2. Given our central preoccupation with the development of increasingly sophisticated conceptions of empirical evidence in 17th century science, appropriate to ask what conception of empirical science is implicit in Newton's short tract on circular motion
3. One thing for certain is that it is far removed from the idea of achieving exceptionally high quality evidence through convergent, precise measures of a fundamental quality like *g*
   a. At the beginning of this class we listed three ways in which Huygens extended the conception of empirical evidence prevailing at the time
   b. All three are clearly present in Huygens's efforts on uniform circular motion
   c. The only one of the three remotely present in Newton's efforts is the idea that theory is a vehicle for evidence, for Newton is adopting various theoretical claims and using them to draw inferences from observations that could not otherwise be drawn
4. Equally, however, the conception of empirical science implicit in Newton's short tract is far removed from the one Descartes exhibits in his *Principia*
   a. Not putting forward bold hypotheses about underlying mechanisms and then drawing out an explanatory account of phenomena from them
   b. Indeed, no real mention of hypotheses about underlying mechanisms in the tract at all, and as it stands it is at least compatible with a vortex model
5. It is not so far removed from the conception found in Galileo's *Dialogue*, however -- a conception on which one is continually looking for empirical information that can be exploited as evidence to settle questions under dispute
   a. Wilson remarks that, partly through the influence of Barrow, Newton had come to maintain a sharp distinction between conjecture and experimentally established results (p. 139)
   b. On this view the fishing expedition Newton is here on amounts to exploiting Descartes' claim about the importance of the *conatus a centro* in all curvilinear motion to look for potentially compelling evidence for Copernicanism
6. If this is correct, then Newton had not yet begun to fashion a highly sophisticated conception of how to marshal empirical evidence in orbital mechanics at the time he wrote this tract
   a. Indeed, he was still more or less at the level of sophistication exhibited in Galileo's *Dialogue*, and far short of the much greater level Huygens had reached by this time
   b. But then the conception of science implicit here is far removed from that in the *Principia*

IV. Newton on Motion along a Cycloid (versus Huygens)

A. Newton's Two Part Proof of Isochronism

1. In his brief tract on the cycloidal pendulum, Newton provides derivations for three basic results announced by Huygens, but published only in 1673 in the *Horologium Oscillatorium*
   a. The isochronism of the cycloidal pendulum, and as a corollary that of the small arc circular pendulum

b. The construction of a cycloidal pendulum via a curved (cycloidal) constraint along the length of the pendulum string

c. The law of the period for cycloidal pendulums -- $2\pi\sqrt{(4a/g)}$ -- and hence a means of measuring g

2. The derivations exploit two basic features of the cycloid, both of which had been established by geometrical means, while remaining free of Newton's criticism of Huygens's displayed in Appendix

   a. The tangent at any point on the cycloid is parallel to the chord from the horizontally corresponding point to the bottom of the generating circle -- i.e. as YC in the attached figure

   b. The length of the remaining arc to the bottom of the cycloid is 2 times the length of this chord -- a generalization of Wren's famous "rectification" of the cycloid announced to the public by Pascal in 1658

3. The first of two steps in the proof of isochronism is to show that the acceleration at any point on the cycloid is proportional to the arc length to the bottom

   a. The fundamental feature of all isochronously vibrating systems! (here discovered by Newton)

   b. $a_D/a_\delta$ = BC/YC, since the chords are parallel to the tangents and the accelerations are as the lengths of such chords (as Galileo had shown)

   c. But DC/δC = BC/YC, since the arc lengths are twice the chord lengths

4. The second step is to show that the time from any point like P to C is the same as the time from D to C -- this time using an infinitesimal argument

   a. Since $a_D/a_P$ = DC/PC, spaces described in the small unit of time required for Dd and Pp are in the same ratio: DC/PC = Dd/Pp

   b. But then dC/pC = DC/PC, so that accelerations at d and p are in same ratio -- i.e. DC/PC = $a_d/a_p$

   c. Therefore the remaining distances are at all times in the same ratio, "until both simultaneously dwindle to nothing"

   d. Notice also the last paragraph: Newton is not assuming that velocity is proportional to the square root of height in his proof (as Huygens did in his), but is instead concluding that this holds for fall along a cycloid from his analysis

   e. In other words, where Huygens's solution is based on the *vis viva* principle, Newton has found a way to avoid this

5. Newton then gives a geometrical proof that the same cycloid can serve as an evolute to produce a cycloidal pendular arc

   a. The key to the proof is to choose an arbitrary point Q on the horizontal, construct two parallelograms using this point and then show that the line common to these parallelograms is tangent to the evolute cycloid and perpendicular to the involute

   b. The isochronism of the small circular arc pendulum Newton then infers as a corollary to this construction

B. The Relationship to the Theory of Free Fall

1. Finally, Newton addresses the problem of the time of descent from D to C versus the time of vertical free fall from B to C
   a. The solution in effect gives the period of oscillation in terms of the acceleration of gravity $g$
   b. Namely, the law of the cycloidal pendulum: $P = 2\pi\sqrt{(4a/g)}$, where $a$ is the radius of the generating circle
2. The key step in the proof is to find an appropriate geometric measure of time, namely the arc lengths along the generating circle
   a. $t_{Dp} / t_{DC}$ = arc B$\sigma$/arc BC
   b. Thus a geometric measure of just the sort Galileo had to find in *Two New Sciences* and Newton will have to find repeatedly in his *Principia*, so that the proof serves to illustrate a general feature of a kind of argument
3. The proof employs a pre-calculus argument
   a. $v \propto \sqrt{(BV)} \propto BT$ since $BT^2 = BV*BC$
   b. Let Pp be a small element so that Pp and T$\tau$ can be taken as straight and parallel to one another
   c. But then T$\sigma\tau$ and TKB are similar triangles since their respective sides are mutually perpendicular
   d. Hence $BT/TK = \tau T/T\sigma$, so that $BT*T\sigma = TK*T\tau$
   e. But BK=TK and T$\tau$ can be taken as given (T$\tau$ because it is equal to the given infinitesimal Pp)
   f. Therefore $BT \propto 1/T\sigma$; but velocity and time are also in reciprocal proportion, so that T$\sigma$ can represent the time!-- the key enabling result
   g. Since each part in time corresponds to an arc in the generating circle, the time required for fall from D corresponds to the arc of the circle
4. Then time to fall from D $\propto (\pi/2)\sqrt{(2BC/g)}$
   a. Since $BT^2 = BV*BC$, time to fall BV can be represented by BT
   b. At outset, descent from D and vertically from B are the same, and this implies that the two representations of time are commensurate with one another (see p. 206)
      (1) For each individually always bears the correct proportions to the length representing the first increment of time
      (2) And hence each always bears the correct proportions to the other
   c. But then $t_{DP}/t_{BV}$ = arc BT/chord BT
   d. Result follows from arc BC/chord BC
5. Corollary: can measure acceleration of gravity by determining period of oscillation and inferring $g$
6. Also notice the parting remark about motion in a cycloid resembling the harmonic motion of a point on the wheel -- a remark developed in more detail by Huygens (and later by Newton too)

C. Newton's Analysis Contrasted With Huygens's

1. While Newton thus obtains Huygens's principal results for the cycloidal pendulum, his derivations are distinctly different from those in the *Horologium Oscillatorium*
   a. Newton does not develop a general theory of evolutes, as Huygens does, but develops the result only for the rather special case of the cycloid
   b. His derivations of isochronism and for relating the period to vertical fall in 1 sec are much briefer than Huygens's, and throughout use the true smooth curve, not an approximation to it
   c. Newton manages to give a necessary and sufficient condition for isochronism (viz. that acceleration be proportional to the distance to the point of maximum velocity) -- something Huygens later discovered without publishing
   d. And Huygens shows full appreciation for the need to physically tune the clock -- i.e. to include the results on the physical pendulum -- to measure $g$ with high precision
2. Still, Newton's work on the cycloid is the one place in his early work in mechanics, most of which parallels Huygens's work in one way or another, in which Newton displays a scientific style at all like that of Huygens (and that of Galileo in *Two New Sciences*)
   a. Some of the Galilean results employed -- e.g. $s$ varies as $t^2$ and the inclined plane results -- are given in the *Dialogue*, along with claims about the pendulum that would have made it evident to Newton how Huygens's efforts were tied to Galileo's
   b. But no where else does Newton try to tie his efforts in mechanics to Galileo's in a way that will open avenues of evidence
3. There are three possible datings of the brief tract on the cycloidal pendulum, all based on the assumption that some external stimulus provoked him to look at the topic since he had shown no interest in it or in any related topic earlier other than the geometry of the cycloid
   a. After receiving his copy of the *Horologium Oscillatorium* -- but then to what end, and why different proofs in Latin, unless showing how to get around his critique of Huygens
   b. After receiving James Gregory's "stimulating essay on pendular and projectile motion" -- "Tentamina Quaedam Geometrica De Motu Penduli and Projectorum" -- in 1672, as Whiteside suggests
   c. Or still earlier -- e.g. around 1670, as Newton said to David Gregory -- perhaps after hearing of Huygens's announced results through the grapevine
4. Regardless, the impressive thing about this short tract is the appreciation it displays for Huygens's style of science, and hence for the conception of empirical evidence implied by Huygens's work from late 1650's and 1660's
   a. Either Newton had managed to form such a conception independently for himself, and simply not evinced it in his other work in mechanics
   b. Or Newton was very quick to pick up such a conception from mere hints of it (or from *Horologium Oscillatorium*)

c. Only thing missing is Huygens's refinement for the center of oscillation for the circular pendulum, removing an element of idealization

5. This absence makes the third option above more credible because of the emphasis Newton was always inclined to place on virtually exact agreement between experiment and theory
   a. Did not have the sort of cavalier attitude about the limits of experimental precision displayed by Galileo, Descartes, and Mersenne
   b. Hence he was always looking to achieve high quality evidence through precise measurement
6. We can thus have reason to think that Newton fully appreciated the conception of empirical evidence implicit in Huygens's work after reading the *Horologium Oscillatorium*
   a. Newton could have used this conception as a point of departure when he began developing a still more sophisticated conception while fashioning the *Principia*
   b. And this would explain his seeming expectation that Huygens would appreciate the full thrust of the evidential argument in the *Principia*
7. Nonetheless, the parallels and overlaps between Huygens and early Newton notwithstanding, it is important to see that Newton was not embarked on a project of developing a full mechanics before 1684 in the way Huygens was
   a. Newton was working on isolated problems, generally in response to Descartes and with an eye toward developing a compelling argument for Copernicanism -- to complete the work of Galileo's *Dialogue*
   b. Huygens, by contrast, showed no interest in arguing for Copernicanism at the time; he was developing a general mechanics, extending the work of Galileo's *Two New Sciences*
   c. The question, What started Newton into developing a general mechanics?, will be answered in the next two classes

V. Newton's Fragment on Hydrostatics (vs. Descartes)

A. Background: the Anti-Cartesian Introduction

1. The fragment, "De Gravitatione et Aequipondio Fluidorum," is presumably an introduction to a book on hydrostatics which stops abruptly after making just a few claims about pressure in liquids
   a. 35 pages long (in Hall and Hall, 40 handwritten), the last 4 of which deal with deforming fluids, and the first 31 of which comprise an anti-Cartesian introduction to the rest in which Newton is endeavoring "to dispose of his [Descartes'] imaginings" -- "figmenta"
   b. Date uncertain, though thought for a long time to be around the same time as the "Lawes" if only because it too is clearly a response to having read Descartes' *Principia* and much of the physics in it seems far removed from that of Newton's *Principia*
   c. But Betty Jo Dobbs put forward an argument that it dates from the early 1680's -- probably 1684 -- and as such represents Newton's final break with Descartes; from manuscript evidence Rob DiSalle and I have now concluded that it does not date from 1685