4. Proposition X: impetus at impact is impetus resulting in vertical fall from a height equal to the sublimity + the altitude -- thus relating the speed (and impact effects) of a projectile to that of same projectile in free fall
5. Propositions XI and XII relate altitude and amplitude for a given initial impetus, yielding a table of (relative) amplitudes and hence ranges as a function of θ and a table of (relative) altitudes as a function of θ
   a. Tables 1 and 2, [304], with amplitudes normalized to an amplitude of 10000, and altitudes normalized to an altitude of 5000, respectively for θ = 45 deg
   b. Tables can be used to determine relative magnitudes under the assumption of same impetus, but different θ
6. Proposition XIV relating altitude and sublimity for a given range, yielding a table of (relative) altitudes and sublimities as functions of θ
   a. Table 3, [307], with distances normalized to an amplitude of 10000 and hence a range of 20000.
   b. Tabulated values slightly inexact -- Galileo does not bother to carry enough significant figures; better calculations would preserve the strict symmetry that the theory entails

C. The Form and Content of Galileo's Table

1. As the tables and the accompanying Propositions attest, the mathematical theory allows a large number of problems to be solved, extending beyond those that can only be solved geometrically
   a. Problems of the form, given certain quantities, determine others
   b. I.e. the theory again yields a reasonably rich question-answering device
2. Since tables are given in the form of relative quantities, will in general need to know something in order to obtain a specific result from them
   a. Values tabulated represent such things as *range*(θ)/ *range*(45) for a fixed impetus and *altitude*(θ)/ *range*(θ) and *sublimity*(θ)/*range*(θ) for a fixed impetus
   b. Hence, can perform calculations as soon as know e.g. range(45) for the impetus in question -- something that will have the effect of pinning down the impetus in a preferred way
3. The obvious question is why not table with absolute, rather than relative quantities
   a. Answer: doesn’t have precise values of g -- i.e. fall in first sec -- or initial horizontal speed, v_0
   b. The precise value of g, as we have seen, is a little hard to come by, but poses little problem when put alongside that of determining the precise value of muzzle velocity
   c. If could ignore resistance effects, could infer muzzle velocity (and g) from actual trajectories
   d. Question: conduct repeated experiments to see if obtain uniform value of g via measurement of range, θ and time, along with values of muzzle velocity; if not, then theory offers a basis for reaching some conclusions about resistance effects
4. One practical virtue of giving relative values in the tables is that error associated with resistance effects tends in practical applications to be canceled out to some extent

a. *True-range*(θ)= *theory-range*(θ) – *resist-loss*(θ)

b. *True-range*(45) = *theory-range*(45) – *resist-loss*(45)

c. Suppose true ranges in both of above cases 10 percent lower than the theoretical ranges -- or, more generally, that *resist-loss* distance is proportional to theo*ry-range*

d. Then *true-range*(θ)/*true-range*(45) would be exactly equal to *theory-range*(θ)/*theory-range*(45)

5. The point generalizes still further: the relative values in the table tend to be more empirically correct than the theory itself is, once resistance effects intrude

a. For effects of resistance tend to cancel out in the relative values as a consequence of their having the same direction of effect in both the numerator and the denominator

b. The percent errors in the table and in calibrated calculations based on the table will be smaller than the errors that would be obtained from using true values of g and muzzle velocity!

6. This is an instance of a general practice in engineering for achieving predictions: use *calibration* of a theory in order to compensate for effects not taken into account, thereby achieving better predictions than the theory itself can yield

D. "Real World" Deviations from the Theory

1. In sum, the theory of projectile motion makes some predictions, raising the issue whether additional empirical evidence accrues to the overall theory from any direct evidence for it

a. The theory of projectile motion is derived from the theory of naturally accelerated motion, supplemented by two further assumptions

(1) A principle of horizontal inertia, at least over short distances

(2) A claim that separate horizontal and vertical patterns remain in effect when compounded

b. Consequently, any evidence of projectile motion is evidence for the main thesis of the overall theory -- uniform vertical acceleration

2. In real world resistance deceleration depends on a viscous effect and a kinetic (pressure) effect

$$R = C_1 * r * v / mass + C_2 * (r^2) * (v^2) / mass$$

where in air $C_1 \approx 0.00031$ kg/m∗sec and $C_2 \approx 0.87$ kg/m^3, where deceleration always in opposite direction from motion; note: this, as we shall see, is only an engineering approximation

a. Depends on mass, size, and velocity, with v^2 term dominating once velocity large enough for any given size and mass

b. Effect appreciable for 10 gm pebble, 1 cm in radius: see figure in Appendix (from French)

c. Result shows that some real care is needed in designing experiments in which air resistance effects are minimized

3. Consider an equivalent combination -- a 1 kg sphere of 10 cm radius -- and compare deceleration to g (9.81 m/sec^2)

a. With v = 10 m/sec, R around 0.9 m/sec^2, less than 10 percent of g, so that effect limited

b. With v = 30 m/sec, R around 8 m/sec^2, roughly comparable to g and hence a quite pronounced effect (near terminal velocity)

c. With v = 100 m/sec, R around 87 m/sec^2, overwhelming the effect of g -- Galileo's "supernatural" case

d. An increase of the mass to 10 kg, keeping everything else the same, reduces the effect in the case of 10 m/sec to less than a 1 percent and, in the case of 30 m/sec, to less than 10 percent

4. Two other factors complicate the situation with projectile motion even further

a. Trouble in measuring or effecting reproducible values of initial velocity, needed to compare e.g. different angles

b. v^2 resistance term has the effect of undercutting the claim that the two components of motion remain in effect independent when compounded, for this term produces cross-talk between the two -- i.e. Simplicio's hunch was right, can no longer just superpose orthogonal components

5. Experimental evidence for theory and hence for underlying claims not going to be easy to obtain from projectile motions

a. Need quite dense spheres, with moderate repeatable velocities, and hence probably require special laboratory set-ups

b. E.g., use inclined plane to produce controlled rolling across horizontal, and then measure amplitude for different heights and initial velocities -- "ski-jump" (see below)

(1) A way of testing not just parabola, but which parabola insofar as $a = 2*\sqrt{(h*p)}$

(2) Could have exposed rolling vs. falling, from wrong parabola, for

$$a_{obs}/a_{pred}=\sqrt{(g_{roll}/g_{fall})}=\sqrt{(5/7)}$$

c. Little chance from artillery or other salient ballistic phenomena of interest, for velocities too high

E. Practical and Scientific Concerns Contrasted

1. Galileo, who regards a science of resistance as impossible [p. 275f!], offers no experimental results in the text, yet clearly thinks his idealized theory is the best one can hope for

a. His view is that air resistance has a small effect in some cases, a larger effect in others, and a dominant effect with "supernatural" velocities, where he openly concedes that the true trajectory will not be a parabola

b. Generally his claims about how small the effects will be are greatly exaggerated, though he is right that there are regimes in which they are small enough to allow meaningful empirical tests of the theory

2. The Tables are of some value for practical "technological" purposes, though the domain in question is probably more limited, or at least the accuracy is more limited, than he thought

a. The relative values given in the tables end up masking errors from resistance effects via calibration, using a measurement for a case in which resistance effects are of course present

   b. Predictions from the tables will be good so long as the reference or calibrating measurement has a typical percentage of resistance effect
   c. Thanks to this, tables may yield decent predictions, say from 30 to 60 deg, even when resistance effects are highly pronounced
3. The numbers in the Tables are thus presented in just the wrong form for purposes of bringing empirical data to bear on the theory!
   a. They understate discrepancies, whether from resistance or from any other uniform error, and hence allow experimental results to appear to provide stronger confirmation for the theory than they really do
   b. They **mask** any systematic discrepancies that could be used to argue that the theory is false, or is open to refinement
   c. Better off for purposes of marshaling empirical evidence if discrepancies exposed as completely and clearly as possible, if only to pursue improved, more telling experiments
4. This is a general feature of "engineering" oriented idealized theories and models that are first calibrated before being applied, where the calibration serves to compensate for neglected effects
   a. The fact that they work in the practical realm provides some evidential support for the theory
   b. But generally less support than it appears to, for more often than not the calibrated theory works to the extent it does too much for the wrong reason
   c. And almost nothing is learned from meticulously comparing theory with observation in these cases, except limits of their practical value
5. Galileo shows little sign of seeing this, but others we will be studying saw the difference quite clearly
   a. In 1670's Collins, among others, attempted to provide better tables for artillery, building on Galileo's theory
   b. Clear by then that Galileo's tables were proving to be of limited military value
   c. A continuing question: what is the trajectory a projectile follows in air
   d. A question of importance in the 19th and 20th centuries -- Babbage and Eckart, the designers of digital computers in the two centuries, focused on the ballistics problem

F. Galileo’s Suppressed "Ski-Jump" Experiment

1. Drake (as well as Hahn and Damerow *et al*) have argued from a sheet in Galileo's notebooks that he indeed did perform the "ski-jump" experiment (folio 116v, which Drake assigns to 1608 (p. 129f))
   a. That is, let a sphere acquire its horizontal speed along a table from descending along an inclined plane and then measure the distance -- i.e. amplitude -- covered horizontally versus the height of further descent after it leaves the table
   b. As shown in the figure in the Appendix, taken from Galileo’s notebooks and cleaned up by Hahn
2. My table in the Appendix compares what Galileo measured against what his theory would have predicted: [(*theory* – *observed*)/*theory*]