A TEN-YEAR-OLD CHILD'S REPRESENTATIONS OF MOTION

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submitted by

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Introduction

Ideas about motion begin to develop from very early in a child's life, initially through observations and interactions with the environment and later through formal instruction in school. Likewise, inside and outside of school, children use various external representations—such as drawing, numbers, and writing—to communicate their ideas. Constructivism holds that in order for learning to occur, a learner must build upon the understandings already present in her mind. In keeping with this framework, one of the goals of this analysis is to provide further insight into the ideas about motion expressed by a young learner through invented representations. While professional scientists and other adults can communicate their knowledge about motion using, for instance, Cartesian graphs, these representations are not often part of children's representational repertoires. For younger students, a familiar medium for communicating ideas about motion is through drawings (Gardner, 1982; Scheuer, de la Cruz, & Pozo, 2002). This paper presents the case study of a 10-year-old boy, Aiden, and the various representations he produces to express his ideas about the motion of a car. I conducted three clinical interviews that focused on the creation of pen and paper drawings and also stop-motion animations using SAM Animation (http://www.samanimation.com) software. This software has previously been used for kinematics instruction at the high-school level (Church, Gravel, & Rogers, 2007), as well as to explore elementary-aged students' understanding of air (Gravel, 2011). The goals of this study were to explore the representations of motion produced by a 10 year old boy; how his paper and pencil representations compared to those produced through animations; and how his understandings of time, displacement, speed, and acceleration were expressed through the representations he created.

Previous Research

Understanding of Motion in the Elementary Grades

The first studies of children's understanding of motion can be traced to Piaget, who examined the development of duration, succession, simultaneity, velocity, and acceleration through clinical interviews with elementary-aged children (Piaget, 1927/1969, 1946/1970). He found that 2 to 6-year-olds did not always clearly differentiate between dimensions such as speed and acceleration, and could recognize fast and slow, but perceived acceleration as an almost instantaneous event rather than a steady change (Piaget, 1946/1970). While 6 to 8-year-olds were only capable of analyzing speed and acceleration through the comparison of the simultaneous motion of objects (Raven, 1972), 9 to 11-year-olds were able to understand speed as ratios or proportions of the distance and time quantities (Piaget, 1946/1970) and thus compare successive speeds of objects (Raven, 1972). A study of children aged 6 through 11-years-old showed that the comprehension of the interrelated dimensions of speed, duration, and distance in the form "A times B equals C" increases with age (Acredolo, Adams, & Schmid, 1984). In a different study, fourth and sixth grade students (aged approximately 9-12 years old) asked to make written descriptions of the motions of accelerating objects (such as a pendulum or a ball rolling up, and back down, a ramp) were observed making use of a "snapshot" description of motion: in written prompts, they noted the speed and direction of a moving object with no mention of the object's change of speed or direction, as if only reporting on snapshots of the object's motion (Dykstra & Sweet, 2009). From these studies, one would expect to see a 10-year-old exhibit understandings of motion involving comparisons and ratios of speeds, durations, and distances beyond simply "fast" and "slow", and a recognition of changing states of motion, even if he cannot quantify acceleration as a ratio of speed and time.

Representations of Motion

Since the early 20th century, a developmental progression of children's drawings, from scribbles to realism, has been recognized and studied extensively (Gardner, 1982; Luquet, 1913; Piaget & Inhelder, 1969). By the mid-elementary years, children use drawing, along with numbers and writing, as an expressive medium for communicating ideas to the outside world (Karmiloff-Smith, 1992; Tversky, Kugelmass, & Winter, 1991; Vygotsky, 1978). There is plentiful evidence that children use drawings and proto-graphs for representing scientific observations (Garcia-Mila, Andersen, & Rojo, 2009; Gravel, 2011; Lehrer & Schauble, 2006; Nemirovsky & Tierney, 2001; Rennie & Jarvis, 1995; Vosniadou & Brewer, 1992), and previous studies have looked at children's spontaneous representations of motion from the kindergarten and early-elementary grades (Kahn, 2010) to middle and high school (diSessa, Hammer, Sherin, & Kolpakowski, 1991; Sherin, 2000). The "native competence" (diSessa, 2004, p. 294) of a student to reflect on his own and others' representations has been studied as meta-representational competence (diSessa & Sherin, 2000), which describes, for instance, the ways in which a learner's focus on salient aspects of a representation, such as audience or elegance, can provide criteria for the interpretation and creation of representations. All in all, external representations provide a way to make a student's ideas visible for communication and analysis, and exist both as a tool through which students may learn, as well as a form of knowledge (Pérez Echeverría & Scheuer, 2009).

In the study reported in this paper, I explored representations created by a 10-year-old boy—Aiden—through two media: traditional paper-and-pencil drawings and stop-motion animations. A dynamic representation is one in which the dimension used to represent time is time, what Stenning (1998) terms "evanescence," and animations have been theorized to be better at representing quantities that change over time than static representations, utilizing time the way space on a page is used to represent space in reality (Tversky, Morrison, & Betrancourt, 2002). Dynamic representations are often in the form of computer simulations (Ainsworth & Van Labeke, 2004). Kaput (1998) contrasted computer-based and paper-based representations, saying that the former can be dynamic in ways drawings on paper cannot afford, and arguing that dynamic media can make mathematical subject matter "come alive," providing live linkage between, and even resulting in new, notational systems. Cox (1999) suggests that graphical tools influence visual thinking at least as much as word processing tools influence writing, and Healey, Swoboda, Umata, and King (2007) conclude that the shared development of notational systems is affected by the technology used to develop it. While the results of using animation for teaching are mixed, most studies have only examined students passively viewing animations, rather than designing their own (Tversky, et al., 2002), and the creation of animations of motion has already been recognized to require an understanding of the basic laws of physics, such as conservation of momentum and inertia (Thomas & Johnston, 1981). It could be that dynamic media, such as animation, are well-suited for students to represent their ideas about motion. This study compares Aiden's paper-and-pencil, static representations to his representation of motion through stop-motion animation, a dynamic representational medium.

Framework for this Study

Taking into account this background literature on children's understanding, and external representations, of motion, this study addresses the following research questions:

1. What kinds of static and dynamic representations of motion are created by a 10-year-old child, and how do these representations compare to each other?
2. How are a 10-year-old child's understandings of time, displacement, speed, and

acceleration expressed through the representations he creates?

In order to answer these questions, it is useful to characterize the learner's drawings with terms like figurative and non-figurative, while avoiding the dichotomies figurative versus abstract, or concrete versus abstract, following Healey et al.'s (2007) suggestion that the latter terminology, specifically the definition of "abstract," can be confusing and vague given its multiple meanings. For this paper, "figurative" is used to describe recognizable, iconic drawings for physical objects from the story, while "non-figurative" is used to describe "graph-like representations" constructed from combinations of lines and shapes that were clearly not identifiable as physical objects from the story. Also, I define the term "working convention" to mean the set of figurative and non-figurative elements and resources used in a representation that are intended by the learner to have meaning within the context of the car's motion. That is, one learner's set of working conventions may be quite different from that of another, and from one representation to the next a learner might build upon or revise their working conventions. Within the learner's working conventions for representing motion, many figurative and non-figurative elements show up repeatedly.

Researchers of meta-representational competence have defined "constructive resources" to refer to the "basic ideas for representing" that can be drawn upon for creating new representations (Azevedo, 2000; diSessa, 2004; Sherin, 2000). I believe that any resources students use in the production of representations are necessarily "constructive", and I henceforth refer to these basic ideas simply as resources. In this literature, many of these resources have been identified among student-created representations of motion, including the following: (1) "drawing," or the previously discussed, socially established, figurative icons frequently used by children (such as a car, a sun, or a stick-person); (2) "temporal sequences," or symbols chronologically organized along a linear dimension representing time (and related to standard notational systems, such as left-to-right text; (3) “line segments,” the use of lines or circles, varying for example in size or slope, to represent a quantity (such as using the length of a line or the diameter of a circle to represent the speed of a car); (4) “more is more,” or using a greater quantity of some representational element to stand for a greater quantity of some attribute (such as using more motion lines to represent faster speeds) and (5) written language. The main argument underlying this paper is that examining the use of these resources in Aiden’s representations would allow me to identify specific understandings of motion, such as speed, distance, and duration, that have been described in previous studies. I was also interested in finding what role these resources would play in the shift from static to dynamic media, and whether new resources would emerge.

Method

The interview participant was Aiden, aged 10 years old, who at the time of the interviews attended 4th grade at a public elementary school in the largest school district in the state. Aiden participated in three interviews, one every three weeks, in the researcher’s home during after-school hours. During two of the interviews, Aiden’s younger brother Liam (7 years old and in the 2nd grade) was present, although this paper will only focus on the analysis of Aiden’s representations. The general procedure of each interview is described in the section below. All artifacts created during the interviews, marker drawings on paper, and stop-motion animations were digitally archived, along with video recordings made during the interviews. Each interview session was transcribed the following week and reviewed in order to plan the activities and prompts for the subsequent interviews. After all of the interviews were completed, they were analyzed using open coding and constant comparative analysis (Glaser & Strauss, 1967;

Merriam, 1998) to reveal several themes that corresponded to the two guiding research questions. A comparison of the three unique representations of the story, and a discussion of the understandings of the car's motion within the representations, provide the focus for the findings section.

The Interviews and Representations

The First Interview

The main task addressed in the interviews was introduced by reading the cactus problem, a modification of a scenario presented in diSessa et al. (1991): *A driver in a car is speeding across the desert, and he's very thirsty. When he sees a cactus, he slows down and stops to get a drink [of water] from it. Then he gets back in his car and drives slowly away.* Aiden was then asked to "draw something that shows how the car moves in the story so someone else could understand how the car moves without hearing the story." The words "of water" were added to the cactus problem text after some confusion during the first interview regarding exactly what the driver was drinking. Aiden independently created his first drawing, followed by a discussion about what he depicted, what this meant in relation to the car's motion and the story, and reflection regarding what the drawing did well or poorly.

Aiden's first drawing (see Figure 1) was highly figurative in nature, an approach to representation which has been observed in drawings of motion among younger children (Kahn, 2010) as well as among students as old as high school (Sherin, 2000). This drawing made use of several resources to indicate the car's motion. Line segments (or motion lines) can be seen trailing the car in Figure 1, panels a, b, and e; Aiden's working convention here was that longer motion lines represented faster speeds, confirmed by the fact that the lines in panels a and b are longer than the lines in panel e. In his drawing, Aiden also made use of a "more is more" resource, through correlating the length of the lines with the speed of the car. Also in Figure 1, panel a, is the drawing of a "puff of smoke" that Aiden used to represent only fast speeds, which he explained as dust being "kicked up" behind the car. This same representation has been observed among younger children (Kahn, 2010). The written word "poof" accompanies the cloud of smoke to reinforce the fast speed of the car, while the word "screech," imitating the noise of the brakes can be seen in Figure 1, panel c as the only indication that the car is slowing down. In Figure 1, panel e, Aiden not only included shorter motion lines to represent the slow speed of the car, but also included a drawing of a magnified view of the car's gear shifter set to "slow."

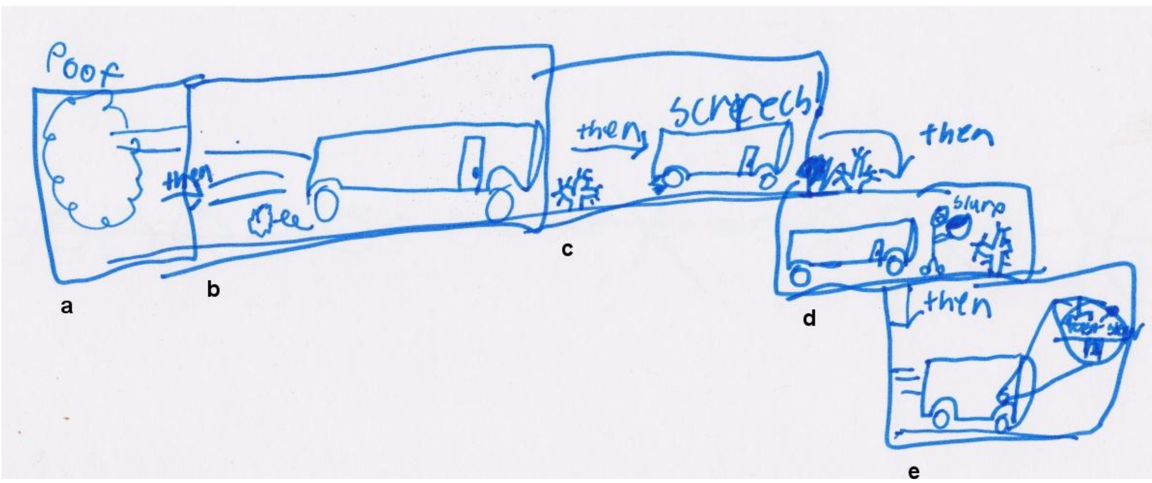

Figure 1. Aiden's first representation: a) and b) the car going very fast; c) the car stopping; d) the car stopped with the man drinking from the cactus; e) the car moving slow, with a magnified view of the gear shifter inside the car set to "slow."

Next in this interview, I presented Aiden with representations of motion (see Figure 2 for one example) previously produced by 6th grade students (diSessa, et al., 1991), with the prompt, "let me show you something and let's see if you think this is an OK way to draw the story." I introduced these representations produced by an outsider because I wanted to see how Aiden would interpret, critique, and appropriate these drawings into his own. I did not have several weeks and a full class of students in which to allow Aiden to invent his own graph-like representations, as in diSessa et al. (1991), and yet I felt that, given the consistency of the use of line segments and temporal sequences for this task across many children in other studies (Sherin, 2000), these proto-graphs would be closer than Cartesian graphs to what Aiden would invent for himself. When first confronted with these representations Aiden was confused, but once the cactus problem was re-read aloud, Aiden immediately recognized how the representation used non-figurative symbols to depict the motion of the car (see transcript lines 1-29 below). Finally, I asked Aiden to create a drawing that he thought was "better than what you did before," leaving "better" undefined. In response to this prompt, Aiden produced a non-figurative representation (see Figure 3). He did not simply copy the symbolic conventions for representing speed from the example drawings, but instead invented a different working convention using circles. In Figure 3, the diameter of the circle indicates the speed of the car, with large diameters representing faster speeds, and stopped indicated by a large "X." Here, Aiden utilized the resources of line segments and "more is more," in the form of circles, as well as a temporal sequence indicated by a linear progression of symbols over time (reading the representation from left to right).

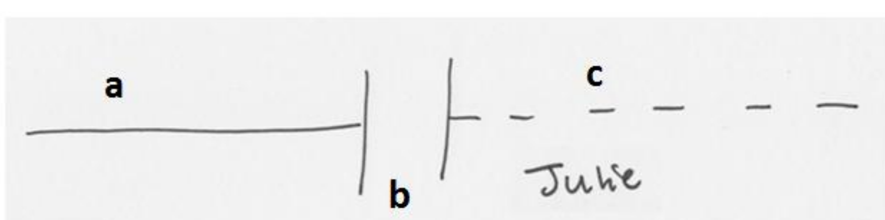

Figure 2. Julie's representation, where part a is moving fast, b is stopped, and c is moving slowly. Adapted from diSessa et al. (1991).

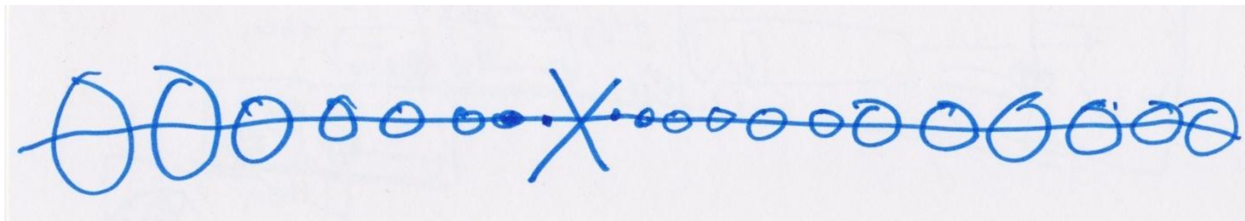

Figure 3. Aiden's second representation: Circles represent the speed of the car, with larger diameters indicating faster speeds, and stop represented by a large "X." The smaller diameter of the circles after the stop indicates that the car is slower than at the beginning.

The Second Interview

In the second interview, three weeks later, Aiden was asked to "show me on paper the motion of the car in this story so that someone else could understand it without hearing the story." This time the instructions explicitly avoided use of the word "draw;" however, "draw" and "drawing" are freely used throughout this study with no intentional bias toward figurative representations. In response, Aiden created another non-figurative representation (see Figure 4) similar to his last drawing from the first interview (see Figure 3), but developed yet another unique working convention for representing the speed of the car, this time using lines more characteristic of slopes and Cartesian graphs, instead of circles:

A: How curved the line is equals how fast. Like there (pointing to his series of symbols in Figure 4).... Fast, fast, fast, slow, slower, slower, slower, slower, sloooower, really slow, stopped. Stopped, stopped, slower, faster, a little bit faster, faster, faster, faster, faster, faster, but not as fast as before.

I: Ok. So, if you could say that there's a rule to your picture, for like the speed of the car, what would that rule be?

A: The more it is like that (producing Figure 5, acute angle a), the faster it is. The more it is like that (producing Figure 5, obtuse angle b), the slower it is. If it's a [vertical] straight line (producing Figure 5, vertical line c), it's completely, um, stopped. If it's a [horizontal] straight line, it's the fastest it can possibly go. It'd be like that, kind of (producing Figure 5, very acute angle d).

Figure 4, and Figure 5, in which Aiden clarifies the conventions he is using, illustrate how Aiden used the acuteness or obtuseness of an angle to communicate the speed of the car in the same fine-grained way as he had used the diameter of the circle in his previous drawing, Figure 3, evoking once again the resources of line segments, “more is more,” and temporal sequences.

In the second interview Aiden returned to a figurative representation (see Figure 6) of the car’s motion (following my questioning of Figure 1) in order to clarify his figurative working conventions. Figure 6 shows the same four states of motion as were represented in Figure 1: (1) fast, (2) slowing down, (3) stopped, and (4) slow; however, the cars are not ordered in any kind of temporal sequence because I asked for him to draw each case separately instead of representing the entire story at once. At this time I asked him to confirm his working conventions for the motion line length/speed relation, and Aiden explicitly stated that, for both his first representation (Figure 1) and this representation (see Figure 6), he varied the length of the motion lines to correspond to the speed of the car, exhibiting use of the line segment and “more is more” resources. Aiden used written language (see Figure 6, car b) with “screech” to show deceleration. He then added car c, with line segments around the wheels to show the sparks emanating from the brakes in case his viewers could not read the word “screech.”

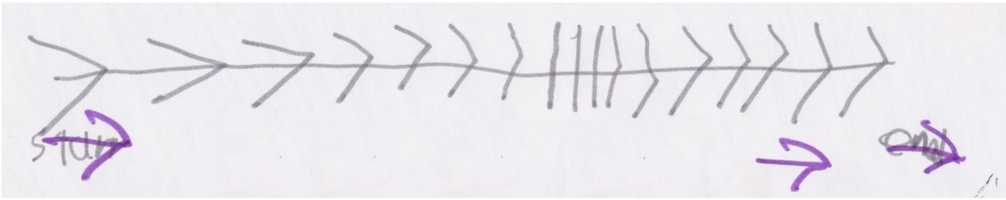

Figure 4. Aiden's third representation: acute angles represent faster motion, and a vertical line represents stopped. "Start" and "end" were replaced with arrows indicating the direction of time in the drawing.

Figure 5. Details of Aiden explaining his third representation (Figure 4). From left to right: (a) fast, (b) slow, (c) stopped, (d) as fast as the car can go.

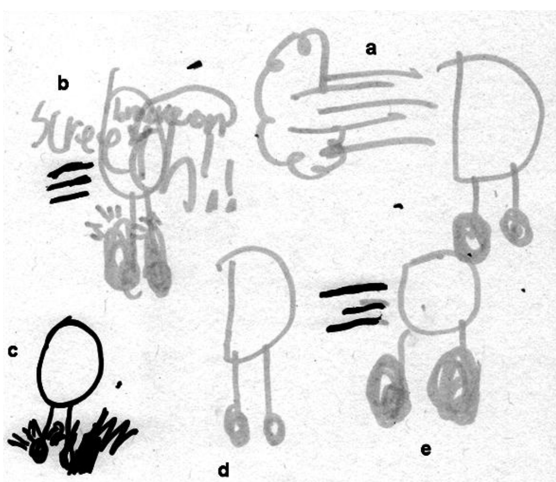

Figure 6. Aiden's fourth representation: a) the car traveling fast; b) the car slowing; c) the car slowing without using words; d) the car stopped; e) the car traveling slowly.

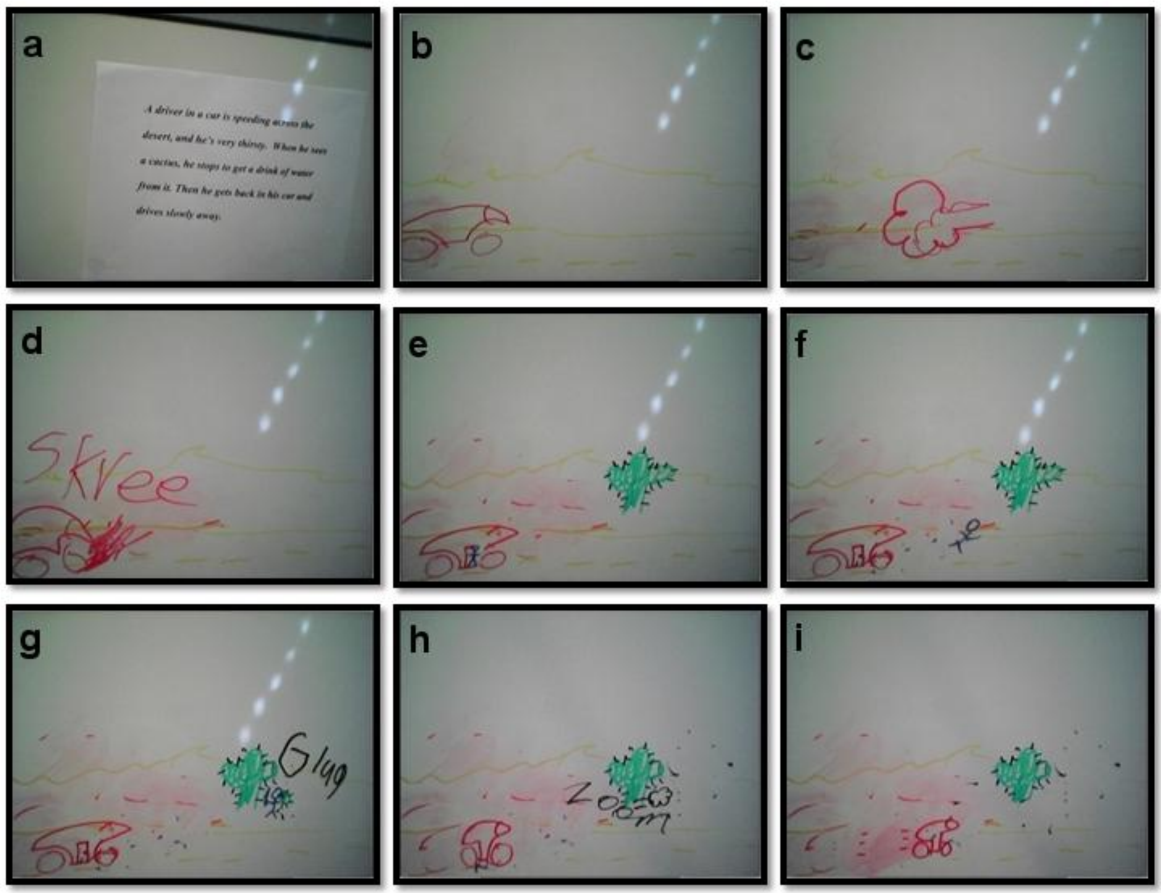

Figure 7. Selected frames from Aiden's animation: a) the text of the cactus problem; b) the car traveling fast; c) a cloud of smoke left behind by the fast car; d) the car braking, with sparks from the wheels and “skree”; e) the man exiting the car; f) the man walking to the cactus; g) the man drinking from the cactus; h) the man running from the cactus to the car with “zoom”, motion lines, and a cloud of smoke; i) the car driving slowly away with short motion lines.

The Third Interview

At the end of the second interview, I briefly introduced Aiden to the SAM Animation software and let him create an open-ended movie. Aiden, a fan of claymation, was eager to create his own examples of animation, and quickly adopted the terminology, such as "frame" and "onion skin," used in the software. I chose to have Aiden create his animations with a dry-erase marker on a small whiteboard, instead of with clay or other three-dimensional props, so that the introduction of time as a variable would be the only major difference between the static and dynamic media. In the third session, three weeks after the second interview, I asked Aiden to make a stop-motion animation of the cactus problem (see Figure 7). His animation adopted figurative elements similar to those used in Figure 1 and

Figure ***6***, from the first and second interview, to depict the motion of the car and its driver. Like his previous drawings, the slow car in Figure 7, frame i has short motion lines to represent its speed; the man running from the cactus to his car in frame h has the word “zoom,” motion lines, and the drawing of a cloud of smoke to indicate his speed; and the word “skree” was used with line segments as sparks at the wheels to indicate the car slowing down in frame d. However, the fast car in Figure 7, frame c, has the drawing of a cloud of smoke but no motion lines, and Aiden animated the man moving from his car to the cactus in frame f using only figurative drawing, with no motion lines or written words.

Results and Discussion

To address my two research questions, I examined the characteristics of the representations Aiden created during the three interviews by classifying them into three categories. Aiden’s initial representation, Figure 1, was static and figurative in nature (along with Figure 6 to clarify his figurative working conventions). Later in the first interview, Aiden created a non-figurative static representation, Figure 3, an approach to representation that was subsequently repeated at the beginning of the second interview in Figure 4 (with clarifications in Figure 5). Finally in the third interview, Aiden created a dynamic representation with figurative elements in the form of an animation, Figure 7. A summary of the characteristics of these three types of representation can be found in Table 1 below, and will be explored in this section.

Type of Representation	**Characteristics of Representation**

Figurative, Static (Ex: Figure 1, Figure 6)	- Student asked to "draw" the story - Initial way of representing the story - Realistic in nature, with a narrative quality - More qualitative in depiction of car's motion, focus on "fast" and "slow" - Large Δt - Snapshot treatment of acceleration - Uses drawing, line segments, "more is more," temporal sequence, and written words
Non-Figurative, Static (Ex: Figure 3, Figure 4, Figure 5)	- Student asked to "show me on paper", or draw a "better" representation of the story - More quantitative in nature, proto-graph - Smaller Δt - Gradient of speed - Able to compare speeds within and between representations - Defines acceleration as $\Delta v/\Delta t$ - Uses line segments, "more is more," temporal sequence, and written words
Figurative, Dynamic (Ex: Figure 7)	- Influenced by cartoons and traditional examples of animation - Student considered it the "best" because it shows "what's going on" and "speed" - Return to the figurative type of representation - Time is explicitly represented by time in the medium - Inconsistent use of resources from static drawings - Use of $A \times B = C$ relationships: $d = v \times t$ and $v = a \times t$ - Uses drawing, line segments, more is more, temporal sequence, and written words

Table 1. Characteristics of the three types of representations created during Aiden's interviews.

Three Unique Representations of One Story

As Aiden progressed through the interviews, the three representational approaches identified above remained distinct, and Aiden seemed to shift effortlessly from one form to the next. However, all three representations made use of the same resources, with the absence of drawing in the non-figurative cases. What inspired Aiden to take three different approaches in representing the same story of the car's motion?

Aiden's initial spontaneous representation of the car's motion (see Figure 1) was figurative, utilizing drawings and words as the qualitative resources for depicting motion. In the interview, I instructed Aiden to "draw" the story so that "someone else" could understand the motion of the car, and even though I was conscious not to call his representation a "picture," the use of the word "draw" could have encouraged him to produce the figurative drawing. I believe it is more likely that his drawing exhibited a high degree of realism because this quality is valued by children in their drawings, even when representing scientific phenomena (diSessa, 2004; Lehrer & Schauble, 2006). Aiden's figurative drawing focused on the narrative of the story more than on the quantification of the car's motion, and emphasized the sequential nature of the four states of motion from the story rather than the durations of each of the states. Aiden had originally linked these "frames" together with arrows and the word "then," leading the reader from left to right and then down following the progression of the car's motion as one would read written English; when describing his drawing later to me, he chose to add the boxes around each frame to emphasize how they were like a comic strip. Aiden does include motion lines behind the car as line segments providing a rudimentary way to quantify the speed of the car, but the variation between long and short lines is not pronounced. When Aiden was asked to revisit the working convention of his figurative drawings (in

Figure ***6***), his use of the length of the motion lines to indicate the car's speed was deliberate, as confirmed through questioning. This refinement of his working convention is either the influence of the prominence of the more is more resource from his non-figurative representations, or his effort to clarify his working convention to his audience in the face of questions about its meaning. This phenomenon, where students become increasingly attentive to simple conventions of correspondence, is called "systematicity" by diSessa (2004, p. 295).

Despite repeating the use of several resources from one representation to the next, Aiden was initially resistant to an outside, non-figurative representation (i.e., Figure 2):

I: What about this (showing them Figure 2)?

A: That's not a really good way.

L: That's not a road!

I: That's not a road? Ok.

A: The road is definitely misplaced. 'Cause if we could do it, that would be inside that, and that would be inside that (indicating how he would move the lines around to make the picture look like a road with dashed lines down the middle).

I: Oh, I see. So you're saying it looks like a road, but the lines are outside of the road.

A: Yeah.

I: Ok. What if I told you that a girl named Julie drew this picture—she did—to represent this story, and she showed in this picture the car speeding across the desert, the car stopped while the guy eats the cactus, and the car slowly driving away.

A: Oh, I see it! It's moving at a steady pace as a line (see Figure 2, part a), and then there's nothing in there (see Figure 2, part b), which means he's stopped, and then totally eating the cactus, and then slowly he moves along, not really stopping, but not moving as steadily, but moving a little slower (see Figure 2, part c).

Sherin and diSessa suggest that the invention and adoption of resources are inseparable (2000). Once Aiden arrived at an interpretation of the non-figurative representation that he was comfortable with, he immediately accommodated its resources into his own representation (see Figure 3), which following Posner, Strike, Hewson, and Gertzog's (1982) theory, suggests that Aiden may have found fault in his previous figurative drawing in order to motivate his adoption of a new non-figurative one.

Aiden spontaneously produces another non-figurative representation (see Figure 4) at the start of the second interview, this time with a different working convention for representing speed. In his critiques of his own drawings, Aiden did not explain the shift from one non-figurative working convention (circles in Figure 3) to the next (angles in Figure 4); he did, at this time, express a preference for the non-figurative representations over figurative ones because he believed they were better at showing the exact speed of the car:

A: [I] thought the second drawings [the non-figurative representations] were a little better, so [I] stuck with those ideas.

I: Why do you think there were better?

A: Because, like, they were a little more descriptive? And, like, someone who couldn't figure out the, um, pictures, like someone who didn't know what a bus was, or a cactus, they wouldn't understand our original pictures. They would understand this one more (pointing to Figures 3 and 4), because they could figure out what the car is *doing*, because in our things [the original figurative drawing, Figure 1] it kind of showed it [the motion of the car], but it was more like pictures. But if somebody never saw a car, or never saw a cactus, this (pointing to Figure 4) would be the better choice. So, [I] stuck with those ideas.

I: So, you don't have to know about cars and cactuses to understand these pictures?

A: You just need to understand speed.

Aiden recognizes quantifying the speed of the car as an important goal of his representation. He temporarily abandons all of the figurative elements (namely, the resource of drawing), but keeps the remaining resources he has used in Figure 1, repurposing them as non-figurative symbols representing the car's speed in Figures 3 and 4. These non-figurative representations utilized line segments, "more is more," and temporal sequences, just like his figurative drawing (see Figure 1) had, in a non-figurative approach similar to the example representation from Julie (see Figure 2), but each with a unique working convention for depicting motion in the cactus problem. Written words were only used once to label the direction in which his representation (see Figure 4) should be read, but were later replaced with arrows. He explicitly mentioned how other people would interpret his drawings, implying that he created the non-figurative drawings with an audience in mind. This may be due to the follow-up questions I asked during the interview regarding whether other people, aliens from another planet or people who could not read, could understand the story from his drawing. This was the stated goal of the exercise each time I prompted Aiden to create a new representation, but he seemed to attend to the audience more after constructing Figure 1.

With only about two hours of work on static representations and a tutorial of the SAM Animation software, Aiden was able to construct a simple animation of the car's motion and describe how he would improve the animation to better quantify the speed of the car, leading to several discussions about the relationship between speed, acceleration, displacement, and duration. Because the animations were essentially also drawings with markers on a 2-D surface, just as his static representations had been, we could expect that Aiden would transfer many of the resources he used in his figurative drawings to his animation, but only did so haphazardly. Motion lines, for example, were sometimes included to represent speed, and sometimes not used at all; it is possible that the less consistent usage of some resources resulted from his inexperience with the medium. Aiden could have chosen to make a non-figurative animation, but perhaps animation was similar enough to drawing that a figurative approach seemed appropriate, or he placed more emphasis on the goal of creating a typical, narrative, “cartoon” animation than the goal of representing the car’s motion. Later in the interview he stated that he prefered the "real life" animations because "you can actually see the car moving" and "it will be easier for people to understand it." This again shows attention to the audience of the representation, and a desire to return to realism in his drawings. In this dynamic representation, however, realism can be employed alongside the dimension of time that is built-in to the medium to provide an opportunity to produce a quantitative depiction of motion.

To answer my first research question, regarding what kinds of static and dynamic representations of motion are created by a 10-year-old child, and how those representations compare to each other, Aiden transitioned from figurative to non-figurative static representations, and then back to figurative for his dynamic representation, all while expressing his preference for quantitative and “more easy to interpret” representations. Aiden was possibly influenced in choosing figurative or non-figurative approaches, or to consider his audience, by the framing of the task when asked to initially “draw” and subsequently “put on paper” the motion of the car so that “someone else” could understand it. It is undeniable that the introduction of an outside, non-figurative representation of the story contributed to Aiden’s esteem of the non-figurative approach. Aiden continually revised his approach to representation, considering the audience for his representation and the affordances of previous approaches; he refinined his figurative representations to reflect the more quantitative nature of the non-figurative representations by explicitly defining the length/speed relationship of his motion lines, and he made several changes to his figurative drawings that made them, in his opinion, easier for other people to understand. In his animation, he expressed a preference for a “real life,” figurative approach, again citing that it was easier to understand. This exhibits several characteristics of meta-representational competence outlined by diSessa (2004), and echoes studies of multiple representations in physics problem solving (Kohl & Finkelstein, 2006, 2008): Aiden is selecting to represent the story with figurative or non-figurative representations based on social or meta-representational factors rather than content-related considerations, for all three representations depcit the same story of a car's motion. This suggests that in three hours of interviews and interaction with static and dynamic media, combined with his competence for understanding and critiquing representations, Aiden is able to produce three unique approaches to representing a car's motion. They all utilize the same set of resources, suggesting it is something other than the resources which defines one approach apart from another.

Representing Ideas about Motion

The other goal of these interviews was to investigate how Aiden's understandings of the motion of a car could be represented through static representations and stop-motion animation. I did not set out to teach Aiden anything about speed or acceleration, but rather to elicit his ideas through the creation of representations. What role did the resources or the medium used to represent motion play in the understandings Aiden expressed in these representations and the interview discussions?

Aiden's figurative representations relied frequently on the resources of drawing and written words; realism in children's representations has already been discussed above, and written-word labels are also a common resource used by children for depicting motion (Kahn, 2010; Sherin, 2000). However, while labeling the speed of the car as "15 mph" would be an example of using numbers and words to quantify the speed of the car, Aiden's words, "poof," "screech," or "then," are only qualitative in nature, merely identifying the car's state of motion, or its transition from one state to the next. Elements of the drawing become quantitative when they offer some way to compare one motion to another, perhaps through the quantity or length of a line segment, and is evidence of a step toward a more “scientific” representation (Sherin, 2000). That is, the “more is more” resource is a rudimentary way to measure the car's speed. Except for the motion lines, which varied in length for different speeds, all of the resources used in Figures 1 and 6 are qualitative in nature and focus on the four states of the car's motion explicitly referenced in the cactus problem: fast, slowing down, stopped, and slow. Aiden presented the discrete motions as snapshots in the order in which they happen with a very large Δt, but does not indicate the duration of each state of motion, or the relative position of the car from its starting point during these states of motion. This focus on the individual states of motion rather than the change between the states is reminiscent of elementary students only attending to "snapshots" of an object's motion, describing only the speed and direction of moving objects (Dykstra & Sweet, 2009; Piaget, 1946/1970).

When I asked Aiden to clarify his working convention for his figurative drawing (see Figure 6), I specifically challenged his more qualitative resources to see how he would revise his drawing to become more quantitative:

I: What if someone couldn't read? Or didn't speak English? Do you think they'd know that your car is stopping?

A: Well, you could put it like that way, cut off that one (see Figure 6, car b, slowing down) and be "beep, beep, beep" (pointing to Figure 6, fast car a; Figure 6, stopped car d; and Figure 6, slow car e only). But like: going.... stop! Sloooower. They would get the main idea. I could put the brake stop to show it’s braked. It’d be like “bonk!”

I: So, what’s that thing you just drew?

A: It’s like a little brake thing. A block in the road. It’d be like, “Shhhhhhhrrrrruuuuu-donk! Bonk, bonk, bonk.” Slow down (indicating with his hands the car moving, running into the block in the road, and bouncing against it, stopping).

Aiden first decided to leave out the "slowing down" car, Figure 6, car b, to make the drawing more understandable, and to give viewers "the main idea." This simplification of the story through truncating the car's deceleration can also be seen in Figure 6, car c, where Aiden decided that if he could not use words, he would represent slowing down with an instantaneous deceleration of the car running into a road block. This is also reminiscent of students of all ages treating acceleration as simply Δv, ignoring the dependence on the time interval Δt in which the change in velocity takes place (Jones, 1983; Piaget, 1946/1970; Trowbridge & McDermott, 1981). Removing the qualitative elements from Aiden’s figurative representations left the large-grained drawings showing motion as abrupt and simplified, distilled down to only what is absolutely necessary to indicate that the car is changing its motion at all, and lacking in quantitative information about acceleration, as well as the duration or distance.

In his non-figurative representations (see Figure 3 and 4), Aiden employed temporal sequence in the form of multiple, repetitive line segments to show the car's speed. Instead of using a separate drawing for slowing down, as he did with his figurative drawings, Aiden produced a gradient of a single symbol from fast to slow to represent the deceleration of the car. In the figurative drawings, the ways in which Aiden represented speed and acceleration are sometimes conflated; a word accompanying a drawing of a car in Figures 1 or 6 could indicate a speed, as in "vroom," just as likely as it could indicate a deceleration, as in "screech." With the non-figurative representations, the symbols strictly represent speed, and it is the slight variation of the symbols as they are repeated that indicates not only that the speed is changing, but also the rate at which it is changing. The car speeding up from stopped to slow was never shown in the figurative drawings (see Figures 1 and 6). However, in both non-figurative representations (see Figure 3 and 4), Aiden did indicate a slight increase in speed at the end of the drawing, rather than an abrupt shift from stopped to moving slowly, shifting from the snapshot view towards a more normative view of acceleration (Dykstra & Sweet, 2009). The interviews never addressed whether he was aware of this shift between representing and not representing this slight acceleration when moving back and forth between the figurative and non-figurative representations; however, I suspect it was not a deliberate choice, but rather having to do with the affordances of the non-figurative approach for focusing on the motion of the car, rather than the extraneous, realistic elements of the story, such as drawing the car, or cactus, or driver. The non-figurative representations' small Δt encouraged Aiden to focus on the micro-steps between the larger states of the car's motion, thus encouraging him to include more information about the car's motion than in a figurative representation, and allowing for a deeper exploration of speed and acceleration, which are rates that necessarily rely on time.

The large-grained nature and narrative approach of Aiden's figurative drawing (see Figure 1) led to a snapshot interpretation of motion (Dykstra & Sweet, 2009; Piaget, 1946/1970), where the states of motion were presented sequentially and qualitatively. The fine-grained approach of his non-figurative representations (see Figures 3 and 4) afforded a more continuous representation of the car's speed through a smaller Δt, thereby allowing for the expression of a more quantitative and sophisticated understanding of the car's changing motion. Healey et al. (2007) noticed similar trends with participants creating representations of pieces of music. The non-figurative representations were fine-grained, semantic models that focused on the internal structure, the melody, tempo, or dynamics, of a piece, while the figurative drawings embodied thematic associations or a narrative for the entire piece. Subjects tended toward the non-figurative approach when they needed to compare their representations with another participant, and the researchers predicted that the participants who created non-figurative representations were attending more closely to the details of a piece of music than those who created figurative ones. This parallels how Aiden preferred the non-figurative approach of representations when wanting to demonstrate the motion of the car to other people, and the non-figurative representations' affordances for representing more quantitative aspects of the car's motion.

After creating his first non-figurative representation (see Figure 3), Aiden realized that his use of the line segments allowed for represented speeds to be compared with one another, something that was possible with the motion lines in the figurative drawings, but not obvious from the representation's construction.

A: I put mine [slow circles] almost as big [in diameter] as the big ones [fast circles], but still not as big [as his].

I: I see what you're saying. So, you're saying that his slow is slower than your slow. Does that make sense?

A: Yes. His slow is way slower, like, it would be moving at three miles per hour.

L: Yeah, you're saying slow. We don't know how slow.

A: But that is way slower than mine.

I: We don't know, do we? The story doesn't tell us, "He moves at fifteen miles per hour."

The diameter of a circle or the degree of an angle were never explicitly quantified by Aiden to produce a precise quantity for speed, but the diameters of two circles could be compared to give a definitive comparison of the car's two speeds, almost in the way a child will compare the simultaneous or successive motions of two balls or cars on a ramp (Raven, 1972). With little prompting, Aiden was very thorough in his definitions of the range of speeds capable of being represented by his symbols (see Figure 5); even without invoking specific angle measurements, an angle did not simply represent fast, but a degree of fastness, allowing for some quantification of speed and the construction of a speed gradient, thereby also defining an acceleration. The angles from his second non-figurative representation, in Figure 4, were a more sophisticated choice for representing the quantity of speed, because they could represent the entire continuum of possible speeds from zero to infinity in one symbol, than Aiden's original circles from Figure 3, which could also represent the complete continuum (although infinite diameters present logistical problems) but where a separate symbol was included for zero speed. This shows improvement in how motion is quantified even within subsequent representations of the same approach.

On the surface, Aiden’s animation looked similar to his figurative drawing, often employing the same resources (drawings, words, line segments, “more is more”) to show the car’s motion. However, the explicit presence of time in the animation meant that he could now use the periodic motion of the car across the screen as a representation of the car's speed. Aiden used only drawings, combined with the dimension of time in the animation, to represent and even quantify the car’s motion, rendering the resources he had used in previous representations, such as motion lines or words like “zoom”, redundant. Throughout the third interview, Aiden discussed the contrast between working conventions in his animation that are, in his words, like "real life," or using only drawing and the dimension of time to show the car’s motion, versus those that are more like a "cartoon:”

I: So this frame (see Figure 7, frame b) doesn't have lines or a puff of smoke.

A: 'Cause that's basically, it's moving along steadily but not slowly. You show it in the picture as you look at it, as it moves from place to place, ‘cause like in real life it doesn't always have to have lines behind it, or a puff of smoke to show it's moving. It could just be moving [literally across the screen in the animation].

I: You didn't show any lines to show that the man was moving [from the car to the cactus] (pointing to Figure 7, frame f). Right?

A: Yeah, it's basically kind of like in real life, because you see him going from place to place [he literally moves across the screen in the animation].

I: If you wanted to show the car moving faster, how would you show that at the end here (pointing to Figure 7, frame i)? Like, what if I were a director and I said, "Oh, at the end, the car there, it's not quite moving fast enough." Like, how would you change it to make it move faster?

A: Um, take it [the pictures of the car] a little farther apart [on the screen] so when you play it together it looks like it's moving faster. 'Cause it's in quicker places. Instead of going "duh-duh-duh-duh-" (moving hand along in short hops) it goes "duh-duh-duh-vroooom!" (moving hand along the table in longer hops, and then flying away).

I: So, each frame the car would be farther apart. That makes sense.

A: Instead of closer together.

Aiden highlights that he does not need to use puffs of smoke or lines to show how fast the car is moving because now, in the animation, he can adjust how far the car moves along the screen and from one frame to the next (thereby controlling the car’s speed), instead of relying on the resources he had used in the static representations. He shows evidence of understanding the "$A \times B = C$" relationship between speed, duration, and displacement (Acredolo, et al., 1984) because he can describe how he would alter his animation to make the car appear faster. Because Aiden never addressed displacement or constant Δt in his previous representations, there was never an opportunity for him to think about these kinds of mathematical relationships between the variables he was representing. The creation of the animation allowed him to focus on the quantitative aspects of the cactus story, relying on the frame rate of the movie to define a consistent duration and Δt, but in the figurative approach of his original static drawing, which he previously claimed to prefer for animation.

Aiden also never addressed how much he explicitly understood the "A x B = C" relationship between acceleration, duration, and speed while constructing static representations. In the third interview, while animating the cactus problem, I asked about the significance of using a certain number of frames to represent one state of motion. His answer showed an understanding of the change in speed given a fixed acceleration over many small time intervals:

I: So, are you saying that lots of [frames] going fast and only a few [frames] slowing down means that, what?

A: Means that, it's not really going exactly really that fast, 'cause if it could slow down really easily, it's not moving that fast.

I: So you're saying only showing three pictures of slowing down means that it's slowing down quickly.

A: Exactly. If it takes just a little while to slow down, it's not moving very fast in the beginning.

Aiden communicated that acceleration depends not only on a change in speed, but also the time interval over which that change takes place, an aspect of the definition of acceleration that eludes many college physics students (Trowbridge & McDermott, 1981). In a more sophisticated utilization of animation, Aiden could have varied the number of frames or distance traveled by the car in each frame to indicate the duration of acceleration, but instead his animation used sparks from the brakes (i.e., line segments) and the word "skree" to show the car slowing down, and did not address the acceleration from stopped to slow at all, much like his first figurative drawing (see Figure 1). As was discussed above, Aiden was unaccustomed to the dynamic medium, and he was just discovering how to represent speed in a "real life" fashion, initially hesitant to discard the resources that had been helpful for conveying motion in the past. Revision of the animation may have led to abandoning these old resources that worked well for figurative static drawings in favor of representing the actual accelerations of the car.

Regarding my second research question, through the three interviews, the understandings of the car's motion exhibited by Aiden's representation evolved from a rudimentary definition of acceleration as a change in speed within the figurative drawings, to acceleration as a ratio of the change in speed and Δt, finally to the animation, where the explicit treatment of the time dimension allowed Aiden to explore the $A \times B = C$ relationships inherent in $d = v \times t$ and $v = a \times t$. This is not to say that Aiden did not understand, for example, the direct relationship between velocity and acceleration before creating his animation. Instead I claim that it was the resources that Aiden chose to use (or not to use) acting in concert with the affordances and constraints of the medium in which he was operating which provided the opportunity to bring his understanding of that relationship into an external representation.

An Interesting Case of Zero Speed

An example of how an external representation can help to explore a student's ideas of motion and representations came while discussing Aiden's understanding about the non-figurative approach towards representing speed, and particularly determining why he used an entirely different symbol for stop instead of modifying the symbols he was already using to represent zero speed in Figure 3. I asked Aiden to comment on a sequence of symbols produced by another student (see Figure 8, on the left), each slower than the last, ending in a symbol for stopped; in that representation, the width of the box got smaller and smaller as the car got slower, but instead of a vertical line (I expected that zero speed would be a box of zero width), Aiden's agreed with the other student's working convention, and suggested switching to a completely different symbol to represent stopped:

A: I'd draw anything for stopped.

I: Anything for stopped?

A: I'd draw... a horizontal line, possibly.

I: Why would you do that?

A: Because it's like that, that, that, that, that (bringing his hands together, indicating decreasing width of the rectangles), and then it will turn into one line. But if you had a horizontal line, that would show that it's stopped; it's not moving.

I: Mm-hmm.

A: Because it can't be up and down. It's not put on how far up and down it is [varying the height]. It's just put—so a straight [horizontal] line might work.

He recognized that if the speed of the car was getting slower and slower, the width of the box would eventually decrease to just a vertical line. However, he also insisted that a separate symbol, in this case a horizontal line, be used for stopped because the height of the line was not an explicitly defined variable within this representation's working convention (see transcript line 98). In Aiden's second non-figurative representation (see Figure 4) his working convention for representing speed also allowed for a variation of the same symbol to be used for zero speed (the angle would become more obtuse as the car slows until it became a vertical line when stopped). I believed that in this second interview he had finally come to the realization that stopped was not a separate state of motion, but rather a magnitude of speed. However, when I asked him to explain how he determined the symbol for stopped in Figure 4, he still considered the vertical line to be a different symbol altogether from the angles:

I: So, if I wanted you to draw something that was really, really, slow—almost stopped—how would you draw that?
A: I would draw something like... (draws a very obtuse angle, Figure 8, a).
I: What about if I wanted you to draw slower than that?
A: There (draws a more obtuse angle, Figure 8, b).
I: Ok.
A: Maybe I could do it a little slower... (draws a more obtuse angle, Figure 8, c). There. That's the slowest I can get without stopping.

Aiden never seemed to recognize stopped as the quantity of zero speed, even though his “slowest” angle approximated a straight line. In other studies of mathematical representations (Hughes, 1986; Nemirovsky & Tierney, 2001), zero quantity is often represented by children with a special symbol, regardless of their previously defined working conventions for representing quantity. These studies suggest children stray from these otherwise mathematically-consistent conventions as a way to emphasize zero because they see it as a special case rather than just another magnitude, of speed for instance.

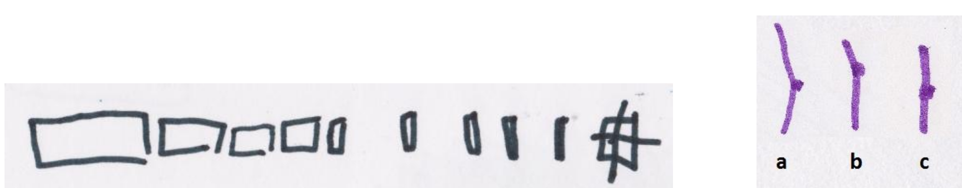

Figure 8. On the left, another students’ attempt at representing slower and slower speeds, ending

in a symbol for stop. On the right, Aiden using the working convention from Figure 4 to show slow speed (a), a little slower (b), and the slowest speed he can draw without the car stopping (c).

Conclusion

Two types of drawings (figurative and non-figurative) and one animation of the motion of a car produced by an elementary student were examined to compare the resources used in their creation and the ways in which they allowed me to explore the students' understandings of motion. Many resources previously observed in other students' works, such as drawings, words, "more is more," temporal sequences, and line segments were present in all three types of representation. The figurative drawings exhibited rudimentary understandings of motion, representing changes in the car's speed as snapshots rather than continuous accelerations. The non-figurative representations depicted slightly more quantitative information about speed and acceleration using almost the same set of resources, but incorporating the idea of acceleration as changing speed. Aiden recognized the potential for demonstrating "real life" motion within the medium of animation, but his actual product shared more similarities with the resources of the static figurative drawings that he had produced than his non-figurative representations, in which he represented quantities more explicitly. However, the animation's explicit treatment of time, and small Δt, provided the basis for the exploration of the "A x B = C" relationships between speed, displacement, and duration, and speed, acceleration and duration. The three forms of representation provided for varied contexts through which his understandings of speed, time, and acceleration articulated in the drawings could be explored in the resulting discussions about his productions. In the end, the resources Aiden used to create his representations seemed to have less influence on the figurative or non-figurative approach and content of those representations than the properties of the media or the context in which the representations were created, in the form of prompts and questions supplied by the interviewer and outside representations used for comparison and critique.

As Nemirovsky and Tierney (2001) write, "one of the motivations for investigating children's representations is to design learning environments that allow for meaningful connections between their approaches and conventional mathematical representations. This poses the 'transition' issue, namely, what are the continuities and discontinuities between children's graphing and conventional graphing?" (p 82). This research has implications for elementary science and mathematics curriculum development because it shows how easily students can be influenced by acts like introducing new external representations and meta-representational discussion about their representations, or by the affordances of the media It also provides an example of a student inventing representations, expressing his meta-representational competence, and exploring many ways of understanding motion in a shorter timeframe than studies such as diSessa et al.'s (1991). Further research might explore children creating representations together, analyzing the influences of peers on the development of representations, and the effects of externalized representations on changing students' understanding of motion (Cox, 1999; Danish & Enyedy, 2007). Follow-up studies could also revisit the creation of these representations weeks later in order to investigate how consistent the student's choices in representing motion are. This work also provides an initial exploration of using animation as an alternative to conventional graphing in the elementary classroom, to quantitatively represent understandings of motion in a form closer to children's spontaneous figurative drawing.

References

Acredolo, C., Adams, A., & Schmid, J. (1984). On the understanding of the relationships between speed, duration, and distance. *Child Development, 55*(6), 2151-2159.

Ainsworth, S., & Van Labeke, N. (2004). Multiple forms of dynamic representation. *Learning and Instruction, 14*(3), 241-255.

Azevedo, F. S. (2000). Designing representations of terrain: A study in meta-representational competence. *Journal of Mathematical Behavior, 19*, 443-480.

Church, W., Gravel, B., & Rogers, C. (2007). Teaching parabolic motion with stop-action movies. *International Journal of Engineering Education, 23*(5), 861-867.

Cox, R. (1999). Representation construction, externalised cognition and individual differences. *Learning and Instruction, 9,* 343-363.

Danish, J. A., & Enyedy, N. (2007). Negotiated representational mediators: How young children decide what to include in their science representations. *Science Education, 91*(1), 1-35.

diSessa, A. A. (2004). Metarepresentation: Native competence and targets for instruction. *Cognition and Instruction, 22*(3), 293-331.

diSessa, A. A., Hammer, D., Sherin, B. L., & Kolpakowski, T. (1991). Inventing graphing: Meta-representational expertise in children. *Journal of Mathematical Behavior, 10,* 117-160.

diSessa, A. A., & Sherin, B. L. (2000). Meta-representation: An introduction. *Journal of Mathematical Behavior, 19,* 285-298.

Dykstra, D. I., & Sweet, D. R. (2009). Conceptual development about motion and force in elementary and middle school students. *American Journal of Physics, 77*(5), 468.

Garcia-Mila, M., Andersen, C., & Rojo, N. E. (2009). The development of scietnific inquiry strategies and representational practices in preadolescents. In C. Andersen, N. Scheuer, M. D. P. Perez Echeverria & E. V. Tuebal (Eds.), *Representational Systems and Practices as Learning Tools* (pp. 167-185). Rotterdam, The Netherlands: Sense Publishers.

Gardner, H. (1982). *Artful Scribbles: The Significance of Children's Drawings*. New York, NY: Basic Books.

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research*. New York, NY: Aldine de Gruyter.

Gravel, B. (2011). *Elementary students' multiple representations of their ideas about air.* Doctor of Philosophy, Tufts University, Medford, MA.

Healey, P. G. T., Swoboda, N., Umata, I., & King, J. (2007). Graphical language games: Interactional constraints on representational form. *Cognitive Science, 31*, 285-309.

Hughes, M. (1986). *Children and Number: Difficulties in Learning Mathematics*. Oxford, UK: Basil Blackwell.

Jones, A. T. (1983). Investigation of students' understanding of speed, velocity, and acceleration. *Research in Science Education, 13*, 95-104.

Kahn, J. (2010). *"I Remember When We Stayed Still and the Computer Still Made Lines:" Young Children's Invented and Conventional Representations of Motion.* Doctor of Philosophy in Science Education, Tufts University, Medford, MA.

Kaput, J. J. (1998). Representations, inscriptions, descriptions and learning: A kaleidoscope of windows. *Journal of Mathematical Behavior, 17*(2), 265-281.

Karmiloff-Smith, A. (1992). *Beyond Modularity: A Developmental Perspective on Cognitive Science*. Cambridge, MA: The MIT Press.

Kohl, P. B., & Finkelstein, N. D. (2006). Effects of representation on students solving physics problems: A fine-grained characterization. *Physical Review of Special Topics - Physics Education Research, 2*. doi: 10.1103/PhysRevSTPER.2.010106

Kohl, P. B., & Finkelstein, N. D. (2008). Patterns of multiple representation use by experts and novices during physics problem solving. *Physical Review of Special Topics - Physics Education Research, 4*. doi: 10.1103/PhysRevSTPER.4.010111

Lehrer, R., & Schauble, L. (2006). Cultivating model-based reasoning in science education. In K. R. Sawyer (Ed.), *Cambridge Handbook of the Learning Sciences* (pp. 371-387). Cambridge, UK: Cambridge University Press.

Luquet, G. (1913). *Le dessin d'un enfant*. Paris: Alcan.

Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco, CA: Josey-Bass Publishers.

Nemirovsky, R., & Tierney, C. (2001). Children creating ways to represent changing situations: On the development of homogeneous spaces. *Educational Studies in Mathematics, 45*(1/3), 67-102.

Pérez Echeverría, M. P., & Scheuer, N. (2009). External representations as learning tools: An introduction. In C. Andersen, N. Scheuer, M. P. Pérez Echeverría & E. V. Teubal (Eds.), *Representational systems and practices as learning tools* (pp. 1-16). Rotterdam, the Netherlands: Sense Publishing.

Piaget, J. (1927/1969). *The child's conception of time*. New York, NY: Ballantine Books.

Piaget, J. (1946/1970). *The child's conception of movement and speed*. New York, NY: Ballantine Books.

Piaget, J., & Inhelder, B. (1969). *Psychology of the Child*. New York, NY.

Posner, G. J., Strike, K. A., Hewson, P. W., & Gertzog, W. A. (1982). Accommodation of a scientific conception: Toward a theory of conceptual change. *Science Education, 66*(2), 211-227.

Raven, R. J. (1972). The development of the concept of acceleration in elementary school children. *Journal of Research in Science Teaching, 9*(3), 201-206.

Rennie, L., & Jarvis, T. (1995). Children's choice of drawings to communicate their ideas about technology. *Research in Science Education, 25*(3), 239-252.

Scheuer, N., de la Cruz, M., & Pozo, J. I. (2002). Children talk about learning to draw. *European Journal of Psychology of Education, 17*(2), 101-114.

Sherin, B. L. (2000). How students invent representations of motion: A genetic account. *Journal of Mathematical Behavior, 19*, 399-441.

Stenning, K. (1998). Distinguishing semantic from processing explanations of usability of representations: Applying expressiveness analysis to animation. In J. Lee (Ed.), *Intelligence and Multimodality in Multimedia Interfaces: Research and Applications*. Cambridge, MA: AAAI Press.

Thomas, F., & Johnston, O. (1981). *Disney animation: The illusion of life* (Vol. 1): Abbeville Press New York.

Trowbridge, D. E., & McDermott, L. C. (1981). Investigation of student understanding of the concept of acceleration in one dimension. *American Journal of Physics, 49*(3), 242-253.

Tversky, B., Kugelmass, S., & Winter, A. (1991). Cross-cultural and developmental trends in graphic productions. *Cognitive Psychology, 23,* 515-557.

Tversky, B., Morrison, J. B., & Betrancourt, M. (2002). Animation: can it facilitate? *International Journal of Human-Computer Studies, 57*, 247-262.

Vosniadou, S., & Brewer, W. F. (1992). Mental models of the earth: A study of conceptual change in childhood. *Cognitive Psychology, 24,* 535-585.

Vygotsky, L. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.