4. A third purpose served by a ramified mathematical development of a theory is that it provides a general question-answering device
   a. Given a question, theory identifies what combinations of information needed to determine answer and provides a way of then determining the answer (a la Bromberger: vicarious experiments): Galileo's solved Problems
   b. A crucial role of theories in science -- most of what you learn when you study sciences
   c. The form in which an indefinite amount of knowledge becomes contained in a fairly simple theory, fairly simple to learn
5. These three purposes, though somewhat distinct, are not unrelated
   a. Question-answering feature plays a critical role in the design of many experiments (as well as in engineering applications)
   b. Testing critical to be able to rely on question-answering when answering novel questions or ones with answers that cannot be readily checked empirically
   c. Explanatory power and predictive power both tied to question-answering power
6. The question-answering aspect of theories in physics more important than is sometimes noted -- look at e.g. Kepler's theory of planetary motion from this point of view
   a. In fact, a tradition dating back to Ptolemy in astronomy, Archimedes in mechanics
   b. Galileo can be thought of as bringing this tradition into a science of motion within mechanics

III. "The Fourth Day": Conceptual Development of the Theory

A. A Parabolic Path -- The Basic Idea

1. Galileo's proposed conceptualization of "projected" motion: suppose uniform motion along a plane which suddenly ends, so that vertical fall commences (with uniform acceleration)
   a. A basic case of a more general type: motion "compounded from two movements; that is, when it is moved equably and is also naturally accelerated." [268]
   b. Uniform horizontal plus uniformly accelerated vertical, "all impediments being put aside"
   c. Idea, then, is to combine the two theories from the "Third Day"
2. Note that Galileo here again seemingly committing himself to a limited version of what we call the principle of inertia -- for equable horizontal motion is taken to be without impediment
   a. I.e. horizontal motion would remain uniform and hence "perpetual" in nature because nothing in the basic natural process to change its speed (since resistance etc. excluded)
   b. Of course, once horizontal ends, then something in nature to change motion, namely propensity (*propensionem*) to fall to ground
   c. Hence a further assumption, open to challenge: uniform horizontal and uniformly accelerated vertical "propensities" continue unaltered when compounded -- no "cross-talk"!
3. Compounding two independent orthogonal motions was in some ways a radical move in the 17th century, however simple it is to us

a. As we noted earlier in the course, thoroughly anti-Aristotelian to think of two motions compounding to form a third motion: one motion always dominant

b. But can be seen in some classical geometry, especially post-Euclidean geometry -- e.g. Archimedes' spiral -- as well as in Ptolemy (compound circles) and Kepler (ellipse out of circle from radial motion)

4. Theorem 1: the resulting trajectory compounded from uniform horizontal and uniformly accelerated vertical motions is a semiparabolic line

   a. Galileo's proof employs the converse of lemma 1, which amounts to claim that y proportional to x^2 in parabolas

   b. Basic idea of proof straightforward: x proportional to t, y proportional to t^2, resulting in y proportional to x^2

5. Theorem 1 more limited than it may at first appear to be, for it says nothing about which parabola is described by the motion -- i.e. the dimensions of the parabola

   a. Theorem 1 the counterpart of Kepler's "law" stating that orbits are elliptical (conforming to area rule), without specifying what factors might determine which parameters

   b. Kepler never offers factors suitable for solving the "initial value" problem, but Galileo does

B. A Problem: Local Versus Circular Motion

1. Much as in the "Third Day" the basic claim immediately elicits a series of challenges to the very idea of conceptualizing projected motion in this way [273-274]

   a. Sagredo: surely not a parabola all the way to the center of the earth, and hence why a parabola even over first segment

   b. Simplicio: motion in a straight line along a horizontal is not uniformly removed from the surface of the earth, and hence will experience a deceleration as if going up an inclined plane!

   c. Simplicio: resistance effects cannot be removed, and they will "destroy" the two separate patterns of motion being compounded

   "All these difficulties make it highly improbable that anything demonstrated from such fickle assumptions can ever be verified in actual experiments" -- [274]

2. Galileo's response to the challenge on horizontal motion grants the point -- i.e. true horizontal motion not eternal -- but then invokes the authority of Archimedes to treat the earth's radius as infinite

   a. Idealization that is approximately correct for short horizontal distances, as evidenced by such practical procedures as parallel plumb lines in architecture

   b. Concedes that some correction may be needed when applying to various real motions -- thereby opening problems for the future

3. Galileo's response to the challenge about the path to the center of the earth: while reaching surface of earth, parabolic shape altered "only insensibly, whereas that shape is conceded to be enormously transformed in going on to end at the center" [275]