    e. A much weaker form of Descartes' conservation of motion: if one body retains all of its motion after impact, then nothing will be taken from or added to the motion of the other

5. The main thing to notice about these five announced hypotheses is that most Cartesians accepted all five, and Descartes himself accepted all save perhaps for the principle of relativity
    a. Thus Huygens is here staying (dialectically) within the same general foundational framework as Descartes, though he is avoiding the latter's talk of "forces"
    b. Moreover, by adopting weak versions of some of Descartes' principles and by adding 'uniform' to the principle of relativity, he is staking out a very limited, safe starting point

B. The Initial Theory: Relative Motion Results

1. The "initial" theory, extending to Proposition VII, uses relativity of motion arguments to generate results contrasting with Descartes'
    a. I.e. the "boat-and-shore" arguments, requiring the same basic results to occur for observers in each frame of reference
    b. Arguments simply adjust the speed of the boat so that the conditions of the second, fourth, and fifth hypotheses hold in one frame of reference
2. Propositions I and II pertain to equal bodies, and contrast with Descartes' Rules 6 and 3, respectively
    a. I: If one body at rest, then it will acquire the speed of the other, and the other will come to rest
    b. Argument typical: let boat be moving in opposite direction from moving sphere, but at half the speed, so that from the point of view of the shore the two are moving at equal speeds toward one another, and the result follows from the second hypothesis
    c. II: If at different speeds, then exchange speeds
3. Propositions III, IV, and VI, which drop the equal body requirement, stand in sharp contrast with claims made by Descartes
    a. III: A body however large is moved by impact by a body however small, moving at any speed -- larger body moving toward smaller in one frame of reference has smaller moving toward larger in the other frame
    b. IV: Whenever two bodies collide with one another, the speed of separation is the same, with respect to each other, as the speed of approach -- adjust the speed of the boat so that the conditions of fifth hypothesis hold in one frame of reference
    c. VI: "When two bodies collide with one another, the same quantity of motion in both taken together does not always remain after impulse what it was before, but can either be increased or decreased" -- straightforwardly from Prop. IV, in direct contradiction to Descartes' third law of nature
4. Note that Huygens needs no apologetic explanations for why experience does not accord with these claims, as Descartes does, for experience does accord with them (making allowances for the imperfect hardness of bodies)

a. Huygens's phrasing, applied to full theory, applies even more here: "not alien to reason and agrees above all with experiments"

b. In effect "correcting" Descartes by introducing relativity of motion, dropping the "contest" principle, and replacing the conservation of motion principle by a weak version of it

5. Main upshot of this initial theory: if adopt principle of relativity as a fundamental axiom, then not only forced to reject several of Descartes' Rules, but forced to modify his foundations

a. Abandon competing forces picture of impact, and unable to retain conservation of motion for two bodies interacting

b. Note how effort to obtain agreement with empirical observations is pushing Huygens toward making a distinction between total (scalar) motion and total vector motion -- i.e. momentum

C. Toward the Extended Theory: Proposition VIII

1. Huygens encounters a problem at this point: not enough principles to determine individual velocities in the general case of unequal bodies impacting at unequal speeds

a. Speed of separation = speed of approach is not enough

b. Descartes encountered a similar problem and added a "principle of least change" (without announcing it) to obtain results

c. Our modern approach is to assume just the conservation of momentum and energy; but think of the evidence required for two such strong claims

2. Huygens's solution: invoke (a variant of) Torricelli's principle as a further axiom -- "the common center of gravity of bodies cannot be raised by a motion that arises from their weight" [p. 10]

a. As used in context: velocities after impact cannot be such as to be able to raise center of gravity higher than it was if velocities before impact were acquired through free fall

b. Huygens saw this principle as associated with the impossibility of a perpetual motion device

3. This principle, coupled to two Galilean principles concerning free fall, allows him to prove Proposition VIII, a generalization of the second hypothesis: if two bodies collide with each other from opposite directions at speeds inversely proportional to their bulks (Latin *moles*), then each will rebound at the same speed at which it approached

a. Second hypothesis: equal bodies & equal speeds entails same speeds afterwards

b. VIII: equal motions -- B∗v, v a (positive) scalar -- entails same speeds afterwards

4. Proof of Proposition VIII via reductio ad absurdum, first assuming that speed afterwards is less than speed before, then assuming the opposite

a. Given speed of separation = speed of approach, can show Torricellian principle violated

b. By letting u_a and u_b be acquired in free fall, and then letting v_a and v_b be converted into upward motion, as on inclined plane

c. And finally eliminating the possibilities of $v_a = 0$ and v_a in the wrong direction

5. In order to use modified form of Torricelli's principle in proving Proposition VIII, Huygens also had to assume two of Galileo's principles pertaining to free fall:
   a. Speed acquired independent of weight and bulk, and dependent only on h -- specifically proportional to $\sqrt{h}$
   b. Speed acquired in perfect free fall sufficient to lift body to original height
6. Notice the irony here: replacing Descartes' foundations with Galilean principles that he had complained lacked any foundation
   a. Huygens more preoccupied with having empirically well-founded basic principles than with a small number of universal ones
   b. In this regard, more in the spirit of Galileo, though with a degree of attention to foundations that reflects Descartes

D. The Extended Theory and its Consequences

1. Can thus think of extended theory of hard spherical bodies in head-on impact in either of two ways
   a. As based on four of the original hypotheses plus Torricellian principle and the Galilean principles needed to apply it -- in which case Huygens's theory of impact becomes a subsidiary of a broader mechanics
   b. As based on four of the original hypotheses plus Proposition VIII, with the latter defended by independent arguments from mechanics
2. Either way, can now use relative motion arguments of same sort as before, but now adjusting the motion of the boat so that $B_a * u_a = -B_b * u_b$, so that in that frame of reference $v_a = -u_a$ and $v_b = -u_b$
3. The general solution for the problem of the velocities after head-on impact is given in Proposition IX, which stands in direct contrast with Descartes' Rule 7:

$$v_a = \{(B_a - B_b)u_a + 2B_b u_b\}/(B_a + B_b)$$

   where all velocity vectors are positive in the same direction and B stands for *moles*, i.e. bulk
   a. Same as the calculational rules given by Huygens on p. 15, though his derivation is geometric
   b. The same textbook solution given today, derived from conservation of momentum and energy
   c. Proposition X just a special case, contrasting with Descartes' Rules 4 and 5 for one body at rest
4. Proposition XI states an important new conservation principle: the total of $B * v^2$ remains the same before and after impact in the case of perfectly elastic bodies
   a. In other words, Huygens is here deriving what was in effect the conservation of kinetic energy from his other principles, beginning the process of bringing evidential support for what was two centuries later to become the principle of conservation of energy
   b. And, of course, his theory also entails that what we now call momentum is conserved as well, though he lacked the vector concept needed to express it felicitously, as can be seen from his 1669 statement of it (see Appendix), which did not appear in the 1703 version of the paper

5. Finally, the multiple body results at the end concern what ratio of consecutive B's will yield maximal velocities after impact
    a. Intervening bodies of intermediate magnitude increase velocity acquired from rest
    b. Greatest when intervening B is the mean proportional, and thus for B's in geometric progression
6. Thus end up with our modern theory, save for mass replacing bulk as the measure of the quantity of matter, but obtained from different fundamental principles
    a. A question-answering theory of the same sort as Galileo's theories, ultimately telescoped into a single formula
    b. Two considerations, besides formal adequacy, seem to lie behind the principles from which the theory is derived:
        (1) All are taken from the literature or based on ones that are
        (2) To a reasonable extent, principles are modest as well as "not alien to reason"

E. Empirical Evidence for the Two Theories

1. Rather in the Galilean tradition (and in clear contrast with Descartes) Huygens simply says that the theory agrees with experimental results
    a. In particular, Huygens claims that Proposition VIII can be verified experimentally, so that the appeal to Torricelli's principle etc. serves to show e.g. why it holds, and that it is nomological
    b. Huygens presents no experiments here, but the relevant experiments were definitely performed
2. Good quantitative evidence not so easy to achieve since velocities not easy to measure with precision
    a. Needed indirect, tractable ways of measuring velocities, or inferring them from something that could be measured -- e.g. infer speeds from heights before and after in a ballistic pendulum
    b. And even then had to face confounding effects from imperfect elasticity of balls and air resistance effects
3. During late 1660s papers on elastic impact by Wallis, Wren, Huygens were discussed at Royal Society, including some experiments performed on the spot, and later ones by Mariotte as well
    a. Ballistic pendulum results, measuring height reached by impacted balls as a proxy for velocity -- again implicating claims from mechanics at large
    b. Royal Society data qualitatively supportive, but quantitatively problematic
    c. (Will see a discussion of how to correct these data to achieve more meaningful quantitative comparisons at the beginning of Newton's *Principia*)
4. Available data, though compatible with the key hypotheses, were scarcely enough to confirm that they hold exactly or generally
    a. Principle of relativity based primarily on an a priori argument
    b. The fifth hypothesis similarly relies on a priori arguments
    c. Evidence for Torricellian principle from statics, but here being taken in a much more general way than that evidence automatically licenses

5. Perhaps best possibility for strong supporting evidence lies in the intervening body results at the end
   a. In the Galilean tradition, a salient, unanticipated result that can readily be confirmed qualitatively -- e.g. huge amplifications of speed, as indicated in the last paragraph; e.g. 5 spheres with bulks in 1:2 ratio each yield 68% more motion on impact than with 3 middle ones removed
   b. All fundamental principles from which the overall theory is derived implicated in these results
   c. Huygens undoubtedly included them precisely because they permitted such comparatively telling tests of the theory -- even qualitatively for imperfectly elastic spheres
6. All things considered, then, Huygens's theory of impact is best thought of as a Galilean alternative to Descartes' theory
   a. It employs Galilean principles, it rejects certain Cartesian principles, and it conceptualizes the problem without forces, though (implicitly) with Cartesian version of principle of inertia
   b. Its mathematical development is thoroughly Galilean in style
   c. And the opportunities it presents for evidence have a Galilean character

III. Huygens on Circular Motion and Centrifugal Force

A. The Basic Conceptualization of the Problem

1. As remarked earlier, Huygens had begun work on centrifugal force by 1659 as part of an effort to measure the acceleration of gravity
   a. The version we read, which was published posthumously in 1703, does not bring this goal out so clearly as earlier drafts, nor even as the announcement of most of the results without proofs in an Appendix to *Horologium Oscillatorium*
   b. What it loses in that regard, however, it gains in clarity and polish
   c. (Clarity and polish in part because this version pieced together from several other versions by Huygens's posthumous editors -- as shown by Yoder
      (1) Proofs written to support theorems given in Appendix of *Horologium Oscillatorium*
      (2) A couple of proofs the work of the editors, not of Huygens himself)
2. Problem posed: let us see what [sort of] and how great a *conatus* belongs to bodies attached to a string that revolves (or is restrained by a circular barrier)
   a. Tension in the string a measure of this *conatus*, and thus a measure of the accelerative tendency at the first instant the body would leave the string
   b. Just the move Descartes had proposed in Article III, 59 of his *Principia*, but now carried out by solving for the tension in a string required to maintain uniform circular motion
   c. Initially want to see whether this tendency is of same kind as that involved in gravitational fall
3. Consider in Figure 3 the distances that would form between the object on the circle and the straight line it would follow if it were released -- an indication, indeed a proposed measure, of the *conatus* of the body to move away from the end of the string
   a. BK, KL LN = arc length of BE, EF, and FM since v constant