Three Distinct "Working Hypotheses" at the Core of Ptolemaic Theory

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1) **The Earth is motionless -- in particular, its location does not vary with respect to the stars along the zodiac over the course of the year.**

2) **All zodiacal motion -- that is, motion from one day to the next along the zodiac -- is centered around the Earth.**

3) **All real celestial motion is compounded out of uniform -- or at least equiangular -- circular motions.**

Evans –
pp. 355, 362, 368

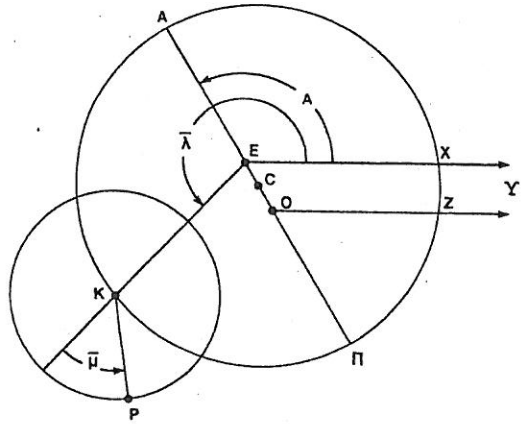


FIGURE 7.32. Ptolemy's final theory of longitudes for Venus and the three superior planets. The Earth is at *O*. *C* is the center of the deferent circle. But the epicycle's center moves at uniform angular speed as viewed from the equant point *E*.

How can we know how large a planet's epicycle is? How can we know how large to make the eccentricity? In this section we demonstrate how the parameters for a superior planet can be determined from observations. We use Mars as an example, but the same procedures could be applied to Jupiter or Saturn. Although the methods demonstrated here do not exactly follow those of the *Almagest*, they show, clearly and simply, the connection of each parameter with the observed motion of the planet. And who knows? It is more than likely that some such rougher method preceded the elegant perfection of Ptolemy.[71]

There are seven parameters to be determined:

1. The mean angular speed of the epicycle's center around the deferent circle—in other words, the rate of change of the mean longitude $\bar{\lambda}$ (see fig. 7.32). This angular speed we denote $f_{\bar{\lambda}}$.
2. The angular speed of the planet on the epicycle. This speed, denoted $f_{\bar{\mu}}$, is the rate at which the mean epicyclic anomaly $\bar{\mu}$ changes.
3. The longitude of the apogee of the deferent, denoted *A*.
4. The eccentricity of the deferent, denoted *e*. This is the ratio *OC*/*R*, or *CE*/*R*, where *R* is the radius of the deferent.
5. The initial value of $\bar{\lambda}$ for some specific date. This initial value will be denoted $\bar{\lambda}_0$.
6. The initial value of $\bar{\mu}$, which we will denote $\bar{\mu}_0$.
7. The radius of the epicycle, denoted *r*. All that matters in Greek astronomy is the size of the epicycle in relation to the deferent, that is, the ratio *r*/*R*.

TABLE 7.4. Modern Ptolemaic Parameters for Venus, Mars, Jupiter, and Saturn

Planet	Mean Motion in Longitude $f_{\bar{\lambda}}$ (°/day)	Mean Motion in Epicyclic Anomaly $f_{\bar{\mu}}$ (°/day)	Radius of Epicycle *r*	Eccentricity *e*	At epoch January 0.5 GMT 1990 = J.D. 241 5020.0: Longitude of Apogee A_0	Mean Longitude $\bar{\lambda}_0$	Mean Epicyclic Anomaly $\bar{\mu}_0$
Venus ♀	0.985 647 34	0.616 521 36	0.72294	0.01450	98°10′	279°42′	63°23′
Mars ♂	0.524 071 16	0.461 576 18	0.65630	0.10284	148°37′	293°33′	346°09′
Jupiter ♃	0.083 129 44	0.902 517 90	0.19220	0.04817	188°58′	238°10′	41°32′
Saturn ♄	0.033 497 95	0.952 149 39	0.10483	0.05318	270°46′	266°15′	13°27′

General precession f_p = 0.000 038 22°/day = 1°23′45″ per Julian Century = 0.838′ per year.

Evidence for Ptolemaic Astronomy

- **Success in predicting salient phenomena: timing of stationary points, timing and extent of maximum elongations (Venus and Mercury), timing and shape of retrograde loops (Mars, Jupiter, Saturn), eclipses of Moon and Sun, and previously unrecognized inequalities in longitude of the Moon in quadrants and octants**

- **This success achieved by "theories" of the seven bodies, employing only five basic parameters (with a model in common for Venus, Mars, Jupiter, and Saturn), thus reducing multiple apparent degrees of freedom in the motions to just a few degrees of freedom in the "theories"**

- **The values of these parameters were determined by means of model-mediated measurements from observations that, when repeated at different times, kept yielding the same values to reasonably high precision, thereby providing evidence that the parameters are constants of nature**

The combination of these, especially the stability over time of the model-mediated measurements of the parameters, gave evidence that there was something fundamentally correct in Ptolemaic theory, notwithstanding the existence of alternative models, by virtue of Apollonius's theorem, that achieve the same as above

"In astronomy, the [Islamic] reactions … ranged from simple corrections of what was thought to be a mistake in the text … of the *Almagest*, to correcting the basic parameters by fresh observations, as in the case of redetermining the better values of precession and the inclination of the ecliptic among others, to critiquing the methods of observation, as was done in the case of the *fuṣūl* method, and finally to casting doubt on the reliability of the very foundations of the Greek astronomical tradition itself when it seemed to violate the principles upon which it was based in the first place.

All these developments … generated a skeptic attitude toward the incoming tradition. In itself this attitude emboldened astronomers to raise deeper and deeper questions as they continued to examine this Greek tradition in the light of their own research. In this environment, it becomes easy to understand why good competent astronomers could not continue to practice astronomy by simply taking the Greek astronomical tradition at its face value….

It was this environment that motivated the research of the new Islamic astronomy. Its main mission, as was enunciated later by Mu'ayyad al-Dīn al-'Urdī (d. 1266) of Damascus, one of the most distinguished astronomers of that tradition, was to create an astronomy that did not suffer from the cosmological shortcomings of Ptolemaic astronomy, that could account for the observations just as well as Ptolemaic astronomy could do if not better, and that did not limit itself to criticizing Ptolemy only, despite all the benefits that one derived from the detailed critique of Ptolemy's mistakes. This urgent need for a higher form of scientific astronomy was almost felt by all serious astronomers *whose work we have come to know only recently*, and who formed a continuous tradition inaugurated toward the beginnings of the ninth century and continued well into the sixteenth century as far as we can know now.

George Saliba, *Islamic Science and the Making of the European Renaissance*, p. 131ff, emphasis added

of the third, on the area of a quadrilateral inscribed in a circle having sides in a given proportion, writing a little treatise of twelve propositions on cyclical quadrilaterals in which the last is the solution to the problem.[14] He then provides solutions to Bianchini's problems, in the seventh showing the trisection of a sixty-degree angle and in the fourth telling Bianchini the news that he has just now discovered in Venice a Greek manuscript of Diophantus that contains six of the thirteen books promised in the preface. If this very beautiful and difficult book could be found complete, he would wish to translate it, for which the Greek he has learned in the house of Bessarion would suffice, and he asks whether perhaps Bianchini could find a complete version in Ferrara where there are a number of people expert in Greek who might have books of this kind.[15] Then he poses twenty-three new problems, some quite difficult, to Bianchini who, we remember, could answer only one of the last eight. Perhaps aware that he has gone too far and may be trying the older man's patience, he concludes:

> But I do not know whither my pen goes, for unless I restrain it the paper will run out. One problem follows upon another, and so many beautiful problems come to mind that I am uncertain which I shall propose. And I had not thought at the beginning that I was about to write so much, especially lest my tumultuous letter be unduly disturbing to your tranquility. I beg you to grant indulgence to my audacious and impetuous pen. For there is no need to grant indulgence to Johannes who is writing since he desires to be looked upon as both more discreet and in all things most obedient to your will.[16]

But that is not the end, for continuing, as it appears, immediately, he devotes the remainder of the letter to the criticism of contemporary astronomy, which we translate here in full.[17]

TRANSLATION

Johannes Regiomontanus to Giovanni Bianchini of Ferrara

I have in my writings many things, believe me, and again many things that I should like to submit to your judgment if there were time, some of which have been determined beyond doubt, but others are suspended in[a] uncertainty and strongly incite my mind to their investigation. That I may begin this discourse with the highest[b] starry sphere, which has thus far afforded the subject of our discussion, I cannot but wonder at the indolence of the common astronomers of our age who, just as credulous women, receive as something divine and immutable[c] whatever they come upon in books either of tables or their canons, for they believe in writers and make no effort to find the truth. For what shall I say of the nature of the motion of the eighth sphere, which our illustrious Ptolemy concluded to move through one degree in a hundred years, but 743 years after him al-Battānī [found to move] through one degree in about 66 years? The former found the maximum declination of the sun 23;51,30°, but the latter 23;35°. After them Thābit found the maximum declination about 23;33°. Accordingly, impressed by the curiosity of this thing, he began to ponder from what cause a variation of this kind arises, and after many observations[d], when he perceived the ecliptic of the eighth sphere to have a variable inclination to the equator, he concluded — that we may pass over many things — that the eighth sphere moves, not on fixed poles, but by the kind of variable motion called the motion of trepidation, and so on. Az-Zarqāl, the compiler of the *Toledan Tables*, also accepted this representation, and even now it is more acceptable to a great many moderns than that nature of the motion of the eighth sphere imparted by the method of computing with the *Alfonsine Tables*.

I, however, am not at all certain which of them is closer to the truth. Nevertheless, I know that both of them (I say this with all due respect to those who judge better) are false. For if the hypothesis of Thābit is to be believed, then the maximum declination of the sun at the time of Ptolemy should have been 23;41°, although he himself found it about 23;51°. Consequently, he would have erred by 10 minutes, and therefore in taking the distance betwee the two tropics he would have erred by 20 minutes, although it is not likely that such a man was so sensibly deceived. For were it so, he would also have been deceived in the entry of the sun into Cancer and consequently in the eccentricity of the sun, and he would have derived other erroneous things. Likewise, in our time the maximum declination of the sun should be 24;2°, although we (my teacher [Peurbach] and I) found it about 23;28° with instruments. I have often heard Master Paolo [Toscanelli] of Florence and [Leon] Battista Alberti saying that they themselves had observed carefully and did not find it greater than 23;30°, which fact also convinces us to correct our tables, that is, the table of declination and others that are based upon it[e]. But, God willing, you will see other conclusive demonstrations of these things.

If, however, we trust the Alfonsine hypothesis, the tables of which, either "orginal" or "resolved", all our moderns make use of, understand what must follow. For first of all, the maximum declination of the sun at the time of al-Battānī should have been 23;45°, which that careful observer nevertheless found 23;35°, an error of 10 minutes which is surely sensible. Moreover, in our time the maximum declination of the sun would be 22;47°, which our instruments show to be 23;28°, a difference of 41 minutes, an intolerable error. Possibly this would not disturb someone if I alone had attempted to find this thing with an instrument, but men learned and worthy of confidence, my former teacher Georg of good memory[f], Master Paolo and Battista mentioned before, provide support.

Next I demonstrate by the most indisputable computations that the distance of the apogee of the sun from the head of Aries in the eighth sphere was 43;35° at the time of Ptolemy. This distance must be invariable if it is true that the motion of the apogee of the sun follows the motion of the eighth sphere. But in the *Alfonsine Tables* this distance is 71;25°. Now there is a difference of 27;50° between the two numbers, from which at times in the computation of the anomaly of the sun we would be in error by 27;50°, to which at the apogee of the eccentric there corresponds about one degree in the equation of the sun. And therefore in the computation of the true motion of the sun there would be an error of about 1 degree, which as unfitting as it is in giving judgments [i.e. casting horoscopes], no one should fail to notice in eclipses and other things. Further, when the sun is in the beginning of Aries in the fixed ecliptic, by computation it will be removed from the equator by about 6 degrees to the north. How, therefore, could we compute the position of the sun in the equator? But enough of these things.

In descending to the lower spheres, it is nevertheless preferable to omit Saturn and Jupiter because their heavens are not so ~~familiarly known~~ obstinate (if I may speak in the manner of some people) as that of Mars. Mars was seen to differ in the heavens and in computation by two degrees in relation to the fixed stars and other observations, at times a difference of this kind of one and a half degrees is distinguished, and sometimes much less. Now some, in ascribing this error to the epochs of the mean motions, have erred beyond reason. For if the error were only such epochs of the mean motions, there should be found a constant difference between the computed position and the true position, which is not found. Hence, with good reason[g] it must be concluded that its eccentricity or the semidiameter of its epicycle have not

been found entirely accurately. It is nevertheless possible that the revolutions of its mean motions introduce some error in a perceptible time, but also it cannot be denied that the epochs of the mean motions increase this error still more. A long time has elapsed from Ptolemy to this day in the course of which, although al-Battānī and others have applied corrections to the motions of the luminaries, they left the other five planets nearly untouched. Further, if the eccentricity of Mars is as much as is assumed by all, and similarly the semidiameter of the epicycle, it follows that the maximum apparent area[h] of Mars to its minimum apparent area[h] is about as 52 to 1. I believe that Mars has never appeared so large to anyone when the air is clear and other things are in the same condition.

In the case of the sun, at last, how could there be certainty of computation if we write one maximum declination of it in the tables and find another in the heaven with instruments? The reason that computations of eclipses are false, both in the time of the duration and the size of the eclipsed part, and likewise in [the times of] the beginnings and ends of eclipses, and consequently in [the times of] true or apparent conjunctions, will be ascribed to both luminaries or principally to the moon.

Finally, I have seen Venus slower in the heaven than the computation had predicted by about three-quarters[i] of a degree, and it is also extremely difficult to avoid falsehood in computing its latitudes. Likewise, following from what has been found of the eccentrics and epicycles, the surface of Venus[k] ought to appear to our sight sometimes as 1, but another time as 45, which condition has never become known to anyone observing. Further, its apparent diameter will sometimes be 0;12,30° that, is, two-fifths the apparent diameter of the moon, which certainly has never been perceived in the heaven.

What shall I say concerning Mercury, which frequently ought to appear at our latitudes if the table of appearances and disappearances that is found among other tables of the moderns reports correctly? But Mercury either never or very infrequently becomes visible to us. In fact this results because the aforesaid table is not of use for our latitudes, for it was composed by Ptolemy in Chapter 10 of Book 13 for the middle of the Fourth Climate, on account of which the ignorance of those who insert it in their tables as though it were suitable to all climates is all the more astonishing.

At last in the case of the moon, a difference so great and so frequent occurs that even ordinary people begin to tear at this divine science of the stars with a sharp tooth. For my part I observed an eclipse in the year 1461 that was in December, the end of which in the heaven preceded the computed end by a full hour. And in order to know the end in the heaven with greater certainty, I took the altitudes of the two stars Alhaioth and Aldebaran at the end of that very eclipse, in as much as one would be evidence for the other. I have also observed other eclipses differing greatly from computation in duration and the size of the eclipsed part, concerning which the proper place for speaking at greater length will be elsewhere. And if the moon has an eccentric and an epicycle in the way that has been claimed, it will follow necessarily that in a particular position the moon appear about four times greater than in another position, other things being in the same condition.

That is now enough concerning these things. I am often troubled by such concerns and am driven to lament the sloth and lethargy of our age. Surely there is at this time abundant subject matter for those wishing to apply themselves to philosophy. We have before our eyes the footprints of our predecessors, by reason of which we can advance more securely provided that we apply intelligence to this matter. If my situation were such that I might pass my life close to you, I would hope to achieve both compensations and profits in innumerable things of this kind. However, my Reverend Lord [Bessarion] is about to go to Greece for the sake of the Christian religion, while I by his order will remain in Italy. Let them go there in order to destroy the Turks; I, with your aid and the aid of other friends shall endeavor to restore the heavens. Let them bring about peace there in earthly things; we shall undertake to remove the rust from the heavenly spheres and guide them back to the royal roads. Let us be granted a life to be passed in peace, with other fears repelled; the leisure for studying philosophy will win[1] us a glory that will last forever, which will come to pass for both of us so much more readily and plentifully as you so kindly, as you are accustomed, show favor to your obedient Johannes. Until you can, through leisure, return an answer to my letter, your virtue will be of encouragement. Now felicitously farewell, and as you have begun, do not cease to love me.

Yours entirely,
Johannes Germanus

COMMENTARY

The Motion of the Eighth Sphere

Regiomontanus's first, and evidently most important, concern is the motion of the eighth sphere, that is, the motion of the sphere of the fixed stars with respect to the equinoxes. Here he accuses astronomers, like "credulous women", of accepting whatever is in their tables without any attempt to ascertain the truth,[18] perhaps an unfair charge since a correct description of this motion was far from easy and was first reached by Tycho more than a century later. The story he tells here is familiar from the *Epitome* and is essentially taken over by Copernicus in *De revolutionibus*. Ptolemy found the stars to move 1° in 100 years, 743 years later al-Battānī found 1° in 66 years, and associated in some way with this motion is a variation in the maximum declination of the sun, for Ptolemy found 23;51,20° and Battānī 23;35°. Thābit ibn Qurra found the maximum declination 23;33°, and further determined that the eighth sphere does not move uniformly nor maintain a fixed inclination, but has a periodic, variable motion called "trepidation", a theory followed by az-Zarqāl in the *Toledan Tables* and still preferred by some to the motion of the eighth sphere in the *AT* (*Alfonsine Tables*). Parts of this account are not true. By "Thābit" is meant the little tract *De motu octavae sphaerae* attributed to Thābit, although not authentic and in fact directly contradicting the uniform motion of the fixed stars in his (presumably) authentic *De anno solis*. The trepidation is adopted by the *Toledan Tables* in that they are arranged to give sidereal longitudes, and the conversion to tropical is done with the tables from *De motu octavae sphaerae*. And while a set of canons for the *Toledan Tables* is attributed to az-Zarqāl, there is no evidence that he actually compiled the tables.[19]

With suitable deference — probably because Bianchini used the Alfonsine motion of the eighth sphere and the obliquity of the ecliptic from the *Toledan Tables* in his tables — Regiomontanus asserts that both Thābit's and the Alfonsine theories are false. His evidence is that the obliquity of the ecliptic that determines the maximum declination of the sun does not agree with the observations of Ptolemy, Battānnī, Peurbach and himself who found 23;28°, and Paolo Toscanelli and Leon Battista Alberti who found not more than 23;30°.[20] The observed and computed values of the obliquity ε are listed in the following table

A BRIEF DESCRIPTION BY NICOLAUS COPERNICUS CONCERNING THE MODELS OF THE MOTIONS OF THE HEAVENS THAT HE INVENTED

1. [INTRODUCTION]

I understand that our predecessors assumed a large number of celestial spheres principally in order to account for the apparent motion of the planets through uniform motion, for it seemed highly unreasonable that a heavenly body should not always move uniformly in a perfectly circular figure. They had discovered that by the arrangement and combination of uniform motions in different ways it could be brought about that any body would appear to move to any position.

Calippus and Eudoxus, attempting to carry this out by means of concentric circles, could not by the use of these[a] give an account of everything in the planetary motion, that is, not only those motions that appear in connection with the revolutions of the planets, but also that the planets appear to us at times to ascend and at times to descend in altitude, which concentric circles in no way permit. And for this reason a preferable theory, in which the majority of experts finally concurred, seemed to be that it is done by means of eccentrics and epicycles.

Nevertheless, the theories concerning these matters that have been put forth far and wide by Ptolemy and most others, although they correspond numerically [with the apparent motions], also seemed quite doubtful, for these theories were inadequate unless they also envisioned certain *equant* circles, on account of which it appeared that the planet never moves with uniform velocity either in its *deferent* sphere or with respect to its proper center. Therefore a theory of this kind seemed neither perfect enough nor sufficiently in accordance with reason.

Therefore, when I noticed these [difficulties], I often pondered whether perhaps a more reasonable model composed of circles could be found from which every apparent irregularity would follow while everything in itself moved uniformly, just as the principle of perfect motion requires. After I had attacked this exceedingly difficult and nearly insoluble problem, it at last occurred to me how[a] it could be done with fewer and far more suitable devices than had formerly been put forth if some postulates, called axioms, are granted to us, which follow in this order:

First Postulate

There is no one center of all the celestial spheres (*orbium*) or spheres (*sphaerarum*).

Second Postulate

The center of the earth is not the center of the universe, but only the center towards which heavy things move and the center of the lunar sphere.

Third Postulate

All spheres surround the sun as though it were in the middle of all of them, and therefore the center of the universe is near the sun.

Fourth Postulate

The ratio of the distance[b] between the sun and earth to the height of the sphere of the fixed stars is so much smaller than the ratio of the semidiameter of the earth to the distance of the sun that the distance between the sun and earth is imperceptible compared to the great height of the sphere of the fixed stars.

Fifth Postulate

Whatever motion appears in the sphere of the fixed stars belongs not to it but to the earth. Thus the entire earth along with the nearby elements rotates with a daily motion on its fixed poles while the sphere of the fixed stars remains immovable and the outermost heaven.

Sixth Postulate

Whatever motions appear to us to belong to the sun are not due to [motion] of the sun but [to the motion] of the earth and our sphere with which we revolve around the sun just as any other planet. And thus the earth is carried by more than one motion.

Seventh Postulate

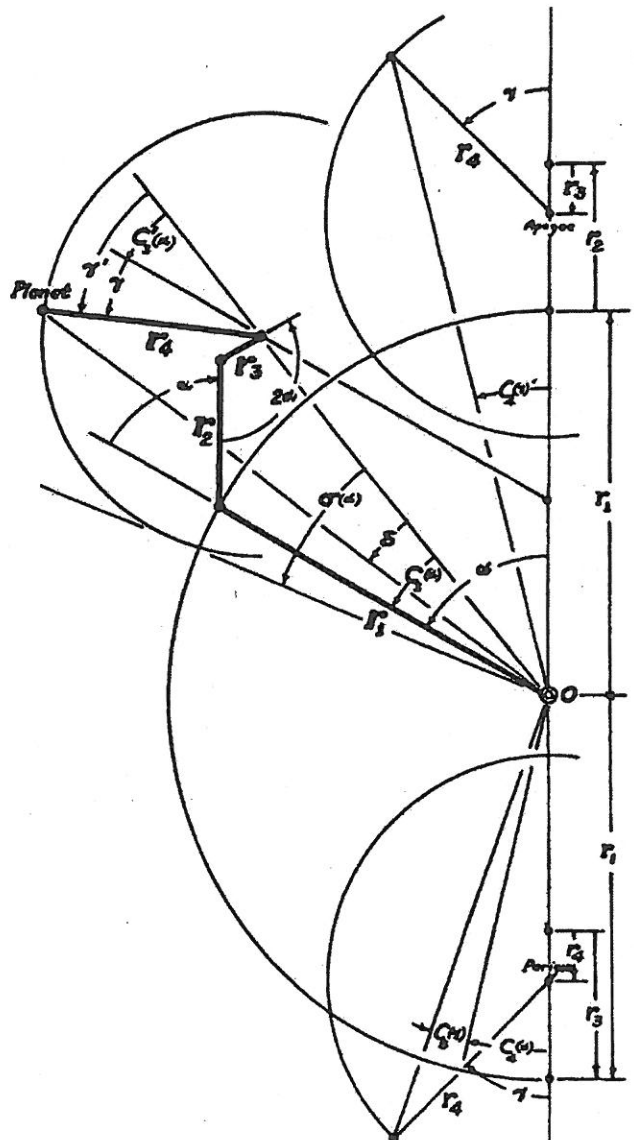
The retrograde and direct motion that appears in the planets belongs not to them but to the [motion] of the earth. Thus, the motion of the earth by itself accounts for a considerable number of apparently irregular motions in the heavens.

Now that these postulates have been set down, I shall attempt briefly to show how carefully the uniformity of the motions may be preserved. I have decided, however, for the sake of brevity to leave the mathematical demonstrations out of this treatise as they are intended for a larger book. Nevertheless, the lengths of the semidiameters of the spheres will be set down here in the explanation of their circles, from which anyone not ignorant of mathematics will easily understand how very precisely such an arrangement of circles agrees with computations and observations.

In the same way, in case anyone believes that we have asserted the movement of the earth for no good reason along with the Pythagoreans, he will also receive considerable evidence [for this] in the explanation of the circles. And in fact, [the evidence] by which natural philosophers attempt so very hard to confirm the immobility of the earth depends for the most part upon appearances. All [their evidence] falls apart here in the first place since we overthrow the immobility of the earth also by means of an appearance.

Ibn al-Shāṭir's Planetary Theory (ca. 1350)

no eccenter, no equant



2η or γ'	$C_3(2\eta)$			$C_4(\gamma')$			$C_5(\gamma')$			$C_6(2\eta)$		
MOON	Ibn al-Shāṭir / Copernicus			$r_2 = \{$ 6;35 / 6;34,55			$r_3 = \{$ 1;25 / 1;25,19					
	Computed	Shāṭir	Copern.	Comp.	Sh.	Cop.	Comp.	Sh.	Cop.	Computed	Ibn al-Shāṭir	Copernicus
30	7;32	7;32	7;34	2;18	2;18	2;17	1;8	1;8	1;12	0;5,7	0;5,0	0;5
60	11;48	11;47	11;50	4;5	4;5	4;5	2;6	2;5	2;6	0;17,55	0;17,39	0;18
90	12;9	12;8	12;12	4;55	4;55	4;55	2;41	2;40	2;40	0;33,17	0;33,8	0;34
120	9;33	9;31	9;35	4;27	4;27	4;27	2;36	2;36	2;35	0;47,11	0;47,8	0;47
150	5;11	5;11	5;11	2;40	2;40	2;40	1;39	1;39	1;37	0;56,42	0;56,39	0;57
α or γ'	**SATURN**	Sh. / Cop.		$r_2 = \{$ 5;7,30 / 5;7,26			$r_3 = \{$ 1;42,30 / 1;42,36			$r_4 = \{$ 6;30 / 6;32		
	$C'_3(\alpha)$			$C_4(\gamma')$			$C_5(\gamma')$			$C_6(\alpha)$		
30	3;6	3;6	3;6	2;42	2;42	2;42	0;17	0;20	0;19	0;3	0;3	0;3
60	5;29	5;29	5;29	4;50	4;49	4;49	0;32	0;35	0;35	0;11	0;12	0;11
90	6;30	6;30	6;31	5;51	5;52	5;52	0;42	0;42	0;42	0;25	0;26	0;23
120	5;48	5;48	5;49	5;21	5;21	5;22	0;42	0;42	0;42	0;38	0;41	0;39
150	3;26	3;26	3;24	3;13	3;12	3;13	0;26	0;26	0;26	0;54	0;53	0;53
	JUPITER	Sh. / Cop.		$r_2 = \{$ 4;7,30 / 4;7,19			$r_3 = \{$ 1;22,30 / 1;22,26			$r_4 = \{$ 11;30 / 11;30		
30		2;31	2;31		4;32	4;32		0;21	0;21		0;3	0;2,50
60		4;24	4;26		8;18	8;17		0;42	0;42		0;12	0;13,10
90		5;14	5;15		10;24	10;24		0;58	0;58		0;26	0;26,37
120		4;40	4;41		9;54	9;54		1;3	1;2		0;43	0;41,50
150		2;44	2;45		6;13	6;13		0;43	0;43		0;55	0;55,15
	MARS	Sh. / Cop		$r_2 = \{$ 9;0 / 8;46			$r_3 = \{$ 3;0 / 3;0			$r_4 = \{$ 39;30 / 39;29		
30	5;15	5;16	5;10	11;9	11;9	11;11	1;26	1;28	1;25	0;2,1	0;2,0	0;2,1
60	9;22	9;23	9;12	21;45	21;46	21;49	3;8	3;7	3;0	0;8,27	0;8,40	0;8,30
90	11;20	11;19	11;5	30;54	30;54	31;0	5;17	5;17	5;5	0;20,4	0;19,26	0;20,8
120	10;20	10;20	10;7	36;23	36;29	36;37	8;29	8;28	8;11	0;35,22	0;35,50	0;36,16
150	6;15	6;15	6;7	31;51	31;51	32;3	13;5	13;4	12;35	0;52,39	0;52,36	0;52,22
	VENUS	Sh. / Cop.		$r_2 = \{$ 1;41 / 1;52			$r_3 = \{$ 0;26 / 0;37			$r_4 = \{$ 43;33 / 43;9		
30		1;0	0;59		12;21	12;24		0;19	0;13		0;2	0;3,38
60		1;43	1;43		24;19	24;24		0;38	0;27		0;14	0;13,32
90		2;1	2;0		35;9	35;21		1;9	0;47		0;28	0;28,28
120		1;45	1;45		43;19	43;35		1;52	1;18		0;44	0;43,10
150		1;2	1;1		42;6	42;34		3;11	2;18		0;55	0;55,0
	MERCURY	Sh. / Cop.		$r_2 = \{$ 4;5 / 4;25			$r_3 = \{$ 0;55 / 1;16			$r_4 = \{$ 22;46 / 22;35; $r_5 = r_6 = \{$ 0;33 / 0;34,12		
30	1;25	1;25	1;24	7;22	7;22	7;15	0;59	0;59	1;16	0;12	0;12	0;8,29
60	2;31	2;31	2;29	13;54	13;55	13;41	2;1	2;0	2;34	0;39	0;39	0;31,39
90	3;1	3;2	3;0	18;26	18;26	18;6	3;4	3;3	3;56	1;3	1;3	0;52,2
120	2;44	2;44	2;44	19;6	19;6	18;42	3;55	3;55	5;2	1;11	1;11	1;0,0
150	1;38	1;38	1;38	13;11	13;10	12;52	3;28	3;28	4;26	1;5	1;5	0;54,25
180	0;0	0;0	0;0	0;0	0;0	0;0	0;0	0;0	0;0	1;0	1;0	0;52,2

Almagest, Book XII (H450-1)
On Calculating Stationary Points

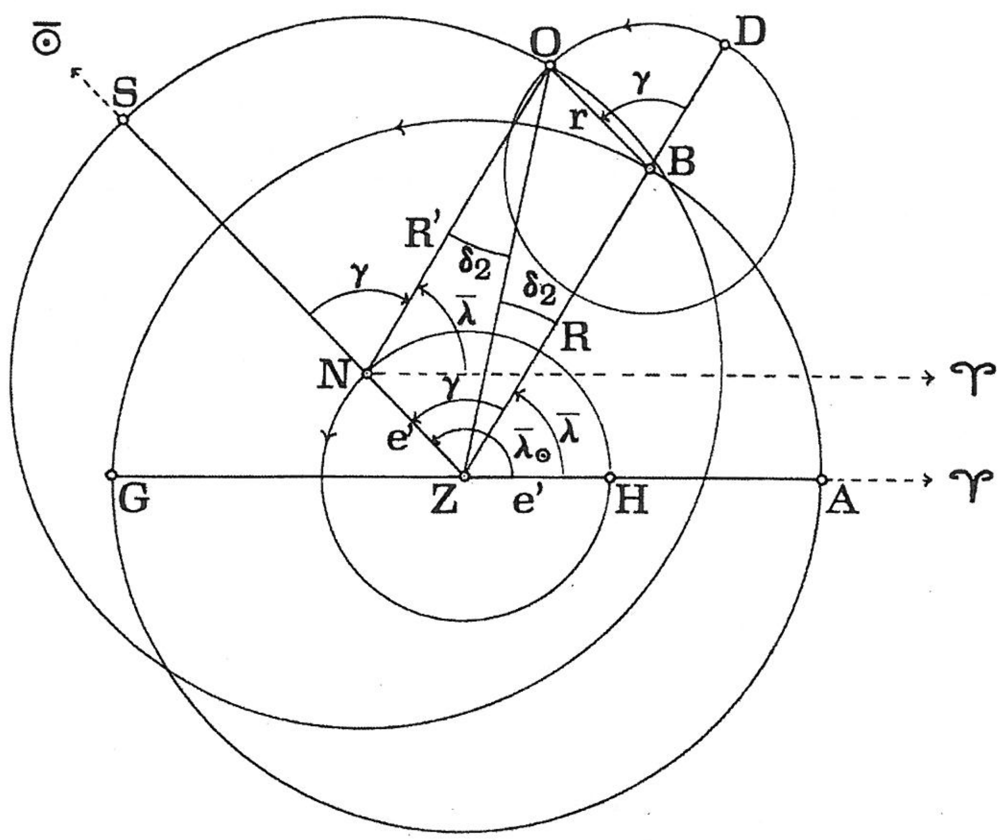
In the definition of this kind of problem, there is a preliminary lemma demonstrated (for a single anomaly, that related to the sun) by … Apollonius ….

If [the synodic anomaly] is represented by the epicyclic hypothesis, …

If the anomaly related to the sun is represented by the eccentric hypothesis (which is a viable hypothesis only for the three planets which can reach any elongation from the sun), in which the centre of the eccentre moves about the centre of the ecliptic backwards along the signs with the speed of the [mean] sun, while the planet moves on the eccentre forwards along the signs with a speed with respect to the centre of the eccentre equal to the [mean] motion in anomaly, …

In other words, retrograde motion (of the outer planets) does not have to be represented by an epicycle, but can be represented via an eccenter.

Ptolemy's Two Apollonian Models for Retrograde Motion (Outer Planets)



Radius Ratios

	Almagest	*De Rev*	modern
Mercury	**0.375**	**0.360**	**0.3871**
Venus	**0.719**	**0.719**	**0.7233**
Mars	**1.519**	**1.520**	**1.5236**
Jupiter	**5.217**	**5.246**	**5.2027**
Saturn	**9.231**	**9.164**	**9.5719**

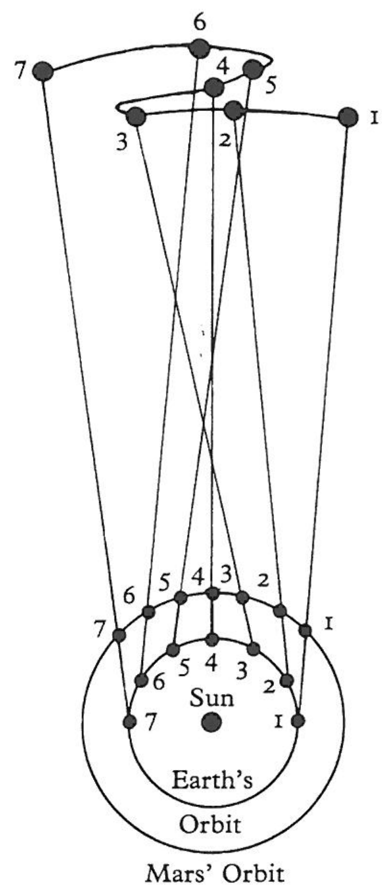
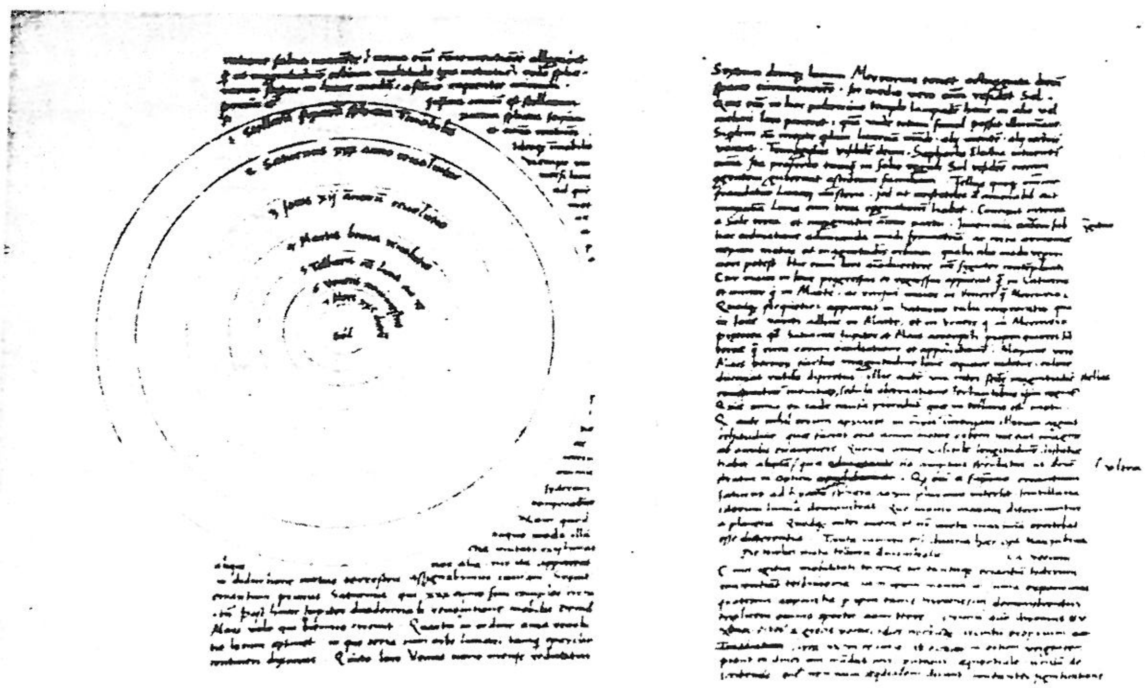


Figure 5: Retrograding of planets.

CONTENTS



Pages from the manuscript of De Revolutionibus *with Copernicus' drawing of the heliocentric system.*

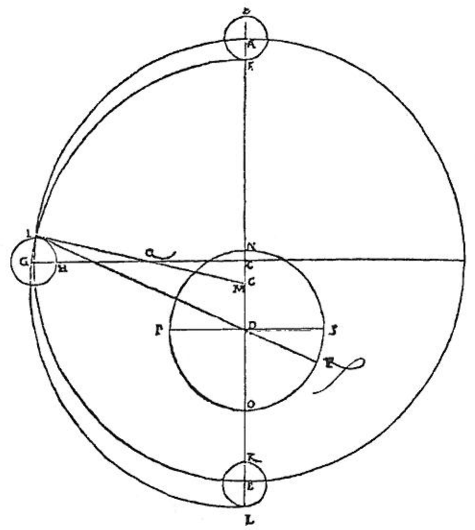


FIGURE 7.63. Copernicus's theory of the superior planets. *NPO* is the orbit of the Earth. *AGB* is the deferent circle of a superior planet, such as Mars. Mars itself moves on a small epicycle which is responsible for producing an anomaly of motion more or less equivalent to that produced by Ptolemy's equant. From *De revolutionibus* V, 4 (Nuremberg, 1543).

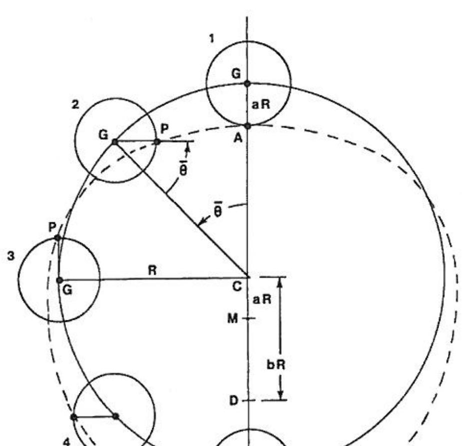


FIGURE 7.64. Copernicus's minor epicycle, a replacement for Ptolemy's equant.

able. While this stress on a coherent system served Copernicus very well in the shift to Sun-centered cosmology, it led him astray in technical matters. For it turns out that the planets really do move nonuniformly and that Ptolemy's equant theory was closer to the mark than Copernicus's "improvement" on it.

Copernican Planetary Theory

A good sense of Copernicus's astronomy can be obtained by examining his theory for the superior planets. Copernicus himself placed a high value on this work, which he believed improved on Ptolemy. Here we must confront not only Copernicus's use of a moving Earth, but also his method of accounting for the planets' nonuniformity of motion.

For the orbit of the Earth, Copernicus chose an eccentric circle: the Earth moves at uniform speed on a circle that is eccentric to the Sun. The model is essentially the same as the solar theory of Ptolemy. For computation of positions it makes no difference whether the Earth or the Sun moves. The essence of the model is uniform circular motion on an off-center circle.

For the superior planets, Copernicus adopted an eccentric circle plus a modified form of the Ptolemaic equant. As we have seen, Copernicus could not abide the equant. But he had, of course, to replace it with something else. He found that a minor epicycle could perform very nearly the same function.

Figure 7.63 is a diagram from the first edition of *De revolutionibus*, illustrating Copernicus's theory of the superior planets. The Earth travels around the annual circle *NPO*, which is centered at *D*. The Sun is therefore located near but slightly displaced from *D*. However, the true Sun does not appear in this figure and plays no part in the theory. For this reason, Copernicus's system has been aptly characterized as merely heliostatic, rather than truly heliocentric. The effective center of the whole system is the center *D* of the Earth's orbit, also called the mean Sun.

In figure 7.63, *C* is the center of the deferent circle *AGB* of a superior planet (let us say Mars). Thus, the center of Mars's deferent circle is eccentric to the mean Sun *D*. So far, this resembles Ptolemy's theory. However, Copernicus does not have an equant point. Rather, he places Mars on a small epicycle, shown in the figure. Further, Mars makes a complete counterclockwise orbit on the epicycle while the epicycle's center travels a complete circle around the deferent. Thus, when the epicycle's center is at *A*, Mars is at *F*. When the epicycle's center is at *G*, Mars is at *I*. When the epicycle's center is at *B*, Mars is at *L*. Finally, the radius *GI* of the epicycle is chosen to be one-third of the eccentricity *DC*.

One thing to note is that Copernicus did not eliminate epicycles from planetary theory. However, the large epicycle of Ptolemy *is* gone. Ptolemy's big epicycle was responsible for retrograde motion. In Copernicus's theory of the superior planets (fig. 7.63), this function is taken over by the circle *NPO* of the Earth's annual motion. The minor epicycle *GI* is Copernicus's substitute for Ptolemy's equant point. Let us study this device in more detail.

Refer to figure 7.64, which elaborates on Copernicus's own diagram. The large solid circle of radius *R* is the deferent of Mars, centered at *C*. The deferent circle is eccentric to *D*, the mean Sun, or center of the Earth's orbit. (For simplicity, the Earth's orbit is not shown in this figure.) The dimensionless eccentricity of Mars's deferent circle is $b = CD/R$.

The center *G* of a small epicycle moves counterclockwise and uniformly around the deferent. The planet *P* moves counterclockwise and uniformly on the epicycle whose radius is aR. (Thus, a is a dimensionless number less than 1.) Further, the two angles marked $\bar{\theta}$ remain equal to one another while increasing uniformly with time. Consequently, while the epicycle's center moves through 180° from position 1 to position 5, the planet revolves through 180° on the epicycle.

The combination of two uniform circular motions for P in figure 7.64 results in a motion that is neither uniform nor circular. The actual path of the planet is indicated by the dashed line. The effective center of the orbit is not C but M, located below C by a distance aR equal to one radius of the epicycle. As Copernicus himself states, the path is not circular but somewhat oblong—the long axis being perpendicular to the line of apsides ΠCA.[143]

Nevertheless, Copernicus's speed rule is virtually indistinguishable from Ptolemy's: the minor epicycle produces a motion that closely approximates equant motion. Refer to figure 7.65. The radius of the epicycle is aR. Let us identify point E on the line of apsides at a distance aR above the center C of the deferent. As already remarked, in Copernicus's model, the rotation of GP is such that angle CGP is always equal to the mean anomaly ACG: both are equal to $\bar{\theta}$. Since also $CE = GP$, it follows that the quadrilateral $ECGP$ is a trapezoid, with sides EP and CG always parallel. Since line CG turns uniformly, it follows that EP turns uniformly, too. In other words, *E is an effective equant point.* The planet P, observed from E, appears to move at uniform angular speed.

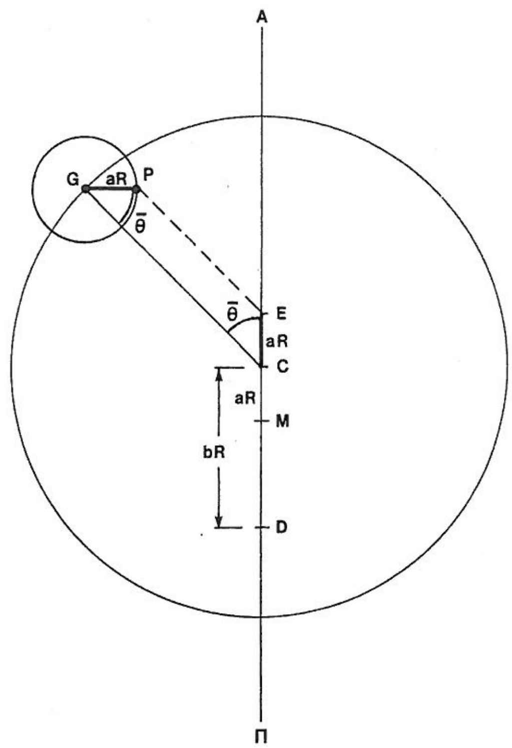


FIGURE 7.65. Copernicus's hidden equant point (E).

Furthermore, Copernicus usually makes the radius of the minor epicycle exactly one-third the eccentricity of the deferent. That is, $b = 3a$. Now, from figure 7.65, $EM = 2aR$, and $MD = bR - aR$, so we get also $MD = 2aR$. Thus, the center M of the effective orbit is exactly midway between D and the effective equant point E. Copernicus, like Ptolemy, bisects the total eccentricity: $EM = MD$ in figure 7.65, just as $EC = CO$ in figure 7.32. An almost perfect equivalence will be established between Ptolemy's eccentric circle with equant point and Copernicus's eccentric circle with minor epicycle if we identify the radius of Copernicus's epicycle with half the Ptolemaic eccentricity e_P; that is, if $a = 1/2\ e_P$. Thus, $b = 3/2\ e_P$.

The combined effect of Copernicus's oblong orbit and hidden equant is illustrated in figure 7.66. M is the center of the solid circle and E represents a Ptolemaic equant point. Thus, if body P moves on the circle according to the law of the equant, $\bar{\theta}$ increases uniformly with time. The dashed curve represents the effective, oblong Copernican orbit. E, then, is also the effective equant point of the Copernican orbit. Thus, when the body is at P according to Ptolemaic hypotheses, it will be at P' according to Copernican principles. For an observer at the equant, P and P' could not be distinguished. But, because of the noncircularity of the Copernican orbit, an observer at D (the center of the Earth's orbit) would see P and P' in directions that differ by a small angle $\Delta\theta$. The eccentricity is greatly exaggerated in figure 7.66. Even in the case of Mars, for which Ptolemy's eccentricity $e_P = 0.1$, the maximum difference $\Delta\theta$ between the directions of P in the two models is only about 3′. Before the work of Brahe and Kepler, the observational consequences of Copernicus's modification of the Ptolemaic equant were nil.

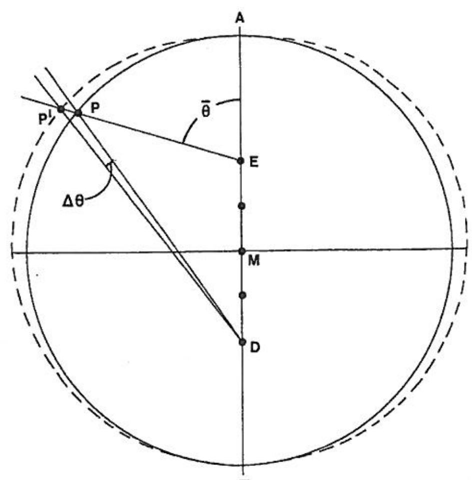


FIGURE 7.66. Comparison of the Copernican model with a Ptolemaic eccentric-with-equant model. The Ptolemaic eccentric circle is drawn in solid line. The oblong Copernican orbit is drawn in dashed line. The Ptolemaic equant point and the hidden, effective equant point of the Copernican model coincide at E. At the same moment (and therefore at the same mean anomaly $\bar{\theta}$) the position of the planet in equant theory is P and the position in Copernican theory is P'. As viewed from the Sun D, there is a small difference $\Delta\theta$ in the directions predicted by the two theories.

Moreover, Copernicus's values for the eccentricities of the superior planets were borrowed from Ptolemy, as may be seen in the following table:

Eccentricities of the superior planets

	Ptolemy			*Copernicus*	
	e_P	$1/2\ e_P$	$3/2\ e_P$	a	b
Mars	0.10000	0.05000	0.15000	0.05000	0.14600
Jupiter	0.04583	0.02292	0.06875	0.02290	0.06870
Saturn	0.05694	0.02847	0.08541	0.02850	0.08540

Column e_P gives Ptolemy's value of the eccentricity for each planet. The columns headed $1/2e_P$ and $3/2e_P$ give the appropriate fractions of Ptolemy's eccentricity. As shown above, Copernicus's theory for the superior planets

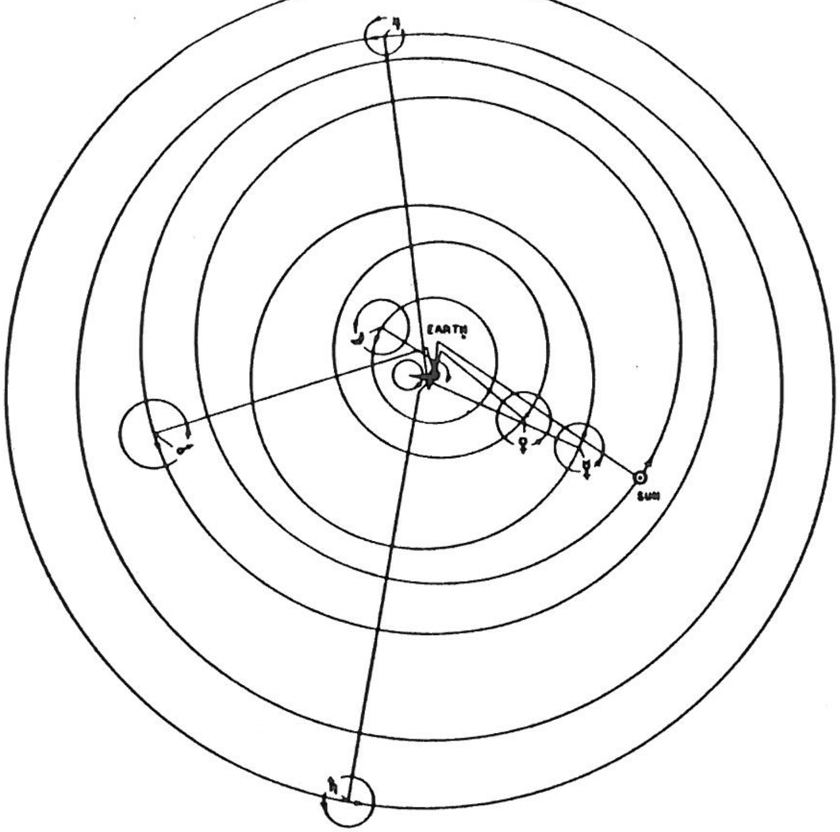


FIG. 1.—THE PTOLEMAIC SYSTEM

These drawings have been designed to point out how similar in complexity were the Ptolemaic and Copernican systems. Even a cursory glance convinces one that neither system is essentially simpler geometrically than its competitor. Drawings cannot be made accurate in radial dimensions, but special care has been taken properly to orient the centers of the planetary orbits relative to the zodiac. Thus, if one traces in the Ptolemaic diagram the radial line from the Sun to the point under "A" in "EARTH," the point which is the center of the Sun's orbit, it is seen to be between the centers of rotation of Venus and Mars, precisely as Ptolemy's geocentric theory requires. The relative senses of rotation of the epicycles on their deferent circles and the planets on the epicycles are indicated by the arrows. The planetary distances remain arbitrary, which is not so in Copernicus.

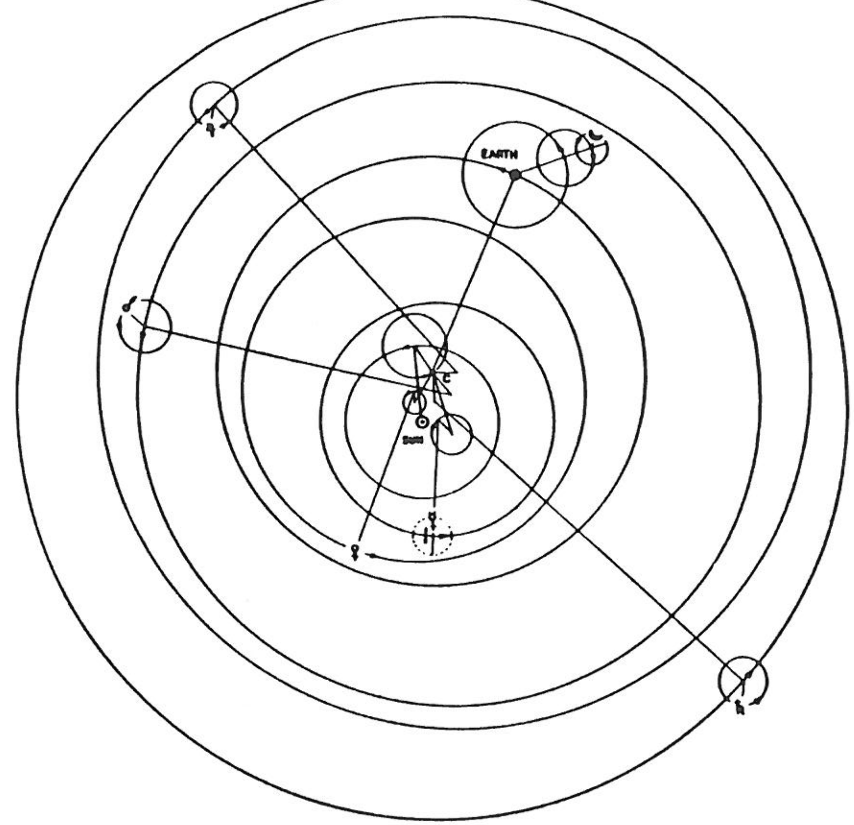


FIG. 2.—THE NEW SYSTEM AS CONCEIVED BY COPERNICUS

In the Copernican system the Sun appears in the center of the stage, but the actual momentary centers of rotation of the planets cluster around the momentary center *C* of the Earth's orbit. In this system Mercury was handled in a unique fashion, librating on the center of an epicycle instead of traveling on the epicycle. The planetary symbols are as follows:

☉	Sun	⊕	Earth
☿	Mercury	♂	Mars
♀	Venus	♃	Jupiter
☾	Moon	♄	Saturn

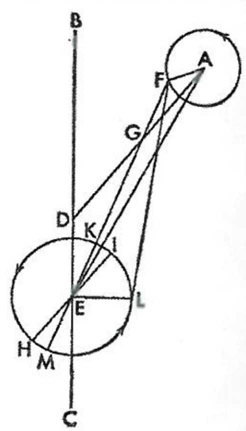
Drawings are by the courtesy of Dr. W. D. Stahlman (see also p. 68).

which occur in the case of Saturn are, as we said, parallaxes arising from the annual orbital circle of the Earth, since, as the magnitude of the Earth in relation to the distance of the moon causes parallaxes, so too its orbital circle, in which it revolves annually, should in the case of the five wandering stars cause [parallaxes] which are far more evident in proportion to the magnitude of the orbital circle. Now such parallaxes cannot be determined, unless the altitude of the planet—which, however, it is possible to apprehend through any one observation of a parallax—becomes known first.

We have such [an observation] in the case of Saturn in the year of Our Lord 1514 on the sixth day before the Kalends of May 5 equatorial hours after the preceding midnight. For Saturn was seen to be in a straight line with the stars in the forehead of Scorpio, namely with the second and third stars, which have the same longitude and are at 209° of the sphere of the fixed stars. Accordingly the position of Saturn is made evident through them. Now there are 1514 Egyptian years 61 days 13 minutes [of a day] from the beginning of the years of Our Lord to this time; and according to [149ᵃ] calculation the mean position of the sun was at 315°41′, the anomaly of parallax of Saturn was at 116°31′, and for that reason the mean position of Saturn was 199°10′ and that of the highest apsis of the eccentric circle was at approximately 240⅓°.

Now in accordance with our problem, let *ABC* be the eccentric circle: let *D* be its centre, and on the diameter *BDC* let *B* be the apogee, *C* the perigee, and *E* the centre of the orbital circle of the Earth. Let *AD* and *AE* be joined, and with *A* as centre and ⅓ *DE* as radius let the epicycle be drawn. On the epicycle let *F* be the position of the planet; and let

$$\text{angle } DAF = \text{angle } ADB.$$



And through *E* the centre of the orbital circle of the Earth let *HI* be drawn, as if in the same plane with circle *ABC*, and as a diameter, parallel to *AD*, so as to have it understood that with respect to the planet the apogee of the orbital circle is at *H* and the perigee at *I*. Now on the orbital circle let

$$\text{arc } HL = 116°31'$$

in accordance with the computation of the anomaly of parallax; let *FL* and *EL* be joined, and let *FKEM* produced cut both arcs of the orbital circle.

Accordingly since by hypothesis

$$\text{angle } ADB = \text{angle } DAF = 41°10',$$

and

$$\text{angle } ADE = 180° - ADB = 138°50';$$

and

$$DE = 854$$

$$\text{where } AD = 10{,}000:$$

whence in triangle *ADE*

$$\text{side } AE = 10{,}667,$$

$$\text{angle } DEA = 38°9',$$

and

$$\text{angle } EAD = 3°1':$$

therefore by addition

$$\text{angle } EAF = 44°12'.$$

So again in triangle *FAE*

$$\text{side } FA = 285$$

$$\text{where } AE = 10{,}667,$$

$$\text{side } FKE = 10{,}465,$$

and

$$\text{angle } AEF = 1°5':$$

accordingly it is manifest that

$$\text{angle } AEF + \text{angle } DAE = 4°6',$$

which is the total difference or additosubtraction between the mean and the true position of the planet. Wherefore if the position of the Earth had been at *K* or M, the position of Saturn would have been apparent as if from centre *E* and would have been seen to be at 203°16′ from the constellation of Aries. But with the Earth at *L*, Saturn is seen to be at 209°. The difference [149ᵇ] of 5°44′ goes to the parallax in accord with angle *KFL*. But by calculation of the regular movement

$$\text{arc } HL = 116°31',$$

and

$$\text{arc } ML = \text{arc } HL - \text{add. } HM = 112°25'.$$

And by subtraction[1]

$$\text{arc } LIK = 67°35':$$

hence

$$\text{angle } KEL = 67°35'.$$

Wherefore in triangle *FEL* the angles are given, and the ratio of the sides is given too: Hence

$$EL = 1{,}090$$

$$\text{where } EF = 10{,}465,$$

$$\text{and } AD = BD = 10{,}000;$$

but

$$EL = 6^p32',$$

$$\text{where } BD = 60^p,$$

by usage of the ancients;

and there is very little difference between that and what Ptolemy gave.

Accordingly

$$BDE = 10{,}854,$$

and, as the remainder of the diameter

$$CE = 9{,}146.$$

But since the epicycle when at *B* always subtracts 285 from the altitude of the planet, but adds the same amount, *i.e.*, its radius, when at *C*; on that account the greatest distance of Saturn from centre *E* will be 10,569, and the least 9,431, where *BD*=10,000. By this ratio the altitude of the apogee of Saturn is $9^p42'$, where the radius of the orbital circle of the Earth $= 1^p$; and the altitude of the perigee is $8^p39'$: hence it is quite evident by the mode set forth above in the case of the small parallaxes of the moon that the parallaxes of Saturn can be greater. And when Saturn is at the apogee,

$$\text{greatest parallax} = 5°45';$$

and when at the perigee,

$$\text{greatest parallax} = 6°39';$$

and they differ from one another by 44′—measuring the angles by the lines coming from the planet and tangent to the orbital circle of the Earth. In this way the particular differences in the movement of Saturn have been found, and we shall afterwards set them out simultaneously and in conjunction with those of the five planets.

[1]Arc *MLIK*=180°.

Grounds for Copernican over Ptolemaic: A Principle from Philosophy of Science

A theoretical proposal put forward as an answer to some one why-question gains support when it provides, as corollaries, answers to why-questions regarding other phenomena.

Harman: "inference to the best (total) explanation"

<u>A possible rationale for this interpretation of the principle</u>: Any complex phenomenon exhibits many *prima facie* distinct features. A (total) explanation of it is less satisfying (1) the more of those features require explanations that are independent of the explanations of other features, and in this respect can be said to *ad hoc*; and (2) the more of those features that are attributed to mere coincidence, and in this respect can be said not to be providing information about why the phenomenon occurs at all. Conversely, therefore, the more *prima facie* distinct features of the phenomenon that become explained for free, so to speak, by an explanation of any one of them, the more satisfying an explanation is of the overall phenomenon. This rationale, as stated, is a comment about what we want in the way of explanations of phenomena, not a comment about the world. One way to link it to the world is through some version of the thesis that "nature is simple," in particular a version claiming that phenomena in nature do not arise from manifold, independently acting causes. Notice, however, the extent to which this thesis amounts to *wishful thinking* on the part of those engaged in research, for the more independent causes contributing to phenomena, the more difficult it is to develop decisive empirical evidence establishing theoretical claims about them. A good reason, accordingly, for researchers to respond favorably to a proposed explanation meeting the stated principle is the *prima facie* promise any such proposal offers that the relevant phenomena are going to be amenable to sustained empirical investigation.

Popper: more opportunities to falsify the proposal

For as long as histories of astronomy have been written, heliocentrism has been regarded as the hallmark of modern astronomy. In accordance with this tradition, Nicholas Copernicus (1473-1543), as the effective originator of heliocentric doctrine, has been hailed as the founder of modern astronomy. In fact, however, except for the motion of the Earth, the revolutionary element in Copernicus's work is very small; in most respects his *De Revolutionibus* (1543) follows Ptolemy's *Almagest* so closely that he can equally well be regarded ... as the last great practitioner of ancient astronomy. On this view, it was the seventy-year period following Copernicus's death in 1543 that actually saw the transition to modern astronomy. And insofar as any such development can be attributed to the influence of one person, that transition was wrought by the ideas and efforts of the Danish astronomer, Tycho Brahe.

(Thoren, *Cambridge History*, p. 3)

NOVA MVNDANI SYSTEMATIS HYPOTYPOSIS AB AUTHORE NUPER ADINUENTA, QUA TUM VETUS ILLA PTOLEMAICA REDUNDANTIA & INCONCINNITAS, TUM ETIAM RECENS COPERNIANA IN MOTU TERRÆ PHYSICA ABSURDITAS, EXCLUDUNTUR, OMNIAQUE APPARENTIIS CŒLESTIBUS APTISSIME CORRESPONDENT.

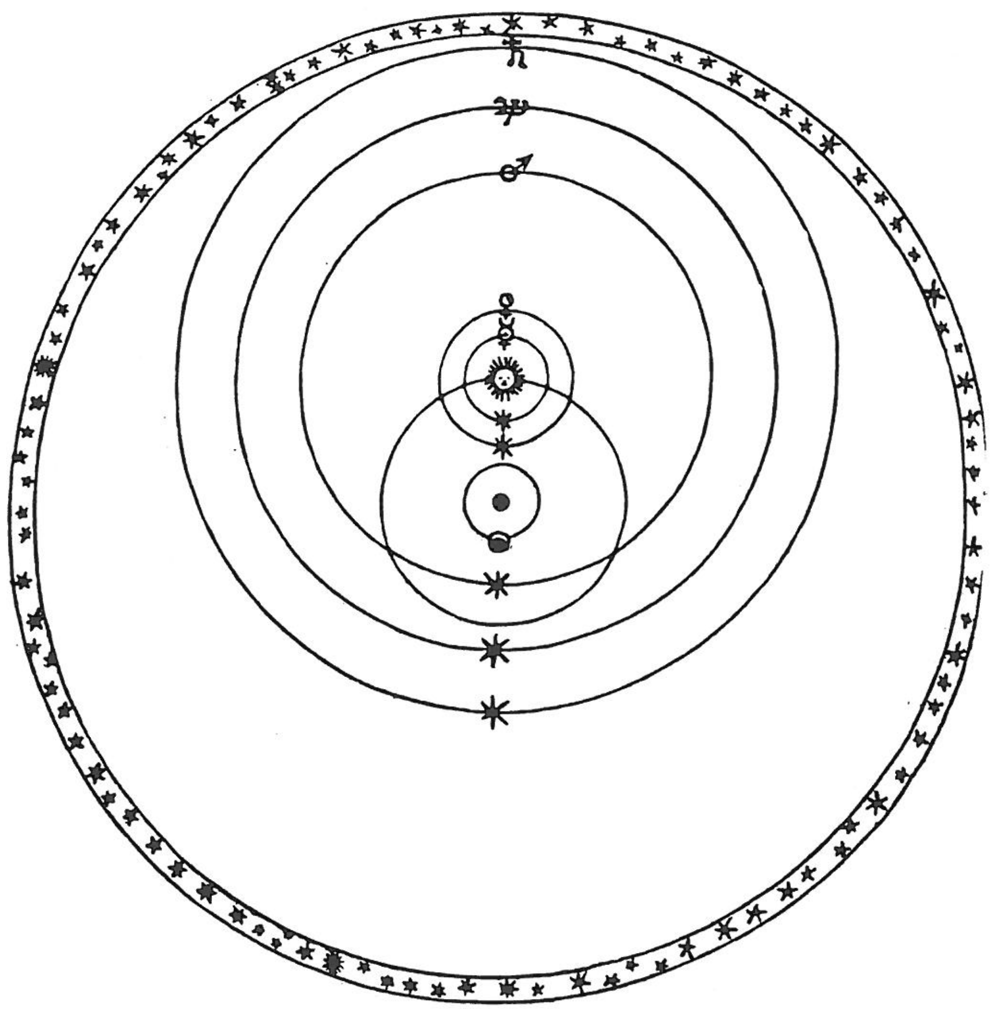
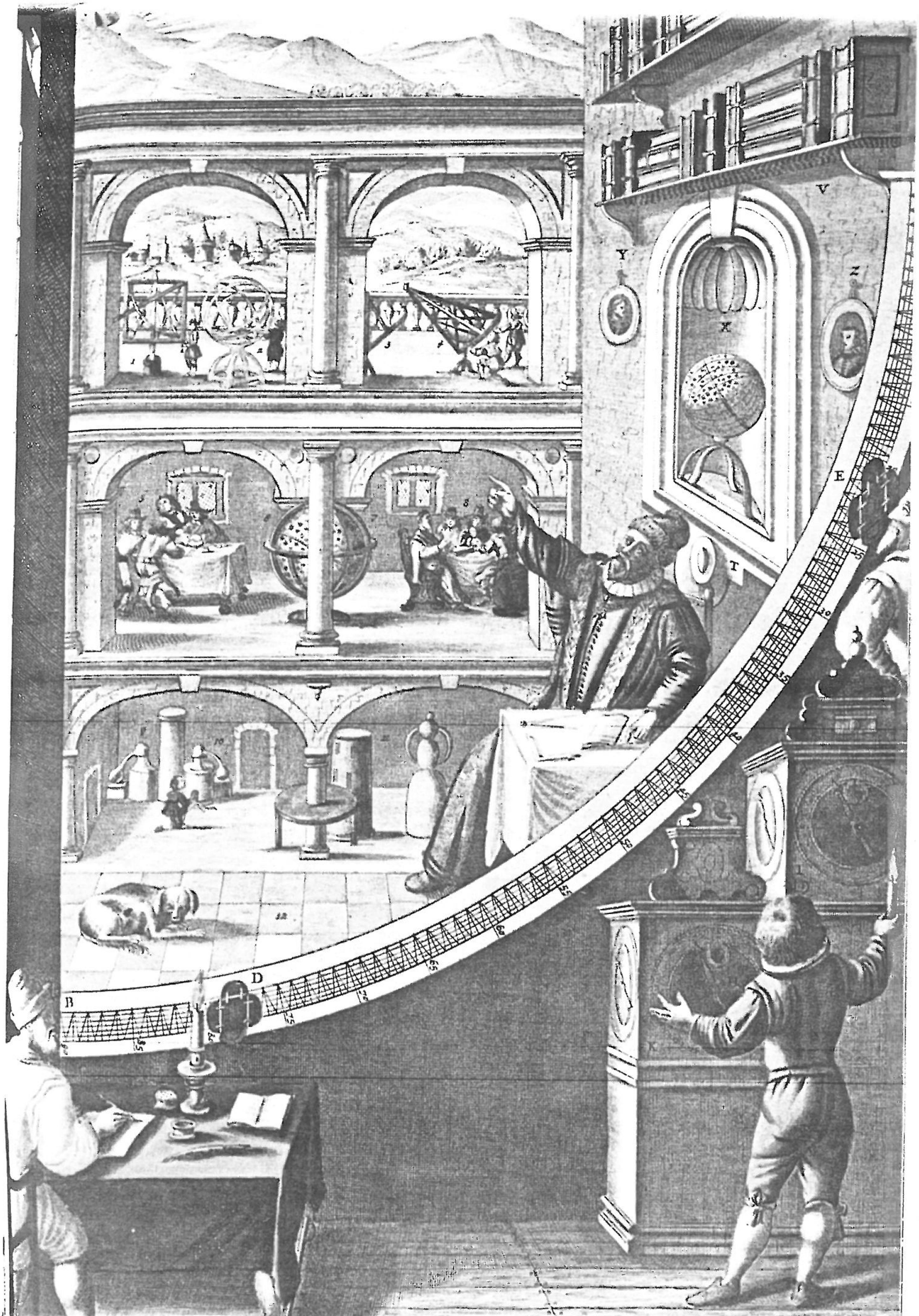


Figure 8.9. The Tychonic System of the world.



V
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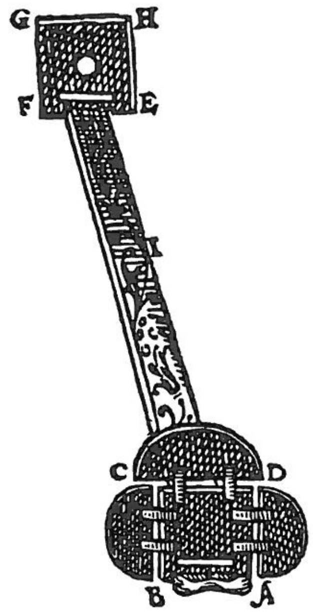


Figure 5.5. Detail of Tycho's sighting mechanism.

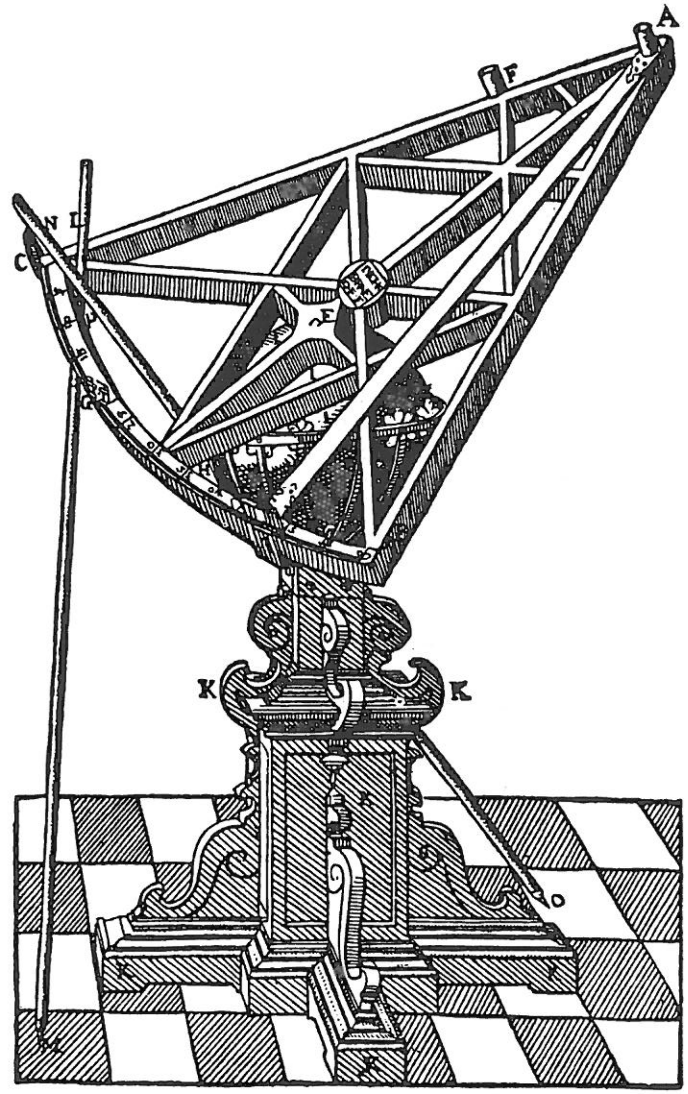
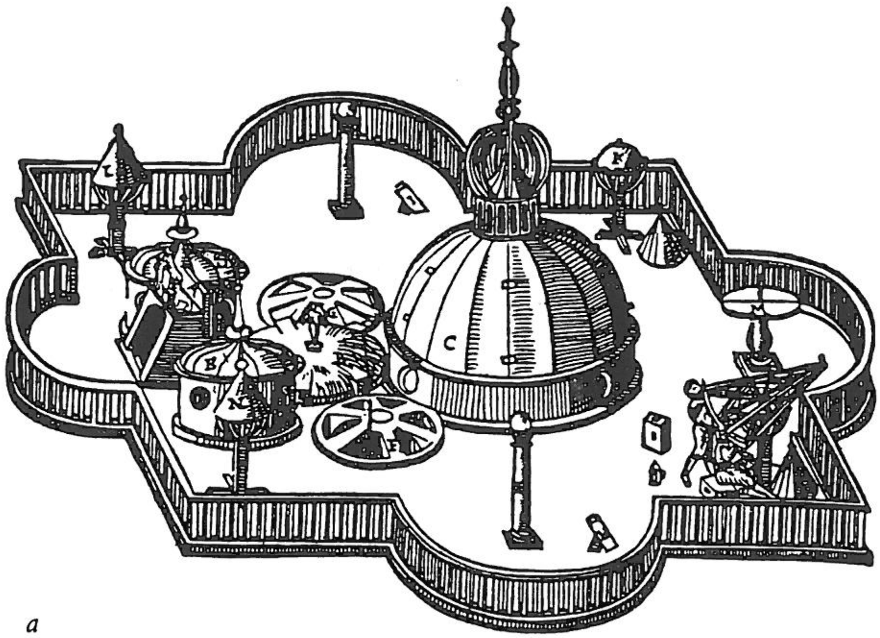


Figure 5.12. The mature Sextant (ca. 1582).

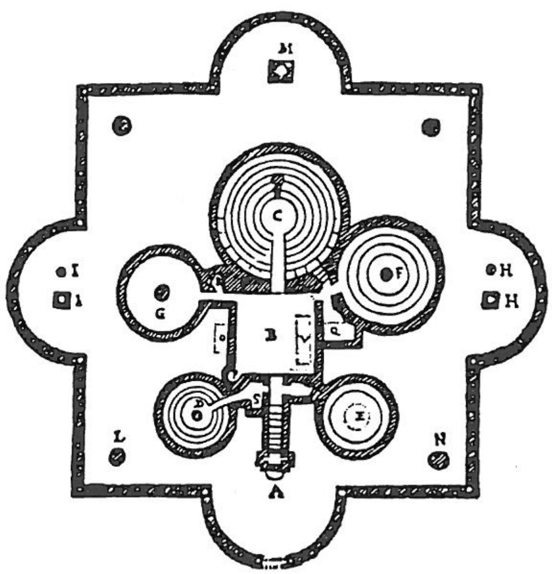
ORTHOGRAPHIA STELLÆBVRGI

EXTRA ARCEM VRANIÆ SITI.



a

ICHNOGRAPHIA STELLÆBVRGI.



b

Figure 5.18. Elevation (a) and plan (b) of Tycho's underground observatory, Stjerneborg; constructed 1584–6.

Tycho Brahe's Mars Observations

Source: Tychonis Brahe Dani Opera Omnia
Imput by: Wayne Pafko (March 24, 2000)
[MS] = Mars Symbol (you know…the "male" sign)

Year	Day	Time	Quote	Volume
1582	DIE 12 NOUEMBRIS, MANE.		Declinatio [MS] 23 7 B	10
1582	DIE 30 DECEMBRIS		Afc. R. [MS] 107o 56' Declin. 26o 36'	10
1582	DIE 27 DECEMBRIS		declinatio [MS] 26o 22 1/3' et Afcenfio]	10
1583	DIE 18 JANUARIJ, VESPERI.		Declinatio 27 18 minus bona	10
1584	DIE 13 NOUEMBRIS, A.M.	H.13 26 P.M.	Declinatio [MS] B. 15 54	10
1584	DIE 27 NOUEMBRIS	H.2 15'	Declinatio [MS] 14 42	10
1584	DIE 20 DECEMBRIS AD VESPERAS.		Decl. [MS] (erat prope horizont.) 14 24	10
1584	DIE 21 DECEMBRIS AD VESPERAS.		Declinatio [MS] 14 21 1/2	10
1584	DIE 21 DECEMBRIS AD VESPERAS.		Declinatio [MS] 14 21 1/4	10
1585	DIE 7 JANUARIJ.		Declin. [MS] I 15 35 II 15 35	10
1585	DIE 9 JANUARIJ.	A.M.	Decl. [MS] 15 50 per Arm. Bor.	10
1585	Die 14 Januarij	H. 16 M. 40 P.M.	Decl. eius B. 16 27	10
1585	Die 22 Jan.	H.14 55 P.M.	Decl. [MS] B. 17 31 0	10
1585	Die 31 Jan. circa mediam noctem.		Decl. [MS] Sept. 18 43 0	10
1585	DIE 3 FEBRUARIJ.	H.9 M.43	Decl. [MS] fept. 19 1 1/6 per Armillas /	10
1585	DIE 3 FEBRUARIJ.	H.9 M.39	Declinatio [MS] per Armillas Boreales 1	10
1585	Die 3 Feb.	H. 6 1/4 P.M.	Declinatio [MS] 19 2 0	10
1585	DIE 4 FEBRUARIJ.	H.9 M.14	Decl. [MS] fept. 19 9 3/4 per Armillas I	10
1585	DIE 4 FEBRUARIJ.	H.8 M.16	Decl. [MS] 19 8 per Armilas Auftrales.	10
1585	DIE 4 FEBRUARIJ.	H.6.40 P.M.	Decl. [MS] B. 19 9 45.	10
1585	DIE 17 FEBRUARIJ.	H.9.45	Decl. [MS] 20 21 45	10
1585	DIE 17 FEBRUARIJ.	H. 9 1/2	Decl. [MS] 20 21 1/2	10
1585	DIE 17 FEBRUARIJ.	H. 9 5/6	Decl. [MS] 20 21 1/2 B.	10
1585	Die 12 Martij	H. 9 1/3 P.M.	Declinatio [MS] B. 20 32 3/4	10
1585	Die 16 Martij	H. 7 5/6	Declin. [MS] B. 20 23 0	10
1585	Die 19 Marij	H. 8 1/4	Declin. [MS] 20 5 30	10
1585	DIE 26 MARTIJ.	H. 8 1/3 P.M.	Declinatio [MS] B. 19 44 0	10
1585	DIE 15 APRILIS.	H. 9 48'	Decl. [MS] Bor. 17o 38 2/3'.	10
1585	Die 15 Aprils	H. 9 50	Declin. [MS] B. 17 38 3/4	10
1585	DIE 26 APRILIS.	H. 9 50	Decl. [MS] B. 16 8 1/2 per Armillas Bo	10
1585	DIE 7 MAIJ.	H. 11 24 1/2	Declinatio [MS] 14 22 1/2 per Armillas	10
1585	DIE 7 MAIJ.	H. 9 1/3	Decl. [MS] 14 22	10
1585	DIE 7 MAIJ.	H. 11 1/4	Decl [MS] 14 22 1/2 B.	10
1585	DIE 12 MAIJ.		declinatio [MS] B. 13o 30 1/4' per Auft	10
1585	DIE 17 MAIJ.	H.11 30	Decl. [MS] B. 12 38 1/2 per Arm. auft.	10
1585	DIE 18 MAIJ.	H.10 40	Decl. [MS] 12 27 B. per Arm. auft.	10
1586	DIE 23 SEPTEMBRIS.	H.5 M.12 P.M.N.	Declin. [MS] B. 18 5 1/2	11
1586	DIE 24 SEPTEMBRIS A.M.	H.3 M.55	Declin. [MS] Bor. 17 56 1/2	11
1586	DIE 10 OCTOBRIS.	H.2 M.32	Declin. [MS] per Armillas 15 3 3/4 B.	11
1586	DIE 10 OCTOBRIS.	H.2 M.32	alt. pinnac. 15 3 1/2	11
1586	DIE 10 OCTOBRIS.	H.6 M.14	Declin. [MS] B. vno 13 0 1/2	11
1586	DIE 10 OCTOBRIS.	H.6 M.14	alt. pinnac. 13 0 2/3	11
1586	DIE 24 OCTOBRIS.	H.6 M.35	Declin. [MS] B. 12 39 3/4	11

1586	DIE 25 OCTOBRIS A.M.	H.5 M.11	Declinatio [MS] 12 29 1/3	11
1586	DIE 25 OCTOBRIS A.M.	H.5 M.16	Repetita Decl. [MS] 12 29 1/3	11
1586	DIE 25 OCTOBRIS A.M.	H.5 M.32	Declin. [MS] vt prius 12 29 1/3	11
1586	DIE 1 NOUEMBRIS A.M.	H.5 M.6	Declin. [MS] Bor. 11 2 3/4	11
1586	DIE 2 NOUEMBRIS A.M.	H.4 M.46 1/6	Declin. [MS] Bor. 11 3	11
1586	DIE 8 NOUREMBRIS A.M.	H.6 M.34	Declin. [MS] Bor. 10 4 1/2	11
1586	DIE 10 NOUEMBRIS A.M.	H.7 M.20	Declin. Bor. 9 32 1/2	11
1586	DIE 10 NOUEMBRIS A.M.	H.7 M.28 1/2	Repetita Declin. [MS] 9 33	11
1586	DIE 11 NOUEMBRIS A.M.	H.4 M.19 S.50	Declin [MS] Bor. 9 25 1/2	11
1586	DIE 11 NOUEMBRIS A.M.	H.7 M.6 45"	Decl. ex alt. 9 25 0	11
1586	DIE 23 NOUEMBRIS A.M.	H.6 M.15	Declin. [MS] B. vno 7 19 3/4	11
1586	DIE 23 NOUEMBRIS A.M.	H.7 M.24	Declin. [MS] B. vno pinn. 7 19 2/3	11
1586	DIE 23 NOUEMBRIS A.M.	H.7 M.24	altero pinac. 7 19 5/6	11
1586	DIE 1 DECEMBRIS.	H.7 M.35 1/2	Declin. [MS] Bor. 6 2 1/6	11
1586	DIE 1 DECEMBRIS.	H.7 M.35 1/2	Alt. pinnac. 6 2 1/4	11
1586	DIE 16 DECEMBRIS, MANE.	H.6 M.4	Decl. [MS] per Armillas 3 53 1/2	11
1586	DIE 16 DECEMBRIS, MANE.	H.6 M.4	alt. pinn. 3 54	11
1586	DIE 27 DECEMBRIS A.M.	H.4 M.8	Declin. [MS] Bor. vno 2 40	11
1586	DIE 27 DECEMBRIS A.M.	H.4 M.8	alt. pin. 2 40	11
1586	DIE 27 DECEMBRIS A.M.	H.7 M.2 S.50	Declin. Martis repet. 2 38 3/4	11
1586	DIE 27 DECEMBRIS A.M.	H. 3 5/6	Declinatio [MS] tis 2 39 1/2 B.	11
1586	DIE 27 DECEMBRIS A.M.	H.4 0	Declinatio 2 39 2/3 B.	11
1587	DIE 1 JANUARIJ A.M.	H.7 M.8	Declin. [MS] per Armill. fubt. 2 11 1/2	11
1587	DIE 1 JANUARIJ A.M.	H.7 M.8	altero pinnacidio 2 12 1/2	11
1587	DIE 9 JANUARIJ A.M.	H.6 M.35 S.56	Declin. [MS] vno 1 39 1/2	11
1587	DIE 9 JANUARIJ A.M.	H.6 M.35 S.56	altero pinn. 1 39 5/6	11
1587	DIE 10 JANUARIJ A.M.	H. 5 M.15	[MS] Decl. Bor. 1 35 bis, bona	11
1587	DIE 11 JANUARIJ.	H.6 M.48	Declin. 1 31 Bor.	11
1587	DIE 11 JANUARIJ.	H.6 M.48	1 31 1/4 dubia	11
1587	DIE 14 JANUARIJ.	H.7 M.44 1/2	Declin. [MS] B. vno pinn. 1 26	11
1587	DIE 14 JANUARIJ.	H.7 M.44 1/2	alt. pinn. 1 25 1/2	11
1587	DIE 14 JANUARIJ.	H.8 M.0	Repetita Declin. [MS] Bor. vno 1 26	11
1587	DIE 14 JANUARIJ.	H.8 M.0	altero 1 25 1/2	11
1587	DIE 15 JANUARIJ, MANE.	H.4 M.45	Declin. [MS] Bor. vno 1 23 1/2	11
1587	DIE 15 JANUARIJ, MANE.	H.4 M.45	altero 1 23	11
1587	DIE 16 JANUARIJ, A.M.	H.4 M.51	Declinatio [MS] 1g 21' B vnico pinnacidi	11
1587	DIE 26 JANUARIJ, MANE.	H.4 M.28	Declin. [MS] B. vno pinn. 1 16 1/2	11
1587	DIE 26 JANUARIJ, MANE.	H.4 M.28	altero 1 16	11
1587	DIE 26 JANUARIJ, MANE.	H.6 M.8	Declin. [MS] B. vno 1 16 1/2	11
1587	DIE 26 JANUARIJ, MANE.	H.6 M.8	altero pinn. 1 16	11
1587	DIE 28 JANUARIJ, MANE.	H.4 M.25	Decl. [MS] Bor. vno pinn. 1 19	11
1587	DIE 28 JANUARIJ, MANE.	H.4 M.25	altero pinn. 1 18 1/2	11
1587	DIE 28 JANUARIJ, MANE.	H.5 M.10	Repetita declin. [MS] Bor. 1 19	11
1587	DIE 29 JANUARIJ, MANE.	H.5 M.12 2/3	Declinatio [MS] Bor. per Armillas 1 21	11
1587	DIE 29 JANUARIJ, MANE.	H.6 M.14	Repetita decl. [MS] Bor. 1 21 1/2	11
1587	DIE 9 JANUARIJ (FEB???)	H.6 M.0	Declin. [MS] 1 39 3/4 B.	11
1587	DIE 9 JANUARIJ (FEB???)	H.7 M.20	Declinatio B. 1 39 3/4	11
1587	DIE 10 JANUARIJ (FEB???)	H.5 M.6	Decl. [MS] B. 1 34 1/2	11
1587	DIE 10 JANUARIJ (FEB???)	H.5 M.17	Declin. Bor. 1 34 30	11
1587	DIE 14 JANUARIJ (FEB???)	H.7 M.4	Declinatio Bor. 1 25 3/4	11

From Raw to Corrected Observations

Parallax Correction

From the observed angular position to the angular position of the object as observed along a line from it to the center of the Earth; depends on location of observer on the Earth and the distance from the Earth to the observed object *in units of Earth-radii*, for which remotely accurate values did not emerge until after 1680

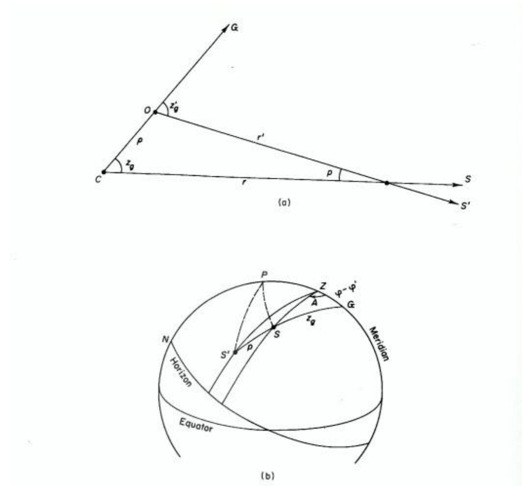


Fig. 10. Geocentric parallax. (a) C, center of Earth; O, observer; G, geocentric zenith; S, geocentric direction; and S', topocentric direction. (b) Z, geodetic zenith; P, celestial pole; and G, geocentric zenith.

Atmospheric Refraction Correction

From observed angular position to the angular position of the object as it would be observed in the absence of the optical refraction from the Earth's atmosphere, as classically estimated from motion (primarily of the Sun) after correcting for parallax; uncertainty about this correction gave reason for preferring observations when object is most nearly directly overhead

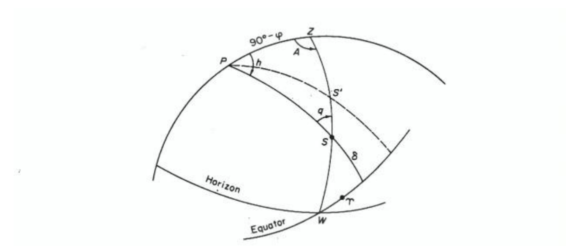
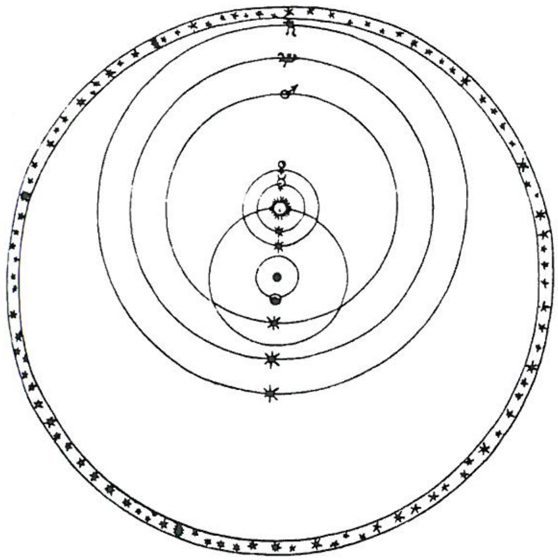
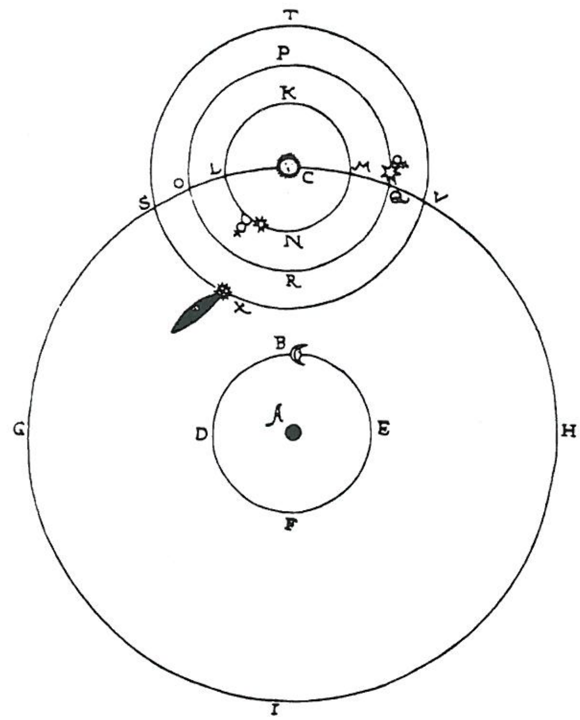


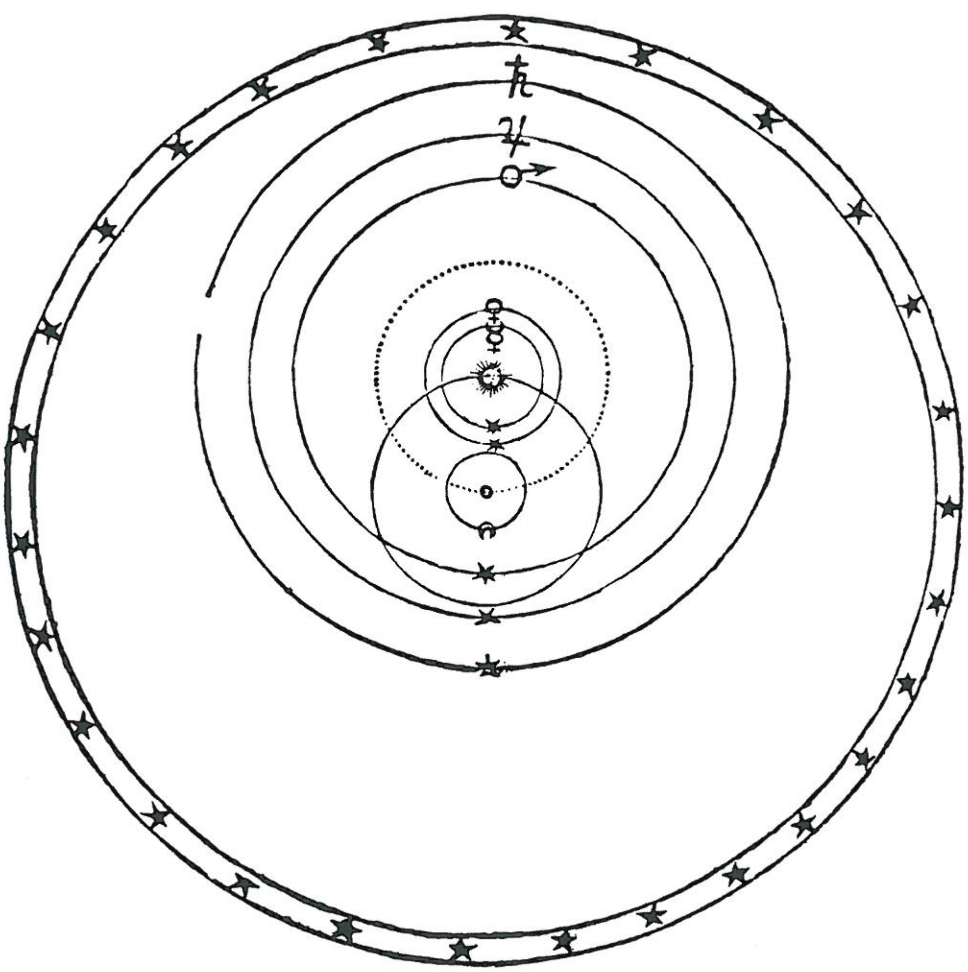
Fig. 15. Refraction in right ascension and declination: S, geometric position and S' position affected by refraction.



1.4. The Tychonic world system. The Earth is at rest at the centre, encircled by the stars. The five planets orbit around the Sun, while the Sun and the Moon orbit around the Earth.



1.5. The comet of 1577 in relation to the inner bodies of the Tychonic system. Mercury, Venus, and the comet orbit the Sun, while the Sun and the Moon orbit the Earth.



3.4. Gassendi's drawing of the Tychonic system, but with the alternative (Copernican) motion of the Earth added as a dotted curve.

The Crisis in Mathematical Astronomy

The sky and the stars have been moving for three thousand years; everybody had so believed, until it occurred to Cleanthes of Samos or (according to Theophrastus) to Nicetas of Syracuse, to maintain that it was the earth that moved, through the oblique circle of the Zodiac, turning about its axis; and in our day Copernicus has grounded this doctrine so well that he uses it very systematically for all astronomical deductions. What are we to get out of that, unless that we should not bother which of the two is so? And who knows whether a third opinion, a thousand years from now, will not overthrow the preceding two?

Montaigne, "Apology for Raymond Sebond", p. 429, (ca. 1580)