

De Motu Corporum in Gyrum

Definition 1. A 'centripetal' force I name that by which a body is impelled or attracted towards some point regarded as its center.

Definition 2. And the force of — that is, innate in — a body I call that by which it endeavours to persist in its motion following a straight line.

Definition 3. While 'resistance' is that which is the property of a regularly impeding medium.

Hypothesis 1. In the ensuing nine propositions the resistance is nil; thereafter it is proportional jointly to the speed of the body and to the density of the medium.

Hypothesis 2. Every body by its innate force alone proceeds uniformly into infinity following a straight line, unless it is impeded by something from without.

Hypothesis 3. A body is carried in a given time by a combination of forces to the place where it is borne by the separate forces acting successively in equal times.

Hypotheses 4. The space which a body, urged by any centripetal force, describes at the very beginning of its motion is in the doubled ratio of the time.

PART II

*The Falling of Heavy Bodies
and Their Motion in a Cycloid*

HYPOTHESES

I

If there were no gravity, and if the air did not impede the motion of bodies, then any body will continue its given motion with uniform velocity in a straight line.

II

By the action of gravity, whatever its sources,¹ it happens that bodies are moved by a motion composed both of a uniform motion in one direction or another and of a motion downward due to gravity.

III

These two motions can be considered separately, with neither being impeded by the other.

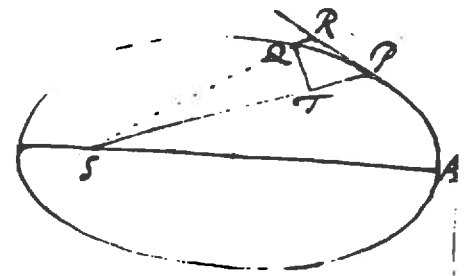
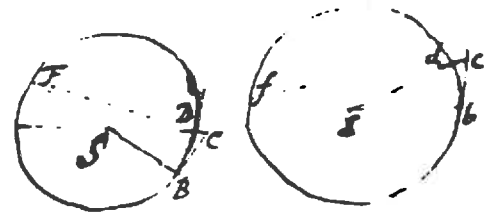
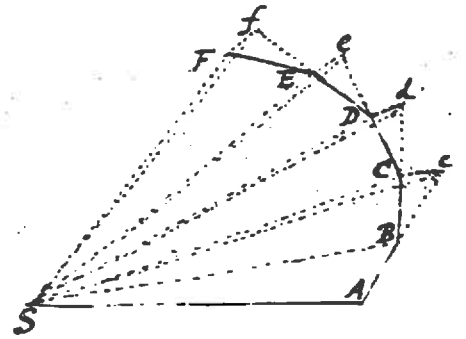
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Theorem 1. All orbiting bodies describe, by radii drawn to their center, areas proportional to the times.

Theorem 2. Where bodies orbit uniformly in the circumferences of circles, the centripetal forces are as the squares of arcs simultaneously described, divided by the radii of their circles.

Corollary 5. If the squares of the periodic times are as the cubes of the radii, the centripetal forces are reciprocally as the squares of the radii. And conversely so.

Theorem 3. If a body P in orbiting around the center S shall describe any curved line APQ, and if the straight line PR touches that curve in any point P and to this tangent from any other point Q of the curve there be drawn QR parallel to the distance SP, and if QT be let fall perpendicular to this distance SP: I assert that the centripetal force is reciprocally as the "solid" $SP^2 \times QT^2 / QR$, provided that the ultimate quantity of that solid when the points P and Q come to coincide is always taken.



Scholium to Theorem 2. The case of the fifth corollary holds true in the heavenly bodies: the squares of the periodic times are as the cubes of their distances from the common center round which they revolve. That it does obtain in the major planets revolving round the Sun and also in the minor ones orbiting round Jupiter and Saturn astronomers are agreed.

A worry. The orbits of the major planets are indeed nearly circular in shape: the largest deviation, for Mercury, has only a 2 percent variation from the largest diameter to the smallest.

Nevertheless, the motions are far from uniform. The ratio of the maximum to minimum velocity is $(1+e)/(1-e)$, which for Mercury is around 1.2/0.8, so that the maximum velocity is 50 percent greater than the minimum, and correspondingly for Mars, the maximum velocity is around 20 percent greater than the minimum.

What sort of conclusion can then be drawn on the basis of the fifth corollary about an inverse-square ratio of the centripetal force holding the planets in orbit around the Sun?