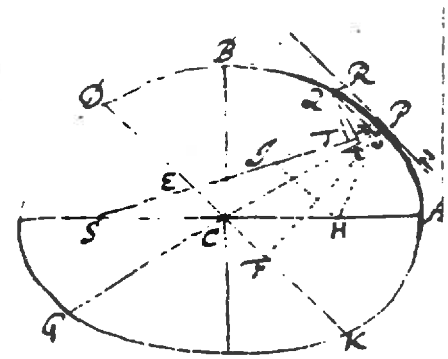
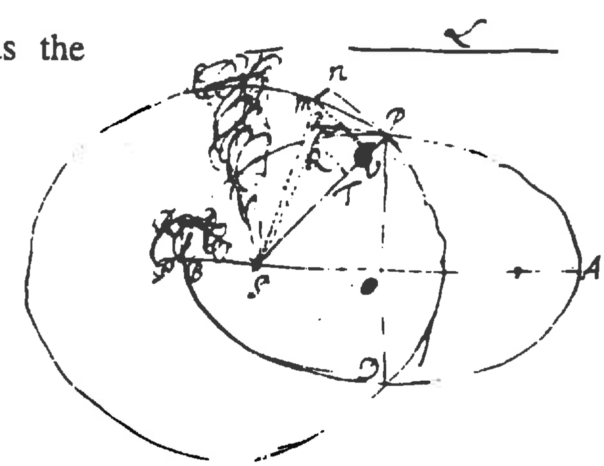
De Motu Corporum in Gyrum

Problem 3. A body orbits in an ellipse: there is required to find the law of centripetal force tending to a focus of the ellipse.



Scholium. The major planets orbit, therefore, in ellipses having a focus at the center of the Sun, and with their *radii* drawn to the Sun describe areas proportional to the times, exactly as Kepler supposed.

Theorem 4. Supposing that the centripetal force be reciprocally proportional to the square of the distance from the center, the squares of the periodic times in ellipses are as the cubes of their transverse axes.



Theorem 4. 3/2 Power Rule Holds for Confocal "Keplerian" Ellipses

From Problem 3 and Stipulation

- $L \times QR = QT^2$; $2SP \times MN = MV^2$
- Force at P is the same, so effect of force in same time the same: $QR = MN$
- Latus rectum $L = PD^2/AB = PD^2/2SP$

Proof

- $L/2SP = QT^2/MV^2$
- So, $QT/MV = PD/2SP$
- Area-SPQ/Area-SPM = PD/2SP =
  $= (\frac{1}{4}\pi AB \times PD)/(\pi SP^2) =$
  = Area-ellipse/Area-circle
- **But then *incremental times* always in this ratio, so that the *period* for the ellipse = the *period* for the circle, and hence the conclusion follows from Corollary 5 of Theorem 2**

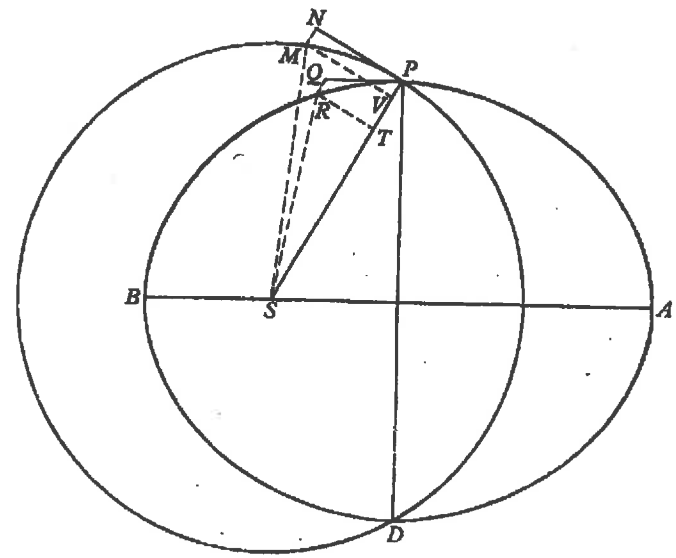


Fig. 63

Note: Q and R reversed as above in the original

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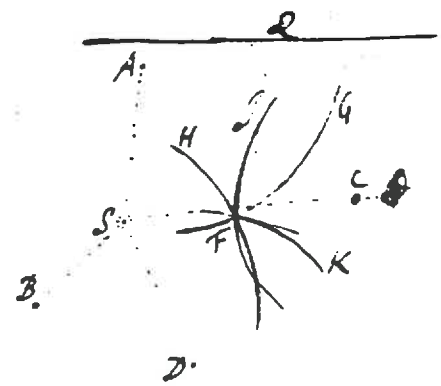
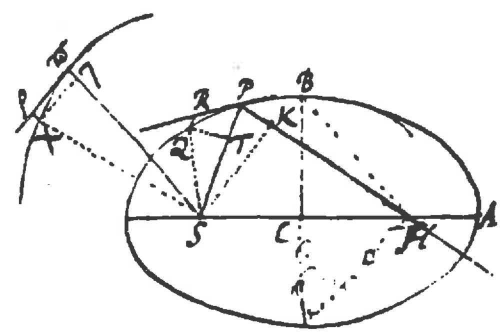
Scholium. Hereby in the heavenly system from the periodic times of the planets are ascertained the proportions of the transverse axes of their orbits. It will be permissible to assume one axis: from that the rest will be given. Once their axes are given, however, the orbits will be determined in this manner.



Problem 4. Supposing that the centripetal force be reciprocally proportional to the square of the distance from its center, and with the quantity of the force known, there is required the ellipse which a body shall describe when released from a given position with a given speed following a given straight line.



$$F_{cent} \propto \frac{[a^3/P^2]_S}{r_{SP}^2}$$

Scholium. A bonus, indeed, of this problem, once it is solved, is that we are now allowed to define the orbits of comets, and thereby their periods of revolution, and then to ascertain from a comparison of their orbital magnitude, eccentricities, aphelia, inclinations to the ecliptic plane, and their nodes whether the same comet returns with some frequency to us.

Problem 4. Solution for the "Initial-Value" Problem

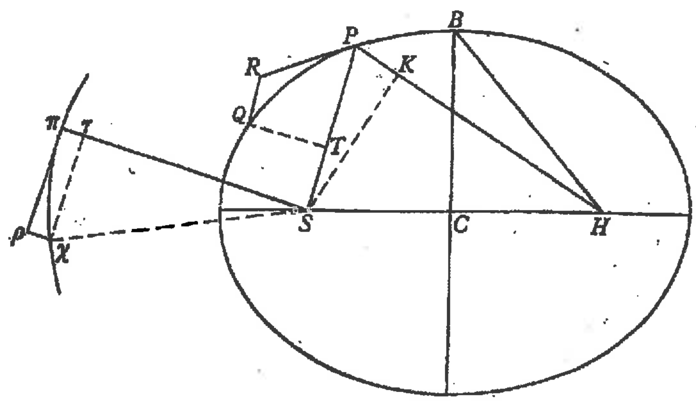


Fig. 66

Given: The velocity PR, in direction and magnitude, at P; the strength of the inverse-square centripetal tendency toward S: $(a^3/P^2)_S$. That then gives the areal velocity of P about S and the uniform motion in the circle $\pi\chi$ about S.

From Prop. 3 then: $QT^2/QR : \chi\pi^2/\chi\rho :: L : 2S\pi$, and so L, the latus rectum of the trajectory, is given

From the geometry of the ellipse, ∠ RPH = 180° – ∠RPS, and so the line PH, from P toward the other focus H, is given in direction, leaving only the problem of finding its length.

From a series of steps,

$$(SP + PH)/PH = (2SP + 2KP)/L$$

and so the length PH is given, determining the location of the other focus H, and hence too the length of the major axis = (SP+PH) and its direction relative to S, P, and PR.

If L = (2SP + 2KP), then the trajectory is a *parabola*;
L > (2SP + 2KP), then the trajectory is an *hyperbola*.