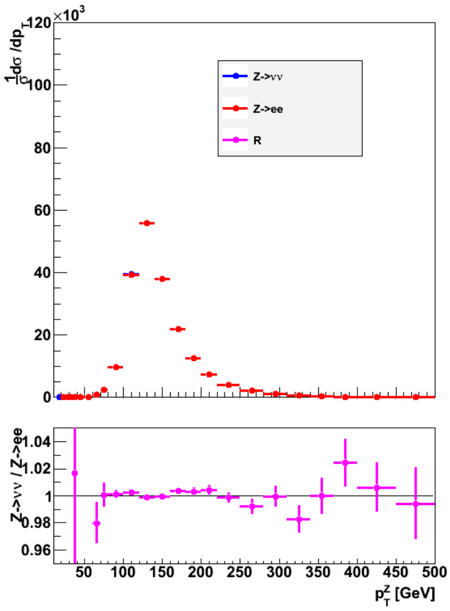


Figure 8: $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$, made flat with no γ^* and born-level Z^0 reconstruction and jet collections (exclusive jet cuts)

(a) Ratio $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$



(b) Ratio $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$



These histograms are flat (within statistical variation), just like the histograms created when using ntuples where QED radiation is suppressed completely. This is expected because born level leptons and jets should, in principle, completely reconstruct the Z^0 decay products and jets in the absence of any QED radiation. As a result, it is reasonable to assume that QED radiation's effects on the ratio can be quantified by observing its effects on Z^0 reconstruction and jet clustering separately, and then considering the combined effect.

While QCD sensitivity was examined for the case where QED radiation was suppressed entirely, it is useful to determine if any additional QCD sensitivity was introduced to the ratio when QED radiation is enabled again(even though, in principle, born-level clustering and Z^0 reconstruction ought to remove any effects that QED radiation might have). Below, the ratio histograms for born-level Z^0 reconstruction and jet clustering with QED enabled for both lepton species and both exclusive and inclusive jet cuts are pictured:

Figure 9: Ratio of born-level $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios for the AZ tune to the 4C tune, with QED enabled but no γ^* effect, and with inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios ($\frac{AZ}{4C}$) (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios ($\frac{AZ}{4C}$)

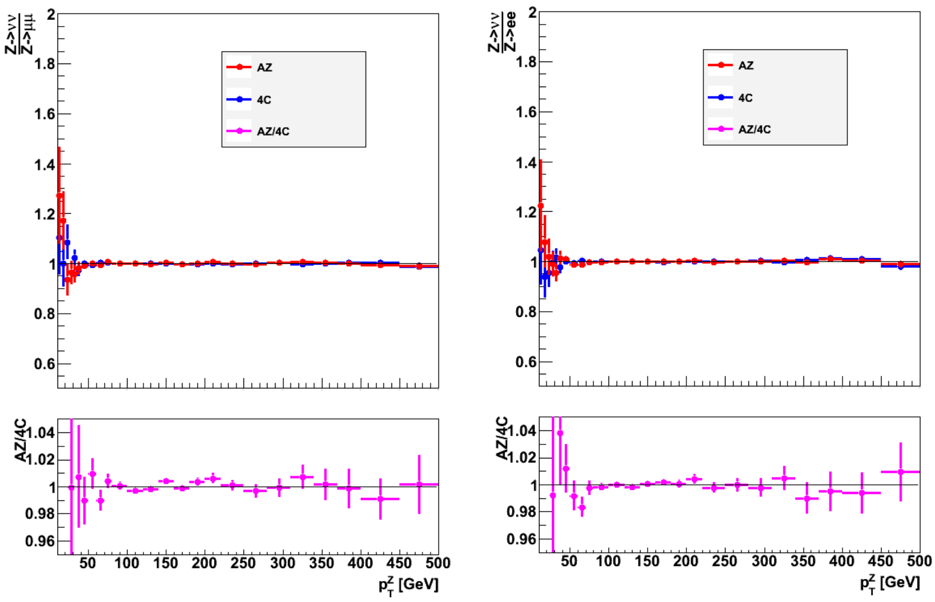
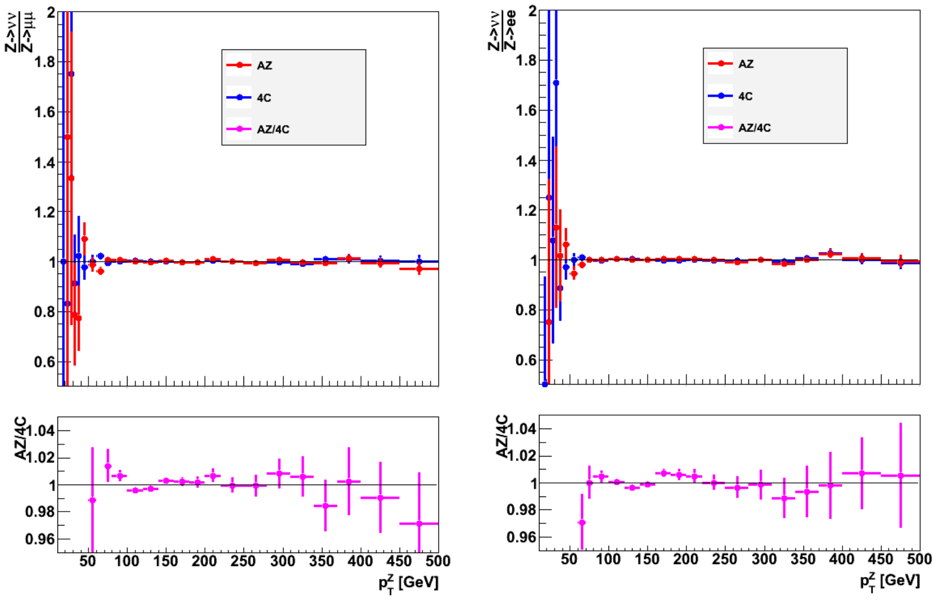


Figure 10: Ratio of born-level $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios for the AZ tune to the 4C tune, with QED enabled but no γ^* effect, and with exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios $(\frac{AZ}{4C})$ (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios $(\frac{AZ}{4C})$



As anticipated, the tune ratios here are quite similar to the tune ratios when QED radiation is entirely suppressed, with significant deviations from 1 within statistics. For the muon ratios, the ratio with inclusive jet cuts has a best-fit value of 1.00013 +/- 0.000771679, a χ^2 of 29.957, and a probability of 57.03%, while the ratio for exclusive jet cuts has a best-fit value of 0.999748 +/- 0.000991047, a χ^2 of 38.8019, and a probability of 15.83%. For the electron ratios, the ratio with inclusive jet cuts has a best-fit value of 0.999521 +/- 0.000771178, a χ^2 of 32.7648, and a probability of 42.93%, while the ratio for exclusive jet cuts has a best-fit value of 1.00015 +/- 0.000991527, a χ^2 of 30.9406, and a probability of 41.83%.

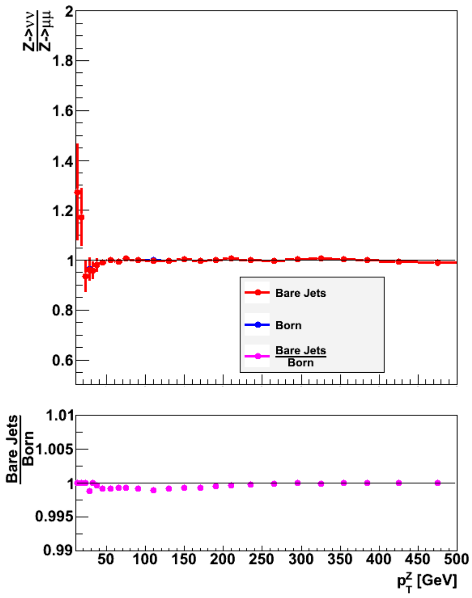
4.2.1 Hadronic Recoil (Inclusive)

In order to determine the ratios' sensitivity to QED effects, the sensitivity of each the two regions that this radiation can impact, namely hadronic recoil and Z^0 reconstruction, should be examined independently. First, the effect of electromagnetic radiation on hadronic recoil is explored, using the ratio of decay channels produced when the jet clustering includes photons radiated from the decay product, but the sensitivity of the Z^0 reconstruction to QED effects is suppressed by using born-level leptons.[5]. In order to inspect these effects without the interference of statistical fluctuations, it is useful to employ a ratio of ratios, in this case comparing the $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratio for born-level Z^0 reconstruction with bare-level jets (that is, only excluding the bare hard products from jet clustering, but including any radiated photons) to the ratio produced using born-level ratios and jets. Below is this ratio of ratios with a cut of 110 GeV/c on the p_T of the leading jet, but no exclusivity cuts:

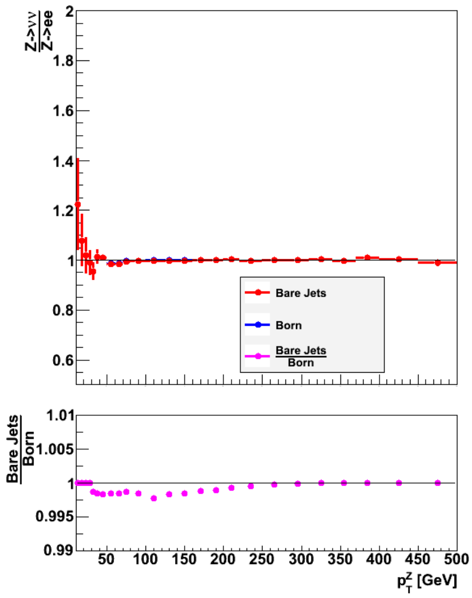
[5] It should be noted that this ratio is not physical, because the photons radiated by Z^0 decay products are double counted (they are added to both the Z^0 reconstruction and hadronic recoil). However, in order to investigate the effect of radiated photons on the jet clustering alone, this artificial case is helpful

Figure 11: Ratio of ratios for the bare-level jet collection to the born-level jet collection, with inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios



As is readily apparent from both histograms, QED radiation does have an effect on this ratio; the ratio produced with bare-level jets will keep slightly more l^+l^- events than the ratio produced with born-level jets, as one would anticipate if radiated photons were contributing to the p_T of the leading jet. In addition, the degree of the excess is clearly correlated to the momentum of the reconstructed Z^0 (which, as was observed with the diminishing statistics of $Z^0_{p_T}$ histograms in the low-p_T region when jet p_T cuts were first applied, is correlated with lower jet p_T), with bins that have $Z^0)p_T$ close to the 110 GeV/c jet p_T threshold being the most strongly affected. Again, this lines up with theoretical expectations: The radiated photons being clustered into a leading jet will be most likely to put a jet that is close to beating the cut over the threshold, while events with very high-p_T jets (and likely high $Z^0_{p_T}$) will pass with or without photon radiation and events with very low-p_T jets (and so likely low $Z^0_{p_T}$) will likely not gain enough momentum from radiated photons to pass the jet p_T cut. In spite of these considerations, however, it is very important to note that the effect of radiation on these ratios is minute (on the order of 0.1% for both electrons and muons), and as a result, any sensitivity to this effect will be overwhelmed by uncertainties in a real experiment. In conclusion, then, for all intents and purposes, the effect of QED radiation on the leading jet p_T can be safely ignored in this analysis.

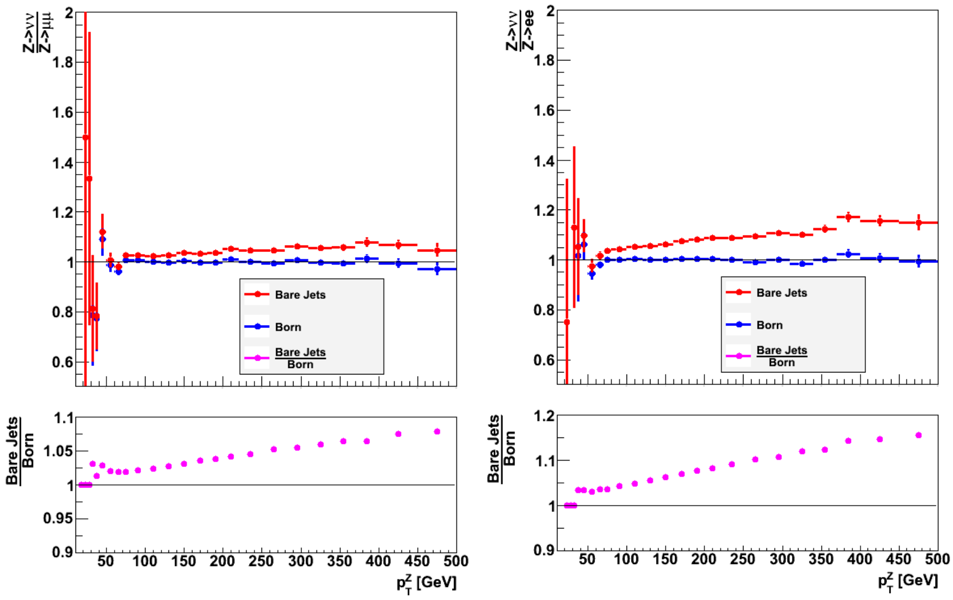
4.2.2 Hadronic Recoil (Exclusive)

In spite of the lack of sensitivity of the ratio $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+l^-}$ to QED radiation when an inclusive jet cutting scheme is applied, it is necessary to also observe the effects of radiation on the exclusive scheme, where events with more than one jet of $p_T > 30$ GeV/c are cut from the analysis. In principle, this effect may be much more significant than the impact of radiated photons on the leading jet: Since secondary jets are likely far less energetic than the leading jet, it is more likely that a radiated photon would be able to significantly add to or even seed such a jet. In addition, unlike the leading jet (which, due to momentum conservation, is highly unlikely to be collinear with the Z^0, and hence unlikely to be collinear with its decay products), secondary jets may be more inclined to be close enough to a Z^0 decay product to have that decay product's radiated photons clustered into the jet. Finally, with a boosted Z^0, the decay products are more energetic and therefore have a higher probability of radiating a hard photon. With these ideas in mind, the effect of QED radiation on exclusive hadronic recoil schemes can now be observed. Below are pictured the ratios of ratios with bare-level jets and born-level Z^0 reconstruction to the ratios with born-level jets and Z^0 reconstruction with exclusive jet selection:

Figure 12: Ratio of ratios for the bare-level jet collection to the born-level jet collection, with exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios

(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios



There are several important characteristics of these plots to note: First, the effect here nicely agrees with theoretical intuition: The implementation of radiation effects on the jet clustering produces an excess in the $p_T^{Z^0}$ ratio, corresponding to a deficit in the number of l^+l^- events in each bin. This deficit appears intuitively consistent with the intuition discussed above, namely, that radiated photons are seeding secondary jets or pushing existing secondary jets over the 30 GeV/c p_T threshold required to cut an event from analysis. Further strengthening the case for theoretical intuition proving correct, higher-$p_T^{Z^0}$ bins require higher correction factors, which, since high $p_T^{Z^0}$ is correlated with high p_T for its decay products, corresponds with higher-$p_T^{Z^0}$ events radiating more energetic photons, which are more likely to seed or push secondary jets over the p_T threshold required to cut the event.

Second, there is a clear sensitivity in this ratio to lepton species; the less massive electrons tend to radiate more, resulting in the e^+e^- ratio to have a correction due to radiation as high as 20% at $p_T^{Z^0}$ near 500 GeV/c, which is nearly double sensitivity of the $\mu^+\mu^-$ in the same bin. In fact, for each bin here, the sensitivity of the electron channel to these effects is approximately double that of the muon channel.

Finally (and most importantly), it is readily apparent that an exclusive cutting scheme is far more sensitive to electromagnetic effects than its inclusive counterpart; in both the $\mu^+\mu^-$ and e^+e^- channels, the effect here exceeds the effect on the inclusive scheme by more than an order of magnitude. While the effect of the inclusive scheme is so small that it can be successfully ignored, the exclusive effect can reach the order of 5% or even 10%, which is well within the regime of a potentially measurable effect. Of course, the case which is currently being considered is highly artificial, without the effects of radiation on Z^0 reconstruction, phase space cuts on the visible decay products, and the virtual γ^* contribution, all of which could cancel or add to the hadronic recoil's sensitivity to QED, so actually considering measurable quantities must be deferred to a later point in the analysis.

4.2.3 Z^0 Reconstruction

As mentioned above, the implementation of QED radiation doesn't just affect the hadronic recoil; with QED radiation enabled, the visible Z^0 decay products radiate away energy, resulting in a less energetic (and likely lower p_T) reconstructed Z^0. In order to explore this effect in isolation (without the established sensitivity of the hadronic recoil to radiation interfering), producing ratio histograms using bare-level Z^0 reconstruction, but born-level jets. Below, the ratio of these histograms to the ratio for born-level Z^0 reconstruction and born-level jets are pictured, with inclusive jet cuts applied and exclusive jet cuts applied:

Figure 13: Ratio of ratios for bare-level to born-level Z^0 reconstruction, with inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios

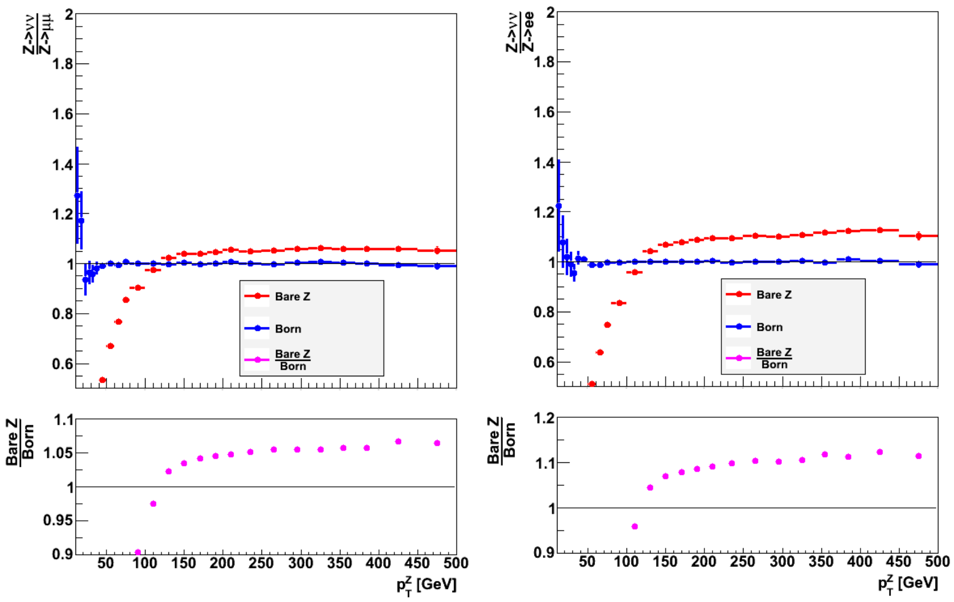
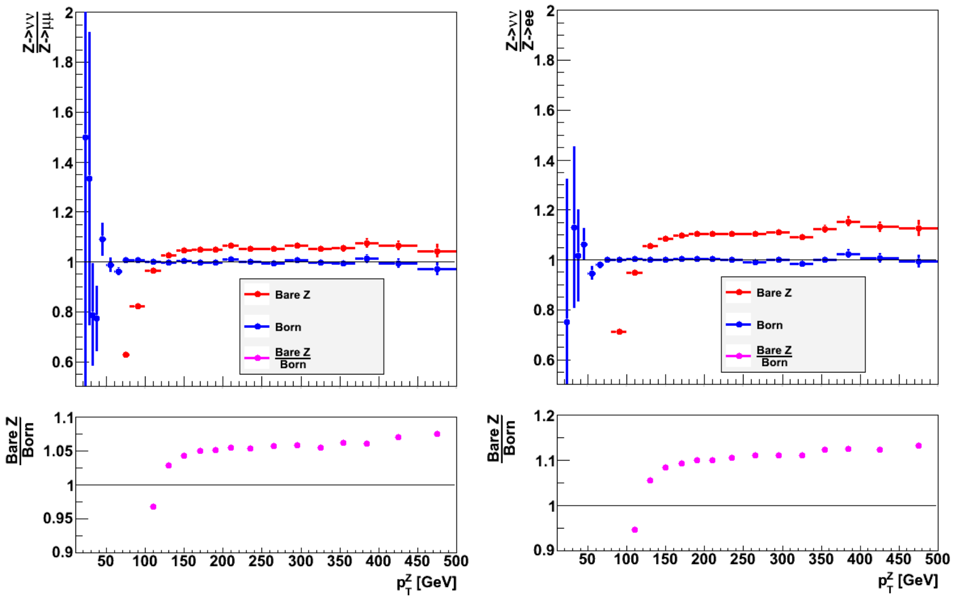


Figure 14: Ratio of ratios for bare-level to born-level Z^0 reconstruction, with exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios



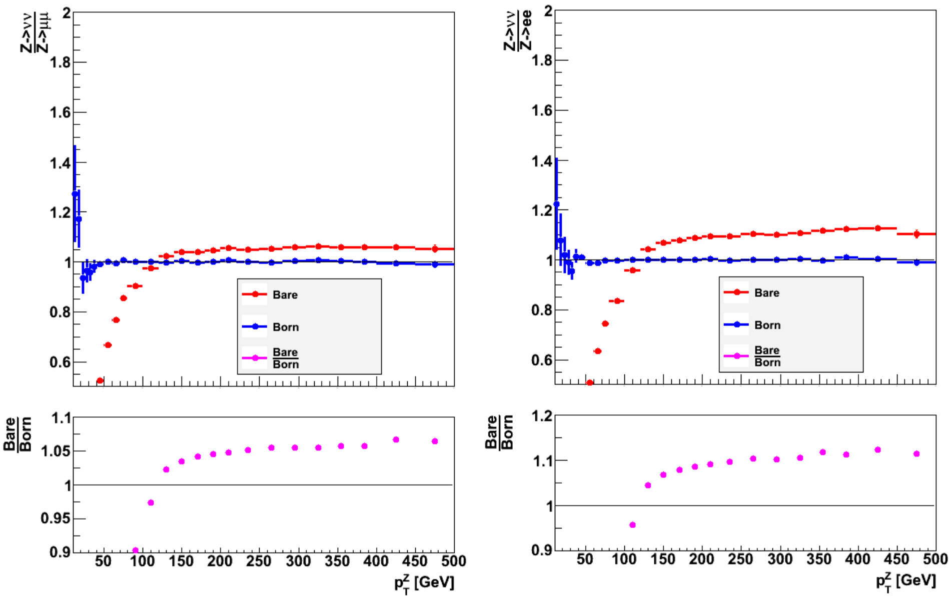
The immediately apparent effect of QED radiation here is that, for both the exclusive and inclusive jet cuts, when bare-level Z^0 reconstruction is used, the $p_T^{Z^0}$ distribution in the l^+l^- is shifted to the left, resulting in a large deficit in the $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+l^-}$ ratio in the low $p_T^{Z^0}$ region (when compared to the born-level reconstruction channel), along with an excess in this ratio when $p_T^{Z^0}$ is higher.This result agrees nicely with theoretical intuition: The born-level decay products for events with QED radiation will tend to have higher p_T than their bare-level counterparts, which in turn will correlate to lower $p_T^{Z^0}$ when bare-level reconstruction is used. In practice, this produces a rising correction factor as $p_T^{Z^0}$ increases, until this factor plateaus at a particular value once $p_T^{Z^0}$ is far enough away from the leading jet p_T cut to ignore the turn-on effect that this cut causes. It is also useful to note, again, that the effects of radiation are more pronounced in the e^+e^- channel than in the $\mu^+\mu^-$ channel, which is again a consequence of the differing masses (and hence differing propensities to radiate) of the two lepton species. Just as in the case with the hadronic recoil sensitivity, the deviation from flatness produced in the electron channel is approximately twice the deviation produced in the muon channel (up to about a 6% deviation occurs in the muon channel, as opposed to the close to 12% deviation found in the electron channel).

4.2.4 Combined QED Sensitivity

While observing the individual sensitivity of the hadronic recoil and the Z^0 reconstruction to electromagnetic radiation effects is instructive, but in both cases, isolating these effects produces an artificial scenario that cannot be duplicated in experiment. In order to produce a more realistic scenario, it is necessary to consider the two effects together. Below are the ratios of ratios comparing bare-level Z^0 reconstruction and jet clustering to born-level reconstruction and clustering, with inclusive jet cuts applied:

Figure 15: Ratio of ratios for bare-level jet clustering and Z^0 reconstruction to born-level, with inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios

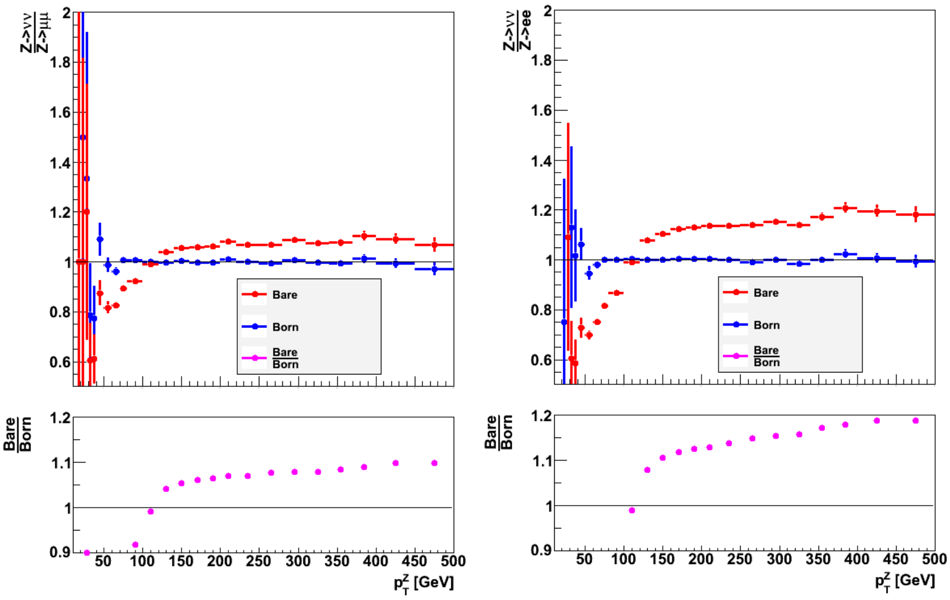


As intuition would anticipate, the full QED sensitivity for the inclusive case is nearly identical to the sensitivity of just the Z^0 reconstruction (which is fully two orders of magnitude greater than the sensitivity of the hadronic recoil); in fact, the difference between the full QED sensitivity and the sensitivity of just the Z^0 reconsruction is so small that it is negligible for practical purposes (on the order of approximately 0.01%).

Unlike the inclusive case, the sensitivity of the hadronic recoil to QED under exclusive jet cuts is of comparable magnitude to the sensitivity of Z^0 reconstruction in this regime. As a result, the total sensitivity of the $p_T^{Z^0}$ ratio to QED in this cutting scheme should differ significantly from the QED sensitivity of hadronic recoil or Z^0 reconstruction alone. Below, these combined sensitivities are pictured:

Figure 16: Ratio of ratios for bare-level jet clustering and Z^0 reconstruction to born-level, with exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios



Here, the sensitivities of the hadronic recoil and Z^0 reconstruction combine, producing larger correction factors that are approximately equal to the sum of the correction factors of both elements individually. As a result, the $\mu^+\mu^-$ channel yields a ratio that has as much as a 10% deviation from 1 when QED is implemented, while the e^+e^- channel yields a ratio that deviates as much as 20%, again demonstrating the rule of thumb that the e^+e^- channel produces ratios that have roughly twice the QED sensitivity of the ratios produced using the $\mu^+\mu^-$ channel.

It is useful, now, to compare this sensitivity to the QCD sensitivity of the ratios; if QCD sensitivities truly cancel, then the effect on the ratio of changing the tune should be far smaller than the effect of introducing QED radiation. The results for tune sensitivity when born-level Z^0 reconstruction and jet clustering are used suggest that this intuitive prediction holds; in these ratios, the fluctuations of tune ratios from flatness are nearly always within one standard deviation from 1, and appear to have no systematic biases. However, the introduction of radiated photons in the jet clustering could have unforeseen effects on the muon and electron Z^0 decay channels, as a result, it is advisable to check the ratios of tunes when QED radiation is fully implemented. Below, the ratios of ratios of the AZ tune to the 4C tune are implemented for both lepton species, and for both inclusive and exclusive jet cutting schemes:

Figure 17: Ratio of bare-level $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios for the AZ tune to the 4C tune, with QED enabled but no γ^* effect, and with inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios $(\frac{AZ}{4C})$ (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios $(\frac{AZ}{4C})$

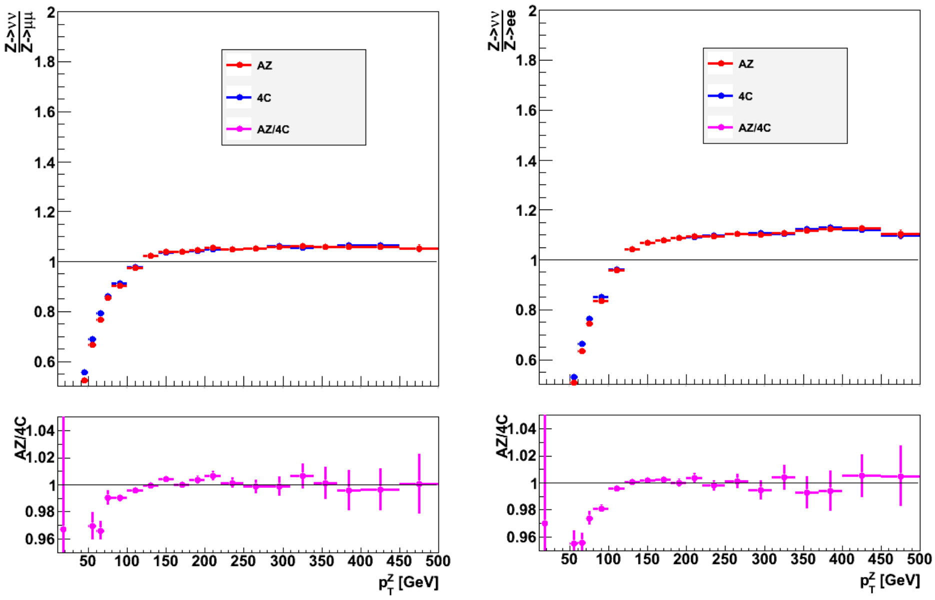
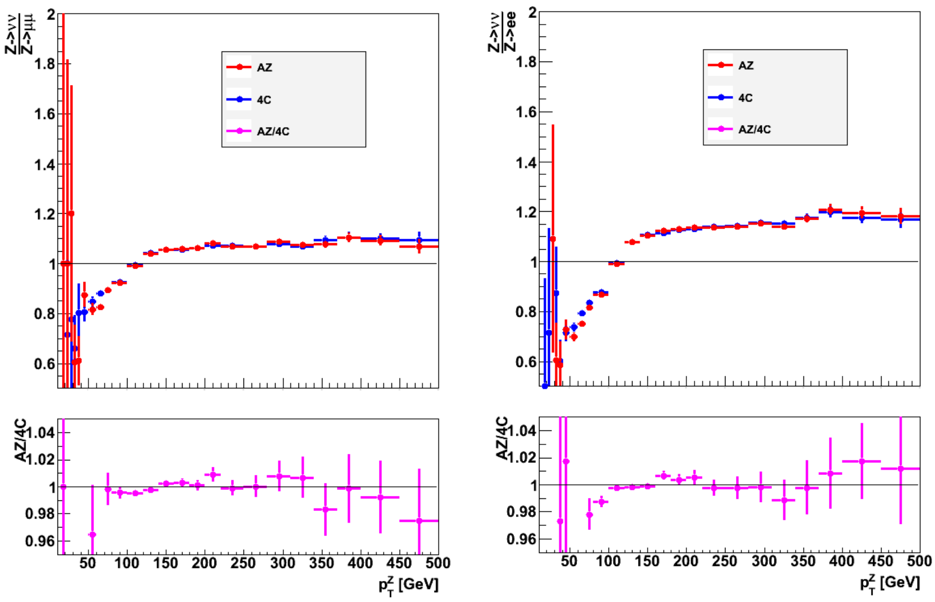


Figure 18: Ratio of bare-level $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios for the AZ tune to the 4C tune, with QED enabled but no γ^* effect, and with exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios $(\frac{AZ}{4C})$ (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios $(\frac{AZ}{4C})$



Here, there is some apparent sensitivity to tune, but only in the regions of exceptionally low $p_T^{Z^0}$; the ratio of tunes only deviates from 1 by more than 2 standard deviations in bins where $p_T^{Z^0}$ is below the jet p_T cut of 110 GeV/c.[6] Requiring a jet cut above the

[6]This effect is much more significant with inclusive jet cuts rather than exclusive ones. As of now,

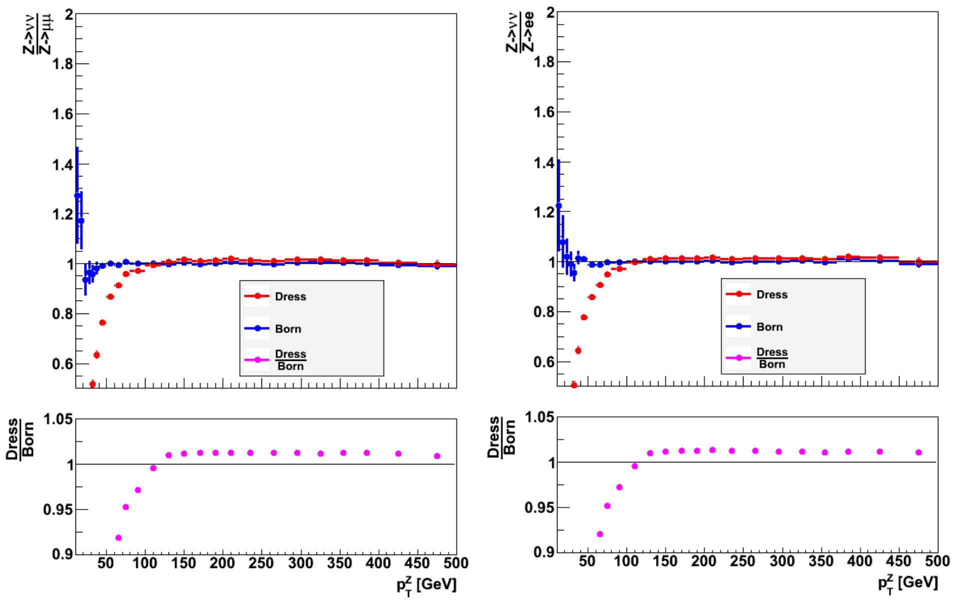
E_T^{miss} trigger threshold (for the invisible Z^0 events) would therefore increase the tune effect, while putting the jet cut below the E_T^{miss} would cause the ratio to become insensitive to QCD effects. It is important to note, however, that the qCD effects here are an order of magnitude smaller than the QED sensitivity, and hence the latter sensitivity dominates the measurement. If experimental uncertainties were sufficiently small, however, it is conceivable that the QCD turn-on effect depicted above could in fact be measured, since it is on the order of a few percent. This result not only agrees with intuition, that the QCD sensitivities of the individual channels would cancel in a ratio, but also bodes well for later use of these ratios to model invisible Z^0 decays with the electron and muon channels; because these ratios are highly insensitive to QCD, theoretical uncertainties in QCD parameters will cause minimal uncertainties in the final prediction this method produces.

4.2.5 Dressing Leptons

Apart from simply considering born or bare leptons when testing the effects of QED radiation on these ratios, it is also useful to consider dressed leptons, which are defined in this analysis as the sum of the bare lepton momentum with all photons within a cone of $\Delta R < 0.2$ around it. In practice, this approximates the detector-level electron (while at the detector level, muons are bare), and should mitigate the effects of QED radiation on both ratios without introducing the magnitude of theoretical uncertainty present in reconstructing the born-level leptons from data. As a result, a complete analysis of these ratios' sensitivity to QED radiation requires an examination of the dressed effect as well. Below, the ratios of the ratios of dressed leptons and jets to the ratios of born leptons and jets for both exclusive and inclusive jet cuts are pictured:

Figure 19: Ratio of ratios for dress-level jet clustering and Z^0 reconstruction to born-level, with inclusive jet cuts

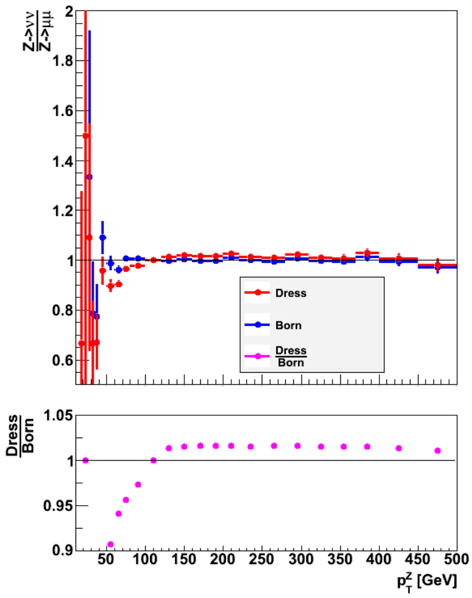
(a) Ratio of $\frac{Z^0\to\nu\nu}{Z^0\to\mu^+\mu^-}$ ratios   (b) Ratio of $\frac{Z^0\to\nu\nu}{Z^0\to e^+e^-}$ ratios



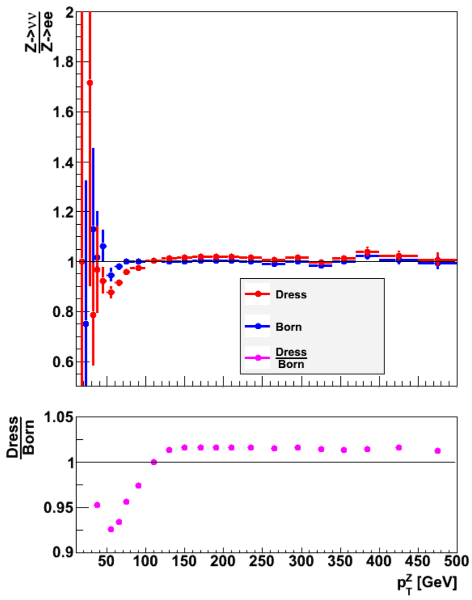
there does not appear to be a satisfactory intuitive explanation for this phenomenon

Figure 20: Ratio of ratios for dress-level jet clustering and Z^0 reconstruction to born-level, with exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios



Dressing the leptons has two major effects on the QED sensitivity of the ratios. First, the overall sensitivity of the ratios to QED effects is greatly mitigated; in both the e^+e^- and $\mu^+\mu^-$, the deviation from flatness is no greater than 2%. While this does not eliminate the sensitivity of these ratios to QED effects (a 2% deviation may still be measurable in an experiment, and the turn-on effect should be clearly visible as a 5-7.5% effect), it certainly greatly mitigates the sensitivity of both channels to these effects. Additionally, the sensitivities of these ratios to both jet cutting scheme and lepton species become small enough that they are essentially negligible. Because of this effect, if one wishes to render data from the electron and muon channels comparable (for a combined analysis to mitigate statistical uncertainty, for example), it is essential to at least dress the leptons (if not reconstruct the born-level leptons in their entirety) in order to compensate for the differing radiative properties of the two particle species.

Because dressing the lepton seriously mitigates the sensitivity of these ratios to QED effects, the size of this effect begins to approach the estimated limits of QCD sensitivity of the ratio that were determined earlier (namely, on the order of 1%). As such, it is necessary to check the sensitivity of the dressed ratios to QCD effects, and determine if these sensitivities are close enough to the QED sensitivities that they must be accounted for in an analysis. Below are pictured the ratios of ratios for the AZ tune to the 4C tune for both an inclusive and exclusive jet cutting scheme;

Figure 21: Ratio of dress-level $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios for the AZ tune to the 4C tune, with no γ^* effect, and with inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios $(\frac{AZ}{4C})$ (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios $(\frac{AZ}{4C})$

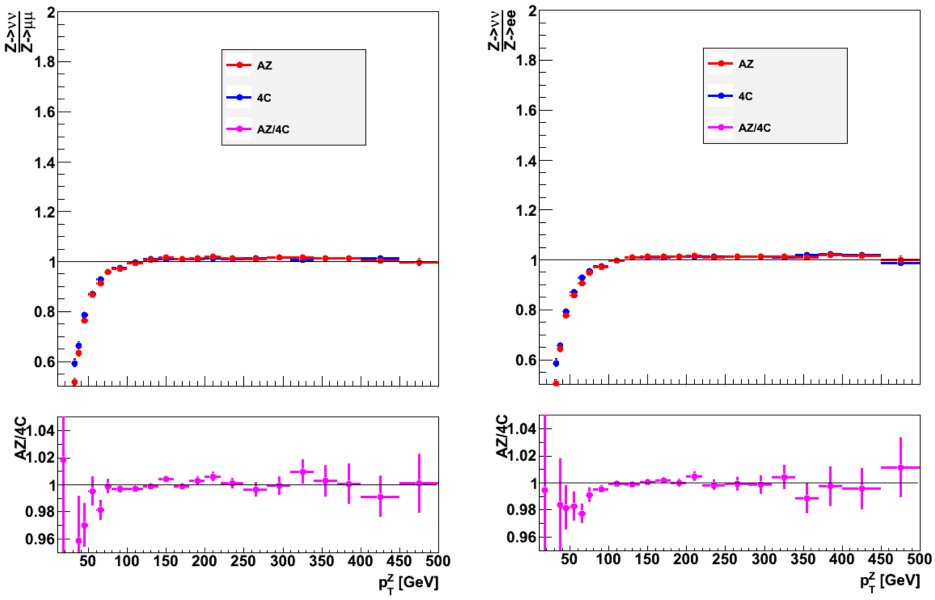
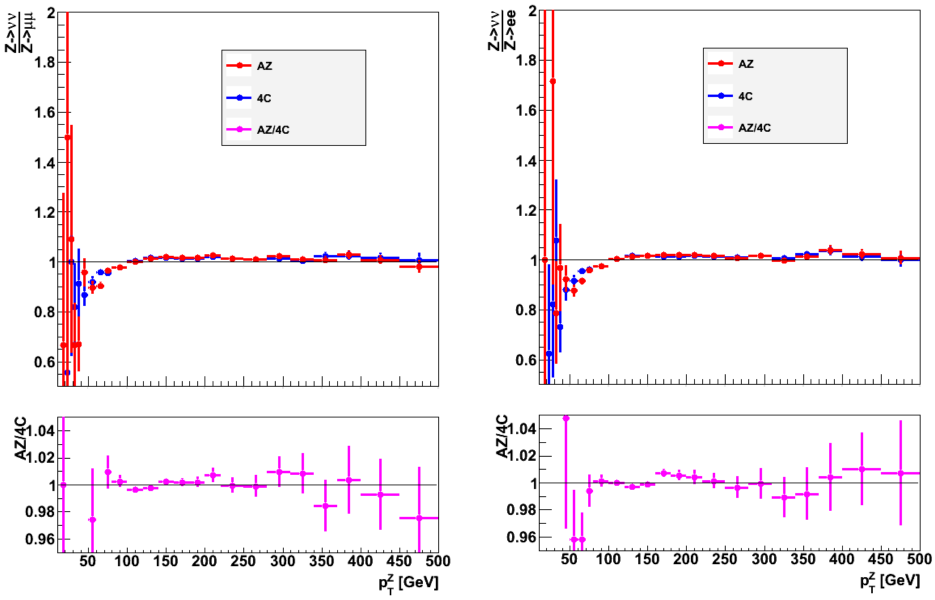


Figure 22: Ratio of dress-level $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios for the AZ tune to the 4C tune, with no γ^* effect, and with exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios $(\frac{AZ}{4C})$ (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios $(\frac{AZ}{4C})$



In these histograms, it becomes clear that the QCD effect, although it can approach close 2%, only does so at points where the statistics of these ratios is already quite poor; for both the $\mu^+\mu^-$ and e^+e^- channels, the sensitivity to tune never approaches the sensitivity to QED until the statistics begin to deteriorate. As a result, it is reasonable to assume that the QCD sensitivity of this ratio is small enough to still have the QED sensitivity consistently exceed it.

4.3 Fiducial Cuts

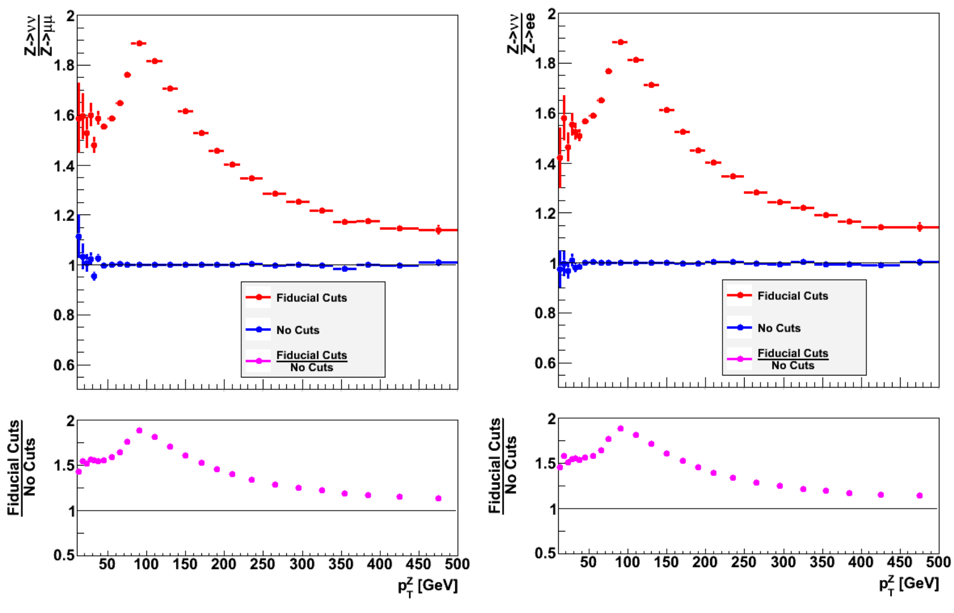
4.3.1 Phase Space Distortion Without QED or Jet Cuts

Thus far, the cases that have been analyzed have all been highly artificial, in that they lacked fiducial cuts on the leptons and realistic accounting for visible Z^0 decay products overlapping the jets. In principle, both these effects represent distortions of the phase space of the visible Z^0 channels without a corresponding distortion in the invisible channels, so the ratio of invisible channels to visible channels should be highly sensitive to both effects. First, fiducial cuts (described in section 3.2.2) are applied. In order to determine the sensitivity of the ratio to this phase space distortion in isolation from other effects, the ratio of ratios with these cuts to the ratios without these cuts are considered, where QED radiation, the γ^*, and any jet cuts are not implemented. These histograms are pictured below for both lepton channels:

Figure 23: Ratio of ratios for fiducial cuts to no fiducial cuts, with no QED, γ^*, or other phase space cuts applied

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios

(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios



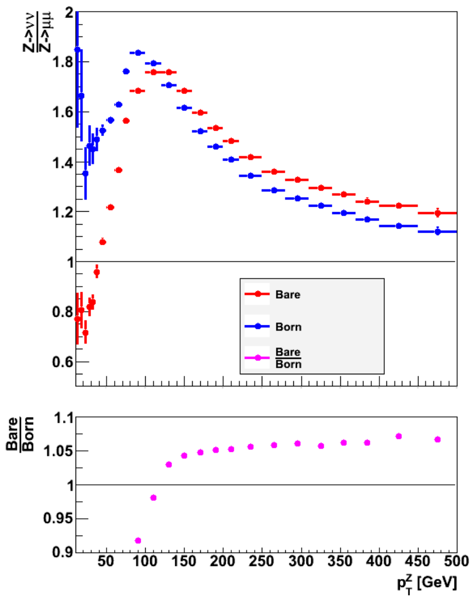
Immediately, it is clear that the effect of fiducial cuts on the lepton is extremely significant, causing as much as a 60% correction to the uncut case, and requiring correction factors on the order of at least 10% in every bin. In order to obtain a flat ratio and produce a correction between visible and invisible Z^0 data, then, this distortion correction must be applied, and the possible effect of this phase space distortion on any previously explored sensitivities must be inspected thoroughly.

4.3.2 Sensitivity to QED Effects

When QED radiation is reintroduced to the system, its effect on the ratios should be re-evaluated: The implementation of these cuts may alter the phase space in such a manner as to change the radiative trends in the visible Z^0 decay pathways. To determine whether or not this occurs, the sensitivity of these ratios to QED effects can be determined in the same manner that was employed in section 4.2: Comparing the born-level ratios with these histograms to the bare-level and dress-level ratios. Below, these ratios for bare-level leptons, with both inclusive and exclusive jet cuts, are pictured:

Figure 24: Ratios of bare $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios to born ratios with no γ^* contribution, but with fiducial cuts and inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios

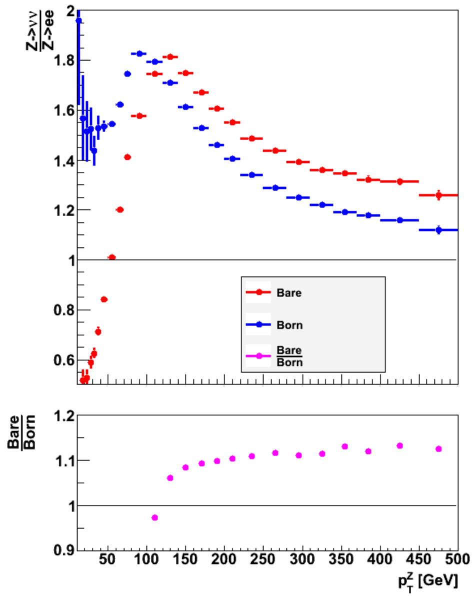
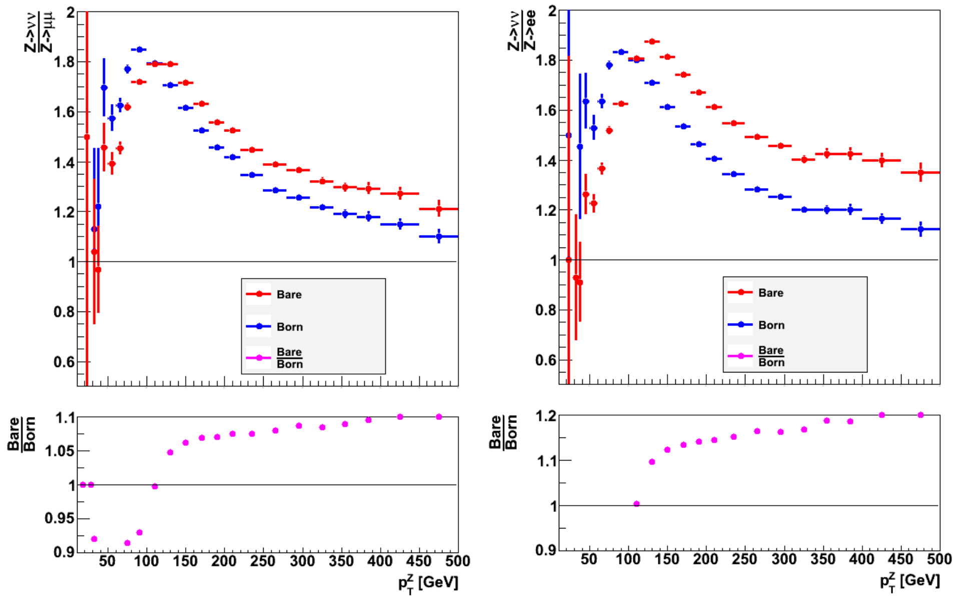


Figure 25: Ratios of bare $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios to born ratios with no γ^* contribution, but with fiducial cuts and exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios

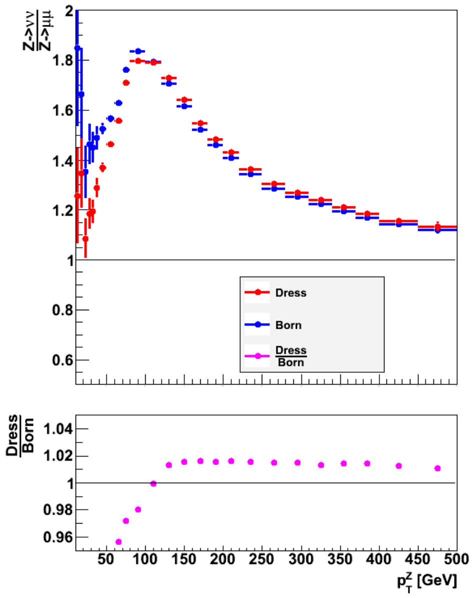


These ratios are practically identical to the ratios comparing bare-level to born-level ratios without fiducial cuts applied; it seems that the QED effects on these ratios are insensitive to phase space cuts on the leptons.

Meanwhile, when the leptons are dressed, the QED sensitivity is greatly diminished, just as in the case without fiducial cuts. Below, the ratios of ratios depicting QED sensitivity with dressed leptons are pictured:

Figure 26: Ratios of dressed $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios to born ratios with no γ^* contribution, but with fiducial cuts and inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios

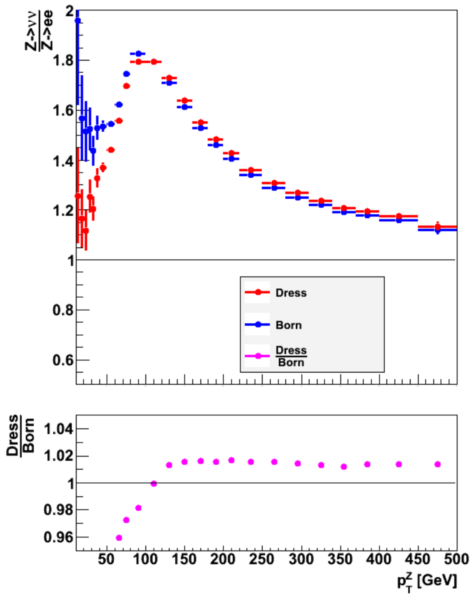
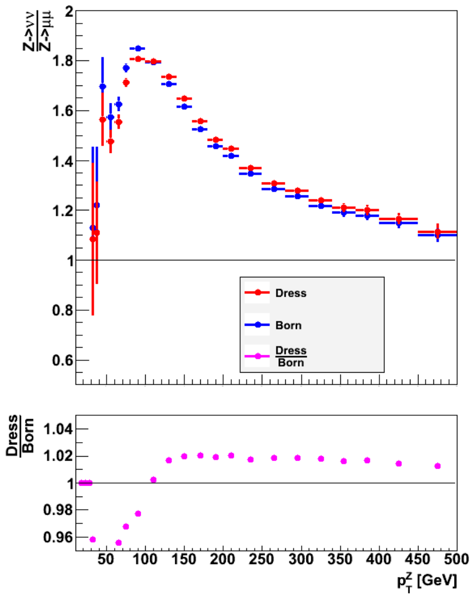
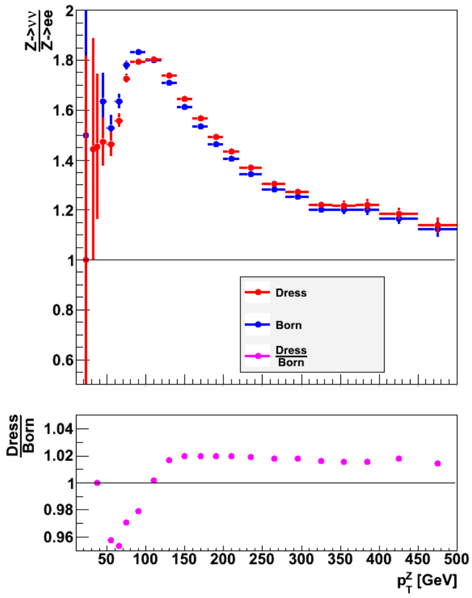


Figure 27: Ratios of dressed $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios to born ratios with no γ^* contribution, but with fiducial cuts and exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios



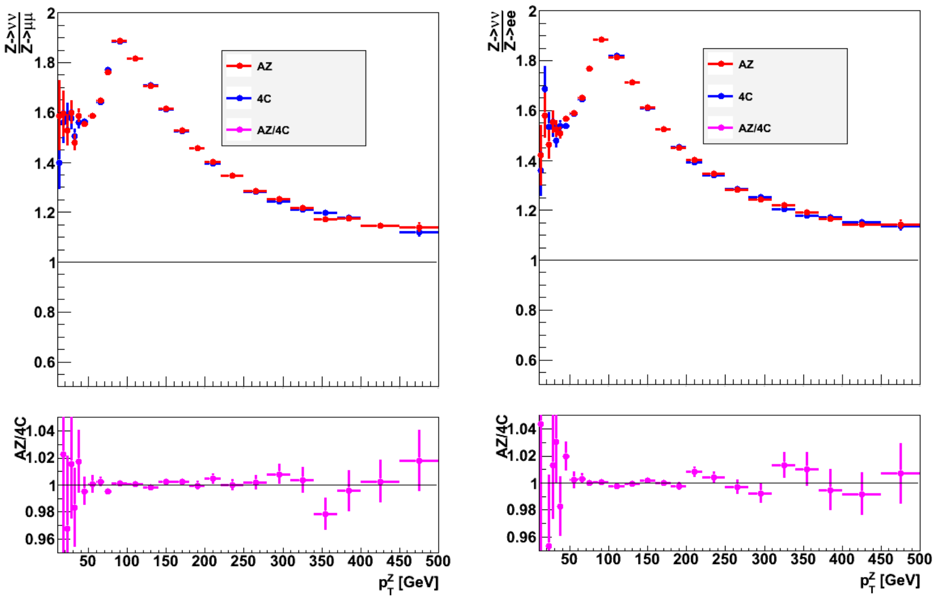
Although a small QED sensitivity is preserved, and, like in the case without fiducial cuts, the dressing process renders the ratio insensitive to lepton species. the remaining QED sensitivity is dangerously close to limits that simulations in the absence of fiducial cuts have placed on the maximum QCD sensitivity of these ratios. As a result, further analysis is needed, the beginnings of which are explored in the following section.

4.3.3 Sensitivity to QCD Effects

Since this phase space cut is applied only to the visible channels in the ratio, and its effects may in turn limit the QCD phase space of these channels, it is possible that the implementation of these fiducial cuts will induce some QCD sensitivity to the ratios. To explore this possibility, the ratio of ratios with fiducial cuts for the AZ and 4C tunes are plotted below, for both lepton species:

Figure 28: Ratio of $\frac{Z^0\to\nu\nu}{Z^0\to l^+l^-}$ ratios for the AZ tune to the 4C tune, with no QED or γ^* effects, and with fiducial cuts

(a) Ratio of $\frac{Z^0\to\nu\nu}{Z^0\to\mu^+\mu^-}$ ratios ($\frac{AZ}{4C}$) (b) Ratio of $\frac{Z^0\to\nu\nu}{Z^0\to e^+e^-}$ ratios ($\frac{AZ}{4C}$)



Despite the restricted phase space, however, the QCD sensitivity of these ratios remains unchanged: For most bins, the sensitivity of these ratios remains in the range of a fraction of 1%, giving a best-fit value for the muon channel of 1.00042 +/- 0.000524216 with a χ^2 of 23.1729 and a probability of 87.28%, while for the electron channel, the best-fit ratio value is 0.999818 +/- 0.000524216 with a χ^2 of 28.3827 and a probability of 65.03%. As a result, it is safe to say that the fiducial cuts on the leptons do not introduce any additional QCD sensitivity to the ratios, and as a result, this ratio remains highly insensitive to these effects.

Now that QCD insensitivity of this ratio in the absence of any jet cuts is verified, exclusive and inclusive jet cuts are reintroduced to this ratio. Below, the results are pictured:

Figure 29: Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+l^-}$ ratios for the AZ tune to the 4C tune, with no QED or γ^* effects, and with fiducial and inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios $(\frac{AZ}{4C})$ (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios $(\frac{AZ}{4C})$

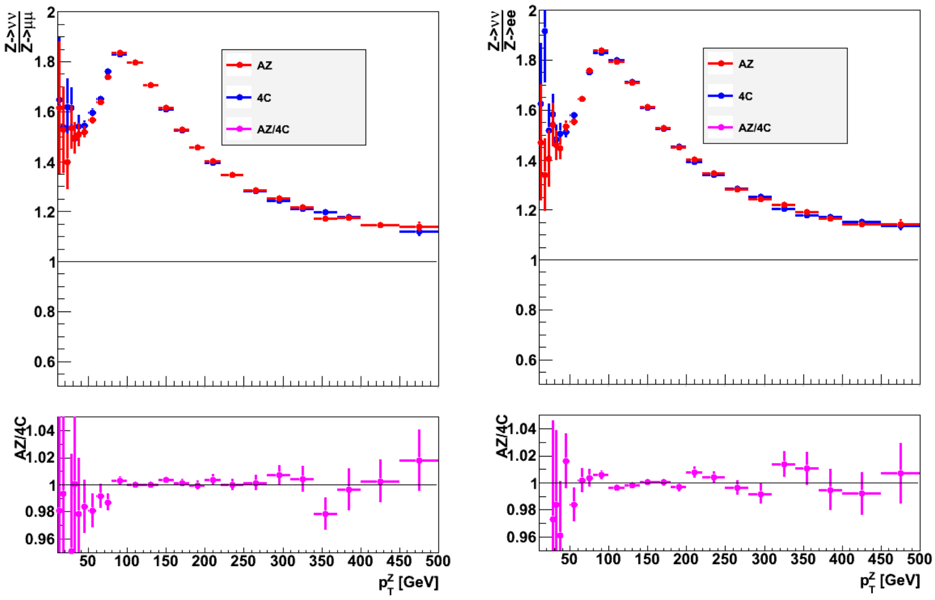
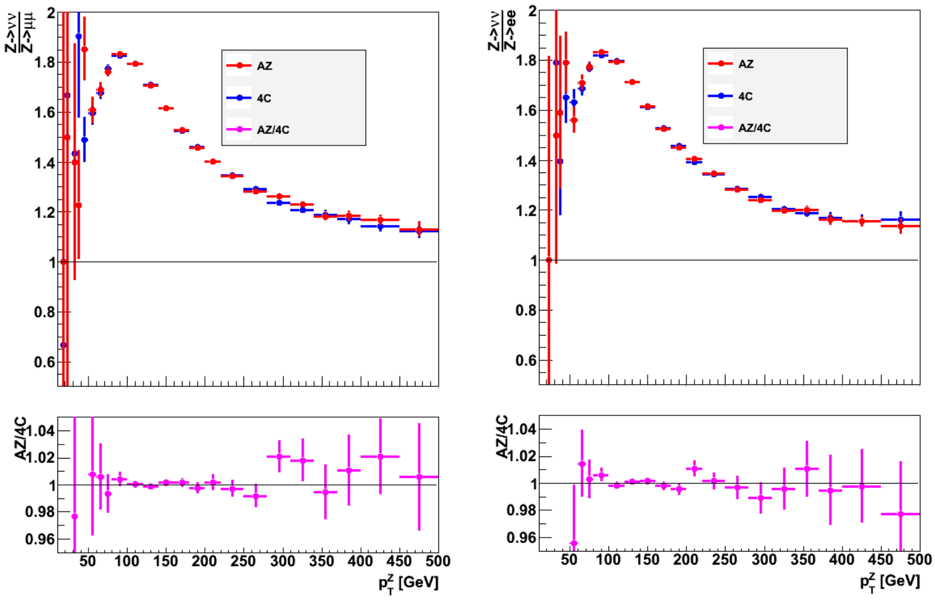


Figure 30: Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+l^-}$ ratios for the AZ tune to the 4C tune, with no QED or γ^* effects, and with fiducial and exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios $(\frac{AZ}{4C})$ (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios $(\frac{AZ}{4C})$

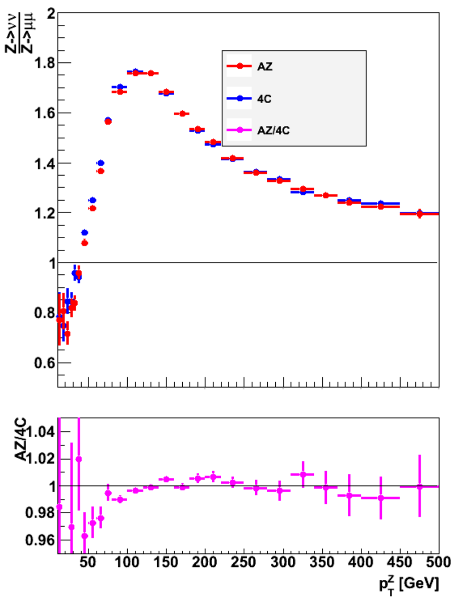


As in the case without fiducial cuts, there is no additional QCD sensitivity in the ratios once jet cuts are introduced; in both the inclusive and exclusive cutting schemes, the deviations of the tune ratios from 1 are still consistent with statistical fluctations.

To complete the exploration of QCD sensitivity, QED effects for the bare and dressed leptons are reintroduced, producing tune ratios that include all effects that have been considered at this point in the analysis. Pictured below are the ratios of the AZ tune ratios to the 4C tune ratios, for both lepton species, bare and dressed leptons, and inclusive and exclusive jet cuts:

Figure 31: Ratio of bare $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+l^-}$ ratios for the AZ tune to the 4C tune, no γ^* effects, and with fiducial and inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios $(\frac{AZ}{4C})$



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios $(\frac{AZ}{4C})$

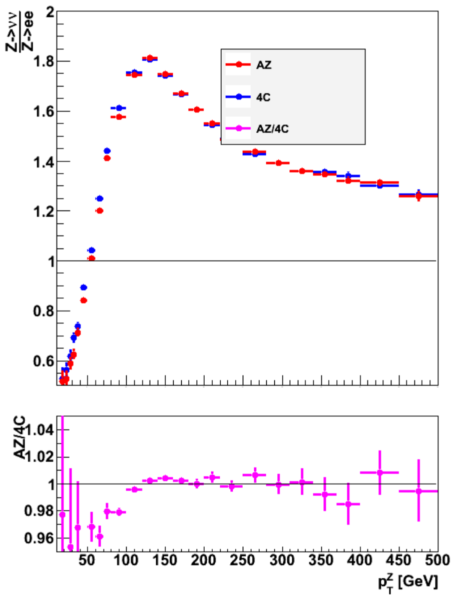
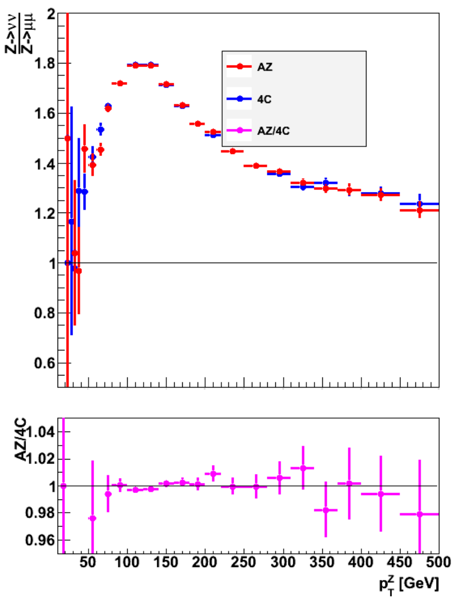


Figure 32: Ratio of bare $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+l^-}$ ratios for the AZ tune to the 4C tune, no γ^* effects, and with fiducial and exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios $(\frac{AZ}{4C})$



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios $(\frac{AZ}{4C})$

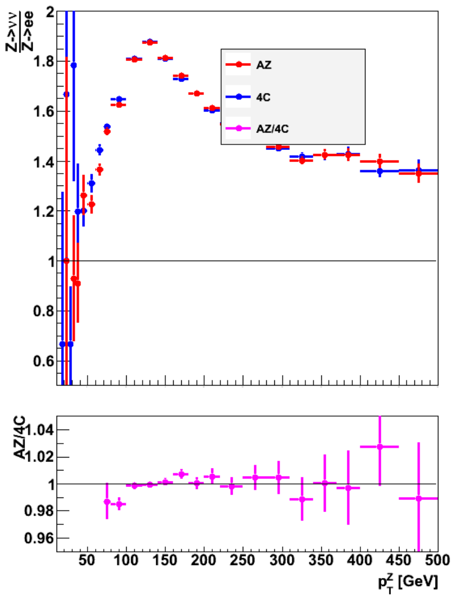


Figure 33: Ratio of dressed $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios for the AZ tune to the 4C tune, no γ^* effects, and with fiducial and inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios ($\frac{AZ}{4C}$) (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios ($\frac{AZ}{4C}$)

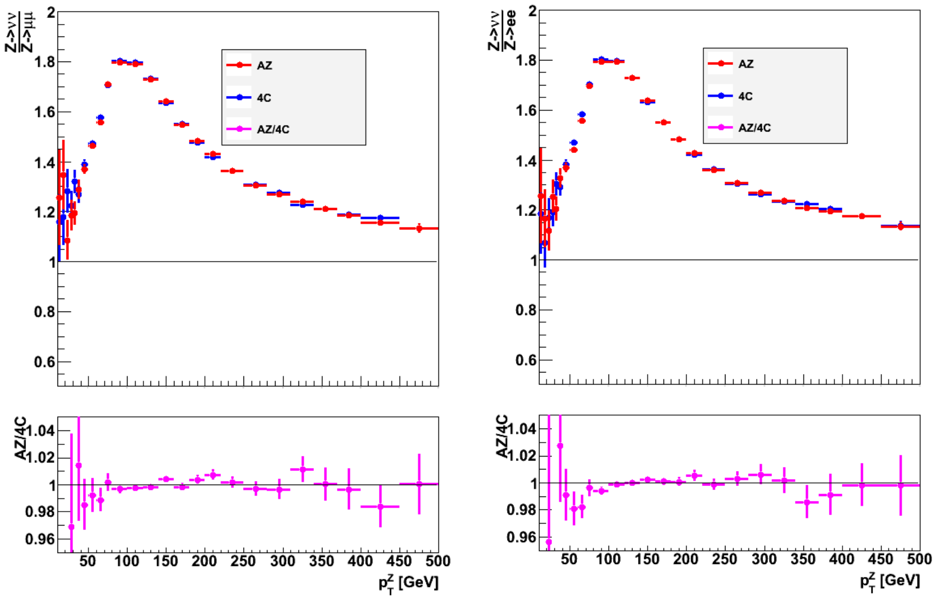
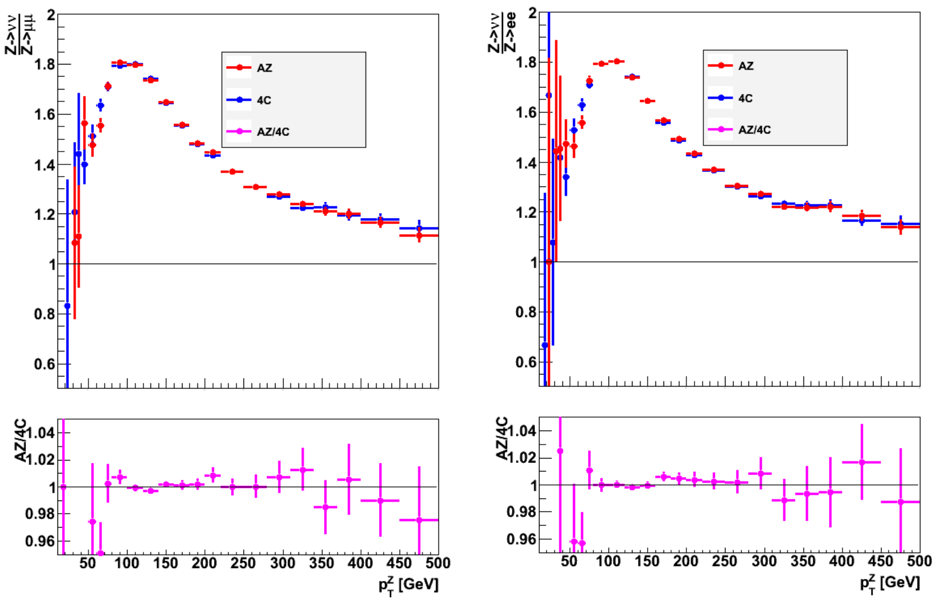


Figure 34: Ratio of dressed $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios for the AZ tune to the 4C tune, no γ^* effects, and with fiducial and exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios ($\frac{AZ}{4C}$) (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios ($\frac{AZ}{4C}$)



Just as in the case without fiducial cuts, the QCD sensitivity is quite small compared to the QED sensitivity, but with a discernible turn-on effect when the E_T^{Miss} threshold is below the jet p_T cut. With dressing, these ratios are still more sensitive to QED than QCD, like in the case without fiducial cuts, but with a much smaller advantage to QED effects, and some residual sensitivity to QCD effects. Dressing leptons, therefore, is not optimal if one wishes to measure QED effects using this ratio.

4.4 Jet Overlap

Thus far in the analysis, a perfect ability to distinguish Z^0 decay products from jets has been assumed. However, at the detector level, electrons yield jets, and some strategy must be implemented to compensate for these particles being clustered into jets. In order to approximate more realistic methods, two strategies are employed: First, for each visible lepton decaying from the Z^0 produced in the event, the jet closest to that lepton is identified. If that jet is within a cone of R=0.4 around the lepton, the jet is removed from the jet collection. This strategy is referred to as "overlap removal" (see section 3.2.3). The second possible strategy also identifies the closest jet to each visible lepton, but then, if the jet is within an R=0.4 cone around the lepton, the lepton's four-momentum is subtracted from that of the jet. This strategy is referred to as "overlap subtraction" (see section 3.2.3). Both of these methods produce a distortion in the phase space of the visible decay channels, which should in turn render the ratio sensitive to overlap schemes. Below, the ratios of ratios with overlap removal implemented to ratios with truth-level omission of Z^0 decay products of jets are pictured, for bare and dressed leptons (since these overlap removal schemes are applied at detector level, where muons and electrons are bare and dressed, respectively, it makes the most sense to consider the effect of this phase space distortion on these lepton definitions):

Figure 35: Ratios of bare $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+l^-}$ ratios with overlap removal to ratios with truth-level jet collections, with QED radiation, fiducial cuts and inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios

(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios

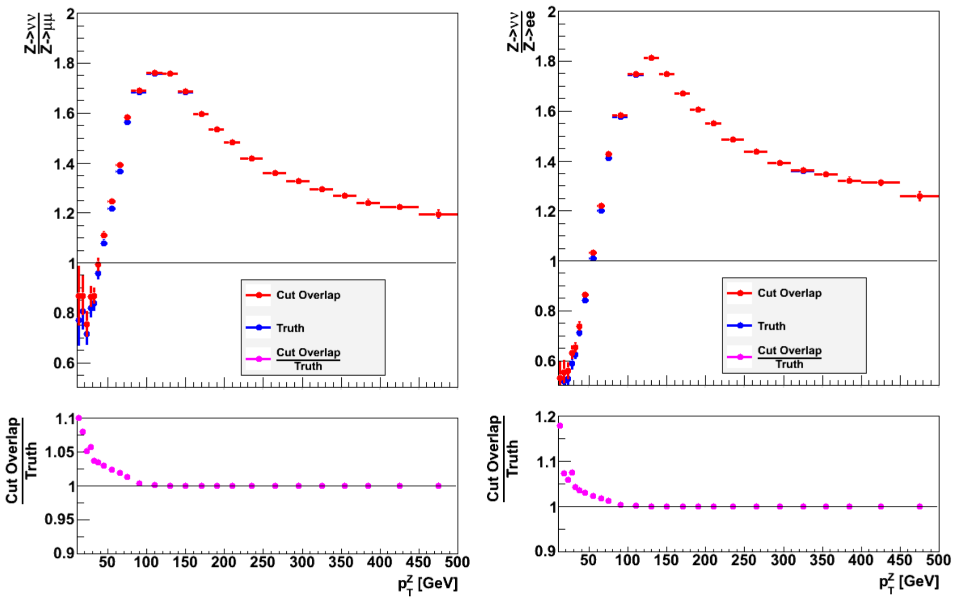


Figure 36: Ratios of bare $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+l^-}$ ratios with overlap removal to ratios with truth-level jet collections, with QED radiation, fiducial cuts and exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios

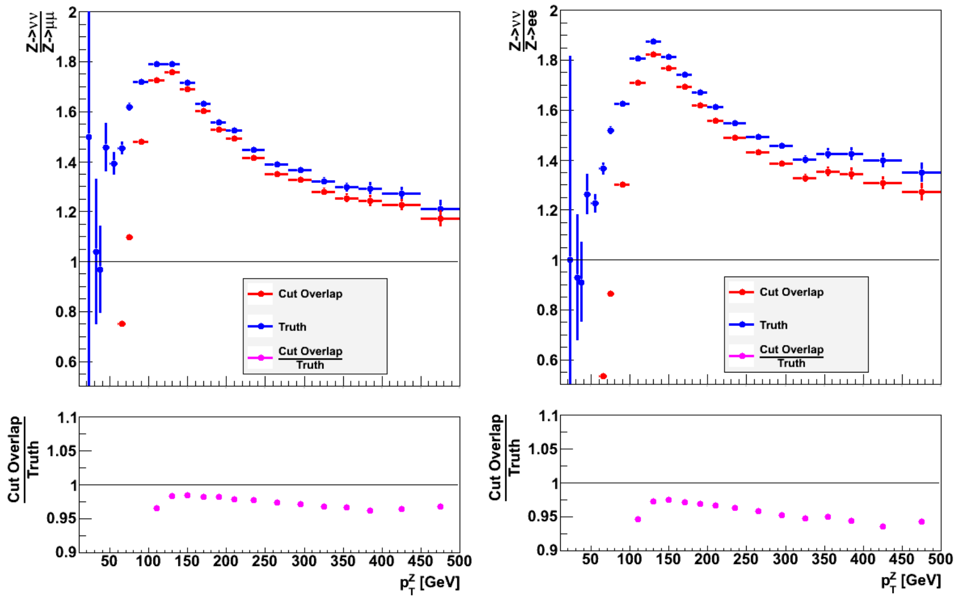


Figure 37: Ratios of dressed $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+l^-}$ ratios with overlap removal to ratios with truth-level jet collections, with QED radiation, fiducial cuts and inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios

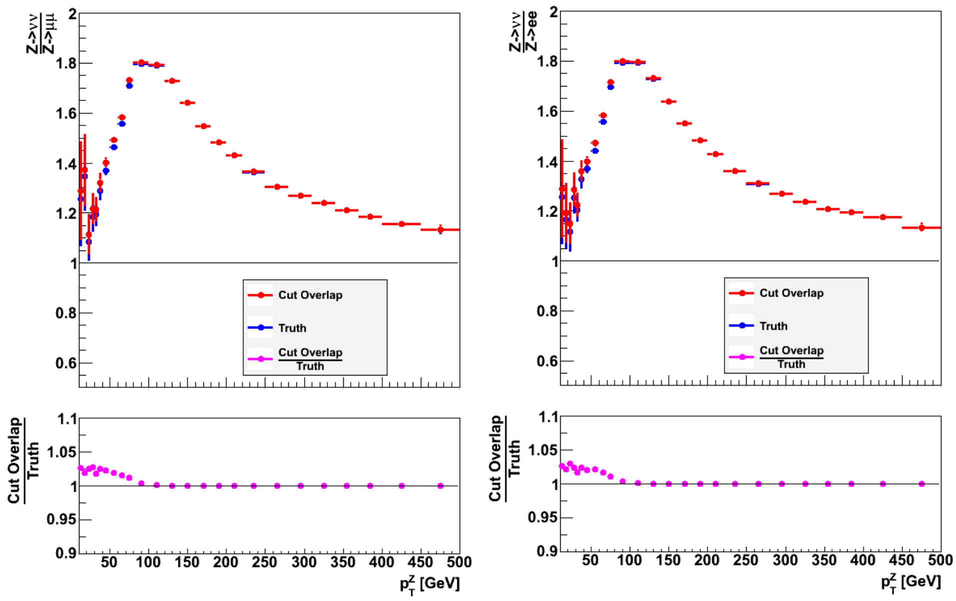
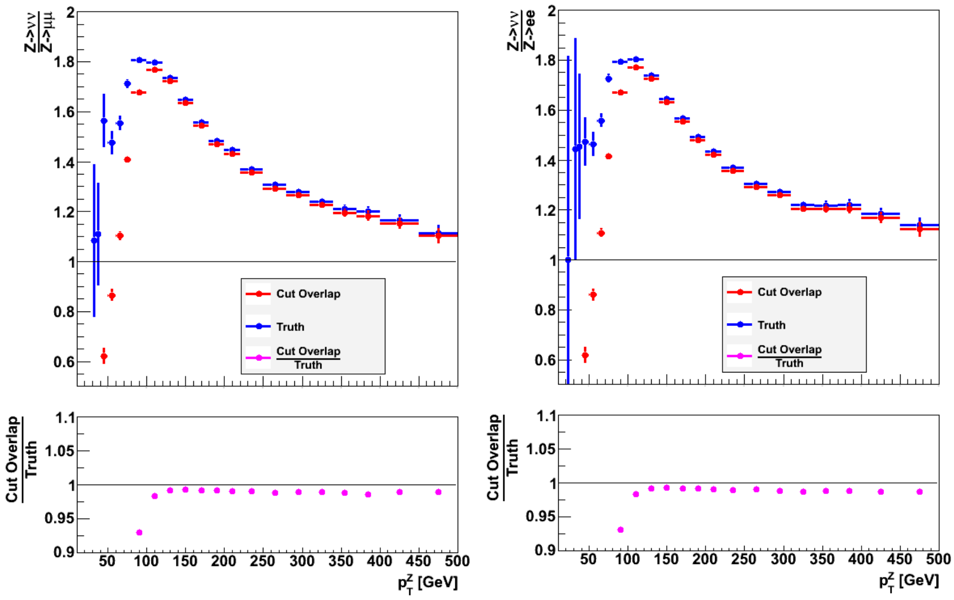


Figure 38: Ratios of dressed $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios with overlap removal to ratios with truth-level jet collections, with QED radiation, fiducial cuts and exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios

(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios

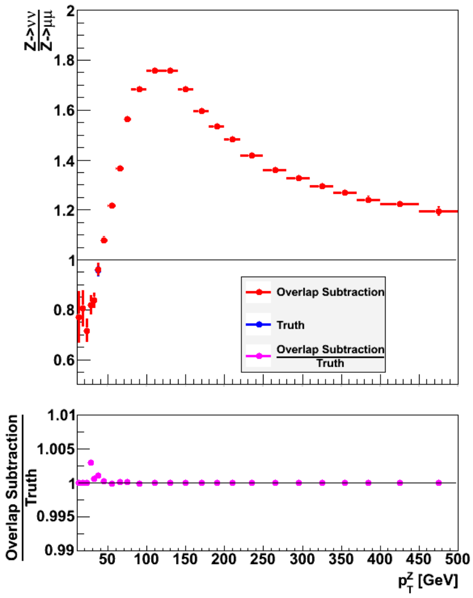


Notably, in the region of high $p_T^{Z^0}$, the sensitivity of the inclusive jet cutting scheme for both channels and both lepton definitions is zero; in this case, a simple overlap removal scheme does not distort the ratio at all, which greatly simplifies any correction factor from detector level data to a flat ratio (a necessary step in using these ratios to model invisible Z^0 distributions). Meanwhile, when exclusive jet cuts are applied, the distortion of the overlap removal technique becomes quite significant, approaching as much as a 6% deviation from flatness in the bare case. Many effects likely contribute to this, one is due to the fact that secondary jets can be collinear to the Z^0 decay products resulting in the removal of that jet and therefore avoiding the cut on any events with secondary jets. Another possible contributor would be cases where the lepton radiation collinear to the lepton direction could seed a jet which could then be removed in the overlap removal approach but not in the truth level. Finally, the Z^0 recoiling a hard jet and a very soft jet will decay into a lepton occasionally flying in the direction of the leading jet, which will result in the leading jet itself being removed from the collection in the overlap removal scheme. It can be concluded, then, that the effect of overlap removal on the phase space need only be considered when an exclusive jet cutting scheme is used or for inclusive jet cut case where $p_T^{Z^0}$ is less than the lead jet p_T.

In lieu of a simple overlap removal scheme, a more complex, but more accurate overlap subtraction scheme may be implemented (as described in section 3.2.3). In this case, because jet clustering in this analysis is performed by adding four-vectors of clustered particles, it can be predicted that the phase space distortion caused by this overlap scheme will be markedly less pronounced than the distortion caused by simple overlap removal. Below, ratios of ratios using this overlap scheme to ratios using truth-level jet collections are pictured, for both bare and dressed leptons and inclusive and exclusive jet cuts:

Figure 39: Ratios of bare $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios with overlap subtraction to ratios with truth-level jet collections, but with fiducial cuts and inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios

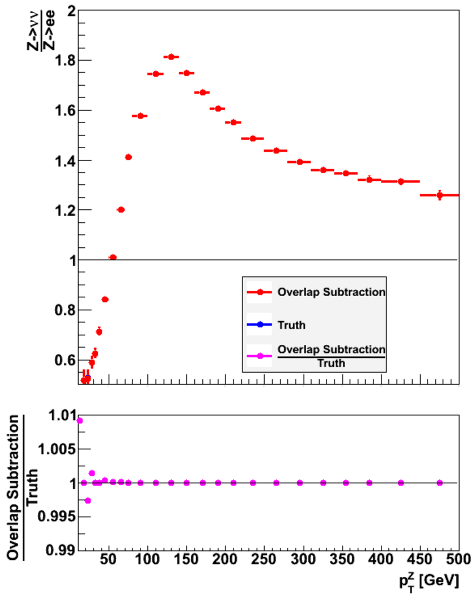
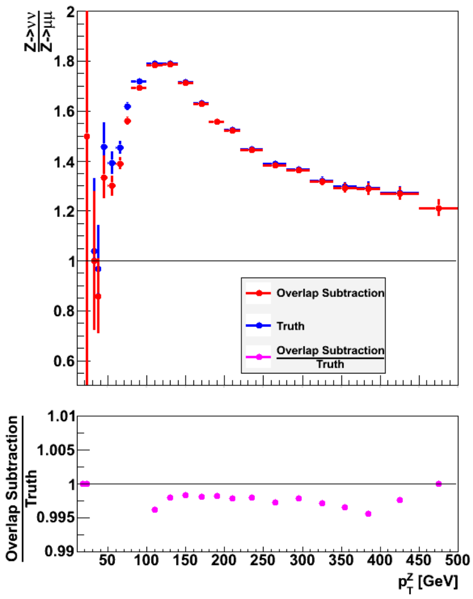


Figure 40: Ratios of bare $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios with overlap subtraction to ratios with truth-level jet collections, with fiducial cuts and exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios

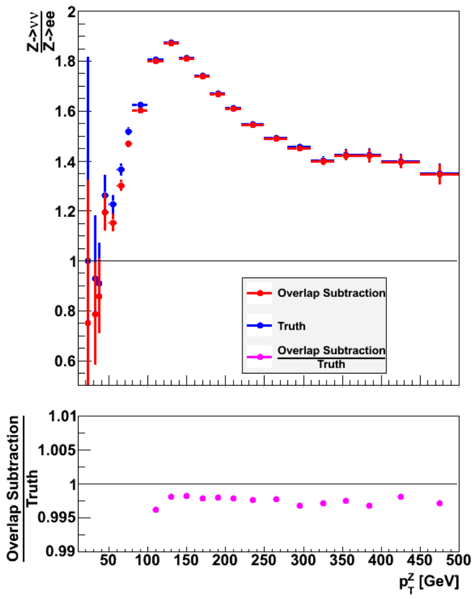


Figure 41: Ratios of dressed $\frac{Z^0\to\nu\nu}{Z^0\to l^+l^-}$ ratios with overlap subtraction to ratios with truth-level jet collections, but with fiducial cuts and inclusive jet cuts

(a) Ratio of $\frac{Z^0\to\nu\nu}{Z^0\to\mu^+\mu^-}$ ratios (b) Ratio of $\frac{Z^0\to\nu\nu}{Z^0\to e^+e^-}$ ratios

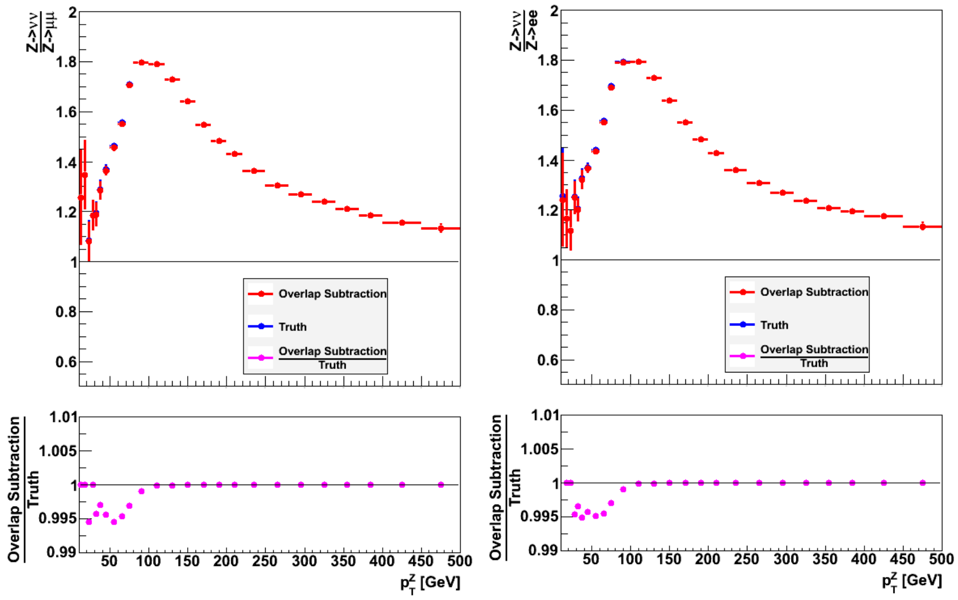
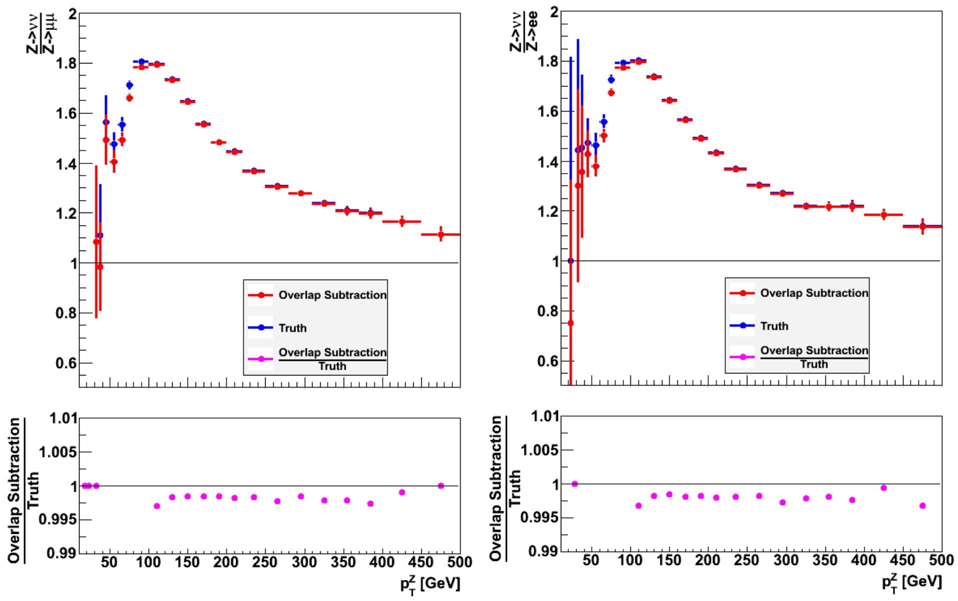


Figure 42: Ratios of dressed $\frac{Z^0\to\nu\nu}{Z^0\to l^+l^-}$ ratios with overlap removal to ratios with truth-level jet collections, with fiducial cuts and exclusive jet cuts

(a) Ratio of $\frac{Z^0\to\nu\nu}{Z^0\to\mu^+\mu^-}$ ratios (b) Ratio of $\frac{Z^0\to\nu\nu}{Z^0\to e^+e^-}$ ratios



The distortion to the phase space caused by the overlap subtraction method is significantly less than the distortion caused by overlap removal; in all schemes, the distortion never reaches above 0.5% in any region of physically interesting $p_T^{Z^0}$. As a result, this overlap scheme is exceptionally useful when trying to maintain a flat ratio; compared to the distortions in the ratio from fiducial cuts and QED effects, any distortion from this overlap scheme is insignificant and can be essentially ignored.

4.5 Virtual γ^*

In order to produce a flat ratio at the beginning of this analysis, the contribution of the virtual γ^* to the amplitudes of the muon and electron channels was suppressed at the generator level, since the neutrino channel had no such contribution. As new effects were added in, the γ^* remained suppressed, so that each new effect could be analyzed in isolation and compared to a flat ratio. However, in an experimental setting, the fundamental laws of quantum field theory stipulate that the γ^* amplitude cannot be suppressed; that amplitude's path from the initial state of the particle beam to the final state observed in the detectors is indistinguishable from the Z^0 amplitude's path, so the two amplitudes must both contribute and interfere in the simulation to accurately reflect physical reality. As a result, it is necessary to quantify the effect of including the virtual γ^* in the simulation, as well as the effect of implementing a cut on the invariant mass of the reconstructed Z^0 (see section 3.2.4 for more detail on this cut) Pictured below are ratios of ratios with the γ^* contribution to ratio without this contribution, for born, bare, and dressed leptons, with inclusive cutting, and overlap subtraction implemented:

Figure 43: Ratios of born $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+l^-}$ ratios with the γ^* contributions to ratios without this contribution, with fiducial cuts, overlap subtraction, and inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$ ratios (b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$ ratios

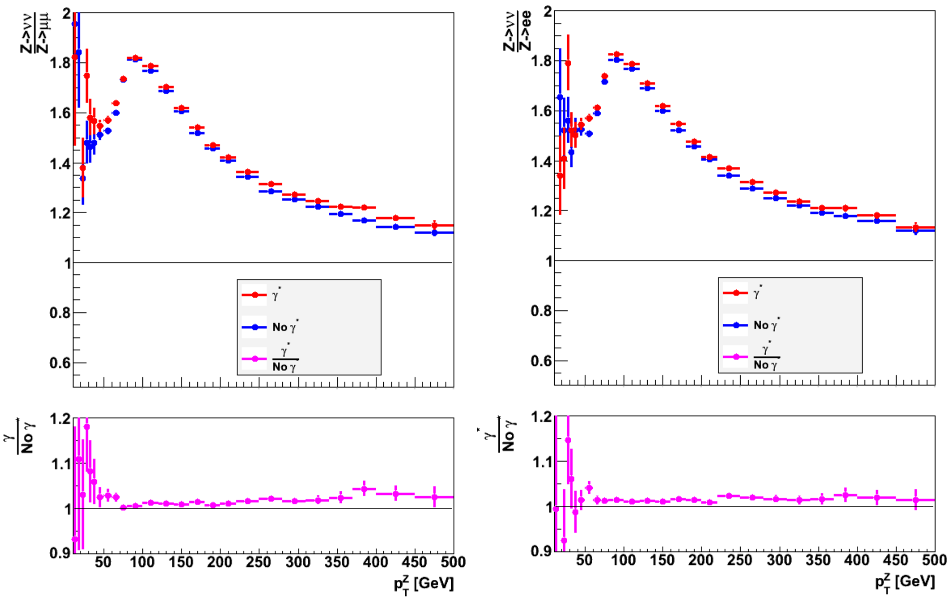
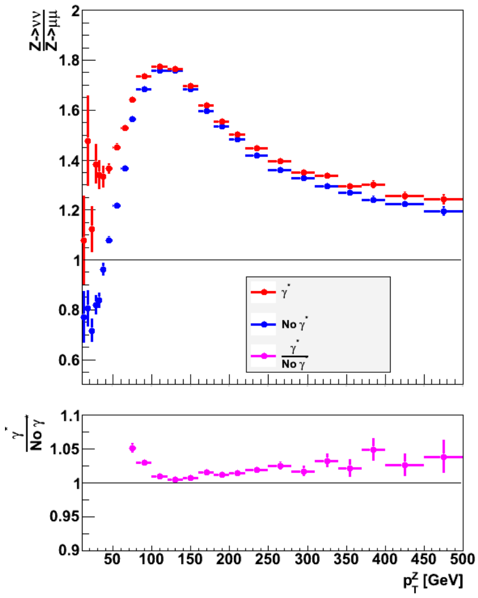


Figure 44: Ratios of bare $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios with the γ^* contributions to ratios without this contribution, with fiducial cuts, overlap subtraction, and inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios

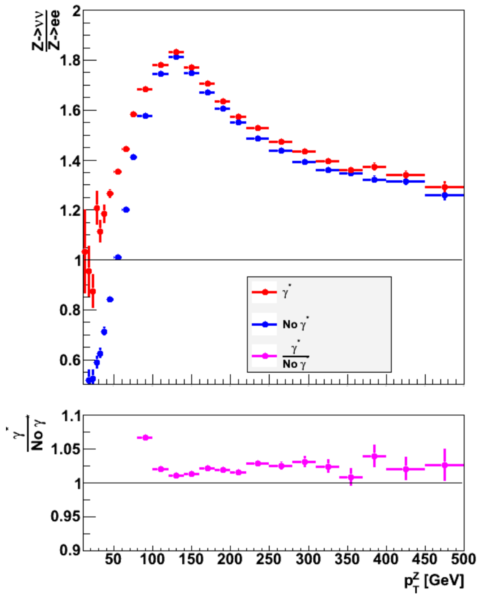
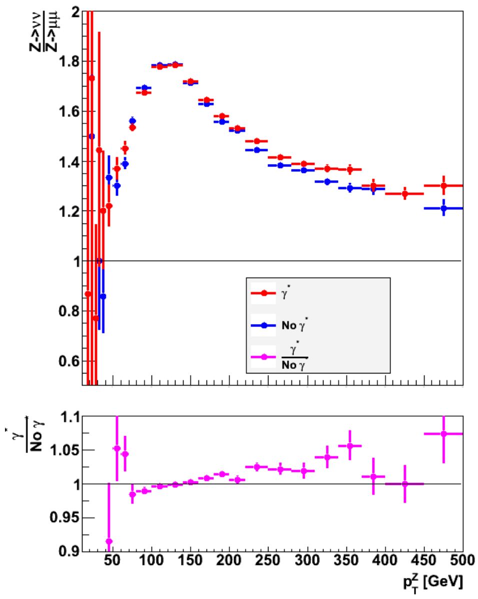


Figure 45: Ratios of bare $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios with the γ^* contributions to ratios without this contribution, with fiducial cuts, overlap subtraction, and exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios

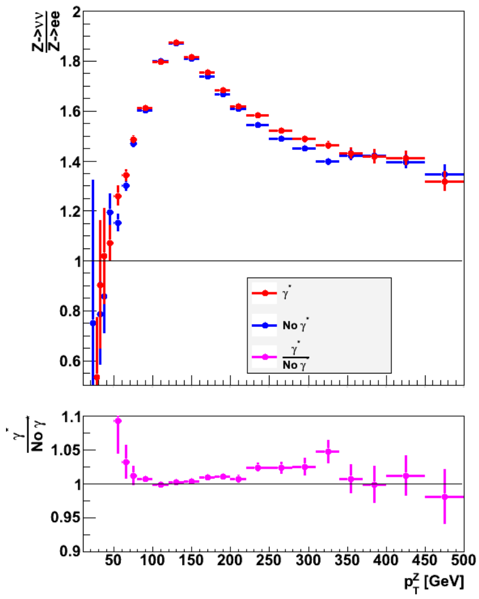
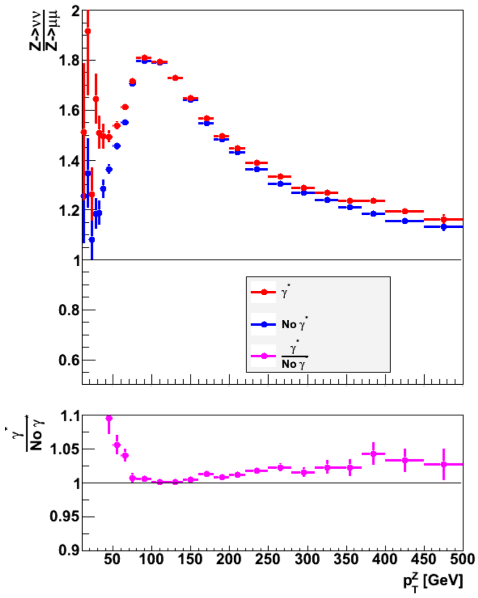


Figure 46: Ratios of dressed $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios with the γ^* contributions to ratios without this contribution, with fiducial cuts, overlap subtraction, and inclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios

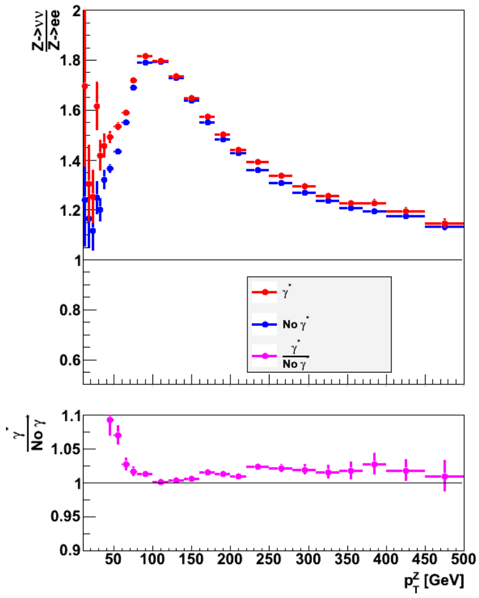
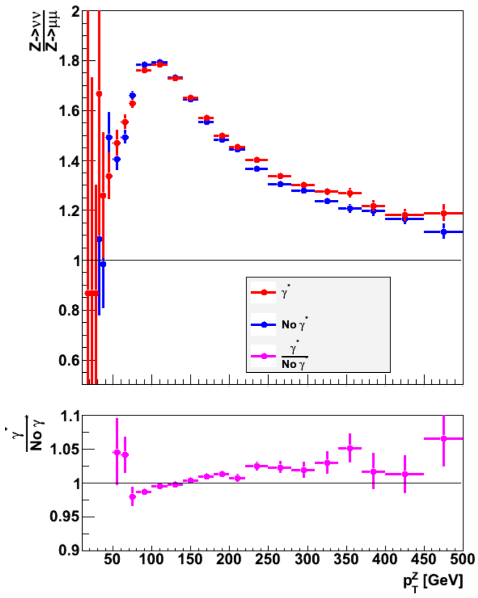
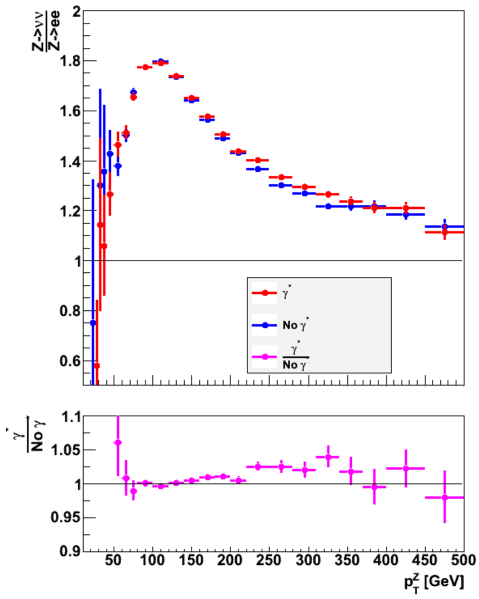


Figure 47: Ratios of dressed $\frac{Z^0 \to \nu\nu}{Z^0 \to l^+ l^-}$ ratios with the γ^* contributions to ratios without this contribution, with fiducial cuts, overlap subtraction, and exclusive jet cuts

(a) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+ \mu^-}$ ratios



(b) Ratio of $\frac{Z^0 \to \nu\nu}{Z^0 \to e^+ e^-}$ ratios



At low $p_T^{Z^0}$, the effect of implementing virtual γ^* is extremely significant. Within statistical error, it seems this correction is insensitive to lepton definition or even jet cutting scheme; in short, in the region of physical interest for this analysis, the effect of implementing the γ^* is consistent with adjusting the norm of the ratio by a factor of about 2%.

5 Discussion

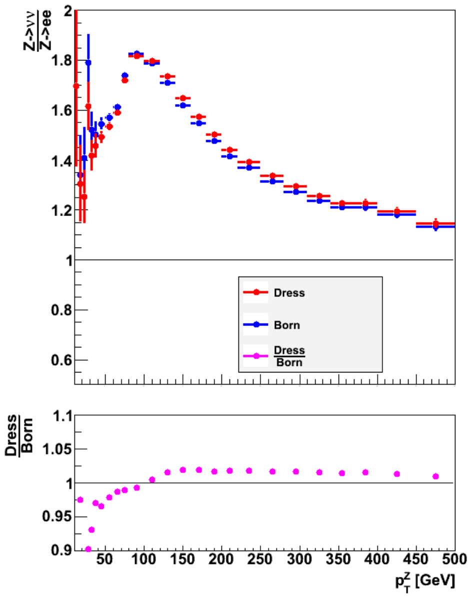
In the above analysis, various potential effects on the ratio of $p_T^{Z^0}$ in the invisible Z^0 channel to $p_T^{Z^0}$ in both the electron and muon channels were explored using Monte Carlo simulation. These include effects such as radiation or virtual photon contributions to the amplitude or lepton fiducial cuts that restrict the phase space, were explored using Monte Carlo simulation. As discussed in section 2, the one of the objectives of this study is to determine which effects have an impact on the shape of the ratio and how to define a measurement that would be maximally sensitive to these effects. This is discussed in section 5.1 below. The other main objective of this study is to produce a factor that will correct detector level data to a flat ratio, for use in probing both Standard Model monojet physics and modeling the Standard Model background to various new physics in monojet events. This correction factor must be maximally insensitive to the various physics effects explored in this study, in order to diminish the uncertainties (particularly QCD uncertainties, which motivate this method of data-driven modeling). In section 5.2

5.1 Testing Validity

As mentioned above, onof the objectives of this study is to render the predictions about the ratios' sensitivity to different physical effects testable. Because the QCD sensitivities in this analysis are within statistical error, it suffices to create a test of the QED sensitivity of the $p_T^{Z^0}$ ratios. First of all, to avoid adding too much uncertainty in the measurement results, fiducial cuts will be applied to the visible Z^0 decay products. The measured ratio will then not be flat, as seen in section 4.3. To test the validity of the model, then, an analysis scheme that maximizes the QED sensitivity of the ratios can be employed, and a measurement of the ratio can be compared to this prediction. If the measured ratio deviates significantly from the prediction made in the simulation, there is likely some other physics sensitivity that has been ignored in this analysis. If instead, the measured results for this ratio agree with simulation, it is reasonable to assume that these simulations have accurately captured the physics of these ratios, and can be used to model $Z^0 \to \nu\nu$ background in monojet searches or other measurements.

In order to be physically meaningful and maximize sensitivity to QED effects, a prediction must employ bare muons and dressed electrons, as well as the γ^* contribution to the amplitude, fiducial cuts on the leptons, and some kind of jet cutting scheme. Because electrons are dressed at the detector level, the maximum achievable QED sensitivity with a detector-level analysis is quite limited; as noted in section 4.2.5, dressing either lepton species reduces the magnitude of the QED correction to approximately 2% in any bin with $p_T^{Z^0}$ of physical interest. As a result, any hope of measuring the QED effect in this channel lies in minimizing any other uncertainties and observing a turn-on effect. Since dressing already removes the dependence of the QED sensitivity on the inclusivity or exclusivity of the jet cut, the inclusive jet cut, which will kill fewer events, is selected, and in order to have minimal distortion of the phase space from overlap effects, the overlap subtraction (rather than overlap removal) scheme is implemented here to deal with visible Z^0 decay products overlapping with jets. Below, the sensitivity of this measurement scheme to QED is depicted, by comparing the ratio produced at the dressed (detector) level and the same ratio produced at the born level:

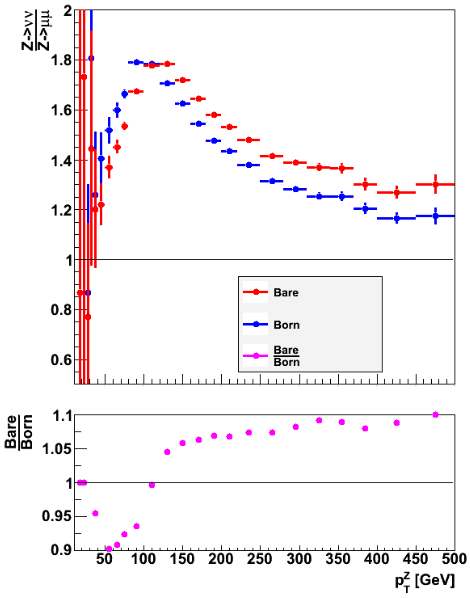
Figure 48: Ratio of dressed electron ($\frac{Z^0 \to \nu\nu}{Z^0 \to e^+e^-}$) ratios to born ratios, with a cutting scheme to minimize theoretical uncertainty and maximize QED sensitivity



As noted above, while the QED sensitivity of this channel is quite small, a distinct turn-on effect in the ratio can be observed if the E_T^{Miss} requirement in this ratio (and hence the $p_T^{Z^0}$ requirement is kept below the jet p_T cut); in this case, the presence of an upward slope in the $p_T^{Z^0}$ curve at the threshold will provide another measurable prediction, helping to mitigate the handicap caused by low QED sensitivity here.

Meanwhile, in the muon channel, the fact that the bare particle appears at the detector level allows for a much stronger QED effect to be observe; in this case, as much as a 10% correction may be achieved. Additionally, use of the bare muon renders the ratio sensitive to the choice of jet cutting scheme; as noted in section 4.2.2, use of the exclusive jet cut increases the effect of QED on this ratio. The implementation of overlap removal when exclusive cuts are used, however, causes a correction to the ratio that partially cancels the effect of QED on the ratio, as noted in section 4.4, where the use of overlap removal decreases the norm of the bare-level ratio substantially more than the decrease it causes in the dress-level ratio, implying that the distortion to the phase space caused by overlap removal will mitigate the positive effects of QED on the ratio's norm in the region of physically interesting $p_T^{Z^0}$. As a result, maximizing QED sensitivity requires the implementation of overlap subtraction, rather than overlap removal, to preserve the increased QED sensitivity caused by an exclusive jet cut. Below, the ratio of ratios comparing the fiducial bare-level to born-level ratios for this cutting scheme is pictured, displaying the QED sensitivity of this ratio:

Figure 49: Ratio of bare muon ($\frac{Z^0 \to \nu\nu}{Z^0 \to \mu^+\mu^-}$) ratios to born ratios, with a cutting scheme to minimize theoretical uncertainty and maximize QED sensitivity



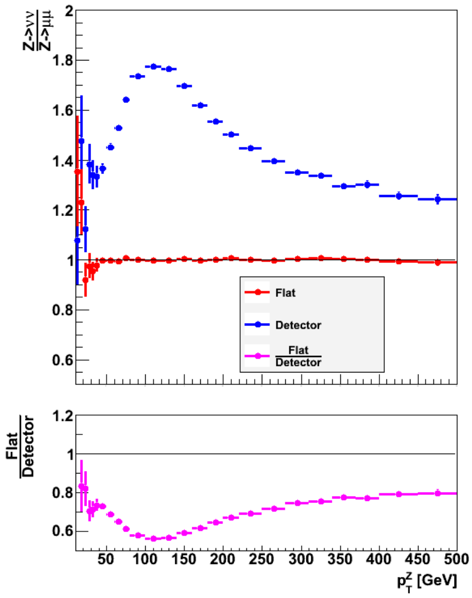
Assuming great enough experimental accuracy, the validity of the simulations in this analysis can be tested by measuring the ratios $\frac{\sigma(Z^0 \to \nu\nu)}{\sigma(Z^0 \to \mu^+\mu^-)}$ and $\frac{\sigma(Z^0 \to \nu\nu)}{\sigma(Z^0 \to e^+e^-)}$ with the above analysis schemes, and comparing the results to the simulated predictions. If the ratios are accurate, then the simulations' depiction of the physically relevant effects on this ratio (namely, QED and phase space distortion) can be considered a faithful reproduction of physical reality.

5.2 Modeling $Z^0 \to \nu\nu$

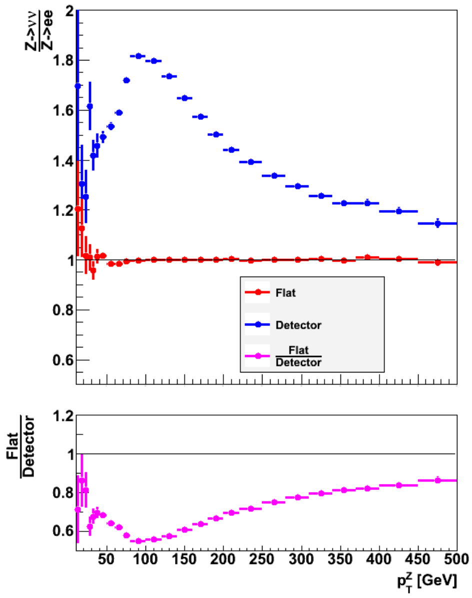
If the predictions made in the previous section are experimentally adequate, the original purpose of this investigation can be realized: These Monte Carlo simulations may be used to generate correction factors to relate the electron and muon channels of the Z^0 to the invisible channel, and as a result, be used to estimate the Standard Model background for monojet events or extract a measurement of the invisible decay width of the Z^0. Using information gained in the earlier analysis of each possible sensitivity of the ratio of Z^0 decay channels, generating this correction factor is simple: The results of the exploration of each sensitivity indicate that flat ratios can be produced for both the ratio of the invisible Z^0 decay to the electron channel and to the muon channel by born-level events, overlap subtraction to eliminate phase space distortion (as long as inclusive jet cuts are applied), no fiducial cuts, and the suppression of the virtual γ^* contribution to the amplitude, so correction factors for both $\mu^+\mu^-$ and e^+e^- ratios can be produced by taking the ratio of these cutting schemes to the bare (for muons) or dressed (for electrons) data with fiducial cuts and inclusive jet cuts implemented. In short, these correction factors will compensate for the effect of the virtual γ^*, fiducial cuts, and QED radiation (from bare-level to born-level for muons, which are bare at the detector level, and from dress-level to born-level for electrons, which are dressed at the detector level). Below, the results of this procedure for both channels is depicted, with the flat ratio being produced with born-level leptons, overlap subtraction, no fiducial cuts, no virtual γ^* contribution, no fiducial cuts, and inclusive jet cuts:

Figure 50: Correction factors from the detector level to a flat ratio (no fiducial cuts or γ^*, with born-level reconstruction, overlap subtraction, and inclusive jet cuts) from $Z^0 \to \mu^+\mu^-$ and $Z^0 \to e^+e^-$ to $Z^0 \to \nu\nu$

(a) $Z^0 \to \mu^+\mu^-$ Correction Factor



(b) $Z^0 \to e^+e^-$ Correction Factor

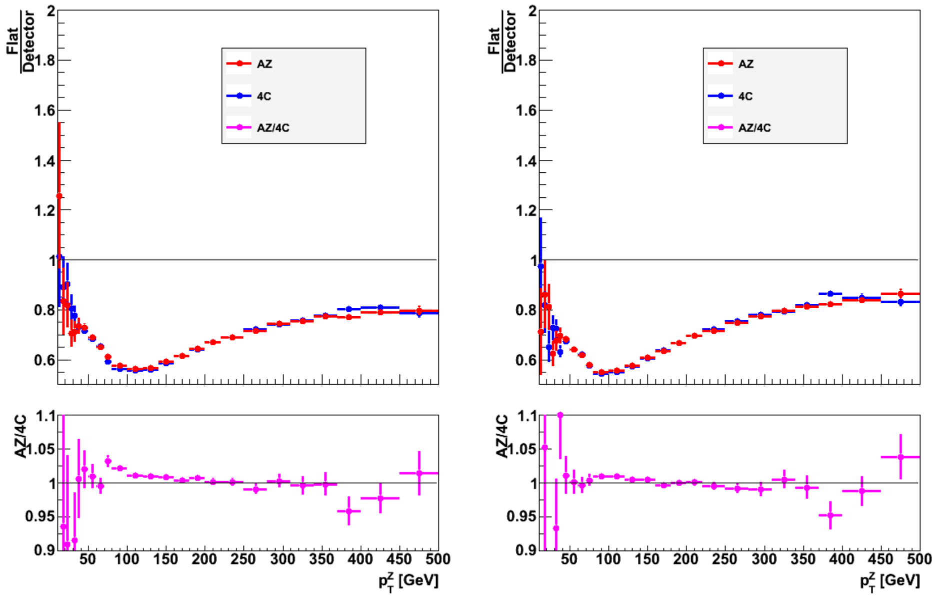


Now, a rough estimate of the QCD sensitivity of these corrections can be achieved by comparing the correction factors produced using different tunes (namely, the AZ tune and the 4C tune). Below, the ratio of correction factors for these two tunes are depicted:

Figure 51: Ratios of correction factors for the AZ tune to the 4C tune, for both $Z^0 \to \mu^+\mu^-$ and $Z^0 \to e^+e^-$

(a) $Z^0 \to \mu^+\mu^-$ Correction Factor Ratios

(b) $Z^0 \to e^+e^-$ Correction Factor Ratios



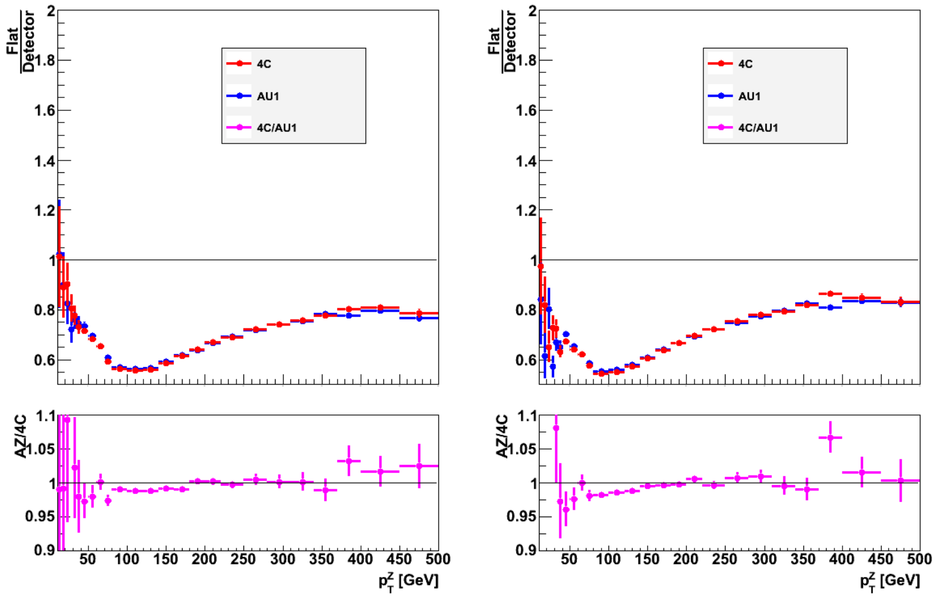
Notably, the tune sensitivity appears extremely small. To estimate the sensitivity of the tune, a linear fit from a 120 GeV p_T^Z (i.e. E_T^{miss}) threshold is used and the deviation of the fit result to the straight line at 1 is taken as the QCD radiation dependence of the acceptance factor, considered as a source of systematic uncertainty. In the muon channel, this deviation is only about 0.5%, while in the electron channel, the deviation is as low as 0.07%. As a result, the benefit of using this correction factor (along with data from visible decay channels of Z^0) to estimate the $Z^0 \to \nu\nu$ background to monojet events is clear: Use of this technique virtually eliminates QCD uncertainties (keeping them on the order of, at most, a few percent) in the estimate, greatly increasing the potential accuracy of this background estimate when compared to a traditional entirely Monte-Carlo driven estimate of this background.

In the interest of thoroughness, the sensitivity of this correction factor to another type of QCD tune (in this case an underlying event tune) is now explored as well. By taking the ratio of the correction factors determined using the 4C tune and the AU1 tune (see section 3.1.3), the QCD sensitivity of this correction factor may be explored in slightly greater depth. Below, the tune ratios for these two underlying-event tunes are pictured:

Figure 52: Ratios of correction factors for the 4C tune to the AU1 tune, for both $Z^0 \to \mu^+\mu^-$ and $Z^0 \to e^+e^-$

(a) $Z^0 \to \mu^+\mu^-$ Correction Factor Ratios

(b) $Z^0 \to e^+e^-$ Correction Factor Ratios



Again, only a very slight QCD sensitivity is observed; using the same technique to estimate uncertainty described for the parton-shower tunes, the muon channel for these tunes has an uncertainty of approximately 0.6%, while the electron channel has an uncertainty of approximately 0.5%. Again, then, the correction factor produced using this method proves nearly completely insensitive to QCD effects.

6 Conclusion

In searches for new physics, monojet events represent a number of intriguing avenues for further study, particularly because of the channel's presence as a final state in a number of different models of particle dark matter. Additionally, even in the absence of significant new physics signals, the fact that the only irreducible SM background for these events is the invisible decay channel of Z^0+jets allows for careful measurement of this decay channel in an environment with limited other backgrounds. However, properly probing this channel relies heavily on Monte Carlo simulations to estimate backgrounds, which will then be subject to large theoretical uncertainties, stemming from uncertainties in QCD parameters. Instead, however, this thesis establishes that a correction factor to translate $Z^0 \to \mu^+\mu^-$ and $Z^0 \to e^+e^-$ data into predictions for $Z^0 \to \nu\nu$ data can be generated, and this correction factor is extremely insensitive to QCD effects. In addition, the validity of this modeling technique to be experimentally tested before it is applied in a true monojet analysis, simply by testing the sensitivity of the measured ratio of visible Z^0 decays to the invisible Z^0 channels to QED effects, and that this sensitivity should be large enough to be observable in an experimental setting. As a result, it seems that a data-driven method to estimate the standard model backgrounds for monojet events is both plausible and desirable in future analyses.

7 Future Research

While the most salient next step would be to perform experimental tests on this model, other further theoretical analysis can be performed as well. Particularly, it would be useful to perform this analysis on larger ntuples, which time and computing constraints (particularly memory limits and a desire not to monopolize the cluster) prevented in this project, to diminish the role of statistical fluctuations in explorations of tune sensitivities and the effect of the virtual γ^* contribution on the ratio. Additionally, in exploring tune sensitivities of these ratios, only parton shower and underlying event tunes were employed, namely AZ, 4C, and AU1. It would be useful to also explore different PDF's, in order to establish that these ratios are truly insensitive to QCD effects. Similarly, exploring more than two parton shower and underlying event tunes could help further confirm QCD insensitivity, since by comparing only two tunes, there is the possibility that these tunes might have a set of parameters that line up closely enough to give similar phase spaces, and hence similar ratios, without the ratios being truly insensitive to QCD.

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