5. Books IX - XIII: on the planets
    a. IX-XI on longitudinal motion
    b. XII on stationary points, retrogradations, and maximum elongations: the phenomena of primary classical interest, comparable to eclipses
    c. XIII on latitudes
6. Throughout the work the mathematics needed to calculate quantities of interest, including calculating parameters of celestial orbits from observations
    a. In principle, able to determine values of parameters from preferred observations (specified by Ptolemy) -- if need be, correcting those given by him
    b. Then can calculate position (geocentric longitude and latitude) of Sun, Moon, any planet, or any star at any time whatever
    c. I.e. a complete calculational system
    d. With supplementary easy to use tables to aid in the calculations
7. Time and again the Almagest reminds the reader of Apollonius's theorem and hence of the possibility of representing various non-uniform motions either by an epicycle or an eccenter
    a. No one who has read the book should ever have thought that the representations Ptolemy in fact adopted were uniquely determined
    b. Still, the parameters of the orbits remained essentially the same either way, again by virtue of the equivalence Apollonius had established

B. General Remarks on the System

1. Basic system, ignoring irregularities of sun and moon and variations in patterns of retrograde motion
    a. Mercury and Venus tied to (mean) Sun; moon and outer planets not
    b. Location of outer planets on their epicycles tied to (mean) Sun
2. No planetary distances, save for distance to moon (via parallax) and (inaccurate) estimate of distance to sun via lunar eclipses
    a. In each case, set radius of each primary circle R=60 (using Babylonian sexagesimal number system with units or degrees, firsts, seconds, thirds etc. not fractions or modern decimals)
    b. Nothing in the system that allows radii to be determined from observations on any uniform basis, for planetary parallaxes not observable
    c. Sequence from Moon to Saturn conjectured on basis of time required for each to return to the same place along the zodiac, and not on any more direct basis, using observations
3. Primary circle for planetary orbits called the deferent, and secondary circle the epicycle
    a. Ratio of radii of epicycle to deferent, r/R, determinable for each outer planet, in effect from (average) angular size of retrograde loops (see Swerdlow in Appendix)
    b. Even more directly for Mercury and Venus, from (average) maximum elongations (via another theorem of Apollonius)

4. Ptolemy separates problem of longitudes from that of latitudes, the latter of which he tries to handle through deferents and epicycles inclined to ecliptic
    a. Not a fixed inclination through center of earth or any other center
    b. Because observations (mostly made during retrograde loops) defy such a fixed inclination
    c. Instead deferents and epicycles tilting over time in an effort to capture shapes of loops
5. Here going to ignore account of latitudes, which was much less successful, except at oppositions
    a. No so salient a pattern in latitudes like the ones for longitudes
    b. I.e. basic pattern of stationary points in case of longitude and pattern of systematic variations in them; the shape of the retrograde loops the sole counterpart for latitudes
    c. Nothing comparable in case of latitude, and hence nothing to help build theory around
6. Theory of latitudes turned out not to be the worst glitch in Ptolemy's astronomy
    a. Adopted too slow a value for the precession of the equinox -- Hipparchus's lower bound estimate
    b. As a consequence, his origin, the vernal equinox, fell progressively behind over the centuries, introducing a systematic, though easily corrected, error in all longitudes (though not in relative positions of celestial objects)
    c. A lesser small, but accumulating error (again easily corrected) from using Hipparchus's difference between the tropical year and the sidereal year, which was about 5 min off

C. The Account of the Sun

1. Essentially Hipparchus's account, including the decision to use the tropical year, and not the sidereal year, as the basic unit of time
    a. Ptolemy gives credit; indeed the main source for our knowledge of Apollonius and Hipparchus
    b. May well have adopted without extensive critical review, for appears not to have thought that he had to do everything
    c. But did emphasize Apollonius’s theorem and did feel free to go back and forth between eccentric and epicyclic models of the first inequality of the sun
    d. That in turn raises a question about just what the status of epicycles and eccentrics were, merely alternative computational devices to describe the apparent motion, or something more
2. Mean motion, with principal anomaly via eccenter (1/24 R) corresponding to 94 1/2 days from vernal equinox to summer solstice, and 92 1/2 days from summer solstice to autumnal equinox
    a. Thus Sun not really speeding up and slowing down on this model, but only appears to do so because observer located off center -- (second evidence problem again!)
    b. Contrast with all of the other orbiting bodies, for which the variations in longitudinal motion are part apparent and part real
3. "Equation of center": relation between true sun and mean sun; “equation of time”: relation between variable length of time from one noon to the next and mean solar day
    a. "Mean Sun": where Sun would appear to be if its apparent angular motion were uniform

b. "Mean solar day": length of day were it uniform; tied to sidereal day, which (as far as they could tell) is uniform (23 hours 56 minutes and 4 seconds: $365\frac{1}{4}/366\frac{1}{4} \times 24$ hours)
c. Both provide basic references throughout the system
d. Equation of center gives angular correction to locate where Sun actually appears
e. Equation of time gives correction versus mean solar day; it involves both apparent variation of sun in speed and angle between the ecliptic and the celestial equator

4. Precession of equinox: 1 deg per century, Hipparchus's lower limit
    a. Correct value near 1.4 deg per century (36,000 years versus 26,000 years)
    b. As remarked above, a systematic error in all absolute longitudes (as measured from the vernal equinox) growing over time, though not in relative longitudes
    c. Error less from blind acceptance of Hipparchus's value than it may at first seem, for Ptolemy obtained supporting evidence from the apparent motion of the apsides of Mercury, which he (wrongly) concluded was 1 deg per century, and then attributed it to precession of the equinox
5. Nothing particularly original in Ptolemy's solar theory, though you should appreciate that it is by no means crude or simple
    a. Does predict variation in angular movement of Sun to reasonably high accuracy -- i.e. the equation of center -- and also the variation in the length of the apparent solar day
    b. As such, predicting patterned deviations from basic regularity
    c. In other words, a "second-order" theory, providing refinements of the first-order uniform circular motion theory

D. The Account of the Moon

1. The principal inequality taken from Hipparchus: a small epicycle, computationally equivalent to eccenter, to account for basic pattern of speeding up and slowing down
    a. Again, perfectly comfortable starting from earlier work
    b. Other considerations favor (minor) epicycle over eccenter
2. Simple theory includes (1) forward motion of apsidal line; (2) common plane of deferent and epicycle tilted with respect to ecliptic; and (3) regression of the line of nodes (at the ends of which the Moon is in the plane of the ecliptic)
    a. First amounts to around 3 deg per revolution on average (9 year cycle)
    b. Second and third needed for account of latitudes, with third amounting to around 1½ deg per revolution on average (18 year cycle)
3. Ptolemy discovered that, while adequate to predict motion in syzygies and hence eclipses, not adequate for motion in quadrants: a further anomaly
    a. A major discovery of Ptolemy, now called the "evection": a significant deviation in longitude when Moon at the quadrants, not apparent from eclipses

b. He ends up handling it via a (circularly) moving eccenter

4. Ptolemy then discovered a still further anomaly -- i.e. discrepancy between theory and observation -- when Moon in octants that is not handled by this moving eccenter
   a. Just as with first anomaly he discovered, found that irregularity correlated with relative earth-moon-sun positions
   b. Handles it by requiring motion on epicycle to be measured from the "mean," not the true apogee
5. One feature of his lunar theory stood out as a shortcoming right down to the time of Copernicus
   a. Theory predicts that the distance between the earth and the moon varies much more widely than parallax and apparent diameter measurements indicate
   b. Specifically, model calls for variation from 64 to 34 earth-radii, which would produce almost a factor of 2 variation in the apparent diameter of the Moon, while the actual variation in apparent diameter is closer to 10 percent
6. In spite of this and other lesser shortcomings, the account of lunar motion the other major advance of *Almagest* over all prior Greek astronomy
   a. Major advances: two newly discovered anomalies, and realization that they correlate with earth-moon-sun positions
   b. Each discovered via deviations from less refined model, leading to further refinement in described motion
   c. Notice that Ptolemy here refining his theory on the basis of systematic deviations from simpler versions of it -- a sequence of successive approximations
7. (An aside: the motion of the moon turns out to be the most difficult of that of any of the naked-eye observable bodies in our planetary system
   a. For this reason we will give little emphasis to it until we get to the *Principia*
   b. And even there, as you will see, it proved more complicated than Newton could handle)

E. The Problem of Planetary Motion for Ptolemy

1. The *Almagest* presents only Ptolemy's finished account, not the way in which he reached it
   a. This gives the impression that Ptolemy simply came up with a hypothetical model and then accepted it because it gave good results
   b. Noel Swerdlow and Jim Evans have each made a compelling case to the contrary, reconstructing steps by which Ptolemy reached his model for Mars from passing remarks in the *Almagest*
   c. Swerdlow's reconstruction presented here (paper in Supplementary Readings); paper by Jim Edwards assigned for next class offers a parallel account
2. From what Ptolemy says, Hipparchus had made the same move in trying to account for the variations in the retrograde loops as he had for the sun
   a. Put the center of the epicycle on a circle eccentric with respect to earth -- an eccenter – to account for the first or zodiacal inequality