    d. The experimentally confirmed solution for centrifugal force in uniform circular motion may help explain why the law of inertia became so quickly accepted in some circles

7. The logic of this evidence is worth noting even though Huygens himself said nothing about it
    a. The principle of inertia, contraposed, says that any deviation from inertial motion requires a "force"
    b. Should be able to characterize the magnitude of the force in any given type of deviation -- e.g. express a measure of the force in terms of the motion
    c. Given such a measure, the question is whether it is correct
    d. Verifying by some other independent way that this measure of the force is correct then legitimates the conclusion that a well-defined force is present, and hence provides evidence for the contraposed form of the principle
8. Huygens's failure to emphasize it notwithstanding, this is an archetypal example of a form of evidence that has been central to physics ever since Newton's *Principia*

IV. Huygens on the Center of Oscillation: Real Pendulums

A. Background: Ideal versus Real Pendulums

1. Huygens's *Horologium Oscillatorium* is divided into five parts, the first of which describes the mechanical features of the clock shown in Appendix and a maritime clock
    a. Part II: theory of free fall, inclined planes, and fall along a curved path, culminating in the isochronism result for the cycloidal pendulum (see Appendix)
    b. Part III: the theory of evolutes, including a justification of the device introduced in the clock to maintain its isochronism (see Appendix)
    c. Part IV: the theory of the real, physical pendulum, in contrast to the ideal pendulum of Part II, plus announcement of cycloidal and small-arc pendulum measures of g
    d. Part V: describes a conical pendulum clock, the bob of which is kept on a paraboloid by a curved plate, and indicates that it too can be used to measure g
    e. Appendix: 13 theorems about centrifugal force, without proofs
2. Theory in Part II employs Galilean principles supplemented by what we now call the principle of inertia throughout -- all the Galilean principles listed in notes for class 7
    a. Three hypotheses at the beginning (see Appendix) license parabolic projection if gravity is taken as uniform and acting along parallel lines
    b. Proposition 1: "derivation" of uniform acceleration of gravity question-begging in assuming same action in each unit of time (Huygens was not always careful about rigor in derivations from listed hypotheses)
    c. Proposition 5: proof of mean speed theorem (via reductio ad absurdum)
    d. Propositions 6-9: generalized pathwise independence principle in ascent as well as descent derived (via reductio ad absurdum) from Huygens's Torricellian principle

e. Proposition 10: universality of speed dictated by height of fall

3. The theories of the cycloidal and small arc pendulum in Part II are idealized in more than one respect
   a. As always in the Galilean tradition, resistance effects are ignored, though these effects can be made small, and of course the clocks themselves can be adjusted to compensate for them (and to compensate for the resistance of the rest of the mechanism)
   b. The pendulum itself is then idealized by ignoring the bulk -- i.e. the quantity of matter -- of its string and by treating the bob as what Euler later called a "point mass"
   c. Friction between the evolute wall and the string also ignored
4. Theory of evolutes in Part III gives a theory of the curvature of a large variety of curves -- i.e. of how these curves are defined as a sequence of osculating circles
5. The section from Part IV develops a theory that drops the idealization of the pendulum itself, but still without taking into account resistance and friction effects
   a. Of practical as well as theoretical interest, because pertinent to way clocks tuned, via small masses added on the end, resulting in multiple bobs
   b. Of course, also yields results for pendular motion involving a rigid body oscillating about a point, with no string
   c. And provides more exact way of calculating the length of simple pendulum with heavy non-point mass on the end
6. As Huygens remarks at the outset, the theoretical issue concerns the center of oscillation, a problem that Mersenne had proposed to him and to others in the 1640's, with no successful respondents
7. I assigned this section, in spite of its greater difficulty, with two things in mind:
   a. To show how far Huygens was able to advance theoretical mechanics beyond the point where it stood in the mid-1640's, leading toward the general theory of the motion of rigid bodies developed by Euler and d'Alembert in the 18th century
   b. As an example of Huygens's willingness to begin dropping idealizations, and hence to work in a realm where small discrepancies from an idealization become evidence for a refined theory and success of refined theory then becomes evidence for principles underlying both theories!

B. The Compound Pendulum and its Significance

1. The problem: given a pendulum with several bobs on a string, what is the length of a simple pendulum with the same total weight and the same period
   a. Mersenne had given the problem to Huygens in his youth (and to Descartes, who failed)
   c. (Cannot be solved from Newton's laws of motion by themselves)
2. Mersenne knew from experiments that the center of gravity of the bodies is not the answer: the required simple pendulum is longer than this
   a. Mersenne, of course, did not know why: some of the motion (energy) is going not into translation, but into rotation of the masses about their common center of gravity