Newton's *De Motu Corporum In Gyrum* (i.e. Version 1)

I. Background To *De Motu Corporum in Gyrum*

   A. Newton's Work in Mechanics Before De Motu

      1. Newton's unpublished work in mechanics of interest to us not as a contribution to the growing field, but because it reveals influences on him -- in particular, ones that did not affect Huygens comparably
      2. Much of Newton's work before 1673 superficially parallels Huygens's -- a theory of impact, a theory of uniform circular motion, and derivations of the key properties of cycloidal pendulums
         a. In contrast to Huygens, however, no growing theoretical network at all, and no pursuit of increasingly high quality evidence
         b. Newton, by comparison, is merely dabbling in mechanics, in response to some glaring deficiencies in Descartes' *Principia*, and, even more so, to problems associated with Copernicanism called attention to in Galileo's *Dialogue*
      3. Newton was conceptualizing motion along the lines of Descartes, in which changes in direction or speed call for an explanation -- i.e. for external intervention of some sort
         a. Uniform motion in a straight line maintained by an internal *vis*
         b. Changes in motion always involve an external cause, sometimes termed an impressed force
            (1) Huygens thought of forces as static -- the force in the string or pressing on a wall
            (2) Newton closer to Descartes in this regard
         c. No less than Descartes, Newton saw this as implying a *conatus a centro* in all curvilinear motion, a *conatus* that had to be resisted by some external force
         d. This is not a Galilean way of conceptualizing motion
         e. But with one notable exception -- the emphasis on impressed forces -- it is the way Huygens was conceptualizing motion
      4. Newton had adopted some of Descartes' laws and principles, often with refinements, but he had rejected others, including conservation of motion, and added some of his own
         a. In addition to inertia, he had come to identify impressed force with change in motion, but along a single direction, thus in effect introducing a vectorial conception of force and motion
         b. Like Huygens, he had shown that the principle of conservation of total motion does not hold under impact of two objects
         c. He had asserted the principle that the center of gravity of a set of interacting bodies does not change motion as a consequence of their interactions -- a principle Huygens announced in 1669
      5. Newton's interest in uniform circular motion was clearly prompted by his interest in "proving" Copernicanism -- something Huygens showed no interest in during this period
         a. His calculation of the ratio of the *conatus a centro* of the moon in its orbit to the force of gravity on the surface of the earth may have been intended to test the hypothesis that terrestrial gravity holds the moon in orbit

b. Though it might have been intended to serve other purposes -- e.g. to establish Kepler's 3/2 power rule for bodies orbiting the earth, allowing then for a decisive argument against Tycho

6. Newton's treatment of the cycloidal pendulum is the one place in which he shows an appreciation for the way in which Huygens had extended Galileo's theory of free fall, in the process developing approaches that were yielding much higher quality evidence
   a. A full appreciation of Huygens's achievement may have come only upon reading the *Horologium Oscillatorium* in 1673/74
   b. But, given Newton's general insistence on maintaining a sharp distinction between conjectural hypotheses and experimental certainty, he surely would have noted the evidentiary implications of Huygens's work

B. The Correspondence with Hooke in Late 1679

1. The six-week correspondence, which Hooke initiated at the end of 1679, came at a time when Newton had little reason to be interested in mechanics
   a. He had been concentrating on chemistry and alchemy for the prior few years, after "renouncing" philosophy to Oldenburg
   b. And he had just returned to Cambridge following the death of his mother in June and the settling of her estate
2. Hooke's initial letter was part of an effort he was undertaking, as the new Secretary of the Royal Society, to solicit interesting papers for the society -- a task which Oldenburg had performed very well from the outset of *Phil. Trans.*, which at that time had ceased publication
   a. Included in the list of topics that Newton might want to address was Hooke's hypothesis "of compounding the celestiall motions of the planetts of a direct motion by the tangent & an attractive motion towards the centrall body" -- NB: two *motions*, not tendencies
   b. Other topics included what we now call Hooke's law and Picard's method for determining precise longitude differences
3. Newton's almost immediate reply instead focused on another topic -- a novel experiment to demonstrate the motion of the earth (Copernicanism again) and with it a conjectured solution to an old problem
   a. An object falling from a height should show an eastward displacement (owing to what became known in the 19th century as Coriolis effects)
   b. The path such a falling body would follow, if it continued on toward the center of the earth, would be a spiral of the sort shown in the figure (see Appendix)
4. Hooke's Dec 9 reply reports the lively interest shown in the idea of the experiment at the Dec 4 meeting of the Society, and then objects to the spiral, claiming instead that the path would resemble an ellipse
   a. A spiral only if medium resistance effects disturbed it from its roughly elliptical path

b. And adds, correctly, that a southward as well as an eastward displacement will occur if Newton's experiment conducted at the latitude of London

5. Newton's almost immediate reply concedes that the path will not be a spiral to the center, but -- supposing gravity to be uniform -- it will "circulate with an alternate ascent & descent made by its vis centrifugal & gravity alternately overballancing one another"
   a. Newton had clearly put some more careful thought into the solution than he had in writing the initial letter, but the trajectory is still not quite the one he derives rigorously in the *Principia*
   b. The idea of alternately overbalancing one another, in a kind of dynamic exchange, undoubtedly comes from a model proposed by Borelli in 1666 to account for any non-circular motion of Jupiter's satellites (Whiteside, p. 1; also in Notes for Class 11)
   c. The multi-lobe figure Newton draws (see Appendix for drawing) is striking first because Newton appears to have drawn the first segment, down to a point where nearest the center, then inverted the sketch and traced it to get the second segment
   d. Michael Nauenberg has argued that he most likely used a curvature method to draw the first segment, with the retracing to get the second producing a somewhat excessive angle (vs. the correct 207.8 deg for projectile motion under a constant centripetal force in the *Principia*)
6. Hooke's reply of Jan 6 grants Newton's solution, but rejects the uniform gravity assumption on which it is based, instead pressing an inverse-square gravity assumption (invoking Kepler as grounds)
   a. Rather than asserting a roughly elliptical shape again, Hooke argues that the exact determination of the curve "will shew the error of those many lame shifts made use of by astronomers to approach the true motions of the planets with their tables."
   b. Though he concedes both that gravity will not hold as the inverse square all the way to the center of the earth, and he remarks on the idealization of treating bodies as points
   c. (Hooke may well have performed a graphical or numerical solution for the trajectory under inverse-square, and found it to resemble an ellipse, but was unable to demonstrate the exact curve)
7. The reply of Jan 6 also announces successful results for Newton's experiment performed out of doors -- three trials, with different eastward displacements, all no less than 1/4 inch
   a. Encouraging, but scarcely compelling results, as Hooke notes
   b. (With great care the experiment would provide evidence for the motion of the earth: in the absence of air resistance, fall from a height of 50 m at latitude 45 deg will yield a quarter of an inch eastward displacement; Hooke's irregular results from far lesser height are excessive)
8. The reply of Jan 6 ends with a discussion of the possibility that gravity may vary with altitude, as suggested by Halley's findings on St. Helena, and, if so, Huygens's proposed universal standard of length will be "spoiled"
9. Hooke's final brief letter of Jan 17, after not hearing further from Newton, reports additional successes with Newton's proposed experiment indoors, and once again implores Newton to address

the problem of the path under an inverse-square center-directed force:

> "It now remaines to know the proprietys of a curve Line (not circular nor concentricall) made by a centrall attractive power which makes the velocitys of Descent from the tangent Line or equall straight motion at all Distances in a Duplicate proportion to the Distances Reciprocally taken. I doubt not but that by your excellent method you will easily find out what that Curve must be, and its proprietys, and suggest a physicall Reason of this proportion."

a. Method to which Hooke refers surely the calculus -- that is, Newton's method of fluxions

b. Well known in science circles that Newton had made a breakthrough in mathematics

C. The Comet(s) of 1680-81

1. Exactly what Newton discovered in 1679-80 in conjunction with this correspondence with Hooke we do not know, for no document remains

   a. Some have speculated that a paper he wrote for Locke after the *Principia* appeared that gives a more simplistic approach to ellipses and the inverse-square represents what he found earlier

   b. But there is no evidence for this, and Newton himself never gave any indication after the dispute with Hooke over credit for the inverse-square emerged in 1686

2. Newton's sole interest in mechanics in the period from 1679 to 1684 for which we have any documents involves an intense episode starting in mid-December 1681 and extending at least to April 1682 in which he was trying to work out the trajectory of the comet (or comets) observed in November 1680 and then December through February 1681

   a. Flamsteed, Newton learned, was proposing, on the basis of symmetry, that this seemingly pair of comets was one comet, seen initially moving from north to south and then from south to north

   b. This proposal initially included a claim that magnetic repulsion as it had approached the sun had caused the comet to be deflected through a large hairpin angle

   c. Included in Newton's letters to Flamsteed in response to this proposal is a denial that (1) that the magnetism of the sun could do this (heat cancels magnetic effects) and (2) that such a hairpin angle could happen through repulsion

   d. In its place Newton offers Flamsteed the alternative that the comet button-hooked around the sun, with the attraction of the latter producing the hairpin angle

3. This supporting proposal notwithstanding, Newton's own efforts on the comet of December centered on a straight line trajectory at uniform speed, with at most slight curvature either from the sun or the "vortex" around it

   a. In particular, the late James Ruffner's (2013) detailed analysis of Newton's recorded observations and calculations from 1681 show him struggling, with less than reliable data, to fit a straight line trajectory of the sort Wren had proposed in 1664 (see Notes, Class 11)

   b. So, while Newton offered suggestions to help Flamsteed with his proposal, he was throughout their correspondence at this time working on an approach expressly denying one instead of two comets