Strains – 3 (processing of field data)

Processing of field data : finding strains from observed data

Observed data   geometry changes for several orientations of material lines

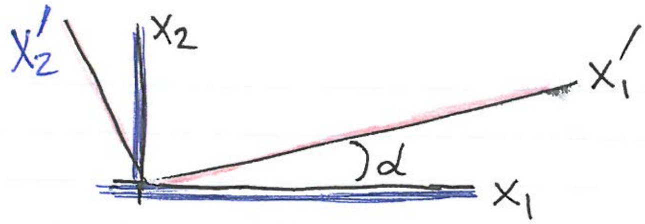
- elongations
- angle changes

Want :   strain tensor

Motivation: analyze material lines of any orientation

Tool needed : given strain components in one coord. system find them in another (rotated) system

in 2-D case



$$\begin{cases} \underset{\sim}{e}_1 = \cos\alpha \, \underset{\sim}{e}_1' - \sin\alpha \, \underset{\sim}{e}_2' \\ \underset{\sim}{e}_2 = \sin\alpha \, \underset{\sim}{e}_1' + \cos\alpha \, \underset{\sim}{e}_2' \end{cases} \qquad \left(= \alpha_{11} \underset{\sim}{e}_1' + \alpha_{12} \underset{\sim}{e}_2'\right)$$

$\Downarrow$ into $\boxed{\underset{\sim}{\epsilon} = \epsilon_{ij} \, \underset{\sim}{e}_i \underset{\sim}{e}_j}$ (in old axes)

$\Downarrow$

obtain $\boxed{\underset{\sim}{\epsilon} = \epsilon_{ij}' \, \underset{\sim}{e}_i' \underset{\sim}{e}_j'}$ (in new axes)

ϵ_{11}' = coeff. at $\underset{\sim}{e}_1' \underset{\sim}{e}_1'$:

$$\epsilon_{11} \cos^2\alpha + \epsilon_{22} \sin^2\alpha + 2 \sin\alpha \cos\alpha \, \epsilon_{12} = \epsilon_{11}'$$

Check : should be same as given by $\dfrac{\overline{\Delta S} - \Delta S}{\Delta S} = \epsilon_{ij} n_i n_j$ in old system

$n_1 = \cos\alpha$, $n_2 = \sin\alpha$

Same for ϵ_{22}'

ϵ_{12}' = coeff. at $\underset{\sim}{e}_1' \underset{\sim}{e}_2' + \underset{\sim}{e}_2' \underset{\sim}{e}_1'$ $= \frac{1}{2}$ angle change between $1', 2'$

$$\epsilon_{12}' = \epsilon_{12} (\cos^2\alpha - \sin^2\alpha) + (\epsilon_{22} - \epsilon_{11}) \sin\alpha \cos\alpha$$

$\Longrightarrow$ we related ϵ_{ij}' to ϵ_{ij}

Note: in formula

$$\epsilon'_{12} = \epsilon_{12}\left(\cos^2\alpha - \sin^2\alpha\right) + $$

$$+\left(\epsilon_{22} - \epsilon_{11}\right)\sin\alpha\cos\alpha$$

ϵ_{11} and ϵ_{22} enter through their <u>difference</u> only

$\Rightarrow$ adding a constant ϵ^0

$$\epsilon_{11} \to \epsilon_{11} + \epsilon^0$$

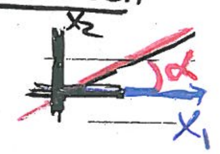
$$\epsilon_{22} \to \epsilon_{22} + \epsilon^0$$

(uniform expansion, such as heating) <u>does not affect angle distortion</u>

transfer info. collected from various orientations to a chosen fixed coord system $X_1 X_2$:

A. Transfer of elongation info.

Let us say relative elongation has been measured $= \delta$ in some direction:



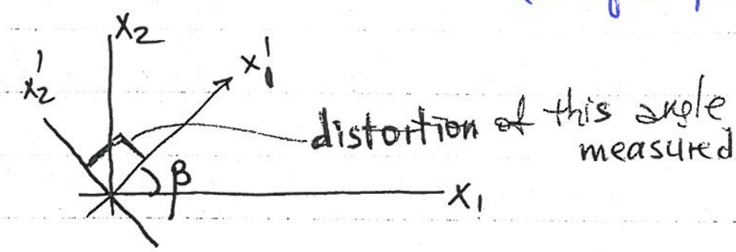
$$\Rightarrow \boxed{\delta = \epsilon_{11}\cos^2\alpha + \epsilon_{22}\sin^2\alpha + 2\sin\alpha\cos\alpha\ \epsilon_{12}} = \epsilon'_{11}$$

measured

one eq-n for 3 unknowns ϵ_{ij}

If δ's are measured in 3 directions – can find all ϵ_{ij} (3 eq-ns for 3 strains)

B. Transfer of angle distortion info



$$\epsilon'_{12} = \left(\tfrac{1}{2}\text{angle change}\right) = \underbrace{(\epsilon_{22}-\epsilon_{11})}_{\text{only the difference enters!}}\sin\beta\cos\beta + \epsilon_{12}(\cos^2\beta - \sin^2\beta)$$

reconstruction of ϵ_{ij} is incomplete – even if data available for several β's (in contrast with info. on elongations)

Note:

Transfer of elongation info. can also be done using basic formula of strain analysis:

$$\underbrace{\frac{\overline{\Delta S} - \Delta S}{\Delta S}}_{\text{measured}} = \epsilon_{ij} \underbrace{n_i n_j}$$

specified the direction $\underline{n}$ where the relative elongation measured

$\Rightarrow$ relation between ϵ_{ij}

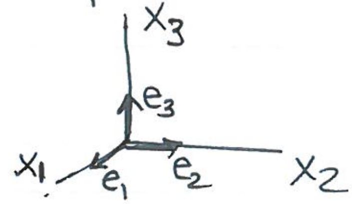
$\Rightarrow$ can find all three of ϵ_{ij} if elong. info. available for three directions $\underline{n}$

<u>However</u>: it cannot be used for transfer of <u>angle distortion</u> info.

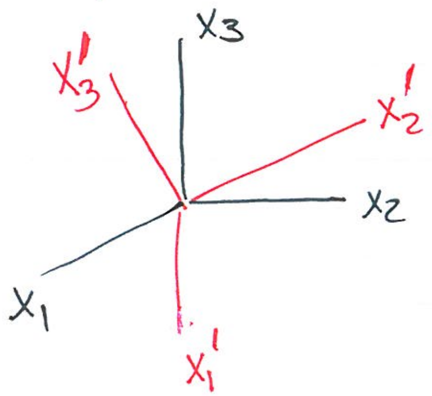
in 3-D:

Strain tensor $\underset{\sim}{\epsilon} = \epsilon_{ij}\, \underset{\sim}{e}_i\, \underset{\sim}{e}_j$

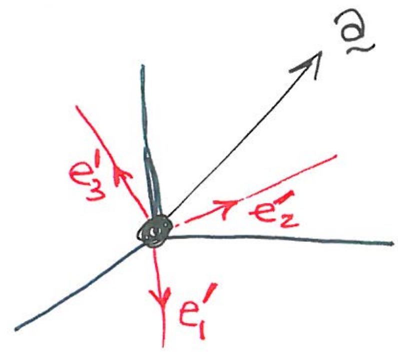
components in system $x_1 x_2 x_3$



Components of $\underset{\sim}{\epsilon}$ in $x_1' x_2' x_3'$?



For vectors:



$\underset{\sim}{a} = a_i\, \underset{\sim}{e}_i$

express in terms of $\underset{\sim}{e}_1'$, $\underset{\sim}{e}_2'$, $\underset{\sim}{e}_3'$

$\underset{\sim}{e}_i = \alpha_{ij}\, \underset{\sim}{e}_j'$

direct. cosines

$\alpha_{23} = \cos(e_2 \,\widehat{}\, e_3')$

$\Rightarrow \quad \underset{\sim}{a} = \underbrace{a_i\, \alpha_{ij}}_{j\text{-th comp. in the (new) system}}\, \underset{\sim}{e}_j'$

$\ldots + (\underbrace{a_i \alpha_{i3}}_{a_3'})\, e_3' + \ldots$

Tensors: similarly

$\underset{\sim}{\epsilon} = \epsilon_{ij}\, \underset{\sim}{e}_i\, \underset{\sim}{e}_j$

express in terms of $\underset{\sim}{e}_1'$, $\underset{\sim}{e}_2'$, $\underset{\sim}{e}_3'$

$\Rightarrow \underbrace{\epsilon_{ij}\, \alpha_{ik}\, \alpha_{jl}}_{\epsilon_{kl}'}\, \underset{\sim}{e}_k'\, \underset{\sim}{e}_l'$