Galileo on Pendulum Isochronism

… So I fell to thinking how one might many times repeat descents from small heights, and accumulate many of those minimal differences of time that might intervene between the arrival of the heavy body at the terminus and that of the light one, so that added together in this way they would make up a time not only observable, but easily observable.

… Ultimately I took two balls, one of lead and one of cork, the former being at least a hundred times as heavy as the latter, and I attached them to equal thin strings four or five braccia long, tied high above. Removed from the vertical, these were set going at the same moment, and falling along the circumferences of the circles described by the equal strings that were the radii, they passed the vertical and returned by the same path. Repeating their goings and comings a good hundred times by themselves, they sensibly showed that the heavy one kept time with the light one so well that not in a hundred oscillations, nor in a thousand, does it get ahead in time even by a moment, but the two travel with equal pace. The operation of the medium is also perceived; offering some impediment to the motion, it diminishes the oscillations of the cork much more than those of the lead. But it does not make them more frequent, or less so; indeed, when the arcs passed by the cork were not much more than five or six degrees, and those of the lead were fifty or sixty, they were passed over in the same times.

First Day, p. [128f]; see also Fourth Day, p. [277]

NON-ISOCHRONISM OF CIRCULAR-ARC PENDULUMS

$$\frac{P}{P_0} = 1 + (\frac{1}{2})^2 k^2 + (\frac{1 \cdot 3}{2 \cdot 4})^2 k^4 + (\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6})^2 k^6 + \ldots$$

where $k=\sin(\alpha/2)$ and α is the arc in descent.

Arc in descent (deg)	P/P_0	Number of full cycles before a 20% discrepancy
2.5	1.00012	1667
5.	1.00048	417
10.	1.00191	105
15.	1.00430	47
30.	1.01741	12
60.	1.07317	3
90.	1.17996	1+

These precise numbers were not calculable before the late 18th century. (It requires the solution of an elliptical integral.) But both Mersenne and Huygens had observed the qualitative results, the former in the 1630s (published at that time) and the latter in the 1650s (published in 1673). A constant-arc circular pendulum would, of course, have a repeatable period, but the question of the trajectory required to maintain isochronism (that is, same period regardless of arc length) was left for Huygens to discover, in 1659.

Why, when two mechanisms are irremediably involved in an actual process, should one be disregarded, focusing exclusively on the other?

- **A theory of the principal or dominant mechanism is possible, and it is needed in order to make empirical investigation of the other "secondary" one tractable**

- **Experiments yielding results of evidential value are possible provided only that the confounding effects of the other mechanism be largely eliminated or controlled**

- **No theory of the other mechanism is possible at all – for example, because too many variables are involved – so that any science becomes possible only by disregarding it and focusing on the one amenable to theory**