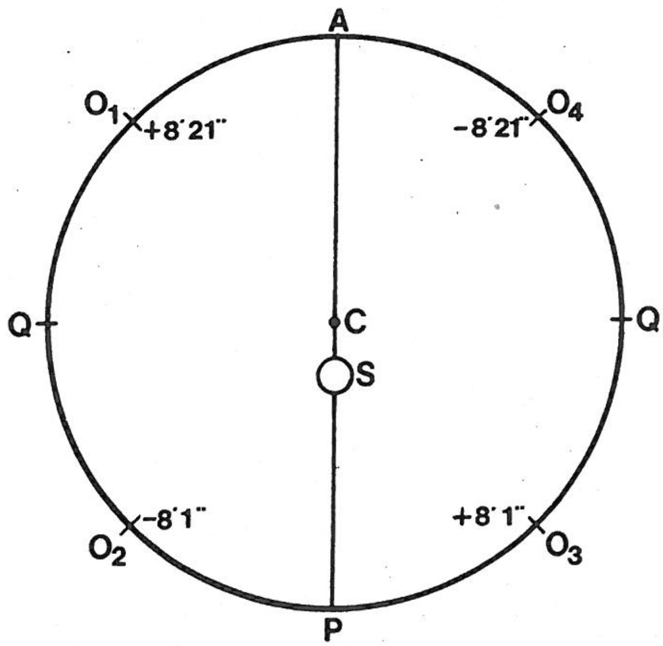
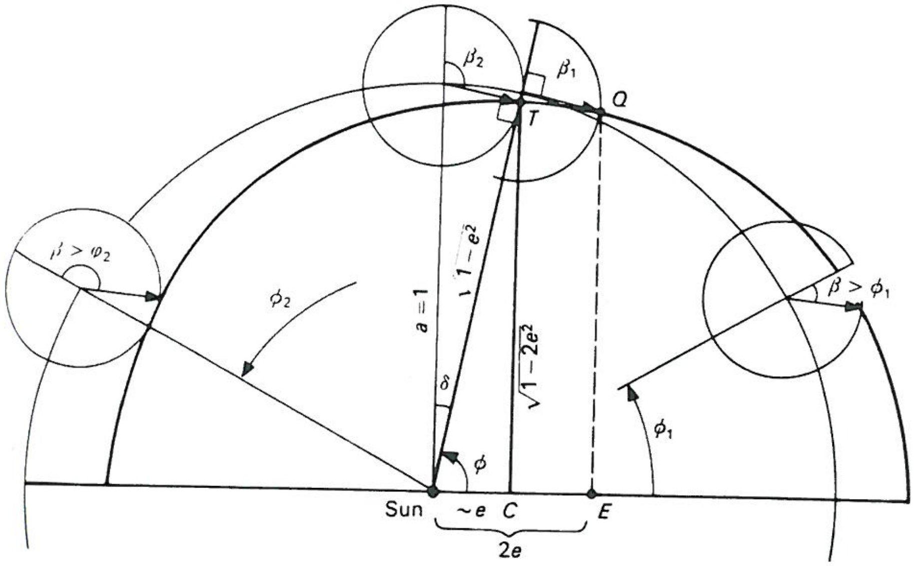
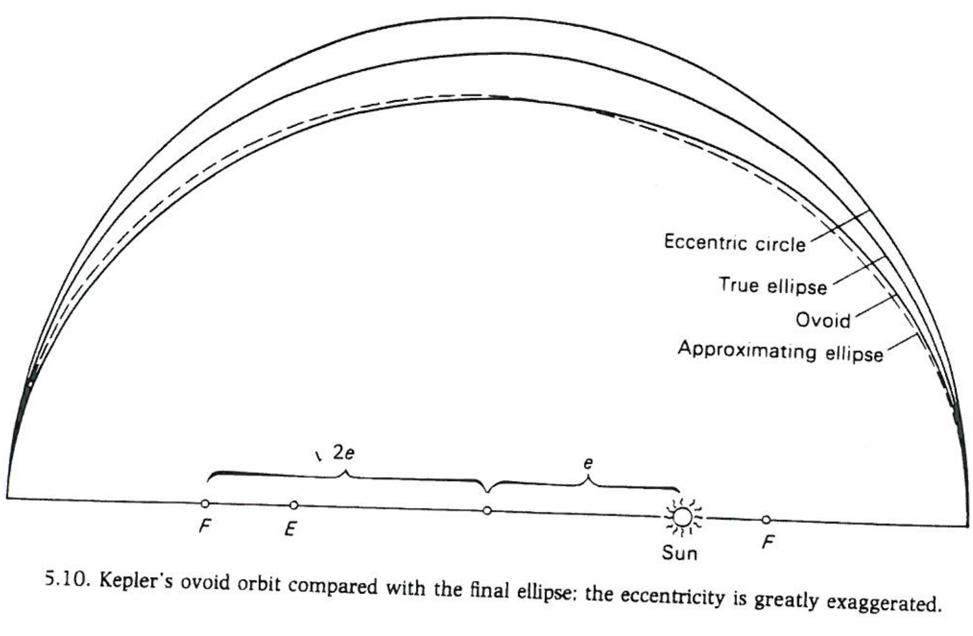
First Trial With Area Rule
Discrepancies for an Eccentric Circle
Versus Vicarious Hypothesis





5.9. The epicyclic construction of Kepler's ovoid (the darker curve). The planet moves along the ovoid at the tip of the epicyclic radius vector. The epicycle has radius e. Angle β moves uniformly with time, whereas ϕ moves non-uniformly in order to satisfy the area law, that is, so that the vector from the Sun to the planet sweeps out an area proportional to time (and proportional to angle β). If this construction had an equant, it would fall $2e$ from the Sun at E and Mars would reach Q in a quarter period: hence β_1 in the epicycle must be very close to 90°. As the epicycle centre moves through the angle δ, the epicycle radius vector will also advance by δ since angle ϕ is close to its mean rate in this part of the orbit. Thus, $\beta_2 = \beta_1 + \delta$, and the angle at T is a right angle. Then the line $Sun - T = \sqrt{(1-e^2)}$ and $TC = \sqrt{(1-2e^2)}$. Since the semi-major axis of the ellipse is $(1-e^2)$, the approximating ellipse to Kepler's ovoid has the eccentricity $\sqrt{2}e$. Kepler, however, was awakened to this relationship in quite another way. He found from an analysis of the errors in longitude that the ovoid gave a maximum departure from a circle too large by a factor of 2; that is, it should have swung in by 660 parts of a semi-diameter (or 0.004 29 for a semi-diameter of 1.000 00) instead of twice that much. He finally realized that an ellipse of eccentricity e gave the required path when he noticed by chance that secant $\delta = 1/\sqrt{(1-e^2)} \approx 1 + \frac{1}{2}e^2$ was 1.004 29; in other words, that it exceeded unity by precisely the width of the lunula between the circle and the non-circular orbit.



5.10. Kepler's ovoid orbit compared with the final ellipse: the eccentricity is greatly exaggerated.

Second Trial with Area Rule Discrepancies for "Egg-Shaped" Oval Versus Vicarious Hypothesis

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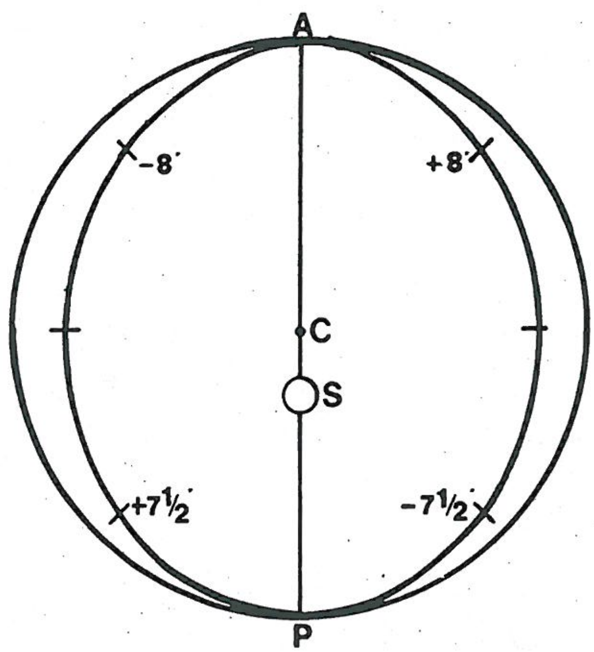

Kepler's Table of Comparisons of Calculated Heliocentric Longitudes

Common mean anomalies	Through a simple eccentricity	Through bisection of the eccentricity and the doubling of the upper part of the equation	Through bisection of the eccentricity and a stable equant point, in the Ptolemaic manner	The vicarious hypothesis using a free division, practically in agreement with truth	Physical hypothesis, through the assumption of a perfect circle	Physical hypothesis through the supposition of the opinion of chapter 45 and of a perfect ellipse
	The various corresponding equated anomalies					
48° 45′ 12″	41° 40′ 14″	40° 45′ 52″	41° 15′ 31″	41° 20′ 33″	41° 28′ 54″	41° 14′ 9″
95 18 28	84 40 44	84 37 48	84 41 22	84 42 2	84 42 26	84 39 42
138 45 12	130 40 46	131 45 0	131 15 31	131 7 26	130 59 25	131 14 5
	Ch. 20 and 29	The excess and defect go in the opposite direction if the lower part is doubled. Ch. 29	Ch. 19	Ch. 16 and 29	Ch. 43 and 29	The present Ch. 47

You will note that the truth is exactly in the middle between these.