THE NEWTONIAN REVOLUTION – Part One
Philosophy 167: Science Before Newton's *Principia*

Class 6

Galileo's *Two New Sciences*: Local Natural Motion

October 7, 2014

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Philosophy 167: Science Before Newton's Principia

Assignment for October 7

Galileo's *Two New Sciences*: Local Motion

Reading:

Galileo, Two New Sciences, "The First Day," p. [105, bottom] to p. [116, bottom] (i.e. pp. 64-76 in Drake edition); pp. [127] to [132, middle] (i.e. pp. 86-91 in Drake edition).

---- "The Third Day," p. [190] through p. [225] (i.e. pp. 147-175 in Drake edition).

Questions to Focus On:

1. Galileo makes various claims about the local motion of heavy bodies falling to the earth in the absence of resistance from air or any other medium. Assuming that no one had a means of effecting a sufficient vacuum at the time to test such claims directly, in what sense were these claims empirical at all?

2. The Treatise from which Salviati reads is a work in mathematics, proceeding from definitions to geometrically proved theorems. What evidence authorizes the further step of concluding that actual objects satisfy the definitions, so that the theorems can be taken to apply to motion in the world?

3. A key postulate in the Treatise is that the velocity acquired by an object moving on an inclined plane depends only on the height from which it starts, and not on the inclination of the plane. What grounds are offered for accepting this postulate? Are the grounds adequate?

4. What exactly does the legendary inclined plane experiment described on pages [212] to [214] show? In particular, what background assumptions have to be adopted to draw Galileo's conclusion from the stated observed results?

5. Galileo insists in the "First Day" that circular pendula of the same length are isochronous -- the time it takes for the pendulum to reach the bottom when started from 60 degrees is exactly the same as the time it takes when started from e.g. 10 degrees. How can he tell this? Why would it be of value in the design of clocks?

Galileo's *Two New Sciences*: Local Motion

I. Local Motion Versus Celestial Motion: The Problem
   A. Astronomy Versus the "Science" of Motion: 1638
      1. 1638, the year in which *Two New Sciences* was published, was the year in which Horrocks was making extraordinarily small changes in the orbital elements of Venus and Earth, to yield an account of Venus's motion within observational accuracy for the first time
         a. I.e. changes Kepler's value of Earth's eccentricity from 0.0180 to 0.0173, of Venus's eccentricity from 0.00692 to 0.00750, and of Venus's semi-major axis from 0.72413 to 0.72333
         b. In the process reducing discrepancies in Venus longitudes from 5 min of arc to less than 2 min, and removing a 0.11% residual difference in Kepler's 3/2 power rule
      2. *Two New Sciences*, by contrast, generally shows little concern for such precise agreement between theory and observation, and at many places outrightly dismisses such concerns
         a. Though a mathematically precise theory, no theoretical calculations to high precision save for the Tables at the end of "The Fourth Day", and this precision scarcely pertinent
         b. And almost no numerically precise results of observations
      3. Galileo working in the context of a 2000 year tradition of predominately quasi-qualitative claims about motion -- e.g. speed acquired in fall varies directly with weight and inversely with the density of the medium (more or less)
         a. Orbital astronomy concerned with comparatively precise numerical claims of one sort or another from the Babylonians on, and surely from Ptolemy on
         b. With the exception of a few theorems on motion by an imitator of Euclid, no mathematical theory of "local" motion remotely akin to the mathematical theories of planetary motion
         c. Nearest thing to it: impetus theory of Buridan and Oresme, and efforts by Mertonians
      4. Galileo himself had held a view of natural motion derived from Archimedean theory in the 1590's (*De Motu*, as per Settle)
         a. Objects have a natural velocity of fall that depends on their density and the density of the medium (our terminal velocity)
         b. This is what one generally observes with bodies falling in water, and also for bodies that are not too heavy in air -- i.e. what happens in nature
         c. Transition to the natural velocity via a gradual decay of non-natural "virtus impressa" after the impressed force ceases to act -- e.g. after what is holding the object is removed
         d. This view gradually gave way to recalcitrant experimental results -- bodies arriving at the ground at virtually the same time and bodies always gaining speed on inclined planes -- which Galileo initially attributed to experimental limitations and imperfections
      5. The science of local motion offered in *Two New Sciences* is thus truly immature science -- a new science at its very beginnings

    a. The physics in Kepler was immature science, but the efforts on trajectories were far from it; indeed, even Ptolemy was well past the point of immaturity at which Galileo was starting
    b. So, here able to see more clearly the process of getting a science off the ground, of the struggle to turn observations into evidence
6. Yet notice that Kepler's orbital astronomy and Galileo's mathematical theory of motion were endeavoring to achieve formally the same sort of thing -- a mathematical specification of motions, trajectories and locations along them versus time
    a. I.e. a mathematical specification of position in space and speed versus time
    b. {Though Galileo imitating in style of presentation not mathematical astronomy, but Archimedes (d. 212 B.C.), whose works (re-published in 1543) he had studied under Ricci at Pisa
    c. Galileo’s “paradigm” not just Archimedes legendary *On Floating Bodies*, but also works in “mechanics” as *On the Equilibrium of Planes*, which announces the principle of the lever}
7. And also the same thing evidentially, namely to establish some (presumably lawlike) generalizations concerning certain sorts of motions
    a. Kepler through numerical agreement within observational accuracy-- or, with Horrocks, through progressively closer agreement -- along with plausible underlying physics
    b. Our question now: how was Galileo’s approach different from that?

B. The Fundamental Empirical Problems Contrasted

1. Although the formal goals were the same, Galileo faced profoundly different evidential problems in trying to formulate an empirical theory of motion
    a. I.e. problems of bringing observations to bear in order to answer questions
    b. Will spell these problems out and examine Galileo's general approach to them before turning to his substantive theory
2. The fundamental difference: no problem in orbital astronomy observing position versus time -- e.g. geocentric longitude and latitude versus time
    a. Relevant unit of time so long -- e.g. a day, or even 0.001 days (more than 86 sec) -- that no problem making observations
    b. Basic evidential problem was need to infer the missing dimension -- e.g. geocentric distance r -- and also to find preferred reference point for motion (relative to fixed stars) -- e.g. Sun -- in order to distinguish apparent motions from ones observable from fixed stars
3. By contrast, objects fall to Earth in too short a time for us simply to observe the sequence of locations versus time and thus to generate data akin to Tycho's
    a. Questions of distance of object from us and preferred reference for motion no problem at all in local motion
    b. But the relevant unit of time much too short to make useful naked eye observations, or even to recognize qualitatively any general pattern in distance versus time

        - c. Galileo needed Doc Edgerton's camera technology for his work on local motion (in contrast to e.g. a telescope)
        - d. Adding to this problem was that of determining velocities, especially when not uniform; observed planetary motions slow enough that could readily determine deg/day variations -- i.e. averaged angular motions across a few days -- as well as mean daily motion
    4. So, from the outset the problem of theory construction and adducing supporting evidence was of an entirely different form
        - a. In mathematical astronomy, had various observed locations versus time, and question was how well theory reproduced these observations; discrepancies could be used as a basis for further refinement
        - b. In the case of local motion, had only some qualitative phenomena, comparable to Greek astronomy before 200 B.C., and nothing like a set of observations theory was to reproduce
    5. Equally, however, those theorizing about local motion had one enormous advantage: they could intervene in the natural process in order to observe what happens under different circumstances
        - a. Use Ian Hacking's word 'intervene' here, rather than 'experiment' because want to preserve latter for the special, narrower forms of intervention in which observation is yielding an answer to a specific question
        - b. Astronomy was not just unable to conduct experiments, but was unable to intervene in the processes about which it was theorizing in any way at all
    6. So, *Two New Sciences* our first encounter with anything remotely resembling experimental science, though Galileo was performing experiments from 1590's on
        - a. In keeping with this, our central concern in this class will be with experimentation as a source of evidence -- experimentation as used by Galileo and some others
        - b. For several reasons -- some of them novel with Galileo -- a far more complicated matter than is often thought

C. A Further Complication: Pertinent Variables

    1. A further complication facing Galileo arose because he was trying to construct a mathematical account of local motion generally, and not just the motion of a handful -- e.g. 6 or 7 -- bodies
        - a. A general account requires decisions about which variables are relevant -- i.e. make a difference -- like *eccentricity* and *longitude of aphelion* in astronomy
        - b. Kepler did not have to give an account of the orbital elements a further new planet would have -- nor even of an account of such things as the different values of actual eccentricity -- in his account of the orbits
        - c. Apollonius had identified key variables in terms of which to parameterize the non-uniform apparent motions, in the process reducing those motions to mathematically manageable uniform motions

    d. Much of *Two New Sciences* devoted to questions of how to “conceptualize” -- i.e. which distinctions to mark -- and parameterize motion near the surface of the Earth

2. Best known decision of this sort by Galileo is that weight of falling objects not a pertinent variable, in contrast to Aristotle's claim that speed is proportional to weight
    a. (And in contrast to Galileo's earlier view)
    b. Famous Galilean thought experiment to support his new claim, involving light and heavy objects falling together [107-109]
    c. Leaning Tower of Pisa experiment story probably a myth, for he never claims to have done it, though others may well have done it and he may well have done parallel experiments
    d. But does report what happens in such an experiment: heavy one lands first, but by small differences versus Aristotle's claim
3. Qualitative result of experiment thus licenses (1) the conclusion that Aristotle's claim is false! and (2) the conclusion that weight at most a minor or second-order variable
    a. Galileo initially contends that the statement that the heavy and light arrive at the same time describes the result to a very close approximation
    b. And that might license saying that they land at the same time, as a description of the phenomenon, even though the heavier always lands first
4. But Galileo does not stop there: the effect of weight is much greater in a medium in which resistance effects are large, like water, than in one where resistance effects are small, like air
    a. Argues that this trend a basis for concluding that no weight effect at all in absence of a resisting medium

        > "Surely a gold ball at the end of a fall through a hundred braccia will not have outrun one of copper by four inches. This seen, I say, I came to the opinion that if one were to remove entirely the resistance of the medium, all materials would descend with equal speed." [116]

    b. Thus weight not only a minor or second-order variable, but more important it is associated with a second-order mechanism or process beyond that of fall itself, something induced by fall -- a profound distinction
5. Galileo clearly attempted to come up with an account of resistance, but ultimately concluded (Day Four) that too many variables were involved for it ever to be amenable to a scientific account
    a. Starts with an account in terms of buoyancy, but adds surface friction and velocity effects
    b. Concludes that velocity effects become strong enough at some point to result in terminal or maximum velocities [137]
    c. The question whether a science of resistance is possible at all will remain a concern right through to the end of the next semester
6. {Notice the transformation from Galileo's earlier view of motion
    a. Acceleration now natural and basic, while terminal speed from a second-order effect

b. Transformation from experiments in which recalcitrant results come to be viewed as fundamental

c. In other words, Galileo's own path to the theory in *Two New Sciences* largely hidden from view there, but does involve a substantial reconceptualization, a la Kuhn

d. Most important element of reconceptualization: motion in the absence of a resisting medium versus motion in its presence; further element: equal increments in time, not space}

D. Galileo's Approach: An "Idealized" Science

1. Galileo sees one notable weakness in the argument that weight is not a pertinent variable: effects of weight may be attributable to resistance over large distances, but what about over small ones

   a. I.e. sees limitations of experiments used so far:

   > "The experiment made with two moveables, as different as possible in weight, made to fall from a height in order to observe whether they are of equal speed, labors under certain difficulties....In a small height it may be doubtful whether there is really no difference [in speeds], or whether there is a difference but it is unobservable." [128]

   b. Note: same type of problem as with stellar parallax: distinguishing between no effect at all and an effect too small to measure adequately

   c. Solution: devise an experiment which would reveal any such small differences through their cumulative effect

   > "...one might many times repeat descents from small heights, and accumulate many of those minimal differences of time that might intervene between the arrival of the heavy body at the terminus and that of the light one, so that added together in this way they would make up a time not only observable, but easily observable." [128]

2. Experiment uses two balls, one of cork and one of lead, at the end of 5 braccia pendulums

   a. Resistance affects the height of the light pendulum more, but time of descent remains the same for both, for they stay perfectly in phase with one another (over 100 cycles) [p. 128]

   b. Key point: lead and cork bobs pass equal arcs in equal times, and hence in equal speeds

   c. Since period independent of initial height and of speed, have grounds for arguing that period, unlike velocity, not affected by resistance at all -- i.e. separate two mechanisms and then confirm that weight shows up only in the second one

3. {Claims made by Galileo about circular pendula in "The First Day" a peculiar mixture of right and wrong, with many of the wrong ones, including this one; Meresenne had denied the isochronism of circular pendulums in print in the mid-1630s, before *Two New Sciences* appeared

   a. Small arc circular pendula are isochronous, and their periods are proportional to the square roots of their lengths, to high accuracy even with air resistance

   b. But large arc circular pendula are decidedly not isochronous, as is evident in trivial experiments, insofar as, for small values of $k^2 = \sin^2(\theta_0/2)$

$$P = 2*pi*\sqrt{(\ell/g)}[1 + (1/2)\, k^2 + (1*3/2*4)\, k^4 + (1*3*5/2*4*6)\, k^6 +.....]$$

        - c. In absence of resistance, period more than 1/6 longer for 90 deg initial half arc, 7 percent longer for 60 deg, and 4 percent for 45 deg than for 5 deg -- readily detectable in a careful experiment after 20 cycles (see table in Appendix)
    - d. Galileo had presented the experiment in a letter to his mentor dal Monte in 1602, which confirms that he did do it, making the isochronism claim all the more mysterious (Bertoloni Meli, p. 70)}
    - 4. Nevertheless, Galileo does have grounds for adopting, at least as a working hypothesis, the thesis that the detailed motion of falling objects results from two distinct physical mechanisms!
        - a. The mechanism of fall itself, and the consequent mechanism of medium resistance to fall
        - b. Weight (and shape and surface roughness) become pertinent in the case of observed falling objects only through the second of these, and can be ignored in an account of the first
        - c. First then depends only on height of fall so far as questions like 'What speed is acquired?' and 'When does falling object reach various intermediate points?' are concerned
    - 5. Such an account is going to comprise an idealized science in its making claims about what would happen in the absence of any resistant medium -- i.e. in a vacuum
        - a. Whether a vacuum was possible even in principle was disputed at the time, though Galileo, in keeping with his corpusculareanism -- i.e. atomism -- thought so
        - b. Hence maybe better to say, in the limit as resistance of medium becomes negligible
    - 6. A radically new approach, raising obvious question: how to assess whether it is right, other than through the long term success of the science predicated on it?
- E. The Evidential Challenge Facing the Science
    - 1. Important to see how radical this approach is: Galileo is proposing to offer not a theory that will approximate observation reasonably well, but a theory that will be exactly true of a situation that at that time could not be observed, for the simple reason that it could not be made to occur
        - a. In contrast to e.g. Ptolemy, who could defend his account as approximating observation reasonably well, Galileo can be taken to be making a much stronger claim of exact truth
        - b. And relative to this claim, how well the account does approximate observation becomes, in some respects, beside the point, vs. how well it would in the absence of a resisting medium
    - 2. Three distinct grounds can be offered for adopting this approach of ignoring one mechanism involved in the actual process and focusing exclusively on the other, even though it cannot be isolated in fact
        - a. A theory of the dominant or principal mechanism is possible, and it is needed in order to make the empirical investigation of the other mechanism tractable: divide and conquer
        - b. Experiments yielding observations of evidential value are possible, provided only that the confounding effects of the other mechanisms be largely eliminated
        - c. No theory of the other mechanism is possible at all, so that a science becomes possible only through such a move -- Galileo

3. The obvious difficulty with adopting this sort of approach at the outset is that it tends to undercut the very possibility of developing evidence for the theory through comparing precise measurements with precise predictions
    a. Of course, in Galileo's case could not easily make precise measurements at the time anyway, because of the time scale of the phenomena
    b. But even if he could, discrepancies between prediction and experiment would not as such have been grounds for falsification or revision
4. Kepler's situation, by contrast, is that he first develops an account, then finds small residual discrepancies, putting him into position to argue that various higher-order mechanisms are at work
    a. For Kepler the question of discrepancies between observation and theory is central to the entire evidence process!
    b. For Galileo, it cannot be once he stipulates that exact agreement with observation not the goal
5. The challenge of developing an idealized science of the sort Galileo is proposing, then, is to find other ways of bringing evidence to bear on the theory
    a. Ways that will allow distinctions to be drawn between errors in the theory and confounding effects from other mechanisms -- this is the key concern
    b. Challenge doubly difficult in Galileo's case since the observations themselves were often so difficult to make
6. And, of course, the danger lurking in the wings is a science that is immune not just to falsification, but to progressive refinement on the basis of empirical evidence
    a. A non-empirical, philosophical theory all over again, though mathematically motivated
    b. Ultimately justified on grounds of elegance etc.
7. The move to such an idealized theory made here by Galileo will not be the last time in the history of science that such a move is made; indeed Truesdell claimed it has been dominant in mechanics
    a. Whenever it is made, the evidence problem becomes complicated in special ways
    b. Want to look carefully this week and next at how Galileo and his contemporaries tried to deal with it and problems related to it

II. "The Third Day": Some Conceptual Obstacles
  A. Galileo's Conceptualization of Motion
    1. Galileo faced another difficulty that anyone must face at the beginning of a new science: no reason at all to think that the concepts and distinctions with which you initially describe the phenomena are effective, and not systematically misleading
        a. By 'concepts' here, I primarily mean sets of distinctions and the basis for making them -- e.g. his weight versus heaviness (see Drake glossary)
        b. He was acutely aware that Aristotle's concepts had proved an impediment -- e.g. lightness as a correlative property of heaviness -- and hence he had profound reason for concern

   c. But on top of this, his situation one in which hard to bring empirical considerations to bear, and empirical considerations are the preferred basis for revising and refining concepts
   d. I.e. prefer distinctions empirical world forces on us
2. The concepts and distinctions Galileo starts with are quite remote from ours; we might even say that he was thoroughly confused
   a. One of Kuhn's key points is the need to read works in history of science from the perspective of the conceptual schemes of the time rather than ours -- prerequisite for understanding
   b. This is the main problem in reading Galileo -- the way in which he conceptualizes motion is foreign to us
   c. We need to understand him first, so that we can later see how empirical considerations helped shape our way of conceptualizing motion out of his
3. Some of the conceptual differences are owing to his use of classical Euclidean and Eudoxean mathematics, where quantities are geometric magnitudes, not numbers
   a. Everything stated as proportions within the theory of ratios, with no fractions, divisions of unlike quantities, etc., so that cannot even talk of velocity as distance per unit time
   b. Galileo chose not to use the rapidly growing body of algebraic methods, and of course had no calculus to use; stayed mostly within Euclidean strictures
   c. Geometric quantities do offer one advantage over numerical ones: exact in spite of “incommensurabilities,” as illustrated by mean and third proportions (see Appendix)
4. But, over and above this, some of his key physical concepts -- most notably that of speed or velocity (*velocitas*) -- are very different from ours
   a. Speed a scalar "intensive" property of bodies in motion, akin to heaviness -- something that a moving body possesses to greater and lesser degrees
   b. In other words, speed and direction are two separate, independent quantities
   c. Uniform motion defined not in terms of speed, but in terms of equal distances being traversed in equal times [191]
   d. Axioms III and IV, which (like Axioms I and II) express qualitative comparative relations, relate speed to spaces traversed in equal times, leading to Propositions I-VI spelling out the specific quantitative relations among speed, distance, and time
   e. No quantity like our acceleration, with its units of length per unit time per unit time
5. Other physical concepts that are radically different from ours include his impetus and momenta, the coincidence of the words and some resemblance to our concepts notwithstanding
   a. E.g. impetus or momentum (*momenta*) for him an intensive property of a moveable that correlates with its effects on impact, but also a property that correlates with the effects of a downward weight on a lever arm, as in already well-developed “mechanics” of equilibrium and devices that overcome it

b. See Drake's Glossary, but always be cautious in taking Galilean concepts to be ours, for more often than you would think, they are not – especially so with '*momenta*'

c. For thorough analysis, see Galluzzi's *Momento, Studi galileiani*

B. Abrupt Versus Continuous Change in Speed

1. With some concepts -- e.g. uniform acceleration -- Galileo shows concern over their appropriateness for describing nature

> ...so I may, without offence, doubt whether this definition, conceived and assumed in the abstract, is adapted to, suitable for, and verified in the kind of accelerated motion that heavy bodies in fact employ in falling naturally." [198]

a. Definitions are arbitrary, but something more is demanded here

b. In other words, Galileo was aware of role of empirical considerations in shaping concepts and distinctions

2. One concern: continuous increase in speed, passing through intermediate degrees, versus abrupt change to e.g. full speed

a. Abrupt change in speed what we seem to see at moment when moveable impacts the ground, and have trouble seeing any gradual development of speed with naked eye

b. So, why is concept of continuous change of speed apropos at all

3. Answer: compare the effects of impact -- e.g. the depth of impressions in soft soil -- from dropping the same object from different heights

a. Greater effects from greater heights, but this because of greater speeds (since effects from speed when weight the same)

b. But continuous gradation of effects of impact, depending on height from which object dropped

c. Hence continuous range of speeds, from very small to great, related to distance of fall

4. Galileo here adopting "percussive force" -- i.e. "quality and quantity of impact" -- as an indicator of speed [199]

a. In fact the percussive effects on impact are a function of speed squared, not speed, as he might have realized from their correlation with height of fall

b. But his argument still goes through for concluding that speeds in fall seem to admit of continuous degrees

5. Note also, in passing, how Galileo dispenses with Zeno-like arguments against having to pass through an infinity of different degrees of speed to reach given speed [200f]

> "This would be so, Simplicio, if the moveable were to hold itself for any time in each degree; but it merely passes there, without remaining beyond an instant"

6. Key point though is that he invokes empirical phenomena to show that concept of continuously increasing speed is "suitable" to the task at hand

C. Motion Versus Rest: Impetus and Momenta

1. Because the new science is one about motion, the distinction between rest and motion does not as such play much of a role here
    a. For Aristotle a distinction of kind, not of degree
    b. Still, a pertinent question that one might want to ask Galileo
2. In a brief digression Sagredo offers a physical picture of the cause of motion under which the distinction would be more one of degree than of kind [201f]
    a. A body hurled upwards progressively loses the "*virtu*" impressed on it by the thrower -- the *virtu* that continues to drive the object upward after it leaves the hand of the thrower
    b. Once the remaining *virtu* diminishes to the point where it is in equilibrium with the *virtu* corresponding to its heaviness, "the moveable stops rising and passes through a state of rest"
    c. Impressed *virtu* then continues to diminish, so that the *virtu* from heaviness progressively out-balances it, causing progressive acceleration
3. Most interesting aspect of this conceptualization of the process of deceleration and acceleration is that Sagredo continues by trying to reconcile static with dynamic *virtu*

    > "Thus, when you support a rock in your hand, what else are you doing but impressing on it just as much of that upward impelling *virtu* as equals the power of its heaviness to draw it downward.... The rock always starts with just as much of the *virtu* contrary to its heaviness as was needed to hold it at rest" [202]

4. Notice how elusive the (Newtonian) concept of force is, under which only acceleration and not motion itself requires force; and how difficult it is to relate the force exerted by a heavy static object -- say on a pulley -- to forces causing motion
    a. We will be seeing the concept of dynamic force emerging as we proceed
    b. Kuhn has claimed (I think wrongly) that not even Newton was always clear about how to reconcile his concept with that of static force -- reconciliation not complete until the middle of the 18th century
5. Also notice how Salviati immediately badmouths Sagredo's physical conjecture, in the process marking a distinction between what we now call kinematics and dynamics [202]
    a. One of several passages cited as evidence for the claim that Galileo wanted science to answer "how" and not "why" questions
    b. This in spite of the fact that he elsewhere is quick to address "why" questions -- e.g. his theory of the tides

D. Alternative Concepts of Uniform Acceleration

1. Galileo now in a position to appeal to simplicity arguments to justify his definition of uniformly accelerated motion
    a. In effect, try simplest "rule" first and see how it conforms with *naturalia experimenta* [197]

   b. Trouble: why is uniform in time simpler than uniform in space

$$\Delta(v) = a*\Delta(t) \text{ versus } \Delta(v) = a*\Delta(s)$$

2. Notice that at the point Sagredo raises the issue in *Two New Sciences* [203] this is a conceptual, not an empirical question
   a. According to Drake, Galileo himself for a long time thought the two were equivalent, but could find no way to obtain the 1,3,5, ... progression he had observed on an inclined plane from it
   b. But here he presents the issue as a conceptual one, eschewing the appeal to the 1,3,5,... progression
   c. Save for pointing out that impact effects seem to be proportional to height of fall, and Galileo had said these effects came from the speed acquired (and not the speed squared)
3. The argument Galileo offers has generally been interpreted as trying to show that $\Delta(v)=a*\Delta(s)$ is incoherent
   a. Given two spaces, one twice the other, then velocities acquired must be proportional to these; but times for traversing proportional to the spaces, and inversely proportional to the velocities; consequently times the same
   b. The trouble is that the alternative acceleration rule is not incoherent: $s=c*\exp(at)$ and hence $v=c*a*\exp(at)$
   c. The only real fault of the alternative rule: it requires v not to be 0 at the beginning of motion
   d. Hence, Galileo's argument generally regarded as fallacious, though what he says does hold if t varies as s/v
4. {The question is whether the argument he offers is at least on the track of an underlying conceptual difficulty
   a. Drake's suggestion: argument envisages a one-to-one mapping between speeds in the half-space and speeds in the whole, from which Proposition II entails the same time for each space, implying instantaneous motion
   b. Fallacy would then be that he is in effect integrating with respect to space and time as if they are linearly correlated}
5. Of course, Galileo can scarcely be faulted for not having the mathematics needed to see what the alternative rule entails, and the reasons he gives in *Two New Sciences* notwithstanding, he did find empirical reasons to adopt $\Delta(v)=a*\Delta(t)$
   a. But it does bring out the way in which concepts that seem so automatic and clear to us in fact required great effort before they became that way
   b. And, while much of that effort was ultimately conceptual -- in the above case, mathematical -- the ultimate guiding factor for Galileo seems to have been empirical

E. A Fundamental Result: The Mean Speed Theorem

1. The discussion from [198] to [205] has removed the conceptual obstacles facing the proposed definition of uniformly accelerated motion, but it has not provided the reader any way to conceptualize such motion
    a. As the preceding discussion of the alternative rule for uniform acceleration makes clear, not in a position to conceptualize it as we do today: $dv=a*dt$ and $v= ds/dt=a*t$, and so $s=1/2*at^2$
    b. For not even Galileo could visualize -- i.e. conceptualize -- the difference between the two rules from their mere statement
2. The "mean speed theorem" -- Proposition I -- bridges this conceptual gap by allowing one to think of uniformly accelerated motion in terms of a corresponding uniform motion
    a. I.e. space traversed in a given time is equal to the space traversed by an object moving uniformly throughout at half the speed the accelerating object has at the end of the time
    b. Licenses inferences about spaces traversed and times required that are parasitic on results for uniform motion, including inferences enabling measured values of varying velocities!
    c. I.e. mathematically reduces uniformly accelerated motion problems to uniform motion problems, thereby avoiding any need for reference to instantaneous speed
3. A trivial result once integrals are used as above, for simply substitute the value for v at the end of the time for $a*t$ in the formula for s: $s = [1/2*at]*t = [\text{mean } v]*t$
    a. Galileo had no way of "adding up" all the infinity of speeds to get the distance covered -- i.e. had no way of integrating
    b. {Nor did the Mertonians and others – in particular, Oresme, of whom Galileo must have been aware -- who had discovered the mean speed theorem before Galileo}
4. Galileo's "proof" invokes a one-to-one comparison of momenta of speed in the uniformly accelerated case with the momenta of speed in the uniform motion case [208f]
    a. Once one-to-one correspondence granted, the argument goes through
    b. The lacuna: what assures such a one-to-one correspondence? -- this is the non-geometric step
5. Huygens ultimately achieved a purely geometric proof of the mean speed theorem, though one that uses reductio ad absurdum twice
    a. Galileo was averse to using reductio ad absurdum proofs at all in mechanics
    b. So, he might have continued to prefer his proof

III. "The Third Day": The Evidential Difficulties

A. The Postulate: Empirical Motivation and Grounds

1. What immediately follows the definition of uniformly accelerated motion in the Latin text is a postulate: "the degrees of speed acquired by the same moveable over different inclinations of planes are equal whenever the heights of those planes are equal" [205]

   a. Speed here clearly a scalar quantity -- direction irrelevant
   b. Idea that inclined plane motion is an instance of naturally accelerated motion can be found in Leonardo da Vinci
2. The postulate used to establish Propositions 3, 4, and 5, which, in modern terms, amount to saying that the acceleration along an inclined plane is sin(angle) times the acceleration in vertical fall
   a. I.e. $v=g*\sin(\theta)*t$
   b. A result we get by resolving the acceleration of gravity g into components, whether of accelerations or of forces, something quite alien to Galileo, even allowing for Day Four
3. The obvious value of the postulate is that it provides a means for empirically investigating naturally accelerated motion generally, and hence free fall, that lessens the empirical difficulties
   a. Slows the process down so that it can more readily be observed: time of fall through height varies inversely with $\sin(\theta)$, but speed at every height along the plane the same as speed at same height if instead falling vertically
   b. Time extended without increasing velocity, so that resistance effects certainly not amplified in the process, and perhaps they are inherently lower in this situation too
4. First empirical defense of postulate turns on claim that interdicted pendulum always reaches the same height from which it started, so that same momenta always yield same height
   a. But momenta in descent = momenta in ascent, so that same height always yields same momenta regardless of pendulum arc
   b. Salviati cautions that such reasoning only analogous when applied to inclined plane, where process of acquiring momenta is different from that along a circular arc
5. Second defense of postulate, added in later editions, turns on the empirical fact (Stevin) that the vertical weight required to hold G in equilibrium on an inclined plane = weight of G $*\sin(\theta)$
   a. Lemma: impetus of descent varies as $\sin(\theta)$, for impetus of descent is proportional to minimum force or resistance needed to stop it, which by above varies with $\sin(\theta)$
      (1) Authorization for key step: "it is manifest that..." [[216]]
      (2) Key question: What exactly is "impetus of descent"? Whatever it is, momenta of speeds are proportional to impetuses of descent
      (3) Thus lemma turns on a static-to-dynamic inference!
   b. Postulate in later editions proved as a theorem, using corollary II of Proposition II, the lemma, and Proposition II from the equable motion part of the treatise [[218]]
6. As we will see, the postulate is strictly speaking false, though it is true so long as it is confined to separate types of motion
   a. Lacuna in proof: the momenta (accelerations) in rolling and in sliding not the same, even though impetuses of descent (forces) are the same along the same incline

    - b. In passing, also notice the point just before [[216]]: impossible that a heavy body should move naturally upward; variations of this principle will be see again
  7. Independently of the rolling vs. falling issue, Galileo's postulate introduces a proposal that will remain in the forefront for at least the next 150 years -- e.g. through Lagrange's equations of motion – for it voices a fundamental principle of modern mechanics
     - a. Torricelli (1608-1647), Galileo's protegé, in his *De Motu Naturaliter Descendentium et Projectorum* of 1644 offers a different justification of the postulate, invoking a principle subsequently named after him and deriving the sine rule from it and the postulate from the sine rule (see Appendix)
     - b. And Huygens, adopts a modified form of Torricelli's principle to justify pathwise-independence claims in general, not just for inclined planes, a decade or so later, and then uses it throughout his later work
     - c. It then became a central element of modern mechanics from Huygens (and Leibniz)

B. Proposition II and the 1,3,5,... Pattern
  1. Proposition II is not only the pivotal mathematical result for uniformly accelerated motion, but also the pivotal result from an empirical viewpoint
     - a. Predicts a distinctive *pattern* in uniformly accelerated motion, akin to the pattern of retrograde loops that provided the basis for Ptolemy's bisection of eccentricity
     - b. A pattern that should be observable as a phenomenon of "naturally accelerated motion," even if only to reasonable approximation as a consequence of air resistance etc.
  2. Proposition II simply says that spaces traversed in uniformly accelerated motion are proportional to the squares of the times -- i.e. s is proportional to t^2
     - a. First corollary then says that the increments of distance over a progression of equal times are as the 1,3,5,... progression
     - b. Second corollary nothing more than times being proportional to square root of the distances
  3. Proof of the theorem and the corollaries uses only Proposition IV from the section on equable motion and the mean speed theorem
     - a. Postulate not needed for this proposition at all
     - b. Calculus steps taken care of by the mean speed theorem: $s = (1/2*v_{max})*t$, but $v_{max} = a*t$
     - c. Proof yields the further result that s varies as v_{max}^2 -- i.e. s_1:s_2 as $v_{1max}^2 : v_{2max}^2$
  4. The Scholium [214] asserts that result holds equally for vertical fall and for motion along inclined plane, both of which are taken to be uniformly accelerated (in the absence of resistance)
     - a. Scholium thus authorizing application of mathematical result to physical situations
     - b. But, unlike Postulate, not making claim about how vertical fall and inclined plane accelerations are related to one another

5. {If Drake is correct (pp. 85-90 of *Galileo at Work*) Galileo first came upon the phenomenon of the 1,3,5,... progression in conducting inclined plane experiments (ca 1603), and then subsequently discovered its relation to delta(v)=a∗Δ(t) (sometime before 1615)
    a. Discovered that this rule yields the progression, perhaps from knowledge of Medieval work
    b. And saw no way that delta(v)=a∗Δ(s) could yield it}

C. The Inclined Plane Experiment in the "Third Day"

1. Inclined plane experimental program described in *Two New Sciences* [212f] in response to Simplicio's request to adduce an experiment in support of earlier claim that natural motion conforms with the mathematical theory

    > "Like a true scientist, you make a very reasonable demand, for this is usual and necessary in those sciences which apply mathematical demonstrations to physical conclusions, as may be seen among writers on optics, astronomers, mechanics, musicians, and others who confirm their principles with sensory experiences that are the foundations of all the resulting structure." [212]

2. Inclined plane 12 braccia long and from 1 to 2 braccia in height, with smooth groove covered with vellum, and hard, well-rounded polished bronze ball {1 braccio = 22.99 in (Drake), 22.7 (Settle)}
    a. Measured times of descent via a relatively crude water clock, opening and closing an aperture and weighing the amounts of collected water on a delicate balance
    b. Descent from various fractions of total lengths, at different inclinations -- "experiments repeated a full hundred times"
    c. In particular, time for total length twice that of time for 1/4 of length -- an easy measurement
    d. Result: spaces always as times squared, with variation with inclination as subsequently claimed -- "never a difference of even the tenth part of a pulse-beat"
3. Questions have been raised, especially by Koyré, about whether he could have gotten the claimed results with such a crude device for measuring time
    a. Total time (rolling) for 12 braccia is 4.90 sec when h=1 and 3.46 sec when h=2
    b. Comparison of full length to 3/4 length when h=1 is 4.90 to 4.24 sec, and when h=2 is 3.46 sec to 3.00 sec
    c. Suppose in case of latter a 0.02 sec error, say 3.02 instead of 3.00 sec: then implied space 7.03 cm different from 9 braccia
    d. All this even assuming that ball rolling perfectly, with no bounce, no sag of plane etc.
4. Mersenne, who tried to replicate the experiment (from a much cruder description in the *Dialogue* pp. 23-31, including a figure not at a low angle of inclination), even using a fast pendulum, could not get such compelling results (see Appendix)

    > "I question whether Lord Galileo ever did the experiments of falls along the plane, since he nowhere says so, and the proportion he gives often contradicts experiment." *Harmonie Universelle*, 1636, p.112

        a. Difficult to get the "right" results in the experiment even when know what they are supposed to be
        b. Therefore terribly easy to get "wrong" results, potentially falsifying theory without knowing whether just a shortcoming in experimental design
        c. Mersenne's problem (see Appendix) appears to have been imperfect rolling -- e.g. at angles too high – and not imprecise time measurement, but main point remains either way: experiment not straightforward to pull off, requiring careful design (and fairly small angles of inclination)
        d. The special care and effort needed to get the experiment to yield the "right" result is not unique to this example: a general situation in science
    5. Tom Settle's modern repetition of the experiment, following Galileo's instructions, was far more successful (Appendix)
        a. With practice (and warm-up) measured times from water-clock within 1/10 sec of predicted times (as Galileo said), and most within ± 1/20 sec
        b. Good agreement for 6.86 deg of inclination (vs. Galileo's announced maximum of 9.6 deg), as well as 3.7 deg
        c. The question, then, is how much effort Galileo put into the development of this experiment
    6. The experimental program thus could at least have shown that the observed results were compatible (to within observational errors) with the 1,3,5, ... progression -- i.e. the results did not clearly falsify the claim -- and they may have shown much more (e.g. evidence for Galileo's "postulate")
        a. Quotation of data themselves would have helped us see just how strongly the results supported the claim
        b. But no data quoted, either in *Two New Sciences* or in notebooks, and hence cannot assess this
        c. Nor can assess whether he began to encounter confuting data at higher angles -- e.g. at 30 deg, for which t in theory 2 sec
        d. {For reasons that will become evident below, if not already from Mersenne's data, the 1,3, 5,... progression will almost certainly cease to hold as the angle of inclination is increased}

D. The Earlier Inclined Plane Experiment -- Drake (*Galileo at Work*)
    1. Drake has reconstructed an earlier inclined plane experiment, primarily from a notebook entry with a list of numbers that he interprets to be an observed instance of the 1,3,5,... pattern
        a. Data points: 33, 130, 298, 526, 824, 1192, 1620, 2123 [2104], where latter is taken to be a correction
        b. Compared with: 33, 132, 297, 528, 825, 1188, 1617, 2112
    2. Outfitted inclined planes with frets that would yield slight sound when ball passed, and then moved frets until ball passed each in uniform times

        a. Got around problem of measuring time by using musicians who could detect discrepancy to within 1/64 of a beat, with a half second beat
        b. Musicians maintained beat, and frets were moved until e.g. each one passed in one beat -- e.g. around 4 sec
    3. Only the last tabulated value, when the ball is moving rather fast, falls outside a 1/64 error of beat (see p. 89 of Drake *op. cit*.)
        a. Drake's proposal is that experiment revealed the 1,3, 5,... progression to Galileo – phenomenon drawn from experiment
        b. But equally, then, can be used to argue that it provides evidence for uniform acceleration down inclined plane
    4. Depending on how musicians were used and the steadiness of their beat over time, this experiment could provide more telling support than the one claimed in *Two New Sciences*
        a. If, indeed, Galileo obtained the 1,3,5,... progression from it, then cannot be accused of "cooking" the experiment
        b. But worries arise if have the prediction first and trying to use this procedure to test it
    5. Also, notice that this experiment, unlike the one in *Two New Sciences*, provides no empirical basis for choosing between the square rule and indefinitely many other rules that would yield the specific idealized numerical sequence given above
        a. Square rule the simplest, but other curves can be fit through these data
        b. Other experiment, by contrast, compares a variety of distances along same plane, thus showing in principle that the 1,3,5,... progression holds for numerous different initial increments, and the distances in the progression correlate with the sine of θ across different heights!
    6. In sum, I am prepared to grant that Galileo did perform some sort of inclined plane experiments, obtaining the 1,3,5,... progression to sufficiently high accuracy for small angles to convince him legitimately that it is a real phenomenon
        a. How long it took others to become proficient in this experiment is unclear -- or whether they even tried to become proficient
        b. Also unclear whether Galileo discovered that get the "right" result only so long as angle small and then suppressed this finding
        c. (Would like to know more about the history of inclined plane experiments, for reasons that are about to become evident)

E. A Crucial Lacuna in the Evidential Argument
    1. The truly crucial lacuna in Galileo's evidential reasoning using the inclined plane -- the lacuna that yields a true, systematic error -- lies in the Postulate
        a. Says ball on an inclined plane will acquire the same speed in falling through a height h whether it falls vertically or along an inclined plane

   b. But unless the ball can somehow be made to fall along the plane entirely friction free -- i.e. be made to slide, with no frictional resistance at the point of contact with the plane -- this is false

2. Rolling down an inclined plane is different from sliding friction free down an inclined plane
   a. Rolling of a spherical ball in a groove: $a = 5/7*g*\sin(\theta)$, so that

      $$v_{end} = \sqrt{(10/7*g*h)},\ t_{end} = \sqrt{(14/5*h/g)}/\sin(\theta),\ \text{and}\ l_{end} = 5/14*g*t^2*\sin(\theta)$$

   b. Friction-free and medium resistance-free slide or fall has $a = g*\sin(\theta)$, so that $v_{end} = \sqrt{(2gh)}$, $t_{end} = \sqrt{(2gh)}/\sin(\theta)$ and $l_{end} = 1/2*gt^2\sin(\theta)$
3. In other words, the rate of acceleration for a sphere rolling on its bottom is 5/7 that of an object completely unencumbered by friction, and hence the velocity acquired by a rolling ball through a height h will be 84.5 percent of that acquired by the falling ball
   a. A 28 percent difference in accelerations, a 15 percent difference in velocities, with consequent differences in times etc.
   b. Something Galileo was totally unaware of throughout
4. Given his descriptions of the inclined plane and the bronze ball, his inclined plane experiments almost certainly involved rolling
   a. Extraordinarily difficult to get any object to slide down an inclined plane friction-free, especially a round object, for the least friction will be sufficient to initiate rolling
   b. Comparably difficult to get a round ball to roll vertically downwards
   c. Can maintain rolling on inclined plane so long as inclination limited and ball does not bounce at all, whether using a v-gutter or a curved groove (see appendix for contrast between them)
   d. Once the inclination becomes high enough, however, the ball intermittently bounces from the surface, accelerating at the full rate rather than 5/7 of it briefly before resuming rolling
   e. Result will then be a somewhat random mixture of two different rates of acceleration
5. The reason Galileo never understood this conceptual distinction was, of course, that no empirical observations ever forced it on him
   a. An example in which empirical contrast needed to draw distinction in the first place, after which theorizing can respond
   b. Huygens appears to have been the first to appreciate the difference, in the early 1690s, when he had become concerned with what became known as the principle of conservation of *vis viva* and applied considerations from his work on the center of oscillation, but did not publish (acknowledge Nico Bertoloni Meli, in response to my challenge)
   c. Became theoretically evident with Euler's work of 1765, and to some extent even with his watershed monograph of 1750; question is whether anyone noted it publicly earlier
   d. Quite a comment about experimental practice if 1765 is the first publication where the difference is fully evident, and no prior experimental result announcing a conflict

F. The Effect of the Lacuna on Propositions III-VI

1. The need for a rolling versus falling distinction enters only in comparisons between the two, and not in any comparisons within either regime itself
   - a. Thus, for example, Proposition II and the 1,3,5,... corollary hold perfectly well for all rolling spherical balls (though not for cylindrical tubes) with respect to one another, and for all falling objects, independent of shape
   - b. Therefore, the main consequence of the lack of the distinction is to widen the lacuna in the evidential reasoning in which observations for rolling balls are taken to provide evidence for what happens with falling objects
2. Proposition III, which says that the times of descent from the same height along inclined planes and vertical are proportional to the distance traversed -- i.e. time is inversely proportional to the sine of the angle of inclination -- is false if object is rolling on plane
   - a. But result holds for different inclined planes so long as ball rolling (and same geometrical shape)
   - b. Similarly for Propositions IV and V, giving comparative times of descent for different planes
   - c. And all three propositions would hold in a regime of pure sliding or falling
3. A possible experiment here to reveal the need to distinguish rolling and falling: drop a ball and start a ball rolling down an inclined plane at the same time, with the heights chosen so that theory predicts both balls will reach the ground at the same time
   - a. According to Proposition III, the 12 braccia plane inclined at 30 deg would be matched by a 24 braccia height; but fall would happen in less than 2 seconds, and rolling on a 30 deg incline might be difficult to maintain
   - b. Instead use 2 braccia height incline, which a 72 braccia height (14 story building) would correspond to; in ideal circumstances, rolling would take 3.46 sec and fall, 2.93 sec, a noticeable difference; but in real world air resistance on falling sphere and brief bouncing will move these numbers closer to one another, making the difference difficult to detect
   - c. A conclusive test thus not so easy
4. The beautiful result in Proposition VI, all chords from top of circle or to bottom have the same time of descent, similarly holds only for rolling or falling separately
   - a. Here again have a result that would seem to be the basis for a critical test of the theory that might not require excessively precise measurements
   - b. But carrying through such a test, and thereby revealing the difference between rolling and falling, very difficult
5. Propositions III-VI outwardly provide not only a means for calculating and predicting what will happen, but also a basis for designing experiments
   - a. This was clearly Galileo's intent

  - b. Indeed, notice Galileo’s whole approach here: put forward a hypothesis, develop a mathematical theory yielding striking predictions that are amenable to quasi-qualitative tests!
  - c. But the experiments were almost certainly too difficult to set up in a way that would yield meaningful results at the time

G. Mersenne's Efforts and the Lacuna: 1633-1647

1. Mersenne, perhaps provoked by a remark in Galileo's *Dialogue*, sees a different way of bridging any lacuna in the evidence for claims about free-fall: measure the distance of fall in the first second -- in effect g/2 -- the constant of proportionality in $s \propto t^2$
   - a. Galileo's remark: objects fall 4 cubits in first sec, which Mersenne knew to be way too small
   - b. Galileo himself calls attention to a lacuna in the argument for the Postulate in the original edition of *Two New Sciences* [207]
   - c. If stable value regardless of height, and if it yields reasonable results for total elapsed times, then direct evidence for claim that free fall uniformly accelerated
2. Fr. Marin Mersenne (1588-1648) a professor of natural philosophy at the University of Paris, a close friend of Gassendi and Descartes, and a long time correspondent and admirer of Galileo's
   - a. Deeply committed to experimentation, and hence naturally tried to reproduce Galileo's experiments, as well as to conduct many further ones on his own, in the process discovering such things as the non-isochronism of circular pendula
   - b. Relevant publications: *Les Méchanique de Galilée* (1634), *Harmonie Universelle* (1636), *Les nouevelles pensées de Galilée* (1639), *Cogitata Physico-Mathematica*, *Phenomena Ballistica* (1644), and *Reflexiones Physico-Mathematica* (1647)
   - c. Huge intellectual correspondence: 17 volumes already published
3. Mersenne made the advance of using a 3 and 1/2 ft pendulum, with period near 1 sec (1 sec at 60 deg arc), to measure the distance objects would fall in varying times
   - a. Experiment performed 50 times, yielding 3 (Paris) ft in 1/2 sec, 12 in 1 sec, 48 in 2 sec, 108 in 3 sec, 147 in 3.5 sec (*Harmonie Universelle*, p. 138)
   - b. Note that listed values were perfect; actual values were 110 ft in 3 sec and 146.5 ft in 3.5 sec
   - c. Still, would appear to be fairly compelling evidence for the 1,3,5,... progression in free fall
   - d. A few years later (in 1640s) dropped objects from 300 ft dome of St. Peter's Basilica in Rome, finding times not 5 sec, as implied by above, but between 5 and 6 seconds
   - e. Ultimately concluded that Galileo's principle of free fall is only a rough approximation, even in absence of air resistance
4. Moreover, his earlier positive result is not all that good in retrospect, for implied acceleration is way too small -- around 788 cm/sec/sec, versus correct value of 981
   - a. Indeed, value closer to that for rolling spherical ball (701 cm/sec/sec) than for falling object

   b. Raising the possibility that he first made a measurement on an inclined plane, then let that value "prejudice" his observations in the free-fall experiment
   c. Other possible sources of error: objects too light to minimize resistance; timing inaccuracies
5. Mersenne knew something was going on, for the 3 ft pendulum he switched to later was descending from 90 deg in roughly 1/2 sec, the time his object was falling 3 ft
   a. I.e. constrained pendulum just as fast as free fall, something he knew was wrong -- a paradox
   b. Concludes that experiments are of limited value, for impossible to obtain precise measurements under controlled conditions -- this in spite of repeating the experiments many times

H. Riccioli's Efforts and the Lacuna: 1640-1651
1. Somewhat independently of Mersenne, Riccioli, a Jesuit Priest at Bologna, conducted his own very careful free-fall measurements during the 1640's in response to Galileo's claims
   a. Giovanni Battista Riccioli, S.J., (1598-1671), the last major scientific supporter of Ptolemy and opponent of Copernicus: *The New Almagest* (1651)
   b. {His system an amalgam of Ptolemy and Tycho: Jupiter and Saturn around the Earth, others around Sun -- claiming that evidence not yet sufficient to justify abandoning *Almagest*}
   c. Hence, generally not treated with the respect he deserves in the historical literature
2. Goes to great trouble during the early 1640s to try to construct a pendulum that would yield exactly 1 sec in astronomical (i.e. sidereal) time (see Appendix, and also Koyré)
   a. Repeated tries, with friends counting cycles and comparing to the number of sec from one noon to the next -- e.g. unwilling to accept 86,998 vs. 86,640 seconds (mean solar day, tied to 86,164 seconds of sidereal day)
   b. Finally accepts a pendulum with a carefully measured period of 59.6 thd, the results from which he can then ratio -- 0.63 percent discrepancy
   c. Uses this to define a 1 sec pendulum and then construct some very fast precise pendula -- e.g. 1/2 sec and 1/6 sec periods checked against his 1 sec pendulum
3. Next conducts free-fall experiments from various heights, using chalk covered spheres of the same size, but differing weights
   a. Finds heavier ball lands sooner -- e.g. lighter lags from 12 to 40 ft in fall of 312 ft -- but not proportional to weight
   b. Thus confirms Galileo's claim that Aristotle wrong and weight secondary, though notice that not so secondary as Galileo said (inches)
4. Then finally turns to question of 1,3,5,... progression, first measuring times from different heights of one building using suitably heavy spheres, and then verifying result by dropping balls from precalculated heights off different buildings and comparing times
   a. Announced values: 15 (Roman) ft in 1 sec, 60 ft in 2, 135 in 3, and 240 in 4 -- perfect agreement with 1,3,5,....; from mean speed theorem, v = 120 (Roman) ft/sec after 4 sec

   b. Obviously, we have to conclude that the results were adjusted to meet expectations; but even so, Galileo's theory at least survived the test: not incompatible with the results

5. So, with Riccioli have evidence for 1,3,5,... progression in free-fall comparable to Galileo's evidence for inclined plane, and evidential lacuna somewhat filled
   a. How good was Riccioli's value for fall in the first sec, 15 (Roman) ft?
   b. If the Roman foot is taken to be 29.57 cm (Koyré, Klein), then Riccioli’s implied value for the acceleration of gravity amounts 887 cm/sec/sec, wrong by around 9.5 percent
   c. If instead (as Steven Weinberg proposed to me) his Roman foot is taken to correspond to his claim that the Tower of Asinelli is 312 Roman feet high -- now said to be 97.20 meters high -- it is 935 cm/sec/sec, wrong by a less than 5 percent; this seems far more plausible, given that a uniform error across all the distances must be an error in the distance measure
   d. Either way, it is close enough to reveal a clear difference between rolling and falling if anyone had done the experiment, for the value for rolling would have been less than 12 (Roman) ft whichever the measure, a clear discrepancy from the value expected from Riccioli’s value
   e. Interesting question, given their concerns about the accuracy of their experiments, whether they would have taken the result seriously if they had done the experiment, or whether they would have instead attributed it to some sort of experimental vagary along the lines Mersenne did when confronted by a paradoxical result
6. At any rate by 1651 have not only some empirical evidence for Galileo's law of free-fall, but were also at least in a position to run an experiment showing the need to distinguish rolling from falling
   a. Through substantially improved means of measuring time
   b. Through measurement of a then comparatively easy and stable value to obtain, the distance traversed in the first second
   c. And through quite elaborate experimental programs in Riccioli's case, over a 10 year period

I. The Evidential Difficulties as of 1651: A Recap
   1. Because the experimental evidence for Galileo's "law" of free fall will be important later, let me summarize the difficulties in adducing evidence for it as of the early 1650s
   2. The "law" makes a claim about what happens in the absence of resistance, yet no resistance-free experiments could be carried out at the time
      a. The most that could be done was to take measures that seem to minimize the effects of resistance
      b. But then no experiment could be expected to yield precise agreement, and discrepancies were open to differing interpretations
   3. The best hope for carrying out precise, repeatable tests, the inclined plane, had the lacuna of assuming that inclined plane motion amounts to the same thing as vertical fall

    a. Galileo's postulate tantamount to claim that inclined plane acceleration is g*sin(angle)
    b. Statics evidence to support this in second edition
    c. But this evidence misleading, for unbeknownst to them the postulate conflates rolling and falling
4. Inclined planes at low angles could provide good evidence of a 1,3,5,... progression, but only at low angles
    a. At higher angles, a mixture of rolling and falling, leading to conflicting results that were likely not to have been repeatable
    b. In absence of rolling-falling distinction, no principled basis for discounting higher inclination results
    c. Raises an interesting question: does an experiment that yields confirming results only under certain special circumstances and yields disconfirming results in other circumstances, with no explanation of why, nevertheless provide evidence for a claim?
        (1) A more widespread phenomenon in experimental science than you might think
        (2) Issue at the heart of the "epistemology of experimentation"
        (3) {See the works of Allan Franklin for more on the epistemology of experimentation}
5. Direct measurements of rate of vertical fall -- i.e. distance in first sec -- had promise of allowing a host of follow-on tests once the value was established (as by Riccioli; but efforts fail to stablilize on a single value)
    a. Because of the care in the experiments, Riccioli's result should have taken precedence over Mersenne's
    b. But the difference between them raised questions, including about whether Riccioli's value the final word
    c. No attempt to bound uncertainty at the time, but we can see in retrospect (and Huygens saw in the 1650s) still off by almost 4 percent, if not almost 10 percent
6. The experimental community appears not to have been at all aggressive in looking for converging support of results and in investigating sources of discrepancies in experimental results
    a. As indicated most of all by failure to take vertical-fall values and conduct inclined plane tests using them
    b. Probably reflects lack of confidence in the potential of experiments
    c. Nevertheless, a failure to realize that the way to improve experiments is to carry out complementary ones, looking to improve technology and identify sources confounding the results
7. If you wonder why historians of science have become inclined to discount the importance of evidence in the acceptance of theoretical claims, this episode instructive
    a. Much harder to carry out telling tests of a claim than is generally recognized

        b. Even when have fairly good confirming results, usually also have a host of conflicting results and lacunae

IV. Galileo's Contribution to Science: Some Issues

  A. Galileo's Place in the History of Mechanics

    1. Even this brief glance at the problems Galileo and his contemporaries faced in getting a science of motion off the ground shows how precarious the familiar claim is that Galileo invented a new science of motion, which Newton then combined with Kepler's laws to form modern mechanics
        a. Not at all clear that Galileo has left us with much more than conjectures that others are going to have to straighten out and confirm
        b. And given how differently he conceptualizes motion from us, not clear that his conjectured "laws" are even ours, even though they have somewhat the same mathematical form and the evidence can be interpreted as bearing on our modern form as well
    2. The legend of Galileo diminishes in somewhat similar ways from a careful examination of his historical context, for his work in mechanics did not proceed out of whole cloth; he was the culmination of a 100 year tradition in Italian mechanics
        a. Starting from Leonardo da Vinci, who had strong interests in various practical questions within mechanics
        b. Tartaglia, Benedetti, and Guido Ubaldo developed accounts of such mechanisms as the balance, lever, pulley, and wedge, and challenged Aristotelian accounts of motion
        c. Galileo's unpublished early work -- *De Motu* (1591) and *Le Mechaniche* (1594) -- transparently within this tradition
    3. Galileo also probably used some of the fruits of a late Medieval tradition on uniformly accelerating motion
        a. Mertonians (at Oxford from 1320-1350), and Buridan and his student Oresme at Paris (also during the 14th century)
        b. Buridan, in particular, developed his account in terms of "impetus", and Oresme offered a "graphical" determination of distance traversed that was still being taught at Padua when Galileo was teaching there (see Clagett’s *The Scinece of Mechanics in the Middle Ages* for originals, Dijksterhuis’s *The Mechanization of the World Picture* for commentary)
    4. Moreover, others like Stevin (1548/9-1620) and Beeckman (1588-1637) in Holland were developing mechanics in parallel with Galileo
        a. Stevin: monumental work on theory of inclined planes, and various problems in statics -- that is, on equilibrium and its loss in "machines" like the lever, the pulley, the balance, etc. (what the word "mechanics" meant in the first half of the 17th century)
        b. Beeckman, with Descartes assisting, developed "law" of free-fall independently of Galileo around 1618

5. Galileo unquestionably a leading, central figure in the development of the modern science of dynamics, but his precise contribution to that science is more complicated, and requires more qualification, than legend would have it
    a. Legend, of course, makes it all sound so easy: once Galileo had rid himself of Aristotelian confusions, he proposed the law of free-fall, which, once proposed, was readily seen to be true
    b. In fact, in addition to Galileo's efforts over a 50 year period, we find Mersenne devoting effort over 15 years and Riccioli over 10 years to the experimental verification of Galileo's proposed law

B. Galileo's Contribution to Scientific Method
1. The legend that Galileo was the "father of" modern experimental science also begins to evaporate a little once one looks carefully into what Galileo did
    a. The old common view can be found in Mach's *The Science of Mechanics* (1893), which emphasizes the experiments Galileo describes in support of his claims
    b. But Mach fails to mention, or maybe to realize, that these experiments could not always be carried out with sufficient precision at the time to yield especially meaningful results
2. Galilean scholarship during middle of 20th century, led by Koyré, produced a radically different picture, under which Galileo was scarcely committed to experiment at all
    a. A Platonist who used crude measurements heuristically and as supporting demonstrations -- in contrast to designing experiments where the world would yield an answer to a question
    b. Thought-experiments instead of actual ones, making him seem more like Descartes: rational science
    c. Claims about experimental results so much polemical excess
3. Galilean scholarship during the last 45 years, led by Drake, has produced a more complex, and in some ways more balanced picture of Galileo's methodology
    a. Empirical observations and experiments played a critical role in Galileo's discoveries, especially from 1590 to 1610
    b. This fact somewhat suppressed in *Dialogue* and *Two New Sciences*, where experiments described as yielding rather more impressive results than we have reason to think he usually achieved
    c. Still, this revisionist view contrasts him sharply with the standard picture of Descartes
4. Whether Galileo performed most of the experiments he said he did or not, his verbal commitment to their importance in the development of any scientific theory is beyond dispute
    a. In other words, his insistence on experimentation had its impact independently of what he actually did, as we have already seen with Mersenne and Riccioli, and as we will continue to see with Huygens and Newton

    b. But he may well have misled many into thinking that the experimental confirmation of a theory is much easier and much more straightforward than it is, especially than it is in the early stages of theory construction

5. Galileo had a clear idea of how an experiment could be used to put the lie to theories – Aristotelian theories in particular – and he understood the role of experimentation as a corrective to misleading observations of nature
    a. The latter especially important insofar as it leads to a change of focus from saving the phenomena as they occur in nature to saving the phenomena as they occur in specially contrived circumstances that may never happen in nature
    b. But, granting that Galileo ever did perform any first rate experimentation in mechanics himself, neither it nor the logic behind it is altogether accurately reported in his principal works (sometimes for reasons of which he was not aware)
6. And though it was founded by his followers 15 years after he died, the Florentine Accademia del Cimento (1657-1667) -- as its name says, dedicated to experiment -- was inspired by him and his many statements about experiments (see Boschiero)

C. Philosophical Disputes Over *Two New Sciences*

1. In sum, not just Galileo the person and scientist, but also his chief scientific work, *Two New Sciences*, turns out on closer inspection to be complex and many sided
    a. An unqualified hero to some, a tainted hero to others, with experts on Galileo in each camp
    b. Though notice that still a hero either way, whether because of his substantive contribution or because of his sociological impact
2. Given this complexity and many-sidedness, it is not surprising that Galileo has become a central figure in some of the current philosophical disputes about the "rationality of science"
    a. Kuhn and those who agree with him that evidence plays a much more circumscribed role in the history of science than is customarily suggested offer Galileo as a prime example
    b. And people like Dudley Shapere, who want to defend the rationality of science, offer re-interpretations of Galileo in reply (*Galileo: A Philosophical Study*, 1974)
3. Paul Feyerabend, someone very much in the same camp as Kuhn, has used Galileo to argue in his *Against Method* that the very idea of a scientific method is almost always going to impede the development of science in one way or another
    a. Feyerabend argues that, on the one hand, Galileo violated almost every dictate of scientific method that anyone has ever proposed, yet, on the other hand, is unquestionably one of the greatest scientists of all time
    b. Since his violations of the dictates seem almost indispensable to many of his principal scientific successes, the only reasonable conclusion is that we would all be better off to abandon all such dictates and follow a policy of anarchism in science

4. By looking at Galileo's work on uniformly accelerated motion in a slightly broader context, hope to have shown that the issues being raised here cannot be addressed by looking at Galileo alone, but require attention to his historical context
    a. As Mersenne's and Riccioli's efforts make clear, however brilliant and original *Two New Sciences* was, its theory of uniformly accelerated motion quickly became a community wide project
    b. A project in which it was not blindly accepted and developed, but was subjected to serious critical scrutiny, within the limits that experimental technology at the time would permit
5. But this alone will not answer those who offer Galileo as a paragon of "irrationality" in science, for we have yet to begin showing that the community managed to bring substantial empirical evidence to bear on Galileo's proposed "laws"
    a. Galileo was central to the transition from an immature to a mature science of motion
    b. Kuhn would argue that that transition will invariably have a large irrational element to it -- a conjectural element reaching beyond all the available evidence
    c. Nothing we have said so far would contradict this -- it was extremely hard at the time to bring all that much telling evidence to bear

D. Issues We Must Face and Our Approach to Them
1. Before we finish with Galileo, then, we need to become more clear about exactly what contribution he made first to the substance of science
    a. To what extent did he put forward the cornerstones of the modern physics of motion -- e.g. laws like those discussed this time and next
    b. And to what extent were these laws the product of efforts subsequent to him
2. We also need to become clear about exactly what contribution he made to the methodology of science
    a. What is his conception of an idealized science and how is it open to empirical verification, falsification, and refinement
    b. What is his conception of the logic of experimentation, in contrast to mere observation, and how is experimentation to interact with theory
    c. And, more narrowly, how did Galileo's approach to establishing generalizations about "local" motions – specifically, the four fundamental "Galilean" principles listed in the Appendix – differ from Kepler's approach to establishing generalizations about celestial motions
3. Want to become clear not just about his conception of what science is and how it is limited, but also the conception of the prominent figures around him, like Mersenne and Gassendi, on whom he had so much influence and who then influenced others
    a. Is he – and are they – committed to the idea that the science of motion is inherently imperfectible – that is, can never become and hence ought not to try to be an exact science

   b. Is he – and are they – committed to the idea that science should focus on "how" rather than "why" – that is, should postpone concerns about causes and concentrate on what actually happens

4. In worrying about such questions, we need to distinguish between four quite different things:
   a. What Galileo originally did to "discover" – i.e. to convince himself of – the various results he announced
   b. What Galileo offers in print in the way of support for these results
   c. What others at the time took Galileo to be offering in the way of support for the results
   d. What others at the time did, following Galileo, to develop support for the results
5. Galilean scholars are primarily interested in the first two, while we are primarily interested in the last two
   a. Our ultimate question: from the point of view of Newton, what was and was not known in 1684
   b. This dictated by the evidential arguments in the community
   c. But we should not entirely lose sight of Feyerabend's challenge as we proceed

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Credits for Appendix

Slides 2-4: Archimedes (2002)

Slides 6, 7, 10, 12, 15, 19, 21, 23, 24: Galileo (1989)

Slides 16-18: http://aleph0.clarku.edu/djoyce/java/elements/elements.html

Slide 25: Torricelli (1644)

Slide 27: Galileo (1989, 2001)

Slides 28, 29: Settle (1961)

Slide 30: Hahn (2002)

Slide 31: Huygens (1888-1950, vol. 18)

Slides 32, 33, 36: Riccioli (1651)