Newton's Early Unpublished Work in Mechanics

I. Background: Newton in the 1660's and 1670's

A. Huygens and the Science of Mechanics

1. Huygens's efforts in mechanics from the mid-1650's until publication of *Horologium Oscillatorium* in 1673 yielded a substantial core of our modern science of mechanics

a. Much of the core that is taught in the first semester of introductory physics courses -- viz. results that do not depend on inverse-square gravity, instead taking *g* to be constant and parallel

b. Huygens not only put forward a wide range of results within this subdomain, all of which still stand, but he also provided evidence in support of them that meets modern standards

2. Huygens did more than just extend Galileo's fragment on "natural" motion to such topics as pendulums; his results extended the domain of the field beyond just motion governed by gravitational fall

a. Uniform circular motion and motion under perfectly elastic impact are not as such related to motion governed by gravitational fall at all

b. But Huygens ended up relating them to it by pursuing evidence for them that was tied to -- indeed, parasitic on -- Galileo's fragment of a theory of motion governed by gravitational fall

c. Two examples of this: the use of his version of Toricelli's principle in the treatment of impact, and the use of the conical pendulum as a way of getting evidence on centrifugal forces

3. In the process Huygens also extended the prevailing conception of empirical evidence to such an extent that one can argue not only that he met modern standards of evidence, but that he went a large way toward "discovering" and establishing them in these efforts

a. First, he made clear the extent to which theory is a vehicle for evidence, extending the range of predictions that can be derived from hypotheses and allowing much stronger inferences to be drawn from observations

(1) What allows him to achieve higher quality evidence than Galileo is, more than anything else, his having a larger fragment of a theory than Galileo had

(2) The extension of Galileo's theory to circular motion and the pendulums opened far more tractable ways of developing evidence

b. Second, he displayed the advantages of having a growing network of interrelated theories

(1) Doing so allows increasingly diverse data to be brought to bear on all parts of the network

(2) Evidence can then start accruing to older parts of the network while it is being extended to new topics and areas

c. The clearest illustration of this is the way in which Huygens's account of circular motion and the conical pendulum supplied the first real evidence for the principle of inertia

(1) The difficult evidential problem posed by this principle is that one must have some way of showing that an impediment or force is always required for a body not to continue moving uniformly in a straight line (the contrapose of the principle of inertia)

(2) Solution of this evidential problem: derive a theoretical measure of the force required for departure from such motion, and then confirm this measure experimentally

(3) Precisely how Huygens proceeds in his derivation of v^2/r: a measure of the departure from what the motion would be if it were to continue unimpeded

(4) This is the general form that the evidence for the law of inertia continued to take with Newton and thereafter: develop a theoretical measure of forces required for departure from such motion and then confirm this measure empirically

(5) Inertial motion not confirmed by direct experiment, but by serving as a sort of "null hypothesis" in the development of theories of force

d. Third, Huygens brought out, perhaps unintentionally, how measures of fundamental quantities that occur pervasively in a theory -- e.g. g -- can provide extremely high quality evidence bearing on that theory

(1) Convergent measured values provide stronger support for a theory than simple hypothesis testing does

(2) And small discrepancies in the measured values bring out the limiting bounds of accuracy of a theory and provide a basis for new discoveries, as the variation of g with latitude did

4. Using Kuhn's terms, Huygens established a paradigm for a tradition of normal science in mechanics, in the process transforming it from an immature into a mature science

a. The approach involved defining problems in a way that would make them amenable to mathematical solution -- "the mathematization of nature", to use Yoder's sub-title

b. But the distinctive feature of the approach was to tie these problems into previous theoretical efforts on other problems, in the process constraining solutions to them and opening avenues for empirical evidence

c. The resulting tradition, known as rational mechanics, might be better called theoretical mechanics insofar as its successful emphasis on theorizing went a long way toward making theoretical physics a distinct subfield

5. Huygens's efforts, for all their accomplishment, left a great deal of work to be done within this tradition:

a. Problems to be solved: e.g. the large arc circular pendulum, a fully general theory of elastic impact, vibration on a spring

b. Further mathematical development: e.g. improved methods with infinitesimals, instantaneously varying *conatus*, quadratures

c. Cleaner foundations: e.g. more general principles governing conceptualization of motion, preferred axioms, a unified treatment of forces

d. New and extended generalizations: as illustrated by the conservation of *vis viva* (ultimately leading to modern conservation of momentum, angular momentum, and energy)

   e. New and refined experiments: e.g. other measures of *g*, exposing the contrast between rolling and falling, and improved ballistic pendulum measurements

6. That said, the state of the sciences of mechanics and orbital astronomy had progressed a good deal -- with regard to both what had become known and the capacity to marshall evidence -- from where Kepler and Galileo (and Descartes as well) had left matters
   a. Both of those sciences had come to look much more like modern science
   b. In mathematical astronomy it had become clear that Keplerian theory holds to high enough approximation to set the standard, yet the proliferation of alternatives to it had forcefully raised questions about whether it holds exactly or even essentially exactly, and not just approximately
   c. And in celestial physics, Cartesian inertia as developed in Huygens's treatment of curvilinear motion had focused increasing interest on inverse-square centrifugal forces
7. Questions now: what was happening with Newton while all this was going on? In particular, how did the changes that were taking place affect his intellectual development?

B. Issue: What Difference Did Newton Make?

1. If Huygens's efforts before 1675 established the modern, maturing science of mechanics, and Hooke was already proposing interactive inverse-square central forces in celestial physics, what difference did Newton make?
   a. We have spent all this semester preparing to read Newton, only to find the usual answer to this question undercut -- Newton did not transform the immature science of mechanics into our modern mature science, nor was he the first to propose inverse-square central forces governing orbits
   b. Given that the course is called the "Newtonian revolution," the appropriate question now is, exactly what revolution?
2. This question is really best posed as two distinct questions which we will be answering from here on, beginning next week
   a. How, if at all, would the sciences of mechanics and mathematical astronomy have progressed differently if Newton had published nothing at all on the topic?
   b. And how would they have progressed differently if he had stopped with the tract "De Motu Corporum in Gyrum," and not gone on to the *Principia*?
3. As initial food for thought, consider Descartes' critique of *Two New Sciences*, a critique he would surely have extended to Huygens's work in mechanics: all of this is without foundation
   a. One complaint is that Huygens's theories do not proceed from fundamental, universal axioms -- e.g. Torricelli's principle is a glaringly parochial claim tied to the surface of the earth
   b. Another complaint is that the entire science is predicated on the assumption that gravitational fall and resistance are two distinct mechanisms and hence can be treated separately -- an assumption Cartesians rejected

4. In the spirit of such complaints, notice how little information was available in 1675 on whether Galileo's and Huygens's results would hold exactly in the absence of air resistance, and if not, what sort of approximations they were
   a. I.e. do they hold skewed or in the mean, and -- air resistance aside -- are they idealizations or mere approximations
   b. Air resistance effects make this question hard to get at, although some progress was made -- e.g. on the physical pendulum
5. Huygens himself developed a mathematically correct theory for motion under resistance proportional to velocity in the 1660s
   a. Horizontal, vertical, and projectile motion
   b. Experiments he conducted then showed that resistance appears to vary with v^2, and he was unable to handle this case mathematically
   c. Published his work only after Newton's *Principia*, preferring his mathematical approach
6. Similarly, little information was available on the ranges over which the various results hold and the *ceteris paribus* conditions -- beyond no air resistance -- under which they hold
   a. E.g. does g vary systematically at extreme high altitudes or at great depths below the surface of the earth
   b. Given any observed departures from the results -- e.g. Richer's -- the confounding effects of air resistance and other factors make it difficult to determine what to attribute them to
7. By comparison, a fair amount of information was available to support the claim that the results should be taken to be nomological, both from their derivation from a unified theory and the diverse evidence accruing to this theory, including high quality evidence
   a. Even so, a Cartesian could challenge their nomologicality on the grounds that the split between resistance mechanisms and the mechanisms treated in the results is spurious
   b. More information on underlying mechanisms -- e.g. from a theory grounded on universal rather than parochial axioms -- would strengthen the claim to nomologicality
   c. Still, arguably more claim to nomologicality than Kepler's rules as of late 1670s

C. Newton: A Biographical Sketch (to 1679)
1. Newton's father died two months before he was born, which was on Christmas day, 1642 (old calendar); and after his mother left Woolsthorpe to remarry three years later, he was raised by his maternal grandmother until 1653, when his step-father died
   a. He rejoined a family with three younger children, but two years later left for grammar school in Grantham, where he was most remembered for "his strange inventions and extraordinary inclination for mechanical works" (Westfall, p. 60)
   b. After a year away from Grantham managing his farm, with marked lack of success, his mother was persuaded in 1659 that he should return to school in preparation for university

2. Newton entered Cambridge -- specifically, Trinity College -- in 1661, one of roughly 300 students entering what had become somewhat of a degree-mill for the well-to-do
    a. Newton entered as a "subsizar", a student earning his keep by performing tasks for the fellows
    b. His education was classical -- including Aristotle -- until roughly 1664, when he started branching out on his own, reading extensively and beginning an intense study of mathematics
    c. Newton was elected to a scholarship in 1664, to a fellowship at Trinity in 1667, and he was appointed Lucasian Professor of Mathematics, succeeding Barrow, in 1669
3. Even though he published virtually nothing at the time, Newton was extraordinarily productive in the decade from 1665 to 1675
    a. By 1666 he had invented the calculus, and was de facto probably the leading mathematician in the world
    b. He followed this up with further work in mathematics, especially further development of the calculus, in the late 1660's and early 1670's: *A Treatise of the Method of Fluxions and Infinite Series, with its Application to the Geometry of Curved Lines* (1671, published in 1730s)
    c. He also devoted time to optics in particular, but to mechanics too and, to a lesser extent, theology, and he began his interest in alchemy during these years, when in his own words he was "in the prime of my age for invention"
4. Cambridge closed twice for periods during the plague years of 1665-66, and Newton returned home to Lincolnshire, where he had his "Annus Mirabilis"
    a. In addition to developing the calculus during that time, he developed his theory of colors and did various work in mechanics, including the first "Moon test"
    b. The attached accounts of this year in the Appendix, including Newton's own, attest to the extraordinary productivity that the year away from Cambridge generated, even after allowances are made for embellishments
5. The unpublished material on mechanics assigned this week presumably derives from work he did in the period 1665 to 1675, with the possible exception of "De Gravitatione" (controversial, but on my view at most a few parts of it date from around 1684)
    a. Newton was a pack rat, so that we now have an enormous body of notebooks, manuscripts, annotated books that he read, etc. on every topic: see the “Newton Project”
    b. But he did not generally date this material, so that we have to surmise when various pieces were written on the basis of his handwriting, the content, and ancillary information
    c. Newton's own later remembrances of this period add to our confusion in dating, for his recollections are not entirely accurate, often in ways that seem disturbingly suited to help him defend various claims to priority
6. The work in mechanics assigned tonight is fully representative of Newton's efforts in this field before 1679 -- indeed, before 1684, when for the first time he began to do more than just dabble

a. An earlier notebook -- "The Waste Book" -- and a brief manuscript -- the "Vellum" manuscript -- contain precursors of some of the papers assigned for tonight, as noted below

b. The only other work was a brief, unsuccessful foray into projectile motion under air resistance -- this following publication of James Gregory's work on this topic and pendulums (1673)

D. Newton's Work in Mathematics: 1664-1680

1. Newton seems to have begun educating himself in mathematics in 1663, from an elementary text on arithmetic and algebra by Oughtred (1631) and a more advanced text by van Schooten (1646)
   a. The work that appears to have brought him to the then-current forefront of the field was van Schooten's second Latin edition (1659) of Descartes' *Géométrie*
   b. From there he turned to numerous sources, including Wallis's work on indivisibles and infinite series and Barrow's works and (presumably) lectures at Cambridge
2. He discovered the fundamentals of what we now call the calculus over a two-year period from 1664 to 1666 (see chart in Appendix), culminating in his first tract, "To resolve problems by motion"
   a. General algorithms for solving problems concerning infinite sums, maxima and minima, tangents (see Appendix), quadratures, being worked on by Fermat, Pascal, and Huygens (among others) in France and by Wallis, Barrow, and Gregory (among others) in England
   b. Employing a Barrow device of a curve described by a moving point, and taking what we would now call derivatives with respect to time, which Newton called "fluxions" of "fluents"
3. This was followed by a tract in Latin, "De Analysi per Aequationes Infinitas," in 1669, which Barrow circulated, gaining Newton recognition as the leading figure in mathematics in England, and then a full-fledged treatise, *De Methodis Serium et Fluxionum*, in 1671, for which Newton was unable to find a publisher; in all of these he continued with fluents unfolding over "time"
   a. The range of the problems addressed in the latter is spectacular (see Appendix, where examples on curvature and a table of integrals are included as well): Newton had full control of the algorithmic methods that came to be known as the calculus by 1671
   b. The history of mathematics would have been quite different if that book had been published then (rather than finally in two different English editions during the 1730s)
   c. Leibniz's initial work on the calculus began in the mid-1670's and came to fruition in the mid-1680's, but unlike Newton's it was published in the leading journals and led to a tradition of research involving the Bernoullis, l'Hôpital, Varignon, and later Euler and several others
4. One shortcoming of Newton's early work on the calculus was a lack of perspicuous notation; only after the *Principia* in 1687 did he invent the dot notation that came to be associated with him
   a. For example, in the early work he often used lower-case letters to represent the fluxions (time-derivatives) of fluents represented by the corresponding upper-case letters
   b. By contrast Leibniz had a more perspicuous notation from the outset
   c. So even if Newton's early works had been published, they might not have caught on quickly