THE NEWTONIAN REVOLUTION – Part One

Philosophy 167: Science Before Newton's *Principia*

Class 10

Huygens and the Beginnings of Rational Mechanics

November 4, 2014

Philosophy 167: Science Before Newton's Principia

Assignment for November 4

Huygens and the Beginnings of Rational Mechanics

Reading:

Huygens, Christiaan, "On the Motion of Bodies Resulting from Impact," pp. 1-22.

-------, "On Centrifugal Force," pp. 1-22.

-------, Excerpts from Horologium Oscillatorium sive De Motu Pendulorum, pp. 33, 40-46, 105-118, 118-129 (skimming), 134-141 (skimming), 141-145, 155-157, 162-178.

Leibniz, "A Brief Demonstration of a Notable Error of Descartes and Others Concerning a Natural Law," pp. 296-298.

Questions to Focus On:

1. What fundamental claims does Huygens adopt as axiomatic in his theory of the motion of bodies under impact? Which, if any, of these departs significantly from any claims made by Galileo or Descartes?

2. What does Huygens mean by 'centrifugal force', and what questions is his theory of it intended to address? Which, if any, of the fundamental claims that he takes to be axiomatic in this theory are drawn from Galileo or Descartes?

3. What evidence does Huygens provide for his theory of motion of bodies under impact and his theory of centrifugal force? What, if any, telling experiments might be brought to bear on each of these theories?

4. What question or questions does Huygens intend his theory of the center of oscillation to be addressing? Does this theory require any axioms beyond those of the other theories?

5. What exactly is the issue between Leibniz and Descartes, and what empirical evidence does Leibniz offer for his position?

6. In what senses, if any, do the mechanical theories put forward in the four assigned papers deserve the appellation, "rational"?

Huygens and the Beginnings of Rational Mechanics

I. Science in the Thirty Years after Galileo's Death

A. The Development of Astronomy: 1642-1672 (a brief summary, in anticipation of next class)

1. During the thirty or so years following Galileo's death, orbital astronomy began to take on a distinctly modern form, taking advantage of increasingly precise measurements in order to use discrepancies between observation and theory as a source of further evidence

a. The kind of approach exemplified by Kepler and followed by Horrocks: proceeding by successive approximations

b. But now with a concerted effort by much of the community

2. The standard of accuracy of the Rudolphine Tables -- e.g. allowing for occasional exceptions, within 4 min of arc for Mars -- had become accepted as the minimal standard by all, but it had been more or less met in different ways:

a. Kepler: ellipse and area rule, with 3/2 power law inferred

b. Boulliau: ellipse and special correction to equant at focus (1645, 1657)

c. Wing: ellipse and equant oscillating about focus, and later a geometric construction (1651, 1656, 1669)

d. Streete: ellipse and Boulliau's rule, but using 3/2 power law instead of measurements to determine mean distances (following Horrocks) (1661)

e. Mercator had shown that, with an ellipse for Mars, the rule for location versus time has to approximate the area rule in order to reach Kepler's level of accuracy, and he had then added his own geometric alternative to the area rule (1669, 1676)

f. Horrocks's *Opera Posthuma* published in early 1670s

3. The Royal Observatory of Paris, with state-of-the-art equipment, was beginning to function, and the one at Greenwich was soon to follow, both directed by superior observational astronomers

a. Both committed to replacing Tycho's data with a new body of more precise data, taking advantage of the new technology

(1) Paris focusing on planet positions and related matters

(2) Greenwich on a new star catalog

b. Not just careful observations, but through comparison with one another to cross-check

4. The Royal Academy had taken on the project of pushing Kepler's approach to its limits, pursuing the very best orbital elements in order to determine true residual discrepancies

a. Choice of Kepler (by Picard and Huygens) instead of Boulliau never explained publicly, but none better than Kepler, and also because Horrocks had shown Kepler offered more promise if solar paralax reduced and orbital elements improved (*Venus in sole visa*, published 1662)

b. Approach was first to determine such fundamental quantities as solar parallax and refraction affecting all observations, then turn to elements

5. During the 1670's this approach had begun to yield some notable results, though nothing yet on planetary orbits themselves
   a. Solar parallax definitely less than 12 sec, and probably less than 9.5
   b. Single table of refraction corrections, consistent with Snel's law
   c. Corrected obliquity of the ecliptic
   d. A method of determining longitude differences between points on earth, based on synchronized observations of eclipses of Io, that permits more accurate mapping
   e. From discrepancies in the eclipses of Io, a rough measure of the speed of light, and associated correction rules
   f. The discovery of an apparent movement of the North Star, revealing a further anomaly that would have to be understood before full advantage could be taken of the telescope
6. Meanwhile, primarily in England, attention was focusing on possible physical mechanisms that could account for the curvilinear trajectories of planets and comets
   a. Focus initially on why not a straight line, not on why other than a circle -- i.e. proceed from first approximation
   b. Gravity hypothesis, advocated by Wren and Hooke, led to proposal that gravity varies inversely with distance squared after Huygens' work on centrifugal force became known in 1673
7. In sum, the two great advances of the first half of the 17th century -- Kepler's discoveries and the telescope -- became assimilated during the 30 years following Galileo's death, and a (broadly) Keplerian approach was increasingly being taken to the problem of extracting answers to empirical questions from observations -- via successive theoretical approximations, trying to use discrepancies as an evidential basis for further refinements

B. Post-Galilean Developments in Mechanics

1. Thus, the modern science of astronomy had largely come into being by the mid-1670's; I will be arguing tonight that much the same can be said of the modern science of mechanics
   a. I.e. the modern form and approach of this science had largely come into being by the mid-1670's, primarily through the efforts of Huygens
   b. With the publication of the *Horologium Oscillatorium* in 1673, taking a giant step toward completing Galileo's fragment of a theory on "local" motion
2. The development of mechanics following Galileo's and Torricelli's deaths proceeded slowly until the late 1650's when Huygens made major breakthroughs by extending Galileo's theory of local motion to constrained curvilinear trajectories
   a. Many of the results became known in the 1660's, but publication with some (though not all) of the proofs was delayed until 1673 -- *Horologium Oscillatorium*
   b. Huygens himself was working as much under the inspiration of Mersenne (and indirectly Gassendi) as Galileo, but his efforts filled the most glaring holes in the latter's theory

3. Galileo's principal protegés in Italy, Torricelli (1608-1647) and Viviani, were continuing work on problems he had posed at the new Accademia del Cimento (Academy of Experiment) in Florence
   a. E.g. the effort to detect a finite speed of light mentioned in the reading for next class
   b. The Accademia, perhaps even more so than Galileo himself, was committed to the idea that the empirical world ought to be the ultimate arbiter
4. Torricelli's theoretical efforts on mechanisms had led him to formulate a principle of some importance: "two interconnected heavy bodies cannot put themselves into motion unless their common centre of gravity descends" [Dijksterhuis, p. 362]
   a. An axiom for Torricelli, supported by the argument that two such bodies in effect form a single body with parts and by fact that center of gravity descends with a pulley
   b. Used to derive the law of inclined planes -- an alternative to the argument in *Two New Sciences*
5. Related to the work for which Torricelli is most famous, the creation of the mercury barometer, which he used (following a suggestion of Galileo) to explain why pumps can raise water only so high
   a. Barometer understood as a device in which air pressure was balancing the weight of a column of mercury, especially following the brilliant experiments by Pascal (1623-1662) in 1649
   b. The idea of vertical limits to natural and related processes a common theme to both efforts and to Galileo's other work
6. Torricelli's principle important because it offered an alternative to impetus theory as a means for deriving inclined plane law via pathwise-independence
   a. Though independent of Descartes' concern for foundations, dovetails with this concern, offering a possible basic principle that has the potential for by-passing talk of forces
   b. Independently developed in France by Pascal
   c. (Of course, the principle is just a special, limited form of the conservation of energy)

C. Christiaan Huygens (1629-1695): a chronology until 1673
1. Youth in the great days intellectually and politically of Holland
   a. Born in 1629, son of Constantine Huygens, the "Thomas Jefferson of Holland" (in U.S. eyes): among first major poets in Dutch, composer of music, a more than major figure in politics and in intellectual life of Holland (with such close friends as Rembrandt, Descartes)
   b. Christiaan educated at Leiden (1645-47) and College of Orange (1647-49)
   c. In 1646 derives theory of uniformly accelerated motion independently of Galileo, which father sends to his friend Mersenne, who from afar takes him under his wing
   e. First published paper on quadratures of hyperbolas, ellipses, and circles (1651)
2. 1650s: becomes a major figure in the scientific world
   a. Sometime in 1652-1656: solution for spheres in impact, but not published
   b. Together with his brother Constantine, becomes the leading grinder of telescopic lenses and develops the "Huygens eyepiece" in mid-1650s

c. 1655: first extended visit to Paris, where Gassendi introduces him to others

d. 1656: using his superior telescopes, discovers Titan (satellite of Saturn) and solves the riddle of Saturn's rings

e. 1656: designs first successful pendulum clocks, culminating in publication of his *Horologium* describing the design -- most notably, the escapement -- in 1658

f. 1657: first textbook on probability theory, inspired by his time in Paris and published as an appendix to a Van Schooten book -- Bayesian in spirit, emphasizing "expectation"

g. 1659: publication of *Systema Saturna*

3. 1660-1673: becomes the leading figure in the world of science

a. Late 1659: centifugal force, conical pendulum, cycloidal pendulum, measurements of g, cycloidal pendulum clocks

b. 1660 (and 1663-64) visits to London where he announces his results on impact, cycloidal pendulum, cycloidal pendulum clocks, and measurements of g

c. 1663: first foreign Fellow of the Royal Society

d. 1664: approached (by Colbert) to be a founding member of the French Royal Academy of Sciences, which he joins at its inception in 1666

e. 1669: cause of gravity, analytical and experimental efforts on motion under resistance, and trials of his clocks at sea (for finding longitudes); publication of his results on impact, announcing conservation of linear momentum, *vis viva*, and center of gravity principle

e. 1673: publication of his *Horologium Oscillatorium*

D. Huygens and the Measurement of g (1659)

1. During the last three months of 1659 Huygens became preoccupied with the problem of measuring the strength of gravity (as described in Joella Yoder's *Unrolling Time*)

a. He knew Riccioli's measurement, and he repeated Mersenne's, concluding that it could not be perfected to the extent needed

b. Initially addressed the problem Mersenne had posed -- compare the time for a pendulum to fall through 90 deg to the time of vertical fall through the same height

c. Knew, both from Mersenne and from his development of the basic pendulum clock, that the circular pendulum is not isochronous

2. First effort in an earlier version of De Vi Centrifuga, exploiting the discovery that uniform circular motion involves uniform acceleration just as vertical fall does

a. Leads to theory of conical pendulum, including result that the period of revolution = $2*\pi*\sqrt{(h/g)}$, as presented in our version of De Vi Centrifuga (published posthumously in 1703)

b. Infer g from measurement of period: 15 and 6/10 Rhenish ft "proxime" in the 1st sec (979 cm/sec/sec), by far the best value to date (Riccioli: 935 cm/sec/sec)

c. (Had he carried through his own calculation to more significant figures, would have obtained 15.625 Rh ft, or 980.9 cm/sec/sec)

d. Measurement by designing and building a conical pendulum clock (see Appendix)

3. Undoubtedly looking for a cross-check, turned to the infinitesimal arc pendulum, successfully deriving the comparable relation for it: $P = 2*\pi*\sqrt{(\ell/g)}$ -- versus Galileo's law, which said only that the periods were proportional to $\sqrt{\ell}$

   a. In the process, discovered a geometric relationship that has to hold to maintain isochronism over non-infinitesimal arcs

   b. From this determined cycloid is isochronous trajectory-- i.e. path of a point on a rolling circle

   c. And settled on a still better value of g: 15 Rh ft 7.5 in (i.e. 15.625 ft) fall in 1st sec (980.9 cm/sec/sec)

4. In conjunction with this, designed and built a cycloidal pendulum clock, and developed theory of evolutes along the way; clock from *Horologium Oscillatorium* shown in figure in appendix

5. One lesson learned in all of this is that theory itself can be an indispensable element in achieving accurate measurements and hence in getting the empirical world to answer questions

   a. Infer g from a comparatively simple measurement, combined with a theoretically derived relation, with cross-checking via a sequence of measurements for different arcs, ℓs, etc., and by measurements employing other theory-based devices

   b. Evidence accrues to theory through stability among repeated precise values and convergence of values from complementary theory-mediated measurements of the constant of proportionality

   c. Lack of agreement instructive -- e.g. defect in experimental design, or flaw in theory

6. Huygens's precise measurement repeated by many, usually using a seconds-pendulum -- i.e. a simple, small-arc pendulum of length 3 Paris ft 8.5 lines -- giving a value for fall in first second of 15 Paris ft 1 in (980.7 cm/sec/ sec, versus current value of 980.97 in Paris)

   a. Much stronger evidence for uniform acceleration in vertical fall than anything preceding, for now a stable, convergent, precise value for rate of acceleration (in mid-latitudes of Europe)

   b. A powerful method of evidence for theory that has been central to physics ever since: obtain stable, convergent, ever more precise values of fundamental parameters of theory through theory-mediated measurements

   c. Huygens himself seems not to have taken much notice of this method of evidence, but, as we shall see, Newton took great notice of it

7. Measurement so reliable that Richer able to infer a 0.35% variation in g between Paris and Cayenne from a 1 and 1/4 line difference (2.8 mm) in the length of seconds-pendulum (see Appendix)

   a. I.e. able to extract evidence from discrepancies between one measurement and another

   b. Will see the importance of having such a precise measure of g in subsequent developments

II. Huygens on Motion Under Perfectly Elastic Impact

A. Hypotheses Underlying the Initial Theory

1. Looking at the Huygens work less from the point of view of the contributions it made to mechanics -- although the assigned readings made major contributions -- than from that of the impact Huygens had on scientific methodology
   a. Huygens one of two key figures (Newton of course is the other) between Galileo and Descartes, on the one hand, and 18th century mechanics -- i.e. J., J., and D. Bernoulli, Euler, Lagrange, etc.
   b. In two weeks will be comparing Huygens's way of dealing with such things as centrifugal force and impact with the way Newton devised in his early, pre-*Principia* efforts
2. Huygens's paper on motion under perfectly elastic impact was written sometime between 1652 and 1668, probably by 1656, but published only posthumously in 1703
   a. Solution plus four key consequences (see Appendix) published (without proofs) in 1669, after he reviewed papers by Wallis and Wren on subject for *Phil. Trans*., and submitted an abridged version of the full paper to the Royal Society, where it remained, unpublished
   b. {All the papers and letters from this episode can be found in a file in Supplementary Material for this class, along with a paper by Murray, Harper, and Wilson}
   c. Experimental checks via ballistic pendulum at Royal Society and (later) by Mariotte
   d. (Huygens held back from full publication because he intended to put out a comprehensive book in mechanics covering all his results, a project undercut by publication of Newton's *Principia*)
   e. (Newton himself came up with the same solution as Huygens had announced, but algebraically and from a different pair of initial principles, in the late 1670s -- see Appendix)
3. Huygens's paper in fact presents two consecutive theories of motion under impact, with the split between them just before Proposition VIII
   a. The initial theory shows that Descartes' rules are incompatible with the principle of relativity that Huygens undoubtedly took him to have espoused
   b. The extended theory then adds a further hypothesis or axiom, beyond any of Descartes', and derives a general solution for perfectly elastic spherical balls impacting head-on
4. The initial theory proceeds from five hypotheses, at least four of which Descartes was surely committed to:
   a. Principle of inertia (applied to moving bodies) -- Descartes' Laws 1 and 2 together
   b. Equal bodies at equal speeds rebound equally -- Descartes' Rule 1, but here in effect a definition of 'hard body'
   c. A (Galilean) principle of relativity (tested by Gassendi): irrelevance of uniform motion of what came to be called the "frame of reference" -- i.e. frame-invariance of laws of impact
   d. Transfer of motion from a large body in motion to a small body at rest -- part of Descartes' third law

    e. A much weaker form of Descartes' conservation of motion: if one body retains all of its motion after impact, then nothing will be taken from or added to the motion of the other

5. The main thing to notice about these five announced hypotheses is that most Cartesians accepted all five, and Descartes himself accepted all save perhaps for the principle of relativity
    a. Thus Huygens is here staying (dialectically) within the same general foundational framework as Descartes, though he is avoiding the latter's talk of "forces"
    b. Moreover, by adopting weak versions of some of Descartes' principles and by adding 'uniform' to the principle of relativity, he is staking out a very limited, safe starting point

B. The Initial Theory: Relative Motion Results

1. The "initial" theory, extending to Proposition VII, uses relativity of motion arguments to generate results contrasting with Descartes'
    a. I.e. the "boat-and-shore" arguments, requiring the same basic results to occur for observers in each frame of reference
    b. Arguments simply adjust the speed of the boat so that the conditions of the second, fourth, and fifth hypotheses hold in one frame of reference
2. Propositions I and II pertain to equal bodies, and contrast with Descartes' Rules 6 and 3, respectively
    a. I: If one body at rest, then it will acquire the speed of the other, and the other will come to rest
    b. Argument typical: let boat be moving in opposite direction from moving sphere, but at half the speed, so that from the point of view of the shore the two are moving at equal speeds toward one another, and the result follows from the second hypothesis
    c. II: If at different speeds, then exchange speeds
3. Propositions III, IV, and VI, which drop the equal body requirement, stand in sharp contrast with claims made by Descartes
    a. III: A body however large is moved by impact by a body however small, moving at any speed -- larger body moving toward smaller in one frame of reference has smaller moving toward larger in the other frame
    b. IV: Whenever two bodies collide with one another, the speed of separation is the same, with respect to each other, as the speed of approach -- adjust the speed of the boat so that the conditions of fifth hypothesis hold in one frame of reference
    c. VI: "When two bodies collide with one another, the same quantity of motion in both taken together does not always remain after impulse what it was before, but can either be increased or decreased" -- straightforwardly from Prop. IV, in direct contradiction to Descartes' third law of nature
4. Note that Huygens needs no apologetic explanations for why experience does not accord with these claims, as Descartes does, for experience does accord with them (making allowances for the imperfect hardness of bodies)

a. Huygens's phrasing, applied to full theory, applies even more here: "not alien to reason and agrees above all with experiments"

b. In effect "correcting" Descartes by introducing relativity of motion, dropping the "contest" principle, and replacing the conservation of motion principle by a weak version of it

5. Main upshot of this initial theory: if adopt principle of relativity as a fundamental axiom, then not only forced to reject several of Descartes' Rules, but forced to modify his foundations

a. Abandon competing forces picture of impact, and unable to retain conservation of motion for two bodies interacting

b. Note how effort to obtain agreement with empirical observations is pushing Huygens toward making a distinction between total (scalar) motion and total vector motion -- i.e. momentum

C. Toward the Extended Theory: Proposition VIII

1. Huygens encounters a problem at this point: not enough principles to determine individual velocities in the general case of unequal bodies impacting at unequal speeds

a. Speed of separation = speed of approach is not enough

b. Descartes encountered a similar problem and added a "principle of least change" (without announcing it) to obtain results

c. Our modern approach is to assume just the conservation of momentum and energy; but think of the evidence required for two such strong claims

2. Huygens's solution: invoke (a variant of) Torricelli's principle as a further axiom -- "the common center of gravity of bodies cannot be raised by a motion that arises from their weight" [p. 10]

a. As used in context: velocities after impact cannot be such as to be able to raise center of gravity higher than it was if velocities before impact were acquired through free fall

b. Huygens saw this principle as associated with the impossibility of a perpetual motion device

3. This principle, coupled to two Galilean principles concerning free fall, allows him to prove Proposition VIII, a generalization of the second hypothesis: if two bodies collide with each other from opposite directions at speeds inversely proportional to their bulks (Latin *moles*), then each will rebound at the same speed at which it approached

a. Second hypothesis: equal bodies & equal speeds entails same speeds afterwards

b. VIII: equal motions -- B*v, v a (positive) scalar -- entails same speeds afterwards

4. Proof of Proposition VIII via reductio ad absurdum, first assuming that speed afterwards is less than speed before, then assuming the opposite

a. Given speed of separation = speed of approach, can show Torricellian principle violated

b. By letting u_a and u_b be acquired in free fall, and then letting v_a and v_b be converted into upward motion, as on inclined plane

c. And finally eliminating the possibilities of $v_a = 0$ and v_a in the wrong direction

5. In order to use modified form of Torricelli's principle in proving Proposition VIII, Huygens also had to assume two of Galileo's principles pertaining to free fall:
   a. Speed acquired independent of weight and bulk, and dependent only on h -- specifically proportional to $\sqrt{h}$
   b. Speed acquired in perfect free fall sufficient to lift body to original height
6. Notice the irony here: replacing Descartes' foundations with Galilean principles that he had complained lacked any foundation
   a. Huygens more preoccupied with having empirically well-founded basic principles than with a small number of universal ones
   b. In this regard, more in the spirit of Galileo, though with a degree of attention to foundations that reflects Descartes

D. The Extended Theory and its Consequences

1. Can thus think of extended theory of hard spherical bodies in head-on impact in either of two ways
   a. As based on four of the original hypotheses plus Torricellian principle and the Galilean principles needed to apply it -- in which case Huygens's theory of impact becomes a subsidiary of a broader mechanics
   b. As based on four of the original hypotheses plus Proposition VIII, with the latter defended by independent arguments from mechanics
2. Either way, can now use relative motion arguments of same sort as before, but now adjusting the motion of the boat so that $B_a * u_a = -B_b * u_b$, so that in that frame of reference $v_a = -u_a$ and $v_b = -u_b$
3. The general solution for the problem of the velocities after head-on impact is given in Proposition IX, which stands in direct contrast with Descartes' Rule 7:

$$v_a = \{(B_a - B_b)u_a + 2B_b u_b\}/(B_a + B_b)$$

   where all velocity vectors are positive in the same direction and B stands for *moles*, i.e. bulk
   a. Same as the calculational rules given by Huygens on p. 15, though his derivation is geometric
   b. The same textbook solution given today, derived from conservation of momentum and energy
   c. Proposition X just a special case, contrasting with Descartes' Rules 4 and 5 for one body at rest
4. Proposition XI states an important new conservation principle: the total of $B * v^2$ remains the same before and after impact in the case of perfectly elastic bodies
   a. In other words, Huygens is here deriving what was in effect the conservation of kinetic energy from his other principles, beginning the process of bringing evidential support for what was two centuries later to become the principle of conservation of energy
   b. And, of course, his theory also entails that what we now call momentum is conserved as well, though he lacked the vector concept needed to express it felicitously, as can be seen from his 1669 statement of it (see Appendix), which did not appear in the 1703 version of the paper

5. Finally, the multiple body results at the end concern what ratio of consecutive B's will yield maximal velocities after impact
    a. Intervening bodies of intermediate magnitude increase velocity acquired from rest
    b. Greatest when intervening B is the mean proportional, and thus for B’s in geometric progression
6. Thus end up with our modern theory, save for mass replacing bulk as the measure of the quantity of matter, but obtained from different fundamental principles
    a. A question-answering theory of the same sort as Galileo's theories, ultimately telescoped into a single formula
    b. Two considerations, besides formal adequacy, seem to lie behind the principles from which the theory is derived:
        (1) All are taken from the literature or based on ones that are
        (2) To a reasonable extent, principles are modest as well as "not alien to reason"

E. Empirical Evidence for the Two Theories

1. Rather in the Galilean tradition (and in clear contrast with Descartes) Huygens simply says that the theory agrees with experimental results
    a. In particular, Huygens claims that Proposition VIII can be verified experimentally, so that the appeal to Torricelli's principle etc. serves to show e.g. why it holds, and that it is nomological
    b. Huygens presents no experiments here, but the relevant experiments were definitely performed
2. Good quantitative evidence not so easy to achieve since velocities not easy to measure with precision
    a. Needed indirect, tractable ways of measuring velocities, or inferring them from something that could be measured -- e.g. infer speeds from heights before and after in a ballistic pendulum
    b. And even then had to face confounding effects from imperfect elasticity of balls and air resistance effects
3. During late 1660s papers on elastic impact by Wallis, Wren, Huygens were discussed at Royal Society, including some experiments performed on the spot, and later ones by Mariotte as well
    a. Ballistic pendulum results, measuring height reached by impacted balls as a proxy for velocity -- again implicating claims from mechanics at large
    b. Royal Society data qualitatively supportive, but quantitatively problematic
    c. (Will see a discussion of how to correct these data to achieve more meaningful quantitative comparisons at the beginning of Newton's *Principia*)
4. Available data, though compatible with the key hypotheses, were scarcely enough to confirm that they hold exactly or generally
    a. Principle of relativity based primarily on an a priori argument
    b. The fifth hypothesis similarly relies on a priori arguments
    c. Evidence for Torricellian principle from statics, but here being taken in a much more general way than that evidence automatically licenses

5. Perhaps best possibility for strong supporting evidence lies in the intervening body results at the end
    a. In the Galilean tradition, a salient, unanticipated result that can readily be confirmed qualitatively -- e.g. huge amplifications of speed, as indicated in the last paragraph; e.g. 5 spheres with bulks in 1:2 ratio each yield 68% more motion on impact than with 3 middle ones removed
    b. All fundamental principles from which the overall theory is derived implicated in these results
    c. Huygens undoubtedly included them precisely because they permitted such comparatively telling tests of the theory -- even qualitatively for imperfectly elastic spheres
6. All things considered, then, Huygens's theory of impact is best thought of as a Galilean alternative to Descartes' theory
    a. It employs Galilean principles, it rejects certain Cartesian principles, and it conceptualizes the problem without forces, though (implicitly) with Cartesian version of principle of inertia
    b. Its mathematical development is thoroughly Galilean in style
    c. And the opportunities it presents for evidence have a Galilean character

III. Huygens on Circular Motion and Centrifugal Force

A. The Basic Conceptualization of the Problem

1. As remarked earlier, Huygens had begun work on centrifugal force by 1659 as part of an effort to measure the acceleration of gravity
    a. The version we read, which was published posthumously in 1703, does not bring this goal out so clearly as earlier drafts, nor even as the announcement of most of the results without proofs in an Appendix to *Horologium Oscillatorium*
    b. What it loses in that regard, however, it gains in clarity and polish
    c. (Clarity and polish in part because this version pieced together from several other versions by Huygens's posthumous editors -- as shown by Yoder
        (1) Proofs written to support theorems given in Appendix of *Horologium Oscillatorium*
        (2) A couple of proofs the work of the editors, not of Huygens himself)
2. Problem posed: let us see what [sort of] and how great a *conatus* belongs to bodies attached to a string that revolves (or is restrained by a circular barrier)
    a. Tension in the string a measure of this *conatus*, and thus a measure of the accelerative tendency at the first instant the body would leave the string
    b. Just the move Descartes had proposed in Article III, 59 of his *Principia*, but now carried out by solving for the tension in a string required to maintain uniform circular motion
    c. Initially want to see whether this tendency is of same kind as that involved in gravitational fall
3. Consider in Figure 3 the distances that would form between the object on the circle and the straight line it would follow if it were released -- an indication, indeed a proposed measure, of the *conatus* of the body to move away from the end of the string
    a. BK, KL LN = arc length of BE, EF, and FM since v constant

b. But for small angles -- i.e. for the first instants in time -- EK, LF, and MN differ negligibly from EC, DF, and MS

c. EC, DF, and MS are proportional to $r*(1/\cos(\theta) - 1)$

d. For small θ, proportional to θ^2 and hence to t^2

4. Therefore the accelerative tendency of a ball in uniform circular motion is of the same general type as that of a ball in free fall -- i.e. distances traversed are proportional to t^2; in other words the accelerative tendency is uniform and hence akin to Galilean motion

   a. Since the two accelerations are of the same type, the way opens to comparing them and to determining the strength of one via a measure of the strength of the other in suitable circumstances!

   b. Specifically, under circumstances where the two strengths are known to be same in magnitude

5. Finally, just as the tension in a string on which a body is suspended vertically is proportional to the weight or bulk of the body as well as to its accelerative *conatus*, so the tension in a string on which a body is suspended in uniform circular motion is proportional to the weight or bulk of the body as well as to its accelerative *conatus*

6. Problem in the original 1659 effort at this point was first to determine what the accelerative *conatus* of a body revolving uniformly is proportional to, and then find some circumstance in which the accelerative *conatus* of gravity is of the same magnitude

   a. Would then be able to measure acceleration of gravity -- i.e. the distance of fall in the first second -- by determining the centrifugal accelerative *conatus*

   b. In effect reformulating the former measurement problem in a more tractable domain

7. Of course, in this later draft had the pendulum methods of determining g, and so end up with different, though related goals -- i.e. giving an account of centrifugal force in terms of g

B. The Basic Results and their Significance

1. Propositions I through III, which concern "equal bodies", develop the fundamental, classic result: the centrifugal force -- i.e. the tension in the string -- is proportional to v^2/r

   a. I: if same period, and hence same angular velocity, in two different circles, then centrifugal force proportional to r

   b. II: if same radii, but two different velocities, then centrifugal force proportional to v^2

   c. III: if at same v, but in two different radii, then centrifugal force proportional to 1/r -- via combining I and II

2. Once the fundamental result is at hand, a variety of other results can be obtained -- more easily by us using algebraic means than by Huygens using geometrical arguments

   a. E.g. Proposition IV: if equal centrifugal force, then period of revolution proportional to $\sqrt{r}$

   b. A result Huygens did not bother to deduce, but that Newton, among others, did: if the square of the period is proportional to the cube of r, then the centrifugal force is proportional to $1/r^2$

c. I.e. Kepler's third law entails inverse square centrifugal forces for concentric circular orbits!

3. Huygens's interests lie in the direction of Propositions V and VI; V states that the centrifugal force is equal to the weight of the body -- the tension in the revolving string is equal to that in the vertical string -- when the velocity is that acquired through a fall of r/2

   a. In effect, a Galilean "sublimity" result, providing a commensurate basis for comparing centrifugal and gravitational accelerations

   b. Algebraically because $v = \sqrt{(2gh)}$, so that $v^2/r = g$ when $h = r/2$

   c. VI then solves a problem: Given the acceleration of gravity, can determine the radius of the circle for which centrifugal force = weight and period = 1 sec: 9.5 Rhenish inches since $P = 2\pi\sqrt{(r/g)}$ when $v^2/r = g$, and g = 375 in/sec/sec

4. Huygens is here flirting with a notion of mass, as distinct from weight, insofar as he is identifying weight with the tension in a vertical string, which is proportional to $B*g$

   a. He is also flirting with the modern form of Newton's second law: weight = force in string = $B*g$

   b. But to extract a notion of mass, he would need to compare the effect of B on centrifugal tension with a body of unit weight, at the very least

5. Notice, however, that result given in Proposition VI is useless as it stands for determining g, for to invoke the conditions of the proposition and hence to infer g from P given r, must have centrifugal force equal to the weight

   a. I.e. need uniform circular motion in which the centrifugal force is known to equal the weight

   b. Will then be able to infer g from P given r on the basis of Proposition VI

C. A Theory of the Circular Conical Pendulum

1. Proposition VII gives the first result allowing such a determination of g, namely from equilibrium conditions for a ball on a rotating parabolic conoid

   a. At the latus rectum, the slope of the conoid is 45 deg, and hence the equilibrium position will automatically have the centrifugal accelerative tendency = gravitational accelerative tendency via Lemma I (from Stevin)

   b. Via a geometrical argument and Lemma II, the very same rate of rotation will then put a ball anywhere else on the conoid into equilibrium

      (1) An isochronism result: the same period for all equilibrium positions on a parabolic conoid

      (2) For a given parabola, one distinct angular velocity and hence period at which equilibrium occurs everywhere

   c. Finally, an algebraic argument exploiting Proposition V yields the fact that this period = the period of a small arc pendulum of length 1/2 the latus rectum (i.e. r at the latus rectum)

   d. Thus can measure g by determining the equilibrium period for a parabolic conoid: $g = (\text{latus rectum}/2) * (2\pi/P)^2$

2. Huygens now switches to the conical pendulum, which provides in some respects a more tractable experimental device than the conoid
    a. The tension in the string of a conical pendulum is from gravity and the centrifugal *conatus*
    b. Equilibrium when centrifugal accelerative tendency produces a tension in the string, the vertical component of which exactly balances the gravitational accelerative tendency -- i.e. when $v^2/r = g*\tan(\theta)$; see Huygens's static equilibrium sketches for conical pendulums in the Appendix
    c. The relationship between $\ell=r/\sin(\theta)$, $h=r/\tan(\theta)$, v, and r is thus fixed for a conical pendulum
3. Less clear than it might be from the De Vi Centrifuga paper is the extent to which Huygens relied on results from statics, especially from his countryman Stevin, in conceptualizing conical pendulums
    a. The figures and results from statics in the Appendix bring this out to some extent
    b. Of particular note is the figure from Huygens's notebook that did not appear in De Vi Centrifuga showing how the conical pendulum amounts to a problem in static equilibrium
4. Propositions VIII through XI give the basic results for a conical pendulum, which prepare the way for the formula, $P = 2\pi\sqrt{(h/g)}$
    a. VIII: If equal heights, equal periods
    b. IX: Period proportional to $\sqrt{\ell}$ when theta constant
    c. X: Period proportional to $\sqrt{h}$ in all cases
    d. XI: Period proportional to $\sqrt{\cos(\theta)} = \sqrt{\sin(ABE)}$
5. The remaining propositions on the conical pendulum now provide a basis of calculation corresponding to the above formula
    a. XII: Period of a conical pendulum approaches that of a small-arc simple pendulum of same length as θ approaches 0
    b. XIII: Centrifugal force = weight when period in conical pendulum = period of simple pendulum of same length
    c. XIV: Period = time for fall through ℓ when $\arcsin(90-\theta)=0.05062$ -- a sublimity result
    d. XV: Centrifugal force in conical pendulum of given h proportional to ℓ
6. As remarked before, Huygens constructed a conical pendulum clock in late 1659 and used it to measure g, obtaining a (rounded-off) value of 15.6 Rhenish ft of fall in the first second (979 cm/sec/sec), as well as using a later clock based on the paraboloid (see Appendix)

D. Comparing Centrifugal Forces with Gravity

1. The remaining results in the paper compare the centrifugal tension in a string with the gravitational tension under various circumstances
    a. So far have a comparison in circumstances where they are in balance
    b. Now looking for results when not in balance – large-arc pendulum and body revolving non-uniformly in a vertical circle

2. Proposition XVI: force in string at bottom of a 90 deg pendulum is 3 times the weight of the object
    a. For, $v = \sqrt{(2gh)}$ in order to conserve Bv^2
    b. Hence centrifugal tension proportional to 2g at bottom, and addition of gravity gives 3g
    c. Huygens's argument approximates the circle with a parabola to get the precise result at the bottom geometrically, instead of via the algebra above
3. Proposition XVII: a vertical rotating body produces at least 6 times the tension that the weight of the body alone does -- 6 times in order to keep the string taut
    a. To keep the string taut at the top, v^2/r must at least be g
    b. But then velocity at bottom must be large enough to yield v^2/r at top after deceleration while going up
    c. "Sublimity" reasoning, converting velocity to height, then yields the result
4. This last proposition has a striking corollary: the specific height at which to intercept a 90 deg pendulum and complete a circle after the intercept is $(2/5)*\ell$ from the bottom
    a. If intercept any higher, will not complete a circle
    b. A testable result related to Galileo's claims about intercepted pendulums
5. Clearly, Huygens was looking to extend his results on centrifugal forces to the large-arc pendulum in these last propositions, but this was as far as he could get
    a. Probably prompted by the 90 deg pendulum Mersenne used in trying to measure g -- a measurement Huygens had repeated more than once with increasing care, before giving up on it
    b. No theory of the large arc pendulum until math for elliptical integrals in second half of 18th century

E. Empirical Evidence for the Overall Theory

1. Once again Huygens, in the tradition of Galileo, says that the results agree with experiments without giving us any experimental data
    a. Since the mathematical arguments involving infinitesimals raise some doubts, empirical evidence an appropriate concern here
    b. And in fact Huygens had performed a variety of relevant experiments in late 1659 and subsequently
    c. So, the decision not to report results reflects a style
2. Outwardly, the theory is making claims about the tension in a string, which at that time could not be measured to any accuracy at all except in certain cases of static equilibrium
    a. Direct measurements become possible only in late 19th century, with the advent of strain gauges
    b. Measurement via determining breaking points not adequate for strength of string too irregular
3. But Huygens himself had managed to obtain an accurate measure of periods of conical pendulums and hence of an implied value for g

a. Evidence for overall theory via close comparison with independently measured value (979-981 cm/sec/sec) or by using the independently measured value of g to design e.g. a precise 1 sec conical pendulum

b. Care is needed with experimental approaches using the conical pendulum -- but Huygens had shown that care is enough

4. Other Galilean-type opportunities present themselves for testing the overall theory by verifying some of its salient results

a. E.g. the equilibrium claims about a rotating parabolic conoid, which will also permit further measurements of g if the period can be measured with sufficient precision

b. The intercepted pendulum results at the end, which are directly about the variable tension in the string (though must be careful to keep resistance effects to a minimum)

c. Note that here again the paper ends on a comparatively easy to test salient result

5. Again, therefore, Huygens's theory of centrifugal forces is best thought of as a Galilean response to questions raised by Descartes

a. Notice the extent to which Huygens has incorporated his theory into the context of Galileo's results on free fall, allowing these results to bring supporting evidence for the theory, and allowing evidence for the theory to support Galileo's results

(1) I.e. by incorporating this theory into the broader context, allowing diverse, hopefully converging evidence for the whole framework

(2) Same true more narrowly when he ties results for circle to parabolic conoid, conical pendulum, and then to small arc pendulum -- a unified theory admitting of diverse evidence through cross-comparisons and alternative measures of same quantity

b. Outwardly, paper displays Galilean approach to evidence too, focusing on verification of striking "quasi-quantitative" predictions

c. But in fact the theory offers much richer evidential approaches via comparison of alternative measures of g, or use of a preferred measure to make testable predictions about e.g. periods

(1) Thus, to a far greater extent, this theory provides a basis for looking for small systematic discrepancies that might provide added empirical information

(2) As remarked above, Huygens seems not to have given much notice to these richer possibilities; he seems to have been content with tested striking predictions

6. Finally, notice that here, for the first time, obtain substantial empirical evidence for law of inertia

a. As Huygens's figure (see Appendix) displays, law presupposed in derivation of v^2/r result and hence in every consequence in paper

b. All evidence for Huygens's theory thus evidence for law of inertia

c. This may be the only way to provide significant evidence for the law of inertia: embed it crucially in a theory and develop evidence for that theory

d. The experimentally confirmed solution for centrifugal force in uniform circular motion may help explain why the law of inertia became so quickly accepted in some circles

7. The logic of this evidence is worth noting even though Huygens himself said nothing about it

a. The principle of inertia, contraposed, says that any deviation from inertial motion requires a "force"

b. Should be able to characterize the magnitude of the force in any given type of deviation -- e.g. express a measure of the force in terms of the motion

c. Given such a measure, the question is whether it is correct

d. Verifying by some other independent way that this measure of the force is correct then legitimates the conclusion that a well-defined force is present, and hence provides evidence for the contraposed form of the principle

8. Huygens's failure to emphasize it notwithstanding, this is an archetypal example of a form of evidence that has been central to physics ever since Newton's *Principia*

IV. Huygens on the Center of Oscillation: Real Pendulums

A. Background: Ideal versus Real Pendulums

1. Huygens's *Horologium Oscillatorium* is divided into five parts, the first of which describes the mechanical features of the clock shown in Appendix and a maritime clock

a. Part II: theory of free fall, inclined planes, and fall along a curved path, culminating in the isochronism result for the cycloidal pendulum (see Appendix)

b. Part III: the theory of evolutes, including a justification of the device introduced in the clock to maintain its isochronism (see Appendix)

c. Part IV: the theory of the real, physical pendulum, in contrast to the ideal pendulum of Part II, plus announcement of cycloidal and small-arc pendulum measures of g

d. Part V: describes a conical pendulum clock, the bob of which is kept on a paraboloid by a curved plate, and indicates that it too can be used to measure g

e. Appendix: 13 theorems about centrifugal force, without proofs

2. Theory in Part II employs Galilean principles supplemented by what we now call the principle of inertia throughout -- all the Galilean principles listed in notes for class 7

a. Three hypotheses at the beginning (see Appendix) license parabolic projection if gravity is taken as uniform and acting along parallel lines

b. Proposition 1: "derivation" of uniform acceleration of gravity question-begging in assuming same action in each unit of time (Huygens was not always careful about rigor in derivations from listed hypotheses)

c. Proposition 5: proof of mean speed theorem (via reductio ad absurdum)

d. Propositions 6-9: generalized pathwise independence principle in ascent as well as descent derived (via reductio ad absurdum) from Huygens's Torricellian principle

e. Proposition 10: universality of speed dictated by height of fall

3. The theories of the cycloidal and small arc pendulum in Part II are idealized in more than one respect
   a. As always in the Galilean tradition, resistance effects are ignored, though these effects can be made small, and of course the clocks themselves can be adjusted to compensate for them (and to compensate for the resistance of the rest of the mechanism)
   b. The pendulum itself is then idealized by ignoring the bulk -- i.e. the quantity of matter -- of its string and by treating the bob as what Euler later called a “point mass”
   c. Friction between the evolute wall and the string also ignored
4. Theory of evolutes in Part III gives a theory of the curvature of a large variety of curves -- i.e. of how these curves are defined as a sequence of osculating circles
5. The section from Part IV develops a theory that drops the idealization of the pendulum itself, but still without taking into account resistance and friction effects
   a. Of practical as well as theoretical interest, because pertinent to way clocks tuned, via small masses added on the end, resulting in multiple bobs
   b. Of course, also yields results for pendular motion involving a rigid body oscillating about a point, with no string
   c. And provides more exact way of calculating the length of simple pendulum with heavy non-point mass on the end
6. As Huygens remarks at the outset, the theoretical issue concerns the center of oscillation, a problem that Mersenne had proposed to him and to others in the 1640's, with no successful respondents
7. I assigned this section, in spite of its greater difficulty, with two things in mind:
   a. To show how far Huygens was able to advance theoretical mechanics beyond the point where it stood in the mid-1640's, leading toward the general theory of the motion of rigid bodies developed by Euler and d'Alembert in the 18th century
   b. As an example of Huygens's willingness to begin dropping idealizations, and hence to work in a realm where small discrepancies from an idealization become evidence for a refined theory and success of refined theory then becomes evidence for principles underlying both theories!

B. The Compound Pendulum and its Significance

1. The problem: given a pendulum with several bobs on a string, what is the length of a simple pendulum with the same total weight and the same period
   a. Mersenne had given the problem to Huygens in his youth (and to Descartes, who failed)
   c. (Cannot be solved from Newton’s laws of motion by themselves)
2. Mersenne knew from experiments that the center of gravity of the bodies is not the answer: the required simple pendulum is longer than this
   a. Mersenne, of course, did not know why: some of the motion (energy) is going not into translation, but into rotation of the masses about their common center of gravity

b. The equivalent simple pendulum, which has all of its motion (energy) going into translation, must therefore have more translational motion (energy) than the compound pendulum has

3. Huygens's Proposition V gives the correct solution for the center of oscillation of the compound pendulum: $x = \Sigma(B_i r_i^2)/\Sigma(B_i r_i) = \Sigma(B_i r_i^2)/(r_{cg} * \Sigma(B_i))$
   a. This theoretical result is thus in full accord with the known fact that the required simple pendulum is longer than the length to the center of gravity
   b. The obvious practical significance of the result is that it provides a principled basis for tuning pendulum clocks by adding small weights, which Huygens proceeded to describe in detail

4. Huygens's proof of this result is based on two hypotheses, one of which is generalized from the collision paper, and the other is from Galileo, both preceded by a string of definitions
   a. Huygens's Torricellian principle reformulated, here stated as follows: "if any number of weights [connected or otherwise] begin to move by the force of their gravity, the center of gravity composed of them cannot ascend higher than where it was located when the motion began."
   b. Galilean: "Abstracting from the air and from every other manifest impediment, the center of gravity of an oscillating pendulum crosses through equal arcs in descending and ascending"

5. A reductio ad absurdum proof: if the velocity of the compound pendulum is greater than (or less than) the velocity of the corresponding simple pendulum, Proposition III, derived from the two hypotheses, is contradicted

   III: "If any magnitudes all descend or ascend, albeit through unequal intervals, the heights of descent or ascent of each, multiplied by the magnitude itself, yield a sum of products equal to that which results from the multiplication of the height of descent or ascent of the center of gravity of all the magnitudes times all the magnitudes"

   a. In effect, a special instance of a conservation of mechanical energy result, for requiring that the velocity acquired be sufficient to raise all of the masses back to their original height
   b. Amounts to requiring the total of $B*v^2$ to be the same in the simple and compound pendulums

6. {The key enabling theorem became a point of controversy from 1691 until Lagrange -- not whether it is true, but whether it is sufficiently transparent to serve as basis for such an important result

   IV: "Assume that a pendulum is composed of many weights, and beginning from rest, has completed any part of its whole oscillation. Imagine next that the common bond between the weights has been broken and that each weight converts its acquired velocity upwards and rises as high as it can. Granting all this, the common center of gravity will return to the same height which it had before the oscillation began."

   a. Jacob Bernoulli in 1691 objected to this principle and proposed replacing it with the "principle of the lever" to obtain the same result from more secure foundations
   b. d'Alembert subsequently proposed the initial version of the principle named after him as still a more secure foundation from which to obtain the result
   c. Lagrange (1788) generalizes d'Alembert's principle as foundation for his equations of motion}

C. The Results on the Center of Oscillation

1. The compound pendulum, with a series of point masses on a massless string, is a special case of a more general problem: given any body fixed at a point and moving through an arc with respect to an axis of oscillation through this point, what is the equivalent simple pendulum
   a. I.e. the simple pendulum that will have the same period of oscillation
   b. Problem asks for the center of oscillation of an arbitrary body with respect to an axis, on a line from the axis through the center of gravity of the body
2. This center of oscillation matches the center of percussion -- e.g. the "sweet spot" on a baseball bat
   a. Period = $2\pi\sqrt{(I/mgh)}$, where h is the distance from the axis of rotation to the center of gravity and I is the moment of inertia of the body vis-a-vis an axis parallel to the axis of rotation, but through the center of gravity
   b. Thus $\ell = I/mh$ gives the distance in question and the length of the equivalent simple pendulum
3. The modern treatment of the problem, besides using the moment of inertia first introduced by Euler in 1750, is developed around the fact that energy is being absorbed both translationally and rotationally by the oscillating rigid body
   a. (The rotational energy in the physical body involves rotation about the center of gravity, and not the axis of oscillation)
   b. Accordingly, this problem is closely related to the contrast between rolling and falling on an inclined plane, for there too the difference arises because part of the energy is going into rotation
   c. {This was undoubtedly why Huygens became the first to appreciate (1693) the difference between rolling and falling in the years immediately after Bernoulli objected to his solution}
4. Rotational energy varies as the product of the angular velocity squared and the moment of inertia
   a. For the moment of inertia is the correlate of mass for rotational motion
   b. Moment of inertia = sum of mass times distance from c.g. squared
5. Thus Huygens is flirting with the notion of moment of inertia and a theory of rotational motion in the problem he is addressing here -- a notion and a theory elaborated in full by Euler around 1760
   a. Given the measurement of g via a cycloidal pendulum, Huygens might well have tried the experiment of measuring g by rolling a ball down a cycloidal trough
   b. If he had, the value of g he would have obtained would have been the equivalent of 701 cm/sec/sec, revealing the difference between rolling and falling
   c. Instead he exposed the distinction between rolling and falling analytically (see Notes, Class 6)

D. The Theory of the Physical Pendulum

1. The problem addressed by Huygens starting, with Proposition XIV, asks for the simple pendulum equivalent to a physical pendulum -- i.e. to a rigid body oscillating about an axis of rotation under the influence of gravity

a. Poses the problem for a completely general shape

b. Clearly thinking of it as a generalization of the compound pendulum problem, but now for an infinity of masses

2. He faces an obvious difficulty attempting such a generalization in the 1660's, before the calculus

   a. The problem is straightforward with the calculus

$$x = \int y^2 dm / \int y dm$$

   b. But it is not at all straightforward without it, requiring quadratures for figures

3. Huygens manages to get solutions for certain standard shapes by generalizing his result for the compound pendulum and arguing to limits for quadratures for these figures

   a. Viz., pyramid, cone, sphere, cylinder, paraboloid, hyperboloid, and semi-cone

   b. Not only sufficient cases to test the theory, but also to allow precise calculation of the length of simple pendulums with solids of various shapes for bobs

4. The results have both scientific and technological significance

   a. Provide basis for designing and tuning clocks using physical pendulums, and for treating the bob of a simple pendulum as other than a point mass

   b. Solves the problem posed by Mersenne for the range of cases of principal interest, leaving to the future the generalization that will yield the full solution

   c. In the history of mechanics, among the first efforts that treat rigid bodies, and not just point-masses or bodies that can be simply reduced to an equivalent point-mass

5. Huygens himself uses the results to argue for a new universal measure of length -- the hour-foot -- corresponding to 1/3 of the length of the pendulum for an exact 1 sec arc -- i.e 1.05518 Rhenish ft, 1.01968 Paris ft, 33.122 cm

   a. Huygens indicates that the center of oscillation of the bob must be taken into consideration to give a precise value of this universal measure

   b. (Shows that when the *Horologium Oscillatorium* was going to press, Huygens was not yet aware of Richer's findings that g not the same in Cayenne as in Paris)

   c. Ends with a discussion of ways of realizing Mersenne's measurement of free-fall g -- i.e. distance of fall in the first second -- more accurately, to confirm pendulum value: 15 Paris ft 1 in

   d. Value presented as including small correction for center of oscillation

6. A disappointment noted at the end of Part IV: no isochronism result for a real cycloidal pendulum

   a. Length to center of oscillation gives only very close to isochronism for cycloidal path, for length changes along the arc of a cycloid

   b. Searched for, but did not find result for real bobs, that is, the isochronous path

   c. Best proposal he found instead: have the axis of the physical bob always remain perfectly vertical by putting the bob on hinges and weighting the bottom of the bob

E. Empirical Evidence for the Overall Theory

1. At first glance one would be inclined here too to say that Huygens is again doing science in a Galilean style
   a. The theory is a direct further extension of Huygens's earlier extension of Galileo's theory of naturally accelerated motion
   b. Its two key hypotheses are Galilean in spirit
   c. And, per usual, Huygens simply announces that the theory agrees with experiment in the manner of Galileo, without presenting any results of experiments: "demonstrated by more certain principles and ... found to agree precisely with experiments"
2. But, to a clearly greater extent than in the two earlier cases, comparatively precise data can be developed in support of this theory with little effort
   a. E.g. by constructing simple pendulums equivalent to various compound ones and comparing their periods -- experiments that a student could perform
   b. By using the theory as a basis for procedures for tuning clocks -- Huygens himself provides a full table for adjusting clocks at the end of Part IV of the *Horologium Oscillatorium*
   c. Evidence from successful "engineering" application in the manner of Galileo
3. In general, resistance effects on P, which tend to be self-canceling anyway, can be held to a minimum in pendulums, as is evidenced by Huygens's success in measuring g: use a comparatively large mass, and keep the velocities small by keeping the arcs small
   a. Resistance effects on P are second-order to begin with
   b. Furthermore, in experiments for testing this theory, resistance effects will automatically be of still higher order
   c. For data represent a before-and-after difference, and resistance effects of nearly the same magnitude are present before and after, even when comparing simple and compound pendulums
4. This brings out an important way in which the experimental evidence for this theory differs from Galilean evidence: the evidence here is being developed from discrepancies!
   a. Initially from the discrepancies between compound pendulums and corresponding simple pendulums with the bob at the center of gravity of the compound
   b. And then from the discrepancy between a clock and its intended measure -- e.g. a 15 sec discrepancy per 24 hours
   c. Thus Huygens is using data from discrepancies as evidence to remove an idealization from his pendulum theory -- a distinctly non-Galilean move
5. Consequently, the evidence for this theory carries more weight in support of the hypotheses from which it was derived -- and the Torricellian principle -- than the evidence for the original theory did
   a. The original theory of the pendulum did not employ the Torricellian principle, though it did employ the other key hypotheses of Galileo's theory of natural motion, plus some idealizations

b. The addition of Torricellian principle and Galileo's principle of ascent=descent now allows the prior theory to be extended to remove idealizations, in the process removing discrepancies between theory and observation associated with them

c. More exacting agreement with observation is being achieved not by adding independent elements to the theory -- as Kepler would or as a theory of resistance would -- but out of largely the same theory, extended only to include Torricellian principle

6. This is the first place we have seen where discrepancies associated with idealizations are being addressed by removing the idealizations without really changing the theory

   a. When successful, an extremely powerful form of evidence for a theory, for not only dispensing with discrepancies, but showing that theory not *ad hoc*, since it can account for things beyond what it was originally intended to

   b. Also showing how effective the theory is in allowing information to be extracted from the world, for original theory the basis for interpreting the discrepancies to be from the idealizations, and not from something else

V. "Rational Mechanics": in the Tradition of Huygens

A. Leibniz versus Descartes on Conservation Laws

1. Leibniz, perhaps the foremost protegé of Huygens, had come to Paris from 1672 to 1676, asking Huygens for tutelage toward becoming a philosopher

   a. Huygens recommended that he concentrate on mathematics, working with him for the four years he remained in Paris, and corresponding with him from then on

   b. Leibniz announced the calculus in the new journal, *Acta Eruditorum*, in 1682, following it in 1684 with the watershed paper: "A new method for maxima and minima as well as tangents, which is impeded by neither fractional nor irrational quantities, and a remarkable type of calculus for this" -- the differential calculus

2. The *Acta Eruditorum* paper of 1686 is famous in part because it challenged a fundamental principle of the leading philosopher of the time, Descartes' third law and more generally the fundamental conservation principle underlying it, conservation of B∗abs(v)

   a. The paper triggered a strong controversy over the next decades

   b. Nowadays, usually (and wrongly) described as a controversy resulting from confusion about the difference between conservation of energy and momentum

3. Leibniz's paper presents the conclusion from Huygens's (not yet fully published) work on impact in a form accessible to the general learned public, at the end citing his solution for center of oscillation

   a. In process driving home force of Huygens's result, which he chose not to emphasize in the paper

   b. But also generalizing the result -- $B*v^2$ (in relation to B∗h) conserved not just in impact, but in all situations, just as in Huygens's center of oscillation solution!

4. Leibniz's argument: Descartes' claim is incompatible with Galileo's results, and latter are to be preferred because they are empirically supported -- ultimately an appeal to empirical considerations
    a. Principle: "force" acquired in descent just sufficient for ascent to the same height
    b. Principle (known to Descartes): "force" needed to lift 4 lbs 1 yard = "force" to lift 1 lb 4 yards -- defensible via statics -- that is, the mechanics of machines
5. Leibniz's "*vis viva*" -- "living force" -- which is proportional to v^2 and not to v, is what is universally conserved; term "vis viva" introduced in Leibniz's "Specimen Dynamicum" of 1695
    a. The height to which a weight is carried is taken to be a basic measure -- again a more general statement of an idea that runs through all of Galileo's and Huygens's work on natural motion
    b. Analysis shows that what is conserved from bodies of one bulk to another is the $B*v^2$ needed to carry them to that height
6. Leibniz's generalization of Huygens's result not a mere philosophical exercise here, for taking a major step toward the general concept of mechanical energy (potential + kinetic) and toward conservation of mechanical energy as a basic principle in mechanics
    a. With ultimate theoretical foundations in the form of generalizations of certain empirically discovered "rational" principles like the principle of conservation of *vis viva*, instead of such narrower principles as Torricelli's
    b. (Notice how informative this more general principle would have been if they had noticed the discrepancy between rolling and falling along an inclined plane, for they would have immediately looked for the missing v^2, and would presumably have found it in rolling -- which seems to be more or less what Huygens did analytically in the early 1690s)

B. The Approach to Science Exemplified by Huygens

1. Huygens's work gives rise to a tradition called "rational mechanics," where the word 'rational' is intended to stand in some sort of contrast to practical mechanics, but also to generalizing inductively from empirical observation: in the spirit of Huygens's phrase, "not alien to reason"
    a. The idea was to develop mathematical theories covering various phenomena in mechanics by employing reason, informed by a qualitative understanding of the empirical world, to settle on appropriate fundamental hypotheses or axioms
    b. Much like constructing an axiomatized mathematical theory, except axioms here are being honed by requiring compatibility with what we generally know about the empirical world
    c. Idea: because precise experiments not possible, must rely on mathematics and especially telling experiments to filter out errors and block us from garden paths when devising empirical theories
    d. Mathematical tractibility a dominant factor, for needed to elaborate theory
2. The three assigned Huygens's pieces, standing alone, certainly provide a paradigm of such an approach, as does Galileo's work on free fall and projectile motion

a. Huygens and Galileo are not conducting a series of experiments or observations and generalizing from them or fitting theory to them, in a manner analogous to that of Kepler
b. Nevertheless, there is much more of an empirical dimension to Huygens's three papers than this description suggests

3. First of all, Huygens sees the elaboration of rich theory not just as an end to itself, but also as a means to higher quality empirical evidence
   a. Problems in direct measurement of g solved by elaborating a theory that provides means for precise measure
   b. Theories in the form of question-answering devices in general can lead to technological devices and experimental set-ups that yield much higher quality evidence
   c. Huygens held a strict hypothetico-deductive conception of evidence in science, as indicated by perhaps the best succinct statement of this approach (see Appendix from Class 7), from the Preface to his *Treatise on Light*, published in 1691 (under the same covers as his *Discourse on the Cause of Gravity*)
4. Second, Huygens's insistence on integrating the narrow problems he is addressing into a broader theoretical context allows diverse evidence to be brought to bear on the whole edifice
   a. So far as possible, proceed from principles used in theories of other phenomena -- preferably weak forms of these principles
   b. Look for ways to tie problem being addressed into other problems for which already have theories -- e.g. centrifugal forces and even motion under impact -- tied to gravity
   c. Can then bring evidence to bear on new theory -- e.g. evidence for centrifugal force claims via free fall theory
   d. And in turn diverse evidence accrues to original theory as it successfully lends itself to extension into new areas
   e. Huygens exploits this as a means for learning -- e.g. conservation of *vis viva* as a potential refinement to the theory
5. Third, Huygens's pursuit of stable, precise complementary measurements of fundamental quantities like the coefficient of proportionality g yields a much stronger form of evidence than does corroboration of salient predictions
   a. Fundamental quantities: those that are ubiquitous throughout the theory
   b. But for just this reason, ought to be open to diverse precise measurement if theory used to design requisite experiments
   c. Stable measure from one experimental paradigm is by itself strong evidence for the fragment of the theory in question, serving to define bounds of precision for it!
   d. Still stronger evidence, and evidence for overall theory, when obtain same value from experiments based on other parts of theory -- e.g. evidence for centrifugal force theory and for free fall

from obtaining same value of g from conical pendulum as from cycloidal pendulum

e. Occasional deviations then a basis for further empirical elaboration and refinement -- e.g. as in response to Richer's results from Cayenne

f. As remarked more than once above, Huygens seems unaware of the power of this further form of evidence, but it is central to Newton's *Principia*

6. Fourth, Huygens's success in extending the theory (without modification) to remove the point-bob idealization adds significant empirical evidence: initial theory an extendable first approximation! -- a move, like the emphasis on measuring fundamental quantities, that goes beyond Galileo
   a. Keep in mind how easy the solution for center of oscillation is to test, with two pendulums in synchrony if solution is correct
   b. Indeed, duration of their synchrony even determines a well-defined bound on accuracy
7. Thus, while others in the tradition of rational mechanics may have managed to limit the detailed empirical dimension of their efforts in subsequent decades, Huygens surely was not trying to do so

C. The Emergence of Theoretical Physics as a Subfield

1. The obvious way of characterizing the progress from Galileo's *Two New Sciences* to Huygens's *Horologium Oscillatorium* is to say that Huygens has extended the original Galileo theory to a much wider range of phenomena, yielding something that can legitimately be called a general theory of uniformly accelerated motion (along parallel lines)
   a. Now have a unified network of theoretical solutions to problems of motion involving gravity in one way or another, all built off a small number of basic principles (or hypotheses)
   b. Evidence for overall theory from salient predictions and from converging diverse measures of the fundamental quantity g
   c. In the process Huygens answered a range of outstanding questions including the twelve listed at the end of the Appendix, all within a unified theoretical framework
   d. And that in turn created a much larger range of opportunities to test, in one way of another, the fundamental principles of that theoretical framework
2. But, at a higher level of abstraction, can also characterize the step from Galileo to Huygens as the emergence of theoretical physics as a distinct subfield, for that is just what theoretical physics typically does, develop rich mathematical theories, leaving extended testing of them to others
   a. I.e. theoretical physics, in contrast to speculative natural philosophy of sort found in Descartes
   b. It has remained a distinct subfield ever since Huygens (which may be a more instructive way of looking at rational mechanics)
3. A subfield involving theoretical conjectures, but within a strict framework, the main elements of which have been empirically motivated and, to some extent, molded
   a. Start from problems and questions that seem to call for a question-answering type of theory, a set of lawlike relations among quantities licensing inferences from one quantity to another

   b. Propose theoretical solutions, proceeding as much as possible from principles -- as weak in form as possible -- devised for prior problems, and adding further principles only under demands of the problem at hand
   c. In process expose new principles, the generalizations of which can sometimes be regarded as empirically based foundations of the sort Descartes wanted
4. Although theoretical in character, such effort is empirically oriented in two crucial respects
   a. The theory is aimed at yielding new means for bringing empirical evidence to bear not just on new parts of the theory, but also on theoretical framework as a whole
   b. Integrating new problems into existing framework allows evidence to be brought to bear on the new, and diverse evidence from different applications to accrue to the whole
   c. Indeed, success-in-being-extended a major form in which evidence accrues to an initial fragment, as illustrated by the enormous increase in evidence for Galileo's fragment resulting from Huygens's efforts!
   d. Notice how what unified the theory is not an underlying mechanism -- e.g. gravity -- for impact and centrifugal force not governed by the underlying mechanism of gravity; what unifies the theory, besides uniform acceleration, is the means for acquiring evidence for the various interrelated parts of it, through tieing them together!
5. One thing that we can see happening as theoretical mechanics emerges in the work of Huygens is a concern for preferred foundations
   a. Huygens being the opposite of bold in his initial hypotheses; the theory itself then yields special cases of potentially bolder principles, and also a potentially simpler set of basic principles
   b. For example, conservation of mechanical energy principle is in process of emerging from Huygens's work, and would have emerged even more strongly if others had noticed the difference between rolling and falling
   c. Approach being taken is allowing the empirical world to mold the foundations -- not just by restricting them, but by justifying generalizations that can serve as alternatives to them
6. Another thing we can see happening is the proliferation of different concepts of force that will have to be reconciled at some point in the future
   a. Not just *vis* and *conatus*, but (1) static balance and elastic deformation; (2) gravity and tension in a string; (3) resistance of media; (4) *vis inertia* (Newton’s phrase) and centrifugal *conatus*; (5) Descartes' change in quantity of motion; (6) forces in impact -- percussion, impetus, and change of motion; (7) *vis viva*, proportional to v^2 and bulk, and to product of height and bulk
   b. As theory becomes elaborated, empirical bases for sorting out concepts become more available, suggesting that the way to go about reconciling them is through further theory development
7. In short, all the empirical advantages of having a subfield of theoretical physics are beginning to become apparent in Huygens's early work in mechanics

a. Rational mechanics therefore need not be construed as a reflection of rationalistic philosophy and Platonism on the continent

b. It can be construed as a sound response to the goal of developing high quality empirical evidence, letting the world be the ultimate arbiter of theory, in the face of difficulties in designing straightforward experiments that control for "externalities" like friction

c. Would have been even more evident if Huygens had published his intended book on mechanics

8. Why with Huygens? Because he was historically situated to build on the work done before him, especially by Galileo and Descartes, but also Mersenne, Gassendi, Torricelli, Riccioli, and Stevin

a. Huygens assimilated the prior work, where "assimilated" means not merely understanding it

b. It means extending it beyond what those before him had done, which in this case resulted in a much richer theory, in part because he was combining Galileo and Descartes

c. Can think of Huygens in relation to his predecessors in mechanics as the counterpart of Horrocks in relation to Keplerian theory -- only Huygens lived to work 40 years, versus Horrocks's 5 years

D. Huygens and the Continental Tradition in Mechanics

1. In a position now to give a picture of the history of the development of mechanics, showing where Huygens (and Newton) fit in

2. Four distinct traditions from the first half of the 17th century involved in the evolution of mechanics (see figure at end of Appendix) -- i.e. of "Newtonian" mechanics, which emerged in the form we know it at the hands of Euler in the mid-18th century

a. One tradition comes out of orbital astronomy, with Kepler as the chief figure, and such other figures as Horrocks, Cassini, and Flamsteed, leading into Newton's *Principia*

b. English "magnetical" philosophy forms a second tradition, coming out of Gilbert and continued by Wilkins, Wren, Hooke, etc., again leading (via Streete) into Newton's *Principia*

c. A third tradition in "rational" mechanics has Galileo as the key figure in the first half of the 17th century, though Beeckman and Stevin deserve mention because of their influence on Huygens; Torricelli, Mersenne, and Riccioli are part of the bridge leading from Galileo to Huygens, the person who, on my view, becomes the key, pivotal figure within this tradition, for it was his work that made clear how solutions for individual problems can be unified into a theory

d. Finally, a fourth tradition, emphasizing comprehensive theories with universal foundations, comes out of Descartes, influencing Newton and Huygens in somewhat different ways, and continuing to exert some influence well into the 18th century

3. A "rational" mechanics tradition grows out of Huygens initially through his direct protegés -- Leibniz, Johann and Jakob Bernoulli, and those working with them in developing mechanics and the calculus, starting in mid-1680's

a. They in turn were followed by Johann's sons, Daniel and Johann, and their boyhood friend Euler, along with a group coming out of the Paris Academy, with d'Alembert the leading figure

b. This group dominated both mechanics and mathematics during the first half of the 18th century, passing on the mantle to such people as Laplace and Lagrange
c. "Rational" because proceeded overwhelmingly from theoretical principles that seemed to be appropriate, and not at all inductively from experimental results
d. Experiments serve to verify salient test results, in the manner of Huygens, but lie mostly in the background: "theory agrees with experience"
e. Opens up a potentially vast array of experimental tests and responses, and hence not inappropriate that experimental ramifications left to others, leading to a split of sorts between theoretical and experimental physics
f. The way mechanics of motion is still taught today, with experiments presented almost as an afterthought

4. The influence of that tradition of "rational" mechanics on Newton's *Principia* comes not directly from Galileo, but from Huygens and his impact on various figures in England
   a. Newton read the *Dialogue*, but unless I am sorely mistaken, he never read *Two New Sciences* (at least before 1700) and most of his knowledge of the theory of "natural" motion comes from *Horologium Oscillatorium* (along with Digby, Charleton, Collins, and, after 1686, Halley)
   b. I.e. he came to know the theory in rich form it had taken after Huygens's work in the late 1650's
5. "Newtonian" mechanics does not come so directly out of Newton's *Principia* as it does out of the influence that book had on the 18th century figures in the tradition of "rational" mechanics deriving directly from Huygens
   a. One of the things they did was to recast the results of the *Principia* into calculus, using them as a check against their own results and, in some cases, building on them
   b. Euler was the one member of this group comfortable with working with forces; he ended up formulating the mechanics they were all generating within a Newtonian framework, using what we now call Newton's three laws of motion (though not the first, Euler explicitly put forward F=ma in 1745 and called it the fundamental principle of all mechanics in 1750)
   c. Others in the tradition tended to avoid forces and instead used global principles like conservation of *vis viva*, but their mechanics was nevertheless fully compatible with Euler's -- as of course was Lagrange's *Analytical Mechanics* (1788), the ultimate culmination of the rational mechanics tradition of Huygens and the Continent
   d. Ultimate culmination of tradition out of Newton: Euler's monographs on pure mechanics (1750-1780s), yielding the Eulerian equations of motion, and Laplace's synthesis with gravitational mechanics in *Mécanique Céleste* (1799-1805)
6. Upshot: Newton's *Principia*, as important as it was, ultimately must be viewed as part of a larger unfolding context that had gained its momentum well before it, and continued to have influence somewhat independently of it indefinitely into the future

Select Sources

Huygens, Christiaan, *Oeuvres complètes de Christiaan Huygens*, Publiées par la sociètè hollandaise des sciences, 22 vols., Martinus Nijhoff, 1888-1950, especially vols. 16-18, 9, and 6.

--------, *The Pendulum Clock or Geometrical Demonstrations Concerning the Motion of Pendula as Applied to Clocks*, tr. Richard J. Blackwell, Iowa State University Press, 1986. (original plus notebook material in vol. 18, pp. 27-438, of *OCCH*)

--------, *Horologium*, (1658), *OCCH*, vol. 17, pp. 41-84.

--------, Notebook material on conical and cycloidal pendulum clocks, 1659, *OCCH*, vol. 17, pp.85-53.

--------, *De Motu Corporum ex Percussione*, first published 1703, tr. Michael S. Mahoney (unpublished). (original plus earlier versions and notebook material in vol. 16, pp. 1-168, of *OCCH*)

--------, *De Vi Centrifuga*, first published 1703, tr. Michael S. Mahoney (unpublished). (original plus notebook material in vol. 16, pp. 235-328, of *OCCH*)

--------, "*A Summary Account of the* Laws of Motion, *communicated by Mr*. Christian Hugens *in a Letter to the* R. Society, *and since printed in* French *in the* Iournal des Scavans *of March 18, 1669*," *Philosophical Transactions of the Royal Society*, vol. 4, 1669, pp. 925-928.

Yoder, Joella, *Unrolling Time: Christiaan Huygens and the Mathematization of Nature*, Cambridge University Press, 1988).

Bos, H. J. M., *Christiaan Huygens*, entry in the *Dictionary for Scientific Biography*, vol. 6, Charles Scribner's & Sons, 1972, pp. 597-613.

Hall, A. Rupert, "Mechanics and the Royal Society, 1668-70," *The British Journal for the History of Science*, vol. 3, 1966, pp. 24-38.

Wallis, John, "*A Summary Account given by Dr*. John Wallis, *of the General* Laws of Motion, *by way of a Letter written by him to the Publisher, and communicated to the* R. Society, Novemb. 26, 1668," *Philosophical Transactions of the Royal Society*, vol. 3, 1668, pp. 864-866.

Wren, Christopher, "*Dr*. Christopher Wrens *Theory concerning the same subject; imparted to the* R. Society Dec. 17 *last, though entertain'd by the Author divers years ago, and verified by many Experiments, made by himself and that other excellent Mathematician* M. Rook *before the said Society, as is attested to by many Worthy Members of that Illustrious Body*," *Philosophical Transactions of the Royal Society*, vol. 3, 1668, pp. 867-868.

Oldenburg, Henry, *The Correspondence of Henry Oldenburg*, vol. 5 1668-1669, ed. A. Rupert Hall and Marie Boas Hall, University of Wisconsin Press, 1968.

Newton, Isaac, *Universal Arithmetick: or a Treatise of Arithmetical Composition and Resolution*, tr. by Ralphson, revised and corrected by Cunn, 1728, p. 80-82; reprinted in *The Mathematical Works of Isaac Newton*, vol. 2, ed. Derek T. Whitside, Johnson Reprint, 1967, p. 46f.

Goldsmith, Werner, *Impact: The Theory and Physical Behaviour of Colliding Solids*, Dover, 2001. (reprint of original published by Edward Arnold, 1960)

Murray, Gemma; Harper, William; and Wilson. Curtis, “Huygens, Wren, Wallis, and Newton on Rules of Impact and Reflection,” in*Vanishing Matter and the Laws of Motion*, ed. Dana Jalobeanu and Peter R. Antsey, Routledge, 2011.

Stronge, W. J., *Impact Mechanics*, Cambridge University Press, 2000.

Stein, Howard, "On Locke, ‘the Great Huygenius, and the incomparable Mr. Newton’," in *Philosophical Perspectives on Newtonian Science*, ed. Phillip Bricker and R. I. G. Hughes, MIT Press, 1990, pp. 17-47.

Smith, George E., "The *vis viva* dispute: A controversy at the dawn of dynamics," *Physics Today*, October 2006, pp. 31-36.

Leibniz, Gottfried Wilhelm, “A Brief Demonstarion of a Notable Error of Descartes and Others Concerning a Natural Law, According to Which God is Said Always to Conserve the Same Quantity of Motion; A Law Which They Also Misuse in Mechanics,” in *Philosophical Papers and Letters*, ed. Leroy E. Loemker, 2nd ed., Reidel, 1969, pp. 296-302.

Dijksterhuis, E. J., *The Mechanization of the World Picture: Pythagoras to Newton*, tr. C. Dikshoorn, Princeton University Press, 1986.

Truesdell, Clifford, *Essays in the History of Mechanics*, Springer-Verlag, 1968.

Dugas, René, *A History of Mechanics*, tr. J. R. Maddox, Central Book Company, 1957; reissued by Dover.

--------, *Mechanics in the Seventeenth Century (from the Scholastic Antecedents to Classical Thought)*, tr. Freda Jacquot, Central Book Company, 1958.

Bertoloni Meli, Domenico, *Thinking with Objects: The Transformation of Mechanics in the Seventeenth Century*, Johns Hopkins University Press, 2006.

Speiser, David, *Discovering the Principles of Mechanics, 1600-1800*, ed. Kim Williams and Sandro Caparrini, Birkhäuser, 2008.

MacMillan, William Duncan, *Statics and the Dynamics of a Particle*, Dover, 1958.

Hooijmaijers, Hnas, *Telling time, Devices for time measurement in Museum Boerhaave: A Descriptive Catalogue*, Museum Boerhaave Communication 311, 2005.

Leopold, John H., "The Longitude Timekeepers of Christiaan Huygens" in *The Quest for Longitude*, ed. William J. H. Andrewes, Harvard University Press, 1996, pp. 102-114.

Baker, Gregory L. and Blackburn, James A., “The Pendulum Clock,” in *The Pendulum: A Case Study in Physics*, Oxford University Press, 2005, pp. 233-257.

Daston, Lorraine, *Classical Probability in the Enlightenment*, Princeton University Press, 1988.

Euclid, *The Thirteen Books of Euclid’s Elements*, 3 vols., tr. Thomas L. Heath, 2nd ed., Cambridge University Press, 1956; reissued by Dover.

--------, “Euclid’s Elements,” an interactive internet site covering every definition and proposition in all thirteen books, created by D. Joyce, aleph0.clarku.edu.

Credits for Appendix

Slide 2: Stein (1990)

Slide 3: Huygens (1888-1950, vol. 18)

Slide 4: Yoder (1988), Huygens (1986)

Slides 7-10: Huygens (1888-1950, vol. 9, translation by Eric Schliesser)

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Slides 28, 30-37, 39, 40: Huygens (1986)

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