

Table I. (Kepler)

	Mean Solar Distance from <i>Rudolphine</i> <i>Tables</i>	Mean Sidereal Motion in 365 days	Mean Solar Distance from Third Law	Difference in Predicted Longitude
Mercury Elongation from Sun	0.38806 22°50'2"	1493.7066°	0.38710 22°46'27"	3'35"
Venus Elongation from Sun	0.72413 46°23'47"	584.7792°	0.72333 46°19'48"	3'59"
Earth	1.00000	359.7469°	1.00000	
Mars Annual Parallax	1.52350 41°1'29"	191.2714°	1.52369 41°1'6"	0'23"
Jupiter Annual Parallax	5.20000 11°5'15"	30.3281°	5.20117 11°5'6"	0'9"
Saturn Annual Parallax	9.51000 6°2'9"	12.2125°	9.53809 6°1'5"	1'4"

Table VI. Some Observations of Horrocks compared with the tables of Kepler and Tuckerman

	Horrocks (H)	Kepler (K)	Tuckerman (T)	(H - K)	(H - T)
1. Venus 15 Dec. 1637 5 : 50 p.m.	314°51'	314°56 1/2'	314°50'	-5 1/2'	1'
2. Mars 13. Jan. 1638 7 : 05 a.m.	225° 4'	225° 4'	225° 2'	0'	2'
3. Jupiter 26 Jan. 1638 5 : 35 a.m.	214°53 1/2'	214°45 1/2'	214°54'	8'	-1/2'
4. Mars 30 Jan. 1638 6 : 30 a.m.	234°12'	234°10'	234°10 1/2'	2'	1 1/2'
5. Jupiter 30 Jan. 1638 6 : 30 a.m.	215° 1 1/2'	214°53 1/2'	215° 2'	8'	-1/2'
6. Mars 12 Feb. 1638 5 : 40 a.m.	240°46'	240°48'	240°48'	-2'	-2'
7. Jupiter 12 Feb. 1638 5 : 40 a.m.	215°10'	215° 1'	215°11'	9'	-1'

Table I), and Horrocks did not find it necessary to change these constants significantly. (He did at length come up against the fact of Saturn's inconstant "mean" motion; he called it "a Gordian knot, which has to be cut because insoluble"⁵²; it once more raised disturbingly for him the question of the perfectibility of astronomy⁵³.) It was to Mercury or Venus that Horrocks had to turn if he was to produce essentially new evidence for the third law, and of Mercury he had difficulty in obtaining sufficient observations⁵⁴. Venus it had to be, then, that led Horrocks, not only to his more famous triumph, the first observation of Venus *sub sole*, but also to his affirmation of the strict exactitude of the third law.

Something of Horrocks' course of thought as well as his observations of Venus can be followed in his correspondence with Crabtree. The reform of the Keplerian parameters for Venus was closely tied up with the correction of Kepler's value for the eccentricity of the sun or earth. In his letter of 3 June 1637, Horrocks points out that the *Rudolphine Tables* have the spring equinox occurring too soon, and he is proposing to remedy the mistake by changing the solar eccentricity from Kepler's 0.01800

⁵² *Opera posthuma*, p. 322.

⁵³ *Ibid.*, p. 325.

⁵⁴ See *ibid.*, p. 17.

to 0.01770⁵⁵. On 23 November 1637 he is admonishing Crabtree to make frequent determinations of the azimuthal differences between Venus and the sun, in order that, having found the place of Venus with respect to the fixed stars, it will be possible to compute the longitude of the sun and thus investigate its inequality⁵⁶. By 19 January 1638 Horrocks has concluded that the earth's eccentricity should be reduced to 0.01730, so that the maximum equation of center becomes 1°59' rather than Kepler's 2°4'⁵⁷.

Among the reasons he puts forward for this change are observations of Venus: for a maximum evening elongation of the planet at about the time of the spring equinox Horrocks finds its place according to the *Rudolphine Tables* to be too far ahead by 10'; and exactly the opposite happens when Venus is near maximum morning elongation at about the time of the autumnal equinox⁵⁸. Both circumstances can be at least partly accounted for if the eccentricity of the earth's orbit is diminished to 0.01730, reducing the earth's heliocentric longitude in the spring and increasing it in the fall. Further correction could be effected by reducing the size of the orbit of Venus, but apparently Horrocks had not yet considered this possibility. A final 3' of correction would be obtained by reducing the earth's eccentricity all the way to 0.01686, the value we find by using Newcomb's 1900 value and rate of change to extrapolate back to 1640. But Horrocks had his reason for choosing 0.01730. He knew that the Keplerian value rested on Tycho's empirical determinations of altitudes of the sun, and that Tycho in correcting the observed altitudes for parallax has assumed the sun's horizontal parallax to be 3'⁵⁹. (This was the ancient value, but for Tycho it had the additional support of Johannes F. Offusius' number mysticism, in that it put the sun just 576 earth-diameters from the earth, 576 being a sacred number⁶⁰.) Kepler, after Tycho's death, found the horizontal parallax of Mars when at or near opposition to the sun, and thus only half as distant as the sun, to be negligible, and therefore took the step of reducing the horizontal solar parallax to 1' in the *Rudolphine Tables*; he did not, however, undertake to alter the eccentricity in Tycho's solar theory in the way this correction would have required. Horrocks' value of 0.01730 for the eccentricity appears by his own account to be derived directly from Tycho's solar theory together with the new, Keplerian correction in solar parallax⁶¹. What Horrocks did not know was that Tycho's correction for refraction was also mistaken, being too small for the equinoctial sun by 40''; the correction of this mistake would have reduced the eccentricity still further. Horrocks' failure to correct for this last mistake causes him to obtain an exaggerated eccentricity for Venus.

⁵⁵ *Ibid.*, p. 288.

⁵⁶ *Ibid.*, p. 296.

⁵⁷ *Ibid.*, p. 301.

⁵⁸ *Ibid.*, p. 17.

⁵⁹ *Ibid.*, pp. 172, 301.

⁶⁰ See Tycho Brahe, *Astronomiae instauratae progymnasmata* (Prague, 1602), p. 472.

⁶¹ See Horrocks, *Opera posthuma*, p. 301.

Horrocks' search for a satisfactory emendation of the Rudolphine parameters for Venus extends through the first eight months of 1638. On January 19 Horrocks is sure that the mean heliocentric longitude of Venus should be reduced by $15'$ ⁶². By March 10 he is retracting this suggestion, and considering the possibility of adding 15° to the aphelion (too much) and subtracting $10'$ from the mean heliocentric longitude; he allows, however, that this proposal does not satisfy all of Tycho's

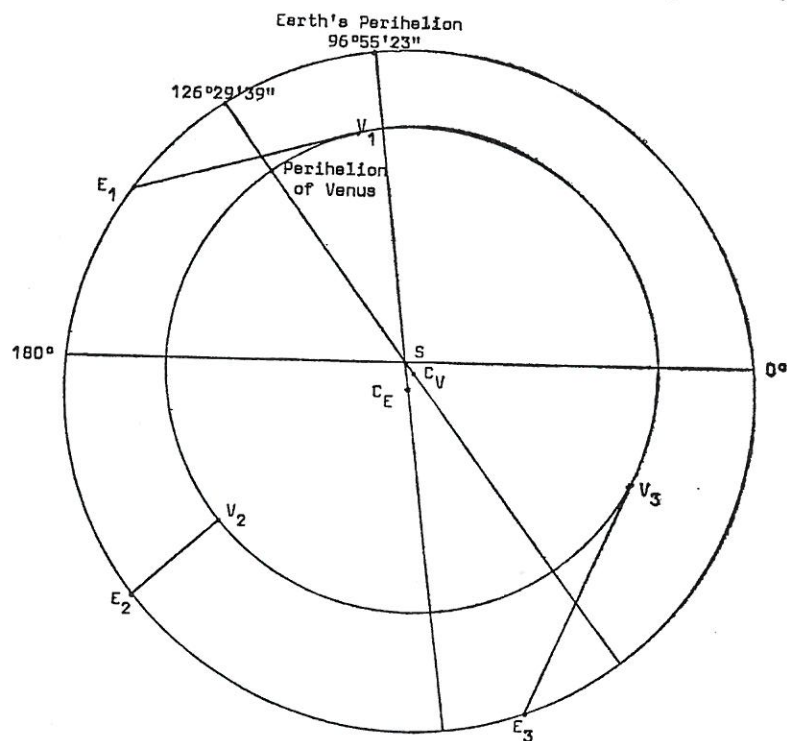


Figure I. Orbits of Venus and Earth in 1640
(C_V = center of Venus' orbit; C_E = center of Earth's orbit; eccentricities exaggerated)

and his own observations. One thing seems certain, he says: if a solar eccentricity of 0.01730 be retained, then the orbit of Venus has a smaller ratio to the orbit of the earth than Kepler gives, causing an observational difference of at least $5'$ when Venus is in maximum elongation⁶³. Venus, we note, had just gone through a maximum evening elongation on or about February 7, when it was near its perigee, and

⁶² *Ibid.*, p. 302.

⁶³ *Ibid.*, p. 306.

it was probably from his observation of this that Horrocks was drawing his conclusion; the geometrical relations are shown in Figure I. On April 10, having been observing Venus as it came on toward its inferior conjunction (to occur on April 21), Horrocks writes that the geocentric longitude of Venus is $8'$ or $9'$ less than Kepler makes it, and that the discrepancy can be eliminated by reducing the solar eccentricity and the size of Venus' orbit as suggested in his previous letter, while also retracting the mean heliocentric longitude by $4'$ or $5'$; but he is not sure whether it is the mean heliocentric longitude or the eccentricity of Venus that should be changed⁶⁴.

He finds an answer to this last question in the summer. On July 25 he writes that he has calculated all the Tyconic observations of Venus very accurately, and that they show the sun's eccentricity to be 0.01730. The eccentricity of Venus, he says, is a little larger than Kepler makes it. He has not yet fixed the exact magnitude of the corrections to be made in the orbital elements of Venus because he lacks observations of Venus near aphelion; but he proceeds in the very next line to report the raw data of two such observations, made on July 4 and 6, "as accurately as I could make them, by repeated checking"⁶⁵. (Venus was in maximum morning elongation on July 1, fairly near its aphelion.) On September 3 he informs Crabtree that he has so corrected the motion of Venus, that he could hardly hope to do better⁶⁶. On September 29 he reports his specific corrections, including an eccentricity of 0.00750 as compared with Kepler's 0.00692, and a mean orbital radius of 0.72333. Of the latter he adds:

This I deduce from Kepler's harmonies and the proportions of equal motions; but observations precisely confirm it. Kepler has 0.72414, hence his *prosthapheresis orbis* is always too large⁶⁷.

No doubt Horrocks was finding exactly what he was looking for, but there are indications that he was obtaining an exceedingly good empirical determination of the mean solar distance of Venus. He had made observations of Venus at maximum elongation near perihelion in February, 1638, and at maximum elongation near aphelion in early July, 1638 (the sightings $E_1 V_1$ and $E_3 V_3$ in Figure I). Now if one imagines determining the eccentricity of Venus' orbit from two such sightings, using Horrocks' solar theory with its exaggerated eccentricity as the basis of calculation, then it can be shown that, to obtain the value that Horrocks obtains, the observations will have to be accurate to within $20''$ of arc. In other words, the error in the eccentricity of Horrocks' theory of Venus is a very accurate reflection of the error in the eccentricity he adopts for the sun. Presumably he used more than two observations and did some judicious averaging. But since the determination of the eccentricity of Venus is independent of the determination of the mean solar distance

⁶⁴ *Ibid.*, p. 307.

⁶⁵ *Ibid.*, p. 311.

⁶⁶ *Ibid.*, pp. 314–315.

⁶⁷ *Ibid.*, p. 320.

— though the same observations are used to determine both, an error in one does not imply an error in the other — it is plausible to suppose that Horrocks is determining the one with about as much precision as he is determining the other, namely to within less than a minute of arc. This would mean that Horrocks' empirically determined value for the mean solar distance fell within the range 0.7233 ± 0.0002 ⁶⁸.

Horrocks, of course, puts down the number as 0.72333, using all five places computed from the periodic times. In brief, he places his trust in Kepler's harmonic law. Horrocks' view of the cosmos, in fact, is very close to Kepler's. He accepts the main tenets of Kepler's celestial physics, while modifying the details. The rotating sun still moves each planet along its orbit by the extended arm of its "magnetic" virtue. But Horrocks objects to the manner in which Kepler accounts for the planet's libration approach to and recession from the sun. Kepler had hypothesized certain magnetic fibers in the interior of each planet, supposing them to maintain their orientation with respect to the stars independently of the axial rotation of the planet's surface, and thus to bring about alternate attractions to and repulsions from the sun. To avoid such a complicated, ad hoc hypothesis, Horrocks brings in the analogy of the pendulum, and proposes a simple attraction to the sun combined with an inertial tendency on the part of the planet. For as he tells Crabtree,

Nature is one, and all things have between them a consensus and harmony. And thus since the motions of the planets agree with the motion of the pendulum both in the figure of the orbits and in the translation of the aphelia, why should not the causes of the two be similar?⁶⁹

That this same proposal might be used to eliminate the role of a solar virtue in effecting the circumgyration of the planets about the sun, does not appear to have occurred to Horrocks; his analogy tells him rather that the hand holding the pendulum suspension must move circularly if the pendulum bob is to move not rectilinearly but in an oval. The new mechanism no more accounts for the sesquialterate proportion between periods and distances than had Kepler's. For Horrocks, as for Kepler, the world is a cosmos beautifully and meaningfully arranged, and so Horrocks like Kepler sees the third law as a pattern directly imposed by a geometrizing creator.

In agreement with this view, Horrocks like Kepler expects that the sizes of the planets will also be found to fit a neat pattern: "Since the sun by its magnetic virtue regulates the motion of the six primary planets. I cannot conceive how it could

⁶⁸ A similar result was obtained a few years earlier by Gottfried Wendelin (Vendelinus). For Venus he found an aphelion distance of 0.72783, a perihelion distance of 0.71903, and hence a mean distance of 0.72343; for Mercury, an aphelion distance of 0.47071, a perihelion distance of 0.30358, and hence a mean distance of 0.38711. Given Wendelin's predilection for "harmonies", there is a likelihood that he was guided to these results by Kepler's third law; the precision of the confirmation is otherwise astonishing. The eccentricities for both Venus and Mercury are much too small; the error can mean that he was using too small a value for the eccentricity of the Earth. See Wendelin's letter to Gassendi of 1 May 1635, in *Petri Gassendi Epistolae* (Lyon, 1658), Vol. VI, p. 428.

⁶⁹ Horrocks: *Opera posthuma*, p. 312 (from the letter of 25 July 1638).

proportion its force to the distance so perfectly, unless the globes themselves be similarly proportioned"⁷⁰. But Kepler's proposal that it is the volumes of the planets that bear the same ratio as the solar distances is no longer tenable, for Gassendi in his observation of the Mercury transit of 28 October 1631 has found Mercury's diameter to be less than 20", while Kepler's formula required that it be greater than 2'⁷¹. The idea comes to Horrocks that the direct proportion may hold rather between planetary diameters and solar distances; and as he indicates in a letter to Crabtree of 26 October 1639, it is the possibility of verifying this idea that especially excites him in the prospect of the approaching Venus transit of November 24⁷².

In the event, Horrocks sees his hypothesis as vindicated; he finds Venus' diameter as it would be observed from the sun to be 28", and is able to argue that the same will be true of the other planets⁷³. One of these planets, of course, is the earth, and Horrocks is thus arguing from the assumed constancy of the ratio that the sun's horizontal parallax (or angle subtended by the earth's radius at the distance of the sun) is just 14"⁷⁴. As we have seen, the effect of reduction in solar parallax on planetary parameters and especially on solar theory had been one of Horrocks' main concerns from the time he obtained the *Rudolphine Tables*; and the new analogy supports his earlier assumption that solar parallax is practically negligible in relation to the attainable observational precision. Just as important, the observational success of the new analogy signifies to Horrocks a triumph for the notion that the solar system is harmonically or architectonically arranged. Thus it joins with observation in supporting the hypothesis of the exactitude of Kepler's third law.

Such, we believe, was the Horrocksian basis for Streete's assertion that, "with the corrected Parallax of the Sun and Aequation of the Earth, the sesquialterate proportion proves most consentaneous unto observation and altogether indubitable".

As already noted, Streete's claims met with opposition. *The Holy Guide* of the astrologer John Heydon (fl. 1661), published in 1662 and claiming to teach "the knowledge of all things, Past, Present and to come; ... and to Cure, Change and Remedy all Diseases in Young and Old, with Rosie Crucian Medicines", included as an appendix an "Advertisement to Thomas Street", in which Streete's solar theory and character are alike attacked;⁷⁵ the objection to the solar theory reduces to a rejection of Streete's or Horrocks' value for the eccentricity, though the author of the appendix (who turns out to be John Gadbury⁷⁶) does not appear to understand this fact.

⁷⁰ Horrocks, *Venus in sole visa* (ed. cit.), p. 141.

⁷¹ *Ibid.*, p. 142.

⁷² Horrocks, *Opera posthuma*, p. 331.

⁷³ Horrocks, *Venus in sole visa*, pp. 137–143; *Opera posthuma*, pp. 160–174.

⁷⁴ Wendelin had reached exactly the same conclusion on the basis of his observations of planetary diameters and through exactly the same appeal to analogy; see the letter cited in note 68.

⁷⁵ John Heydon, *The Holy Guide* (London, 1662), Appendix, pp. 43–55.

⁷⁶ See Thomas Streete, *An Appendix to Astronomia Carolina* (London, 1664), p. 26.