III. De Motu, Version 3 (the "Augmented" Version) -- Two Extended Scholia

A. The "A Priori Proof" of Copernicanism

1. The "Copernican Scholium," added following Theorem 4, makes a series of claims, falling into three consecutive parts; the first part pivots on the claim that the Copernican system can be proved:

   "Moreover the whole space of the planetary heavens either rests (as is commonly believed) or moves uniformly in a straight line, and hence the communal center of gravity of the planets (by Law 4) either rests or moves along with it. In both cases (by Law 3) the motions of the planets among themselves [*inter* se] are the same, and their common centre of gravity rests in relation to the whole of space, and so can certainly be taken for the still centre of the whole planetary system. Hence truly the Copernican system is proved a priori. For if the common centre of gravity is calculated for any position of the planets it either falls in the body of the Sun or will always be very close to it."

   a. Desire to prove Copernicanism, and not worries about multiple centers, may have prompted Laws 3 and 4 in the first place
   b. Regardless, Laws 3 and 4 entail that the appropriate single point to refer the motion of the planetary system is its "communal centre of gravity"
   c. And calculation shows this point to be very near the sun

2. The standard objection to this line of reasoning, voiced by Wilson (and supported by Whiteside), is that it presupposes key unstated elements of the law of gravity

   The term 'center of gravity' is odd -- 'center of mass' would be preferable -- but it is possible that the "notion of center of gravity played a heuristic role, leading Newton's thought from familiar experiences with connected systems of weights to the idea of the solar system as a group of interconnected bodies. That such mutual interaction actually occurs is so far an unsubstantiated assumption..."

   "The other important assumption in the computation is that the attractions exercised at a given distance from these several bodies are as the masses of the attracting bodies."

   a. In other words, the supposed proof turns on calculating the center of mass of the planetary system, which itself requires first a determination of the masses of the bodies, not to mention the concept of mass itself
   b. But the only way of determining these masses is from the strength of the centripetal force "field" around each body being proportional to its mass, an inference licensed only by the law of gravity

3. There is another way of construing Newton's reasoning here, starting from concerns about multiple centers and the need for a single center to refer the motion to
   a. As indicated in Problem 5, Newton had identified a^3/P^2 as a measure of the accelerative strength of the inverse-square centripetal acceleration field around any body known to have such a field
   b. This quantity is a constant for each such body: C_h, C_j, C_s, C_e, the respective constants for the sun, Jupiter, Saturn, and the earth

4. Consider now just the case of the sun and Jupiter -- a two attractive center problem, with the strength of each attraction given

a. By Law 4 there must be a "center of gravity" around which both are revolving

b. And the only way in which Law 4 cannot be violated is if they revolve in a tandem balance with respect to one another, the sun revolving about this center with a radius r_h, and Jupiter with a radius r_j, where r_h/r_j a constant

5. The question then is, what is the (non-varying) ratio between r_h and r_j -- i.e. what are the comparative radii by which each revolves about the proper center of reference

   a. Assuming circular motion, Jupiter's centripetal acceleration is as r_j/P^2, so that $C_h/(r_{jh})^2$ is as r_j/P^2

   b. Similarly, in the case of the sun: $C_j/(r_{jh})^2$ is as r_h/P^2

   c. But then $C_h/C_j = r_j/r_h$!! -- a conclusion that has been reached without any consideration (or notion) of mass

6. But C_h is known to be much larger than C_j, for the diameter of the orbits of Jupter's satellites in astronomical units are known to sufficient accuracy to assure that C_h/C_j is a very large number

   a. But then r_h must be small compared to r_j; hence proper center of reference is very near the Sun

   b. The argument generalizes to Saturn and earth individually paired with the sun, both of which have smaller a^3/P^2 than that of Jupiter

   c. And the conclusion continues to hold even for a "displacement" of the sun corresponding to 6 times the strongest known planetary attraction, that of Jupiter's

7. Thus indeed a proof, not just that the Tychonic system is false, but that a main claim of the Copernican system is basically true

   a. A much stronger inference is being drawn from Kepler's 3/2 power rule than the one I am proposing that Newton was trying to draw in the original "Moon test" -- an inference made possible by Law 4!!

   b. Of course, there are loose ends in the reasoning, and hence some may say that this is not yet a proof, but it nevertheless is an extremely promising line of reasoning toward a proof

B. The Imperfectibility of Astronomy

1. The second part of the "Copernican Scholium" draws a corollary from the proof, leading to the conclusion that astronomy is imperfectible

   "By reason of this deviation of the Sun from the centre of gravity the centripetal force does not always tend to that immobile center, and hence the planets neither move exactly in ellipses nor revolve twice in the same orbit. So that there are as many orbits to a planet as it has revolutions, as in the motion of the Moon, and the orbit of any one planet depends on the combined motion of all the planets, not to mention the action of all these on each other. But to consider simultaneously all these causes of motion and to define these motions by exact laws allowing of convenient calculation exceeds, unless I am mistaken, the force of any human mind. Ignoring those minutiae, the simple orbit and the mean among all errors will be the ellipse of which I have already treated."

   a. The "proof" of Copernicanism has as a corollary that the movements of the planets are exceedingly complex, and hence orbital astronomy is probably not perfectible

   b. Nothing comparable to the above statement anywhere in the *Principia*!

2. The corollary stems from the fact that the forces on the individual planets are no longer strictly centripetal, thus undercutting the whole structure of the scheme of De Motu Version 1
    a. An ellipse with reference to the sun ceases to be an ellipse with reference to the proper center to which motion is referred if the sun is moving in response to the several planets
    b. This independently of any direct interactions among the planets
    c. Therefore the ellipse and the area rule are just approximations, though ones that might still hold exactly if there were only two bodies, say the sun and Jupiter
3. The comparison of the motion of the planets with that of the moon is instructive, for the moon was known to deviate from Keplerian motion vis-a-vis the earth in ways then beyond any calculation
    a. The suggestion is that perturbed Keplerian motion is the general rule, and not just an occasional exception, though the reason given in the case of the planets doesn't extend to the moon
    b. The claim that no two orbits are ever the same has to be overreaching what was known, and hence calls for qualification
4. The claim that the perturbed motion of the planets is beyond the capacity of the human intellect to calculate is less obvious since at most six planets are involved
    a. But once differences in inclination and period are taken into account, the planets will produce an exceedingly complex motion of the sun
    b. Newton here sounds like Galileo (and Descartes) on resistance -- beyond capacity because too many degrees of freedom
    c. Ironic, for shortly after Newton begins to try to calculate
    d. But nonetheless telling, for Newton here qualifying, if not retracting, a main conclusion of De Motu Version 1, *viz.* that orbital astronomy is perfectible so long as no secondary mechanisms are at work or they can be systematically accounted for via superposition!
5. Finally, the claim that the Keplerian ellipse represents the mean motion (and hence the Horrocksian approach is still authorized) is open to more than one interpretation
    a. If 'mean' is interpreted loosely, then only saying that the actual motion never deviates that far from the ellipse, and the ellipse lies somewhere in the middle
    b. But if 'mean' taken literally, then saying that ellipse approximates the actual motion in the mean, and hence making a strong claim, calling for justification
    c. Some justification can be given, for the sun's deviation from the proper center of reference will presumably result from a series of basically harmonic perturbations
        (1) Deviation in the case of interaction with each planet is harmonic
        (2) Superposition of finite number of harmonic deviations, each with its own fixed period, is a complicated deviation with a long period, rather like the Moon's 18 year Saros cycle
6. Note: Newton is invoking strict inferential reasoning here from the prior "proof", and not from the law of gravity, much less universal gravity

a. Not invoking 3-body effects or interactions with comets

b. And inference that Keplerian motion is a mean has some basis in proof of Copernicanism

7. Note finally: Newton's conclusion about the imperfectibility of astronomy is not coming from unaccounted for observed discrepancies between theory and observation -- as worried Kepler and Horrocks -- but from an underlying physics as inferred from the phenomena! – a physics very different from that on the basis of which Descartes reached his conclusion of imperfectibility

C. Implications for Orbital Astronomy

1. The inferred underlying physics thus entails that there must be discrepancies between Keplerian theory and the actual motion even though they may not yet have been observed

> "If anyone tries to determine this ellipse by trigonometrical computation from three observations (as is customary), he will have proceeded without caution. For these observations will share in the very small irregular motions here neglected and so cause the ellipse to deviate somewhat from its actual magnitude and position (which ought to be the mean among all errors), and so there will be as many ellipses differing from each other as there are trios of observations employed. Very many observations must therefore be joined together and assigned to a single operation which mutually moderate each other and display the mean ellipse both as regards position and magnitude."

a. A practical corollary, going with "imperfectibility" corollary

b. And with the warning, a call for a new method of computing orbits, not yet entirely spelled out

2. The basic warning is perfectly clear: do not infer orbital parameters from a minimal or small set of observations, no matter how ideal these observations may be in other ways

a. The orbital parameters are known to be sensitive anyway -- e.g. sensitive to small errors in observation unless highly selective in observations used

b. But now saying that there are always perturbations having the same sort of effect as observational error

c. Consequently, will not get unique or converging values for orbital parameters so long as use small numbers of observations

3. Spirit of the remarks reminiscent of Descartes' view that the actual trajectories are subject to an indefinite number of quasi-random perturbations

a. Also Cartesian in a deeper sense, for analysis and theory are taking precedence over observations -- theory entailing discrepancies beyond any that can as yet be observed

b. Epochs needed to have any hope of using data to clean up data, and hence better off resorting to theory immediately

c. Akin to Kepler's despair in letter to Bernegger -- will not live long enough to have sufficient data to sort out -- but different too, for more basis for settling on Keplerian motion as the underlying mean about which the true motions fluctuate

d. In the process explaining why Kepler might have found the values of some of his elements changing over time

4. Stronger implication: undermine the evidential value of discrepancies between observation and theory, for may be mere transient perturbations!
    a. Version 1: all such discrepancies informative, for provide a basis for identifying and characterizing further forces at work
    b. Version 3: only select discrepancies informative, and burden of argument on anyone drawing inferences from a small discrepancy, for perhaps just do not have mean orbit correct yet
5. Version 3 thus represents a retreat in evidential prospects, and hence standards, to ones more akin to those of Galileo and Huygens
    a. Just as resistance effects may be beyond science, orbital perturbations may be beyond science
    b. Therefore there are limits on empirical evidence -- there will always be room to explain away small discrepancies
    c. Newton in effect being pushed into a position more akin to that of rational mechanics, though what is pushing him into it is a sequence of (theory-mediated) inferences from phenomena
6. The key thing to notice is that all these points in the "Copernican Scholium" are coming from Laws 3 and 4 (and two conclusions of Version 1: multiple centers, and a^3/P^2 as a measure of the underlying attractive strength of each center)
    a. Not from universal gravity, for reasoning neutral with respect to mechanism of centripetal force
    b. Nor from an especially tenuous chain of reasoning; the weakest link in the chain would appear to be Laws 3 and 4 themselves, which is why I say that they are doing the main work

D. Resistance and Planetary Motion
1. The other addition to Version 3, here called the "Resistance Scholium", outwardly appears to be nothing more than a way of smoothing the transition from Problem 5 to Problem 6
    a. First paragraph says that up to this point he has been considering motion in non-resisting media as a basis for drawing conclusions about the motion of celestial bodies in the aether
    b. Aether either non-resisting or insensibly resisting
    c. Notice the Cartesian tie between density and quantity of solid matter: Newton has not yet got his concept of mass!
2. Bottom line of argument of first paragraph: results so far are applicable to planets and comets, for no grounds whatever to conclude that aether offering any resistance
    a. Conclusion stated at beginning of second paragraph: "Motions in the heavens are ruled therefore by the laws demonstrated"
    b. Note: propositions here being termed "laws" in original, not after a revision by Newton
3. Argument: no observed "dissipative" effects, which would have to be present were the medium offering any resistance at all
    a. Argument appealing to claim that resistance proportional to density presupposes that density of aether small -- perhaps justified by negligible effects in measuring densities of bodies