Monetary Policy and Lending Distortion in China

A thesis

Submitted by

Xiaozhou Ding

In partial fulfillment of the requirements for the degree of

Master of Science

in

Economics

TUFTS UNIVERSITY

May 2015

Advisor: Marcelo Bianconi

Acknowledgements

I would like to sincerely thank my thesis advisor, Professor Marcelo Bianconi, for his constant support, guidance, and encouragement throughout the course of my research. My interest in macroeconomics and dynamic models stemmed early from talks with him over department Happy Fridays and later strengthened by his inspiring teaching during Advanced Macroeconomics II course. It was in those models taught in his class that I found the beauty of economics and determined to set a goal for a thesis research. It was also him who has been giving me opportunities more than I can expect to participate in economics conference and reach out to great economics people. I would not have the chance to receive the admission of doctoral program without his invaluable advice and support.

I would also like to thank Professor Daniel Richards for his thoroughly help, which is vital for me in completing my thesis. His patience and kindness was always refreshing. His thorough reading and insightful comments have provided me with ideas for future research.

Professor Steven Block deserves my special thanks. It was him that gave me the opportunity to be a research assistant during the first summer at Tufts and he has been constantly helping me with great advice for thesis and doctoral program application. Professor David Garman as my Stata TA supervisor not only had provided me opportunities to gain teaching skills, but also offered me great value of advice of choosing my goals. Meeting Professor Metcalf as my graduate advisor is one of the most fortunate thing I had at Tufts. He has been encouraging me to exploit myself and motivating me to chase my dream of getting into a doctoral program all the time. I learned a lot from his straightforward comments and I am so grateful for all the help he gave me.

Abstract

The author investigates the effects of monetary policy in a distortionary economy consisting of two types of firms: State-Owned Enterprises (SOEs) and Private-Owned Enterprises (POEs). They mainly differ in the ability of getting loans from commercial banks. By modelling the differences between SOEs and POEs with dynamic stochastic general equilibrium model, the author solves the large equation system by Dynare, a platform running on Matlab.

In the first section, the author builds a simple Real Business Cycle model to examine the effect of a monetary policy shock in an economy where interest rate subsidy is identical to all firms with or without an interest rate subsidy. It verifies the money-neutrality of classical model and an identical subsidy with a technology shock will induce expansion in the economy. In the second section, the author adds sticky price and distortion to the RBC model and the distortion is measured by that only SOEs could be able to get the interest rate below the benchmark rate while POEs have to pay a capital rental tax to use capital. The result shows that an expansionary monetary policy could be able to increase output, consumption, capital and inflation but only last very shortly. Output will increase 4% after the shock and in the medium run, the output and inflation will return to mild level, nearly 0.5%. And to boost the economy you have to introduce more aggressive policies. The heterogeneity problem will not hurt the economy so much so long as the subsidy-favored sector has a larger share of the economy. This looks like the situation in China since state-owned economy still dominates and with the policy bias. It also corroborates the findings of some scholars that the quantity instrument has very limited effect.

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1 Introduction

In recent decades, the New Keynesian Dynamic Stochastic General Equilibrium Model has become the workhorse for analyzing and understanding the dynamic effect of monetary policies. It is built on the famous paper by Kydland and Prescott (Kydland and Prescott 1982) which introduced the Real Business Cycle (RBC) model, and is the cornerstone for modern macroeconomic analysis. After that, a lot of scholars have devoted to develop a more convincing macroeconomic model with microeconomic foundations. Menu cost (Mankiw 1985) and other findings validate the sticky price model and the nominal rigidities have been the key interest of major macroeconomists. Nowadays many central banks are using DSGE models to formulate their monetary policies, including the European Central Bank, Bank of Finland, and the Federal Reserve. Those DSGE models vary from small to medium size and consist of various sectors that are intertwined. Incorporating different frictions and distortions with DSGE models is valuable for making policies and understanding them.

In this thesis, the research question being asked is what the effect of monetary policy is in a lending-distortionary economy where different firms have different cost of renting capital. Does the monetary policy shock generate enough growth in output and persistent inflation? We are very interested in whether the heterogeneity problem will slow or endanger an economy from growing and recovering. The author will use Dynare, a software platform running on Matlab to solve the dynamic system of equations and simulate the DSGE models to see the impulse response functions of major variables and analyze the dynamic effect of a monetary policy shock. As some researchers mentioned, a model that is useful for policy choice need not to fit the data well, and well-fit models necessarily sacrifice economic interpretability. Some scholars do not agree with this idea and try to construct interpretable models with superior fit to the data (Leeper, Sims et al. 1996). In this paper, the author leaves the data-fitting issue behind and will try to deliver the theoretical implication of a monetary policy in a lending distortionary economy.

2 Literature Review

China has witnessed a great institutional change during its marketization process and opening up. But it is still under the financial reform and there exist a lot of frictions among its economic system. Most of them result from the structural problem. For example, China's stock market was initially developed to support state-owned enterprises which induced extreme volatility (Green 2003). China's financial system has been dominated by a large banking system and because of the lower hurdle rate for investment they face, State-Owned Enterprises (SOEs) end up creating more investable assets than Private-Owned Enterprises (POEs). While due to more constrained credit availability, POEs save more and invest less than SOEs (Allen, Qian et al. 2012). In addition to that, the earning ability of POEs is greater than SOEs (Fig 1). However, SOEs have preferential access to credit (Fig 2). All these characteristics make China's economy different from the rest of the world. Not only in the way how it developed recently, but also how the administrations responded to economic structures and made policies. During the 2008 financial crisis, while a lot of major economies faced a depression and deflation situation, China stepped out the recession very quickly (Fig 3) in 2010 and even experienced an inflation pressure and asset price bubbles in a short term (Fig 4). The author thinks that the reason why China performed differently from other countries lies within itself even though the government did no differently than the rest of the world (Fig 5). As we can see, China injected a huge amount of money into the economy with a package of stimulus plan. The monetary policy was very loose during the financial crisis.

The monetary policy instruments applied by the People's Bank of China (PBC) include reserve requirement ratio, central bank base interest rate, rediscounting, central bank lending, open market operation and other policy instruments specified by the State Council. In recent years, especially during financial crisis, PBC has operated frequently with reserve requirement ratio to release liquidity. Unlike most developed countries, PBC sets the growth target of money supply at the beginning of every year after National People's Congress. And in terms of the interest rate, PBC also sets benchmark interest rate at the beginning of each year and then determines the deposit rates and loan rates for the commercial banks based on the benchmark interest rate. Consequently, these interest rates cannot change with the money demand and supply conditions. Only the interbank market has some floating ability around the benchmark rate. That leaves a question for borrowing and lending friction (Chen and Huo 2009). A lot of scholars have examined the monetary policy rules in China. The results vary a lot. Li-gang Liu and Wenlang Zhang use a New Keynesian model evaluating the China's monetary policy framework and demonstrate that a hybrid rule combining both interest rate and quantity of money appears to be more suitable than its alternatives before financial crisis (Liu and Zhang 2010). Kong tests four kinds of monetary policy rules for historic data from 1994 to 2006 and conclude that Taylor rule is better than McCallum rule in evaluating Chinese monetary policy performance (Kong 2008). Xiong develops a new policy stance index and uses an ordered probit model to study monetary policy in China and concludes that inflation plays a key role in determining PBC's monetary policy stance and PBC informally target inflation (Xiong 2012). In respect with asset price, real estate price was the major concern not only by citizens, but also by the policy makers. Iacoviello adds collateral constraints tied to real estate values for firms and nominal debt for households as two key features to financial accelerator (Iacoviello 2005). His results show that monetary authority's response to asset prices yields negligible gains in terms of output and inflation stabilization. Andrés imposes two features on Iacoviello's model: borrowing is subject to collateral constraints and credit flows are intermediated by monopolistically competitive banks (Andrés, Arce et al. 2013).

In practice, the money supply increased substantially in the wake of the global financial crisis. China's M2 growth rate rose sharply from about 15% in 2007 to over 25% in 2009. China's inflation surged from moderate levels to over 6% by 2012, despite increases in the PBOC required reserve ratio from 14% in late 2007 to 21% in 2011, and its maintenance of this policy variable at levels close to 20%. This surge in Chinese inflation during a period when foreign interest rates dropped sharply relative to domestic rates is consistent with a monetary policy trade-off between sterilization costs and price stability (Chang, Liu et al. 2012).

In a theoretical paper, Liu built an economy with four sectors: government, SOEs, POEs, and banks. He argued that the financial repression and ownership discrimination caused various harmful results to Chinese economic development such as moral hazard, bad balance of sheet, and efficiency loss of POEs (Liu 2011). In my thesis, I will not include the possibility that firms will default and hence the finance every SOE and POE get is going to be used as capital market clear in general equilibrium. This delivers the distinctive steady states results. Pesaran and Xu built a DSGE model where firms gain capital from both banking sector and households and discuss the different effect of firms' default in steady states, concluding that even without price rigidities the effect of credit shock will be persistent (Pesaran and Xu 2013). Khramov uses a New Keynesian DSGE model with U.S. data to show that capital accumulation can generate persistency in the dynamic of the main economic variables (Khramov 2012). Other research indicates that models with nominal rigidities do not generate enough persistence in output after a money shock (Chari, Kehoe et al. 2000).

3 A Simple RBC Model with Real Interest Rate Subsidy

We know China is still conducting its interest marketization and under the institutional reform from a central-planning economy to a fully market economy. There are a lot of biases existing in China's economy. And among those biases, the lending discrimination is the major one. The author starts with a simple RBC model to examine the response in an economy with lending distortion. The simple RBC model has the basic components of a DSGE model and is also easy to quantify. In this case, the author tries to capture the quantitative results of a distortionary economy that SOEs are getting lower interest rates loans than POEs. I will assume there is an interest rate subsidy through all the SOEs and no interest rate subsidy in POEs.

3.1 Households

3.1.1 Lifetime Utility

As traditionally setting, we assume the representative households who maximize the lifetime utility.

$$\max \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{{c_t}^{1-\sigma} - 1}{1-\sigma} + \chi \frac{(1-n_t)^{1-\xi} - 1}{1-\xi} + \nu \frac{{m_t}^{1-\zeta}}{1-\zeta} \right)$$

where $\mathbb{E}_t$ is the conditional expectation operator, c_t is the consumption, n_t is labour, m_t is the real money balance, and we also have discount factor β.

3.1.2 Budget Constraint

There are three resources of nominal income for households in time period t, : nominal wage, nominal interest with previous period bond holding, and previous money holdings. Households can use all of the resources to purchase either consumption, newly issued bonds, or hold current period money. Therefore, in nominal terms, the budget constraint can be written in this way:

$$p_t c_t + B_t + M_t = W_t n_t + p_t \Pi_t + (1 + i_{t-1}) B_{t-1} + M_{t-1}$$

where B_{t-1} and M_{t-1} are nominal bonds and money holdings between time $t-1$ and t. t_{t-1} is the nominal interest rate on bonds. W_t is the nominal wage, p_t is price level, and Π_t is profits coming from firms.

For the convenience of calculation and coding, we rearrange it in real terms, which is dividing every term by p_t :

$$c_t + \frac{B_t}{p_t} + \frac{M_t}{p_t} = \frac{W_t}{p_t} n_t + \Pi_t + (1 + i_{t-1}) \frac{B_{t-1}}{p_t} + \frac{M_{t-1}}{p_t}$$

Let $b_t = \frac{B_t}{p_t}$ denote real bonds holdings, similarly $m_t = \frac{M_t}{p_t}$ as real money holdings and $w_t = \frac{W_t}{p_t}$ as real wage. We have different time index with respect to bonds and money, and the way of solving it is to introduce inflation to get the appropriate terms $1 + \pi_t = \frac{p_t}{p_{t-1}}$. Plugging them into the budget constraint, we get:

$$c_t + b_t + m_t = w_t n_t + \Pi_t + (1 + i_{t-1}) \frac{B_{t-1}}{p_t} \frac{p_{t-1}}{p_{t-1}}$$

Simplifying yields:

$$c_t + b_t + m_t = w_t n_t + \Pi_t + (1 + i_{t-1}) \frac{b_{t-1}}{1 + \pi_t} + \frac{m_{t-1}}{1 + \pi_t}$$

3.1.3 First Order Conditions

Set the Lagrangian as following and λ_t is the Lagrangian multiplier:

$$\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma} - 1}{1 - \sigma} + \chi \frac{(1 - n_t)^{1-\xi} - 1}{1 - \xi} + \nu \frac{m_t^{1-\zeta} - 1}{1 - \zeta} \right.$$
$$\left. - \lambda_t \left(c_t + b_t + m_t - w_t n_t - \Pi_t - (1 + i_{t-1}) \frac{b_{t-1}}{1 + \pi_t} - \frac{m_t}{1 + \pi_t} \right) \right\}$$

The first order conditions are:

$$c_t^{-\sigma} = \lambda_t$$

$$\chi (1 - n_t)^{-\xi} = \lambda_t w_t$$

$$\lambda_t = \beta\lambda_{t+1}\frac{1+i_t}{1+\pi_{t+1}}$$

$$vm_t^{-\zeta} = \lambda_t - \beta\lambda_{t+1}\frac{1}{1+\pi_{t+1}}$$

3.2 Firms

Assume that the firm owns capital stock and decides how many labour and investment to use to produce output in each period by maximizing the present discounted profits in real values. The production function is Cobb-Douglas:

$$y_t = e^{z_t}k_{t-1}^{\alpha}n_t^{1-\alpha}$$

Assume that the technology follows an AR(1) process:

$$z_t = \rho z_{t-1} + \epsilon_{z,t}, \epsilon_{z,t} \sim N(0,\sigma^2)$$

Here I adopt Sims' setting for firm's problem. The firm tries to maximize the value, which consists of current profit and discounted value of future profits. Since the SOEs are able to get loans at a lower price, which is distorted real interest rate, in the simple RBC model we assume that all the firms are identical SOEs and there is an exogenous distortion imposed on the interest rate. Let γ denote the interest rate discount ratio:

$$\max V_t = e^{z_t}k_{t-1}^{\alpha}n_t^{1-\alpha} - w_t n_t - I_t$$
$$+ \mathbb{E}_t\sum_{j=1}^{\infty}\prod_{k=1}^{j}(1+(1+\theta)r_{t-1+k})^{-1}(e^{z_t}k_{t+j-1}^{\alpha}n_{t+j}^{1-\alpha} - w_{t+j}n_{t+j} - I_{t+j})$$

The capital accumulation equation follows:

$$k_t = I_t + (1-\delta)k_{t-1}$$

3.2.1 First Order Conditions

Use ϕ_t as the Lagrangian multiplier to solve the first order conditions:

$$\mathcal{L} = e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha} - w_t n_t - I_t$$
$$+ \mathbb{E}_t \left\{ \sum_{j=1}^{\infty} \prod_{k=1}^{j} (1 + (1+\theta) r_{t-1+k})^{-1} \left(e^{z_t} k_{t+j-1}^{\alpha} n_{t+j}^{1-\alpha} - w_{t+j} n_{t+j} - I_{t+j} \right) \right.$$
$$\left. - \phi_t (k_t - (1-\delta) k_{t-1} - I_t) \right\}$$

The first order conditions are:

$$w_t = (1-\alpha) e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha}$$
$$(1+\theta) r_t = e^{z_{t+1}} k_t^{\alpha-1} n_{t+1}^{1+\alpha} - \delta$$

We can see from the first order condition that the real marginal cost of capital is the equal to marginal production of capital minus depreciation rate, which is equal to $(1+\gamma) r_t$. Thus, for $\gamma < 0$, which indicates γ percent discount off real interest rate, and for $\gamma > 0$, which is equivalent to γ percent tax imposed on real interest rate.

Note that another important equation we will use in the full model is Fisher Equation:

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}$$

3.3 Government

Assume that there is a government (central bank) setting money growth rule exogenously. It is easily noticed that in the steady state, the money growth rate is equal to inflation rate. For simplicity, we assume positive inflation rate in steady state, which leads to the following inflation target rule:

$$m_t - m_{t-1} + \pi_t = (1-\rho_m)\pi^* + \rho_m (m_{t-1} - m_{t-2}) + \rho_m \pi_{t-1} + \varepsilon_{m,t}$$

where π^* is the target inflation rate, ρ_m is the parameter of money growth weighted on previous period and $\epsilon \sim N(0, \sigma^2)$.

3.4 Equilibrium

Now we have the full characterised model consisting of the following equations:

$$c_t^{-\sigma} = \lambda_t \tag{1}$$

$$\chi(1-n_t)^{-\xi} = \lambda_t w_t \tag{2}$$

$$\lambda_t = \beta\lambda_{t+1}\frac{1+i_t}{1+\pi_{t+1}} \tag{3}$$

$$\nu m_t^{-\zeta} = \lambda_t - \beta\lambda_{t+1}\frac{1}{1+\pi_{t+1}} \tag{4}$$

$$y_t = e^{z_t}k_{t-1}^{\alpha}n_t^{1-\alpha} \tag{5}$$

$$y_t = c_t + I_t \tag{6}$$

$$z_t = \rho z_{t-1} + s\varepsilon_{z,t} \tag{7}$$

$$k_t = I_t + (1-\delta)k_{t-1} \tag{8}$$

$$w_t = (1-\alpha)e^{z_t}k_{t-1}^{\alpha}n_t^{1-\alpha} \tag{9}$$

$$(1+\theta)r_t = e^{z_{t+1}}k_t^{\alpha-1}n_{t+1}^{1-\alpha} - \delta \tag{10}$$

$$1 + r_t = \frac{1+i_t}{1+\pi_{t+1}} \tag{11}$$

$$dm_t + \pi_t = (1-\rho_m)\pi^* + \rho_m\pi_{t-1} + \rho_m dm_{t-1} + \varepsilon_{m,t} \tag{12}$$

$$dm_t = \ln m_t - \ln m_{t-1} \tag{13}$$

3.5 Steady States

We investigate the "log-log" case where $\sigma = \xi = \zeta = 1$. The Dynare simulation results are shown in the following table based on the calibration.

In this simple RBC model, we examine the symmetric cases that if all firms receive a 20 percent real interest rate subsidy and if all firms face a 20 percent additional capital tax. The steady states results are straightforward and easily understood. We can see that an interest rate subsidy does induce an increase in investment and capital stock, and more importantly, boost the output and lift consumption. But comparing to the percent increase of subsidy case, the absolute value of decrease of tax case falls less than the subsidy case.

TABLE 1-CALIBRATION OF SIMPLE RBC MODEL

Parameter	Calibration
β	0.98
α	0.33
δ	0.05
χ	1.5
ξ	1
ζ	1
σ	1
s	0.007
ν	0.5
π^*	0.02
ρ	0.95
ρ_m	0.95

TABLE 2-STEADY STATE COMPARISON OF SIMPLE RBC MODEL

Variable	Subsidy $\theta = -0.2$	Tax $\theta = 0.2$
Consumption c	2.38%	-2.26%
Labour n	1.33%	-1.15%
Wage w	2.98%	-2.74%
Output y	4.35%	-3.86%
Capital Stock k	10.78%	-9.13%
Real Effective r	-20%	20%
Investment I	10.78%	-9.12%
Money Balance m	2.39%	-2.25%
Nominal Effective i	-20%	20%

3.6 Impulse Response Functions

In the simple RBC model, there are some implications that worth noting: (a) in a fully flexible price model, introducing interest rate subsidy is able to generate positive deviation from steady states of real variables to a technology shock. The technology shock leaves some differences among the three cases. The shock comes as a 0.01 standard deviation from steady states. As we can see, output jumps very quickly and then sluggishly returns to steady state (Fig 7). Consumption and capital moves similarly as a hump-shaped path. But the magnitude of the responses is still very small, even though there are distinct differences between the three cases (Fig 6 and Fig 8). The subsidy case performs best, boosts consumption, real wage, output, and capital both in the short run and long run; (b) money shock does not have effect on real variables. The change in consumption, labour, capital, output, and other major variables are trivial, as shown in Fig 9. The main reason is because we are using a money-in-the-utility function to optimize our model but assuming fully flexible price. Firms can adjust their price due to any nominal monetary changes therefore no real variable change will happen under a monetary policy.

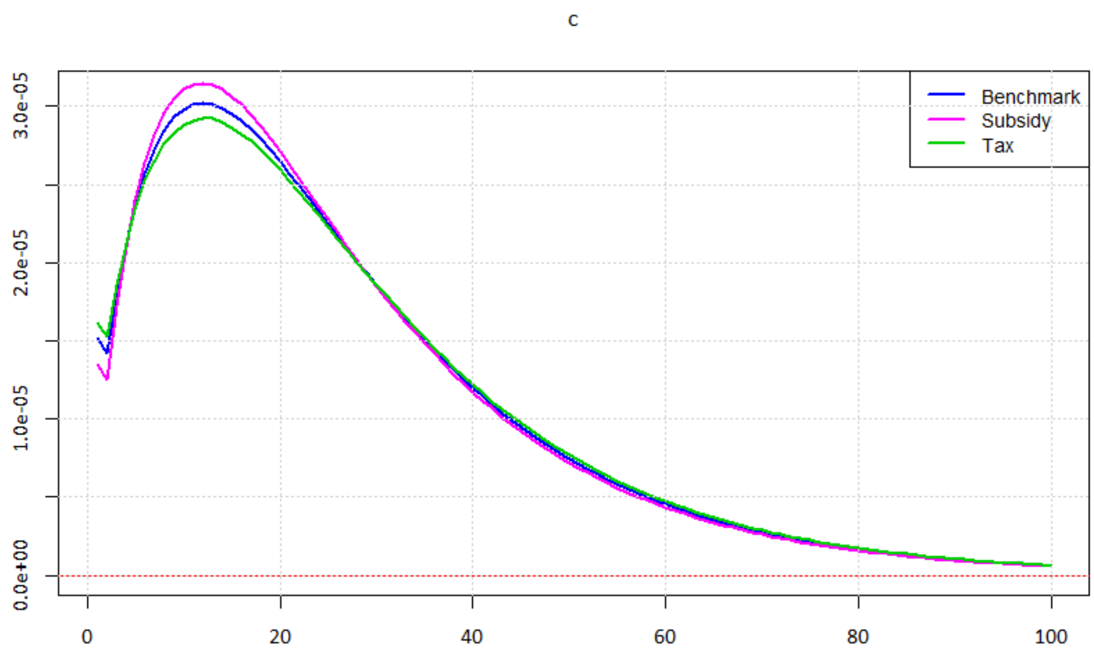

FIGURE 6-IMPULSE RESPONSE OF CONSUMPTION TO A 0.01 TECHNOLOGY SHOCK IN RBC MODEL

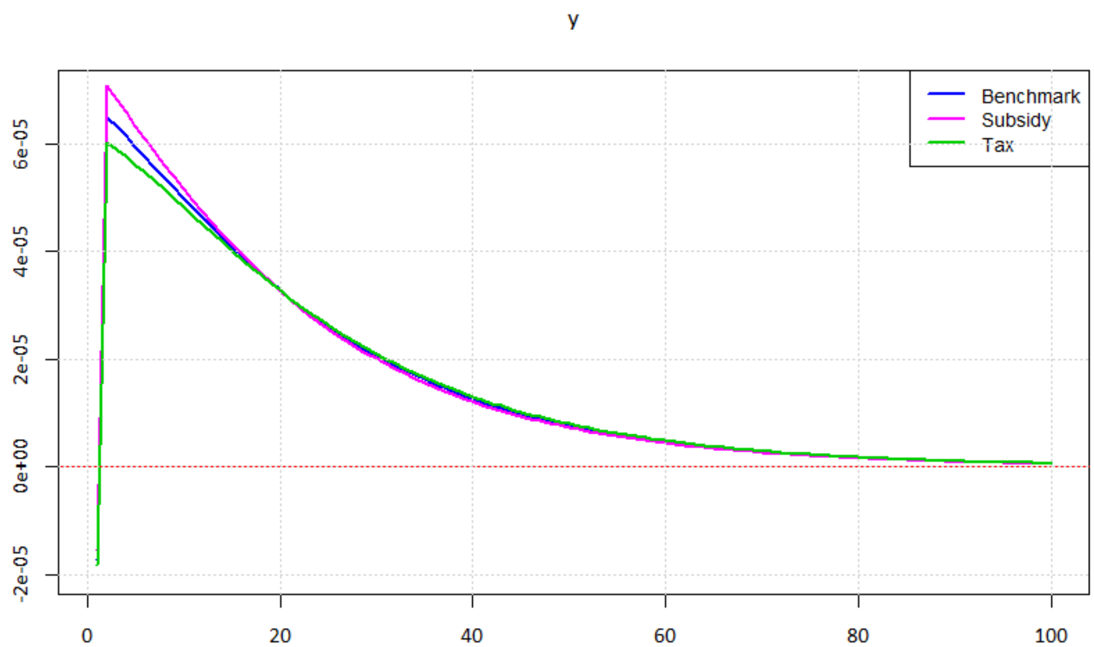

FIGURE 7- IMPULSE RESPONSE OF OUTPUT TO A 0.01 TECHNOLOGY SHOCK IN RBC MODEL

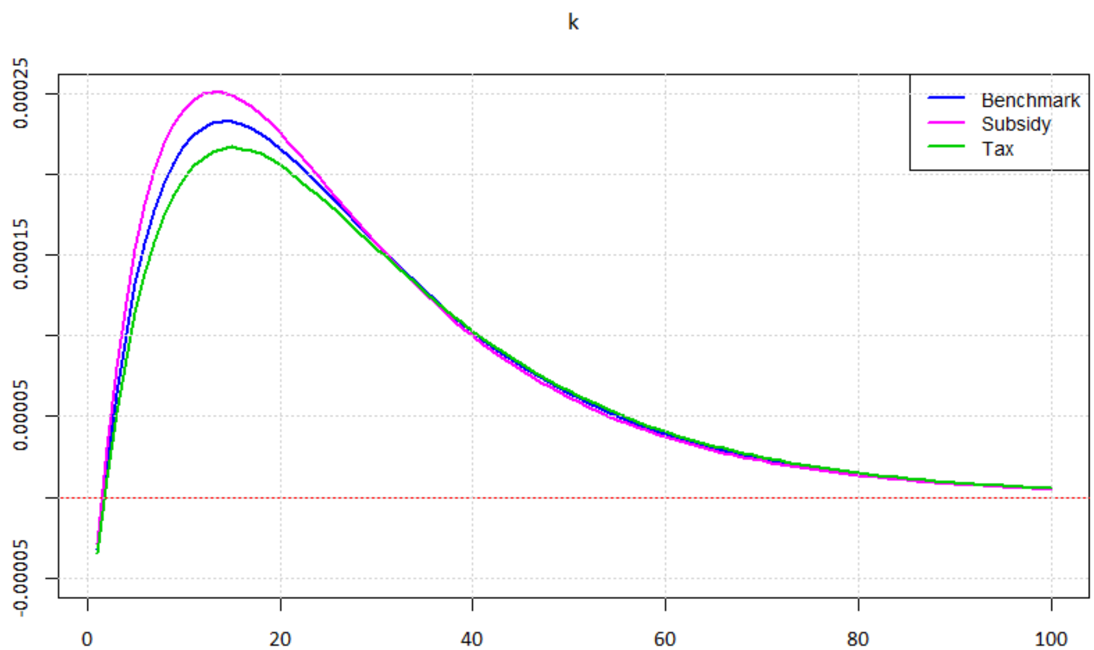

FIGURE 8- IMPULSE RESPONSE OF CAPITAL TO A 0.01 TECHNOLOGY SHOCK IN RBC MODEL

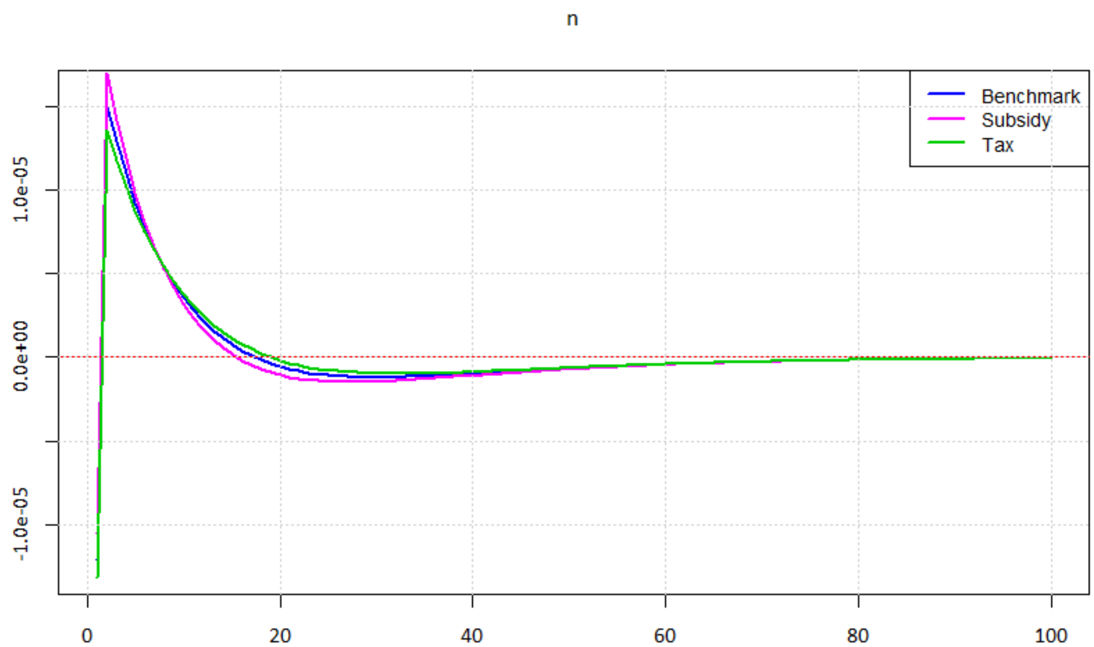

FIGURE 9- IMPULSE RESPONSE OF LABOUR TO A 0.01 TECHNOLOGY SHOCK IN RBC MODEL

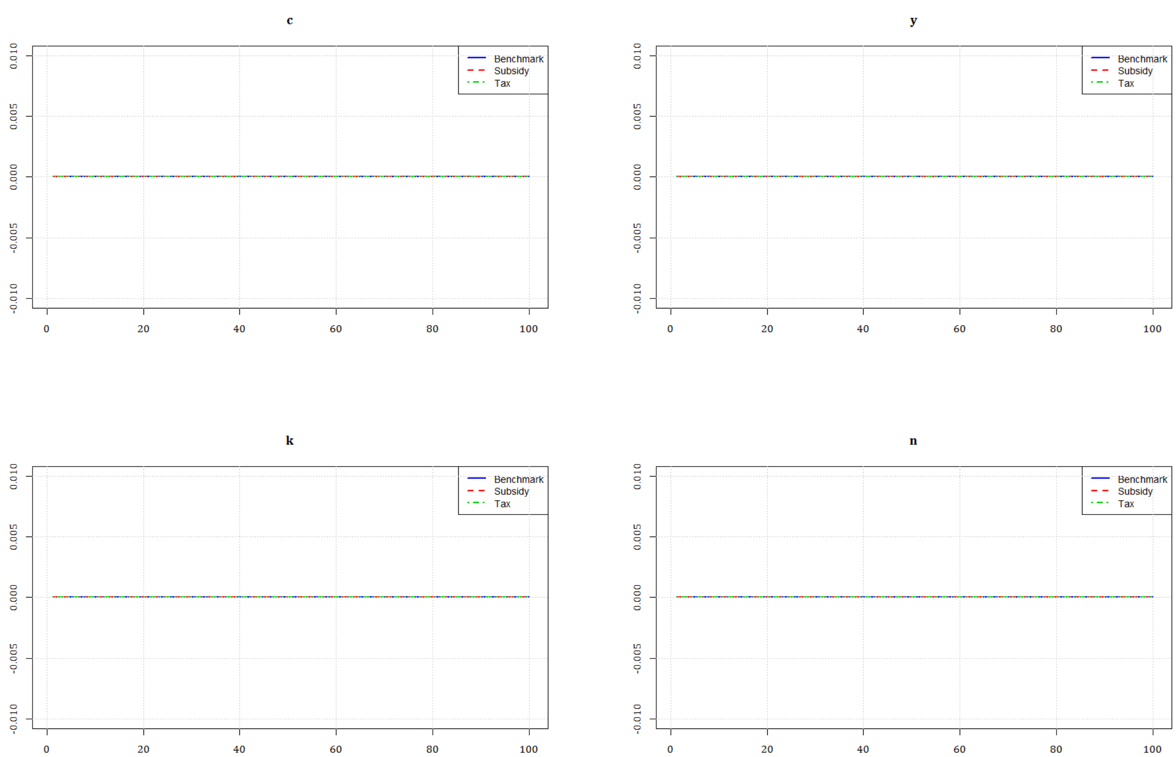

FIGURE 10-IMPULSE RESPONSES OF REAL VARIABLES TO A 0.01 MONEY SHOCK IN RBC MODEL

4 Real Lending Distortions in New Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) Model

The RBC model leaves a great policy relevant issue, which is that the neutrality of monetary policy. The shock of increasing the growth of money supply only has effect on nominal variables, and it does not leave impact on real variables such as consumption, output and capital. This does not accurately picture the real case and a lot of scholars have written tons of papers to introduce the stickiness of. For this reason, I will incorporate Calvo pricing to get the staggered price and introduce the non-neutrality of monetary policy.

In this section, I will go further based on the assumption that there exist heterogeneous firms in China's economy and examine the monetary policy under the nominal rigidity with lending distortion.

4.1 The Final Goods Producers

There are two types of firms in the economy. The final goods firms buy intermediate goods from the market and sell it back to consumers. The final good producers maximize their profits.

$$\max P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

$$s.t. Y_t = \left[\int_0^1 Y_t(i)^{\frac{\psi-1}{\psi}} di \right]^{\frac{\psi}{\psi-1}}, \psi > 1$$

where P_t and $P_t(i)$ are final good price and intermediate good price respectively. Y_t and $Y_t(i)$ are final goods and intermediate goods respectively. $\psi > 1$ is the elasticity of substitution in production.

Differentiate with respect to $Y_t(i)$ gives the FOC:

$$Y_t(i) = Y_t \left[\frac{P_t(i)}{P_t}\right]^{-\psi} \tag{14}$$

Then we can substitute the demand back to the bundle good and get the production function as:

$$Y_t = \left\{\int_0^1 \left[Y_t \left(\frac{P_t(i)}{P_t}\right)^{-\psi}\right]^{\frac{\psi-1}{\psi}} di\right\}^{\frac{\psi}{\psi-1}} = Y_t \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\psi} di\right]^{\frac{\psi}{\psi-1}}$$

simplifying the above equation gives

$$P_t = \left[\int_0^1 P_t(i)^{1-\psi} di\right]^{\frac{1}{1-\psi}} \tag{15}$$

This is the pricing formula we will use later in the aggregation.

4.2 The Intermediate Goods Producers

Intermediate goods firm is the second type firm in the economy indexed by $i \in [0,1]$, each firm produces an intermediate good that is different from that of other firms. The production function is Cobb-Douglas:

$$Y_t(i) = A_t k_t(i)^{\alpha} n_t(i)^{1-\alpha}$$

These firms are price takers and could not set price freely in each period. Labour is paid at the cost of real wage $w_t \equiv \frac{W_t}{P_t}$, capital service has real cost r_t. The intermediate firms will make decisions on how many labour to hire and how much capital to minimize cost.

Suppose there are two types of real interest rates that firms could get: $\theta_1 r_t$ and $\theta_2 r_t$. $\theta_1 \in (0,1)$, $\theta_2 > 1$. That is to say, for firms who are SOEs in our model could get $\theta_1 r_t$ interest rate, their capital rental cost is below the benchmark rate, and on the other side, those firms who are

POEs get $\theta_2 r_t$ thus operate with higher cost than SOEs. Both types of firms face same labour cost[1].

Additionally, assume there are $\gamma \in (0,1)$ firms who are SOEs that get the θ_1 real interest rates. Therefore the cost minimization problem is:

$$\min w_t n_t(i) + \theta r_t k_t(i), \theta = \{\theta_1, \theta_2\}$$

$$s.t. A_t k_t(i)^\alpha n_t(i)^{1-\alpha} \geq Y_t \left[\frac{P_t(i)}{P_t}\right]^{-\psi}$$

Let $\phi(i)$ be the Lagrangian multiplier, and the first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial k_t(i)} = 0 \Leftrightarrow \theta r_t = \phi_t(i) \alpha A_t k_t(i)^{\alpha-1} n_t(i)^{1-\alpha}, \theta = \{\theta_1, \theta_2\}$$

$$\frac{\partial \mathcal{L}}{\partial n_t(i)} = 0 \Leftrightarrow w_t = \phi_t(i)(1-\alpha) A_t k_t(i)^\alpha n_t(i)^{-\alpha}$$

Comparing the two equations and dividing w_t by r_t eliminates $\phi_t(i)$, hence we have:

$$\frac{k_t(i)}{n_t(i)} = \frac{1}{\theta} \frac{\alpha}{1-\alpha} \frac{w_t}{r_t}$$

As we can see, lower θ yields higher capital-labour ratio.

Since the RHS does not depend on i, the capital-labour ratio is identical to all firms if they have same θ.

Combining these two FOCs with production function we can solve for the optimal demand for labour and capital:

$$k_t(i)^* = \left(\frac{1}{\theta} \frac{\alpha}{1-\alpha} \frac{w_t}{r_t}\right)^{1-\alpha} \frac{Y_t(i)}{A_t}$$

$$n_t(i)^* = \left(\theta \frac{1-\alpha}{\alpha} \frac{r_t}{w_t}\right)^{\alpha} \frac{Y_t(i)}{A_t}$$

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[1] From historical data, in recent years, state-owned wage is almost the same as other ownership, which indicates the similar wage among them. But another issue is the wage rigidity. Data shows a stable annual increase of wage in China.

Substituting $Y_t(i)$ into above two equations gives us the total nominal cost:

$$TotalCost = w_t n_t(i) + \theta r_t k_t(i)$$

$$= \theta^{\alpha} \frac{w_t^{1-\alpha} r_t}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \frac{Y_t(i)}{A_t}$$

Differentiating total cost with respect to $Y_t(i)$ delivers the marginal cost:

$$mc_{1,t} = A_t^{-1} \theta_1^{\alpha} \frac{w_t^{1-\alpha} r_t^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} = \theta_1^{\alpha} mc_t$$

$$mc_{2,t} = A_t^{-1} \theta_2^{\alpha} \frac{w_t^{1-\alpha} r_t^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} = \theta_2^{\alpha} mc_t$$

Where mc_t represents the marginal cost for the non-distortion case.

Here we adopt Calvo pricing, following (Christiano, Eichenbaum et al. 2005), (Smets and Wouters 2003) and (Calvo 1983) to implement the price rigidity. Suppose there is a probability ρ that the firm will keep the price to next period and probability $1-\rho$ that the firm could optimally adjust price. The objective for each firm is to maximize the expected profit, which is derived above, given by revenue from selling goods minus the costs of producing that production subject to demand function and the sticky price assumption. Let $\Delta_{t+j} = \beta^j \frac{\lambda_{t+j}}{\lambda_t}$ denote the discount factor, where λ_t is the marginal value of additional unit of income. Usually, $\lambda_t = u'(c_t)$, $\Delta_{t+j} = \beta^j \frac{u'_{c,t+j}}{u'_{c,t}}$.

The intermediate firm is also maximizing its discounted profits by choosing prices optimally. We start our derivation with SOEs and get the result of POEs by replacing the real rental cost parameter θ with θ_2.

$$\max_{P_t(i)} \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta_{t+j} \left[\frac{P_t(i)}{P_t} Y_{t+j}(i) - \theta_1^{\alpha} mc_{t+j} Y_{t+j}(i) \right]$$

$$s.t.\ Y_{t+j}(i) = A_{t+j} k_{t+j}(i)^{\alpha} n_{t+j}(i)^{1-\alpha}$$

Plug the budget constraint into objective function. The problem is equivalent to:

$$\max_{P_t(i)} \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta_{t+j} \left\{ \left[\frac{P_t(i)}{P_{t+j}} \left(\frac{P_t(i)}{P_{t+j}} \right)^{-\psi} - \theta_1^\alpha mc_{t+j} \left(\frac{P_t(i)}{P_{t+j}} \right)^{-\psi} \right] Y_{t+j} \right\}$$

$$= \max_{P_t(i)} \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta_{t+j} \left\{ \left[\left(\frac{P_t(i)}{P_{t+j}} \right)^{1-\psi} - \theta_1^\alpha mc_{t+j} \left(\frac{P_t(i)}{P_{t+j}} \right)^{-\psi} \right] Y_{t+j} \right\}$$

The first order condition with respect to $P_t(i)$ is

$$0 = \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta_{t+j} \left\{ \left[(1-\psi) P_t(i)^{-\psi} P_{t+j}^{\psi-1} - (-\psi) \theta_1^\alpha mc_{t+j} P_t(i)^{-\psi-1} P_{t+j}^{\psi} \right] Y_{t+j} \right\}$$

$$\Rightarrow P_t(i)^* = \theta_1^\alpha \frac{\psi}{\psi-1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta_{t+j} mc_{t+j} P_{t+j}^{\psi} Y_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta_{t+j} P_{t+j}^{\psi-1} Y_{t+j}}$$

Note $P_t(i)^*$ is the reset optimal price for SOEs. For POEs,

$$P_t(i)^\# = \theta_2^\alpha \frac{\psi}{\psi-1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta_{t+j} mc_{t+j} P_{t+j}^{\psi} Y_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta_{t+j} P_{t+j}^{\psi-1} Y_{t+j}}$$

$$= \frac{\theta_2^\alpha}{\theta_1^\alpha} P_t(i)^*$$

Now $\theta_1^\alpha \frac{\psi}{\psi-1}$ is the gross mark-up of the ith state-owned intermediate goods producer price over the ratio of the discounted nominal total costs divided by the discounted real output. We can see that since we add the distortion of interest rate, comparing to the non-distortion case, the reset price for both SOEs and POEs are different from the benchmark case with a coefficient θ_1^α. The result for POEs is similar.

Since $0 < \theta_1 < 1 < \theta_2$, $\alpha \in (0,1)$, we get

$$0 < \theta_1^\alpha < 1 < \theta_2^\alpha$$

and

$$P_t(i)^* < P_t(i)^\#$$

That is to say that under the interest rate distortion, the reset optimal price for SOEs is lower than POEs. Additionally, we can see the pricing equation for firm i does not depend on i, therefore all the intermediate goods producers who are able to adjust price face the same price $P_t(i)^* = P_t^*$ if they have the same marginal cost.

In order to simplify the optimal price equation, we add two auxiliary functions:

$$X_{1,t} = u'(c_t) mc_t P_t^{\psi} Y_t + \rho\beta \mathbb{E}_t X_{1,t+1}$$

$$X_{2,t} = u'(c_t) P_t^{\psi-1} Y_t + \rho\beta \mathbb{E}_t X_{2,t+1}$$

Hence,

$$P_t^* = \theta_1^{\alpha} \frac{\psi}{\psi - 1} \frac{X_{1,t}}{X_{2,t}} \tag{16}$$

4.3 Households

Here we still use the representative households appeared in the simple RBC model, where $\mathbb{E}$ is the conditional expectation operator which collects all the information available at date t, β is the discount factor, c_t is consumption at date t, M_t is nominal money holdings at the beginning of the period t, P_t is overall price level at t, and $\frac{M_t}{P_t}$ indicates the real money holdings. Households will choose consumption, bonds, capital, money holdings, and labour supply based on the budget constraint. The household's problem is then defined as:

$$\max \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\eta}}{1+\eta} + \frac{\upsilon}{1-\zeta} \left(\frac{M_t}{P_t} \right)^{1-\zeta} \right\}$$

At the beginning of each period, households spend income on consumption, bond purchasing, hold some cash balances, and make the decision of investment. The income of households are: wage income, capital gains with real return rate r_t, which is also the real interest rate, bond interest payment with the gross nominal interest $1 + i_t$, firm profits, cash balances of previous period, and they also receive some lump-sum monetary transfer from government. Capital also follows the capital accumulation equation with depreciation rate δ.

The budget constraint thus is:

$$c_t + \frac{B_{t+1}}{P_t} + \frac{M_t}{P_t} + I_t = \frac{W_t}{P_t} n_t + r_t k_t + \Pi_t + (1 + i_t)\frac{B_t}{P_t} + \frac{M_{t-1}}{P_t} + \Pi_t + T_t$$

$$k_{t+1} = I_t + (1 - \delta)k_t$$

Forming the Lagrangian we can solve for the households first order conditions:

$$\chi n_t^{\eta} = c_t^{-\sigma} w_t$$

$$c_t^{-\sigma} = \beta \mathbb{E}_t c_{t+1}^{-\sigma}(1 + i_{t+1})\frac{1}{\pi_{t+1}}$$

$$\upsilon \left(\frac{M_t}{P_t}\right)^{-\zeta} = c_t^{-\sigma} - \beta \mathbb{E}_t c_{t+1}^{-\sigma} \frac{1}{\pi_{t+1}}$$

$$c_t^{-\sigma} = \beta \mathbb{E}_t c_{t+1}^{-\sigma}(r_{t+1} + 1 - \delta)$$

4.4 Government

The government sets the monetary policy rule following inflation targeting. The monetary shock comes from the money supply. We still adopt the same monetary policy rule from simple RBC model. As usually, we add price level in money supply equation and rearrange to get:

$$\Delta \ln m_t = (1 - \rho_m)\pi - \pi_t + \rho_m \Delta \ln m_{t-1} + \rho_m \pi_{t-1} + \varepsilon_{m,t}$$

The lump-sum transfer is defined as following:

$$T_t = M_t - M_{t-1} + B_{t+1} - (1 + i_t)B_t$$

4.5 Equilibrium

4.5.1 Aggregate Output

On the demand side, put intermediate goods production function and Eqa. (14) together, we have

$$A_t k_t(i)^{\alpha} n_t(i)^{1-\alpha} = \left[\frac{P_t(i)}{P_t}\right]^{-\psi} Y_t$$

Integrating over i,

$$\int_0^1 A_t k_t(i)^\alpha n_t(i)^{1-\alpha} di = \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\psi} Y_t di$$

$$A_t k_t^\alpha n_t^{1-\alpha} = Y_t \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\psi} di \tag{17}$$

Define s_t as price dispersion:

$$s_t = \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\psi} di$$

Eqa. (17) becomes

$$Y_t = \frac{A_t k_t^\alpha n_t^{1-\alpha}}{s_t} \tag{18}$$

Since $s_t > 1$, we know due to the nominal rigidity, there is production loss in the economy. You would have produced more if you do not have price dispersion.

4.5.2 Capital and Labour Market

Following the same method we use in previous section, we integrate capital over i.

$$\int_0^1 k_t(i)di = \int_0^\gamma \left(\frac{1}{\theta_1}\frac{\alpha}{1-\alpha}\frac{w_t}{r_t}\right)^{1-\alpha} \frac{Y_t(i)}{A_t} di + \int_\gamma^1 \left(\frac{1}{\theta_2}\frac{\alpha}{1-\alpha}\frac{w_t}{r_t}\right)^{1-\alpha} \frac{Y_t(i)}{A_t} di$$

$$k_t = \gamma \left[\frac{1}{\theta_1}\left(\frac{\alpha}{1-\alpha}\right)\frac{w_t}{r_t}\right]^{1-\alpha} \frac{Y_t}{A_t} + (1-\gamma)\left[\frac{1}{\theta_2}\left(\frac{\alpha}{1-\alpha}\right)\frac{w_t}{r_t}\right]^{1-\alpha} \frac{Y_t}{A_t} \tag{19}$$

Similarly,

$$n_t = \gamma \left[\theta_1 \frac{1-\alpha}{\alpha}\frac{r_t}{w_t}\right]^\alpha \frac{Y_t}{A_t} + (1-\gamma)\left[\theta_2 \frac{1-\alpha}{\alpha}\frac{r_t}{w_t}\right]^\alpha \frac{Y_t}{A_t} \tag{20}$$

By Walras' Law, we only need one equation of the above two to calculate the steady state.

4.5.3 Price Level and Inflation

Let $P_t^* = P_t(i)$ be the adjusted price and follow the assumption that for those ρ non-adjusting firms will keep their prices as last period. The price level can be expressed in this way:

$$P_t^{1-\psi} = \int_0^1 P_t(i)^{1-\psi} di$$

A fraction $(1-\rho)$ of those firms are able to update price to the same reset price, P_t^*. The other fraction ρ will only keep the price same as previous period. Also, assume a fraction γ of those firms who are able to update price level by P_t^*, $(1-\gamma)$ will be able to adjust by $P_t^{\#}$. Therefore we have the following pricing formula:

$$P_t^{1-\psi} = \int_0^{1-\rho} P_t^{adjust^{1-\psi}} di + \int_{1-\rho}^1 P_{t-1}(i)^{1-\psi} di$$

Since P_t^{adjust} has probability γ to be P_t^* and $(1-\gamma)$ to be $P_t^{\#}$, $P_t^{adjust} = \gamma P_t^* + (1-\gamma) P_t^{\#}$. Substituting it into the above equation gives:

$$\begin{aligned} P_t^{1-\psi} &= \int_0^{1-\rho} [\gamma P_t^* + (1-\gamma) P_t^{\#}]^{1-\psi} di + \int_{1-\rho}^1 P_{t-1}(i)^{1-\psi} di \\ &= (1-\rho) \left[\gamma P_t^* + (1-\gamma) \left(\frac{\theta_2}{\theta_1}\right)^{\alpha} P_t^*\right]^{1-\psi} + \int_{1-\rho}^1 P_{t-1}(i)^{1-\psi} di \\ &= (1-\rho) \left[\gamma + (1-\gamma) \left(\frac{\theta_2}{\theta_1}\right)^{\alpha}\right]^{1-\psi} P_t^{*1-\psi} + \rho P_{t-1}^{1-\psi} \qquad (21) \end{aligned}$$

Therefore, the aggregate price level is the convex combination of the reset price and lagged price. Furthermore, dividing the Eqa. (21) by $P_{t-1}^{1-\psi}$ yields the aggregate inflation rate:

$$\begin{aligned} \pi_t^{1-\psi} &= \left(\frac{P_t}{P_{t-1}}\right)^{1-\psi} \\ &= (1-\rho) \left[\gamma + (1-\gamma) \left(\frac{\theta_2}{\theta_1}\right)^{\alpha}\right]^{1-\psi} \pi_t^{*1-\psi} + \rho \end{aligned}$$

where π_t^* is the SOE reset price aggregate inflation rate.

Now let us take a further look at the new inflation equation with the non-distortionary one. The difference is where we have $\left[\gamma + (1-\gamma) \left(\frac{\theta_2}{\theta_1}\right)^{\alpha}\right]^{1-\psi}$ before the reset inflation rate. We can take the group inside the bracket as a convex combination of 1 and $\left(\frac{\theta_2}{\theta_1}\right)^{\alpha}$. And we know it will always be greater than 1.

4.5.4 Price Dispersion

Recall that $s_t = \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\psi} di$, and we follow the same approach when we derive the aggregate inflation rate.

$$
\begin{aligned}
s_t &= \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\psi} di \\
&= \int_0^{1-\rho} \left(\frac{P_t^{adjust}}{P_t}\right)^{-\psi} di + \int_{1-\rho}^1 \left(\frac{P_{t-1}(i)}{P_t}\right)^{-\psi} di \\
&= \int_0^{1-\rho} \left(\frac{\gamma P_t^* + (1-\gamma) P_t^{\#}}{P_t}\right)^{-\psi} di + \int_{1-\rho}^1 \left(\frac{P_{t-1}(i)}{P_t}\right)^{-\psi} di \\
&= \int_0^{1-\rho} \left(\frac{\gamma P_t^* + (1-\gamma)\left(\frac{\theta_2}{\theta_1}\right)^{\alpha} P_t^*}{P_t}\right)^{-\psi} di + \int_{1-\rho}^1 \left(\frac{P_{t-1}(i)}{P_t}\right)^{-\psi} di \\
&= (1-\rho)\left[\gamma + (1-\gamma)\left(\frac{\theta_2}{\theta_1}\right)^{\alpha}\right] \pi_t^{*-\psi} \pi_t^{\psi} + \rho \pi_t^{\psi} s_{t-1} \qquad (22)
\end{aligned}
$$

We also have SOEs reset optimal price Eqa. (16) $P_t^* = \theta_1^{\alpha} \frac{\psi}{\psi-1} \frac{X_{1,t}}{X_{2,t}}$. Define two new auxiliary functions to replace nominal terms,

$$x_{1,t} \equiv \frac{X_{1,t}}{P_t^{\psi}}$$

$$x_{2,t} \equiv \frac{X_{2,t}}{P_t^{\psi}}$$

which can be written in this way:

$$x_{1,t} = u'(c_t) mc_t Y_t + \rho\beta \mathbb{E}_t \frac{X_{1,t+1}}{P_t^{\psi}} \qquad (23)$$

$$x_{2,t} = u'(c_t)Y_t + \rho\beta\mathbb{E}_t \frac{X_{2,t+1}}{P_t^{\psi-1}} \tag{24}$$

In order to get inflation enter the reset price, we need to add P_{t+1} into Eqa. (23) and Eqa. (24).

$$x_{1,t} = u'(c_t)mc_tY_t + \rho\beta\mathbb{E}_t \frac{X_{1,t+1}}{P_t^{\psi}}\frac{P_{t+1}^{\psi}}{P_{t+1}^{\psi}}$$

$$= u'(c_t)mc_tY_t + \rho\beta\mathbb{E}_t \frac{X_{1,t+1}}{P_{t+1}^{\psi}}\frac{P_{t+1}^{\psi}}{P_t^{\psi}}$$

$$= u'(c_t)mc_tY_t + \rho\beta\mathbb{E}_t x_{1,t+1}\pi_{t+1}^{\psi}$$

$$x_{2,t} = u'(c_t)Y_t + \rho\beta\mathbb{E}_t \frac{X_{2,t+1}}{P_t^{\psi-1}}\frac{P_{t+1}^{\psi-1}}{P_{t+1}^{\psi-1}}$$

$$= u'(c_t)Y_t + \rho\beta\mathbb{E}_t \frac{X_{2,t+1}}{P_{t+1}^{\psi-1}}\frac{P_{t+1}^{\psi-1}}{P_t^{\psi-1}}$$

$$= u'(c_t)Y_t + \rho\beta\mathbb{E}_t x_{2,t+1}\pi_{t+1}^{\psi-1}$$

Plugging $x_{1,t}$ and $x_{2,t}$ into Eqa. (16) gives P_t^*:

$$P_t^* = \theta_1^{\alpha}\frac{\psi}{\psi-1}\frac{x_{1,t}}{x_{2,t}}\frac{P_t^{\psi}}{P_t^{\psi-1}}$$

$$P_t^* = \theta_1^{\alpha}\frac{\psi}{\psi-1}\frac{x_{1,t}}{x_{2,t}}\,P_t \tag{25}$$

Dividing both sides by P_{t-1}, we can write the above equation all in inflation terms:

$$\frac{P_t^*}{P_{t-1}} = \theta_1^{\alpha}\frac{\psi}{\psi-1}\frac{x_{1,t}}{x_{2,t}}\frac{P_t}{P_{t-1}}$$

$$\pi_t^* = \theta_1^{\alpha}\frac{\psi}{\psi-1}\frac{x_{1,t}}{x_{2,t}}\pi_t \tag{26}$$

Eqa. (26) represents the reset inflation rate and is related to the mark-up multiplier, the distortionary magnitude, and the auxiliary functions.

4.6 Summary

$$\chi n_t^{\eta} = c_t^{-\sigma} w_t \tag{27}$$

$$c_t^{-\sigma} = \beta \mathbb{E}_t c_{t+1}^{-\sigma} (1 + i_{t+1}) \frac{1}{\pi_{t+1}} \tag{28}$$

$$\upsilon m_t^{-\zeta} = c_t^{-\sigma} - \beta \mathbb{E}_t c_{t+1}^{-\sigma} \frac{1}{\pi_{t+1}} \tag{29}$$

$$r_{t+1} + 1 - \delta = \frac{1 + i_{t+1}}{\pi_{t+1}} \tag{30}$$

$$Y_t = c_t + I_t \tag{31}$$

$$k_{t+1} = I_t + (1 - \delta) k_t \tag{32}$$

$$k_t = \gamma \left[\frac{1}{\theta_1} \left(\frac{\alpha}{1 - \alpha} \right) \frac{w_t}{r_t} \right]^{1-\alpha} \frac{Y_t}{A_t} + (1 - \gamma) \left[\frac{1}{\theta_2} \left(\frac{\alpha}{1 - \alpha} \right) \frac{w_t}{r_t} \right]^{1-\alpha} \frac{Y_t}{A_t} \tag{33}$$

$$Y_t = \frac{A_t k_t^{\alpha} n_t^{1-\alpha}}{s_t} \tag{34}$$

$$mc_t = A_t^{-1} \frac{w_t^{1-\alpha} r_t^{\alpha}}{\alpha^{\alpha} (1 - \alpha)^{1-\alpha}} \tag{35}$$

$$s_t = (1 - \rho) \left[\gamma + (1 - \gamma) \left(\frac{\theta_2}{\theta_1} \right)^{\alpha} \right] \pi_t^{*-\psi} \pi_t^{\psi} + \rho \pi_t^{\psi} s_{t-1} \tag{36}$$

$$\pi_t^{1-\psi} = (1 - \rho) \left[\gamma + (1 - \gamma) \left(\frac{\theta_2}{\theta_1} \right)^{\alpha} \right]^{1-\psi} \pi_t^{*1-\psi} + \rho \tag{37}$$

$$\pi_t^* = \theta_1^{\alpha} \frac{\psi}{\psi - 1} \frac{x_{1,t}}{x_{2,t}} \pi_t \tag{38}$$

$$x_{1,t} = c_t^{-\sigma} mc_t Y_t + \rho \beta \mathbb{E}_t x_{1,t+1} \pi_{t+1}^{\psi} \tag{39}$$

$$x_{2,t} = c_t^{-\sigma} Y_t + \rho \beta \mathbb{E}_t x_{2,t+1} \pi_{t+1}^{\psi-1} \tag{40}$$

$$dm_t = (1 - \rho_m) \pi - \pi_t + \rho_m dm_{t-1} + \rho_m \pi_{t-1} + \varepsilon_{m,t} \tag{41}$$

$$dm_t = \ln m_t - \ln m_{t-1} \tag{42}$$

$$z_t = \rho_z z_{t-1} + s \epsilon_z \tag{43}$$

$$A_t = e^{z_t} \tag{44}$$

We can replace A_t with e^{z_t} so it will reduce to 17 equations with 17 variables. We still adopt Dynare to calculate the steady states and will examine different responses to a technology shock and money shock under different parametrization.

4.7 Calibration

There were many scholars working on measuring the capital stock and accurate depreciation rate of China. Zhang compares various methods and uses his own version calculating that the capital-output ratio is around 2 before 1960 and between 3 and 4 since then (Jun and Yuan 2003). Chen uses maximum likelihood method based on the production function to estimate China's constant and variable capital depreciation rate. Constant capital depreciation rate is about 5.65% and mean value of variable depreciation rate is 5.63% (Chen 2014). Since in this model, the steady state capital stock only relates to the depreciation rate, and some authors argue that a key feature in China's rapid growth is the high depreciation rate. Thus, we will adopt a relative high depreciation rate.

Some scholars argue that there exist some structural shifts in China's money demand function and China witnessed a huge decline of money velocity[2] after 2008 financial crisis (Liao and Tapsoba 2014). From the data I collect from Fred, it corroborates the findings that money velocity was decreasing these years (Fig 11).

[2] Velocity is calculated as the ratio of nominal GDP over nominal money balance.

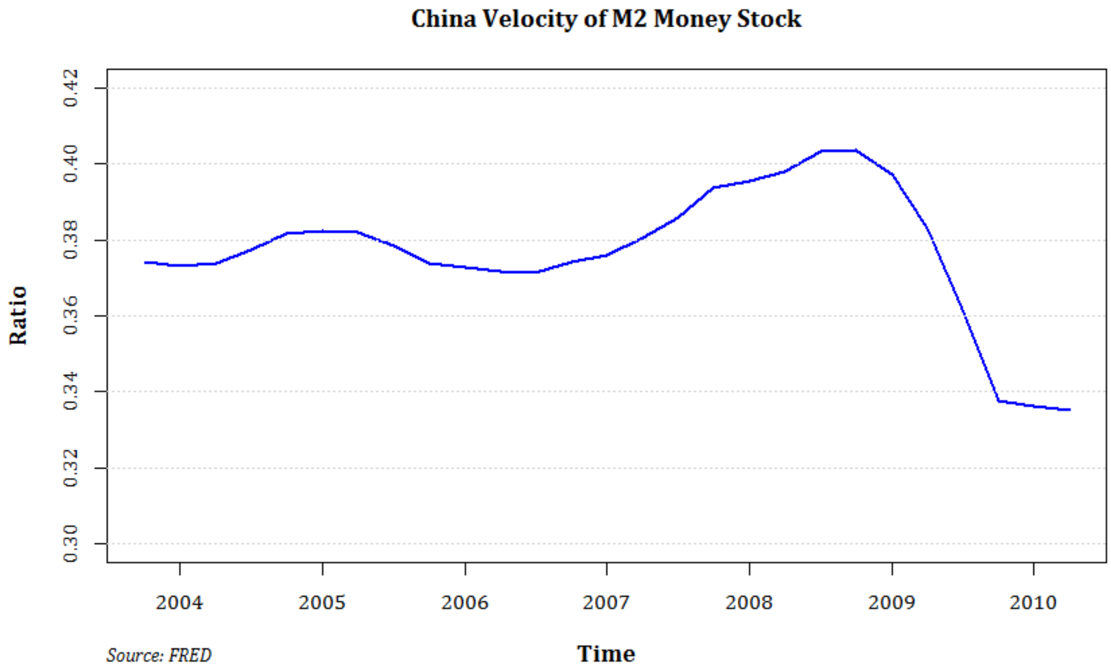

FIGURE 11-CHINA MONEY STOCK VELOCITY

Therefore, since our starting point is to build a model based on quantity monetary rule, thus during calibrating, we aim to give values to related parameters and get the velocity close to 0.3~0.4. Since we normalise labour to unit 1, the labour hired in the economy should be around 0.3, which requires the labour coefficient χ to be 5, and labour elasticity η to reach 1.5.

Another feature of China's economy is the opposite directions of consumption-output ratio and capital-output ratio. Historical data shows the consumption-output ratio in China has decreased more than 10 percentage point and due to the large amount of investment throughout the whole country, the capital-output ratio has increased a lot.

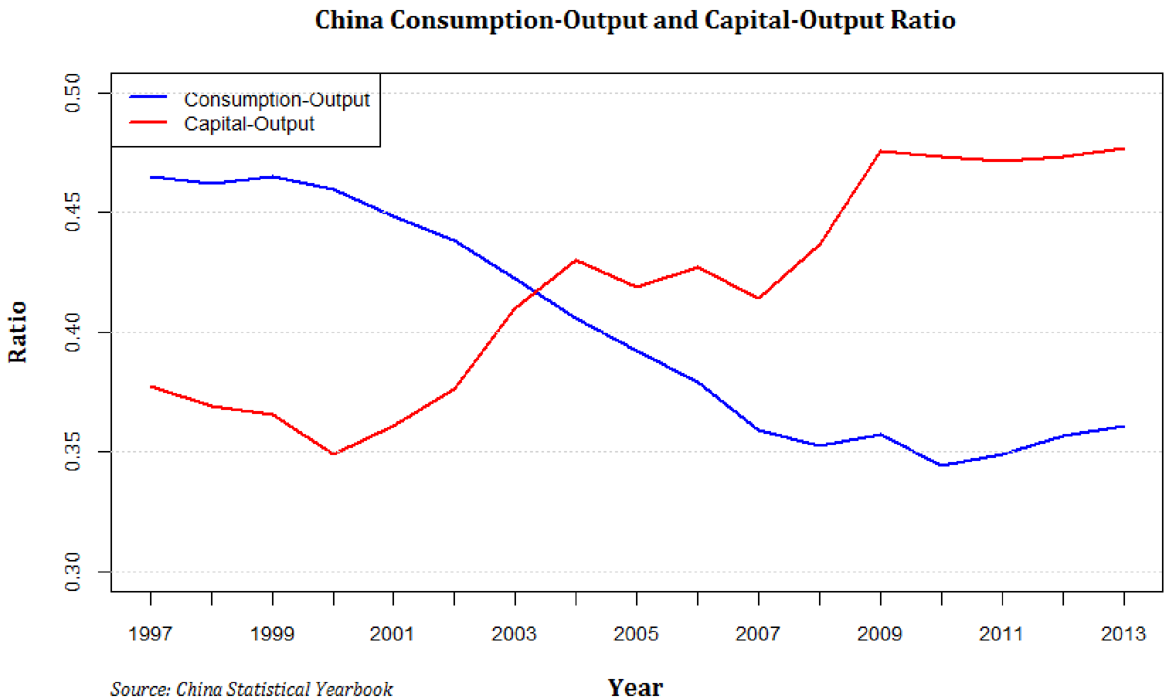

FIGURE 12-CHINA CONSUMPTION-OUTPUT AND CAPITAL-OUTPUT RATIO

Table 3 is the parameterization for the benchmark model with no distortions.

TABLE 3-CALIBRATION OF NK MODEL

Parameters	Values
σ	0.35
χ	5
β	0.995
ζ	1
η	1.5
ν	0.02
δ	0.15
α	0.2
ρ	0.55
ψ	3
π	1.02
ρ_z	0.95
ρ_m	0.95

4.8 Simulation Results

4.8.1 Steady States

If $\theta_1 = \theta_2 = 1$, then the first model turns into the basic New Keynesian model without assumed distortions. The steady states of benchmark model are listed in the column (1). After the benchmark model, we examine another two categories consisting of four cases which are from column (2) to (5). The first category is partial distortion, which only SOEs get real interest rate subsidy but POEs still face the market interest rate. The second category is fully distortion, while SOEs are able to get capital with real cost lower than benchmark, POEs have to pay higher real cost than the market to get capital. In both cases, we take a look at two scenarios: 1) SOEs only take smaller share of the entire economy, i.e. $\gamma = 0.2$; 2) SOEs take larger share of the entire economy, i.e. $\gamma = 0.6$.

TABLE 4-STEADY STATE COMPARISON OF NK MODEL

Variable	Benchmark Model	$\theta_1 = 0.7, \theta_2 = 1$ Partial Distortion		$\theta_1 = 0.7, \theta_2 = 1.3$ Full Distortion	
	$\theta_1 = \theta_2 = 1$	$\gamma = 0.2$	$\gamma = 0.6$	$\gamma = 0.2$	$\gamma = 0.6$
	(1)	(2)	(3)	(4)	(5)
Consumption c	0.2527	0.2282	0.2484	0.1996	0.2322
Labour n	0.3020	0.3128	0.3140	0.3115	0.3132
Wage w	0.5127	0.5216	0.5404	0.4945	0.5256
Real money balance m	0.5042	0.4865	0.5012	0.4642	0.4895
Marginal cost	0.6657	0.6750	0.6944	0.6467	0.6791
Output y	0.2900	0.2651	0.2961	0.2254	0.2724
Capital k	0.2491	0.2461	0.3178	0.1720	0.2679
Price dispersion s	1.0018	1.1247	1.0632	1.2272	1.1143

I convert the steady states into percent change which are shown in Table 5, hence providing an easier way to compare the four cases with benchmark model. There are several things to be noticed: (a) consumption declines in all four cases, this is fundamentally due to substitution between consuming and saving. Since in this economy, firms are owned by households and decide how much to invest and consume based on the real return rate, when the real interest rate subsidy enters, it creates the incentives for saving and investing, and hence reduces consumption; (b) labour and wage rate increases in all four cases except (4) where output, investment, and consumption all strikingly drop; (c) money holdings are lower than the benchmark case since people are unwilling to keep their money balance and put them into investment; (d) output only increases in case (3); (e) (3) performs best and (4) performs worst among all the five cases.

TABLE 5-STEADY STATE CHANGE COMPARISON OF NK MODEL

Variable	Benchmark Model	$\theta_1 = 0.7, \theta_2 = 1$ Partial Distortion		$\theta_1 = 0.7, \theta_2 = 1.3$ Full Distortion	
	$\theta_1 = \theta_2 = 1$	$\gamma = 0.2$	$\gamma = 0.6$	$\gamma = 0.2$	$\gamma = 0.6$
	(1)	(2)	(3)	(4)	(5)
Consumption c	0.2527	-9.69%	-1.70%	-21.02%	-8.10%
Labour n	0.3020	3.60%	3.99%	3.14%	3.70%
Wage w	0.5127	1.75%	5.41%	-3.55%	2.53%
Real money balance m	0.5042	-3.51%	-0.60%	-7.93%	-2.91%
Marginal cost	0.6657	1.40%	4.31%	-2.85%	2.02%
Output y	0.2900	-8.60%	2.07%	-22.30%	-6.09%
Capital k	0.2491	-1.20%	27.56%	-30.96%	7.53%
Price dispersion s	1.0018	12.27%	6.13%	22.50%	11.23%

The result is not difficult to interpret. As we expect, in case (3), which is also the closest situation as China, the state-owned economy control much more resources and get better benefits than private-owned and collective-owned. Nonetheless, its share over the whole economy also exceeds the other ownerships. Estimated that by 2001, the state-owned assets still hold 50% or more practical capitals of whole society (Ping 2003). Under this circumstance, when the bias exists and with SOEs, it somehow increases the investment and the capital stock. And since the gap between distortion and benefits is large enough, it is still able to grow. But column (4) is totally a different story. Not only are POEs having a bigger distortion, but also taking the larger share of the economy, which makes the situation worse than just partial distortion case in column (2).

4.9 Impulse Response Functions

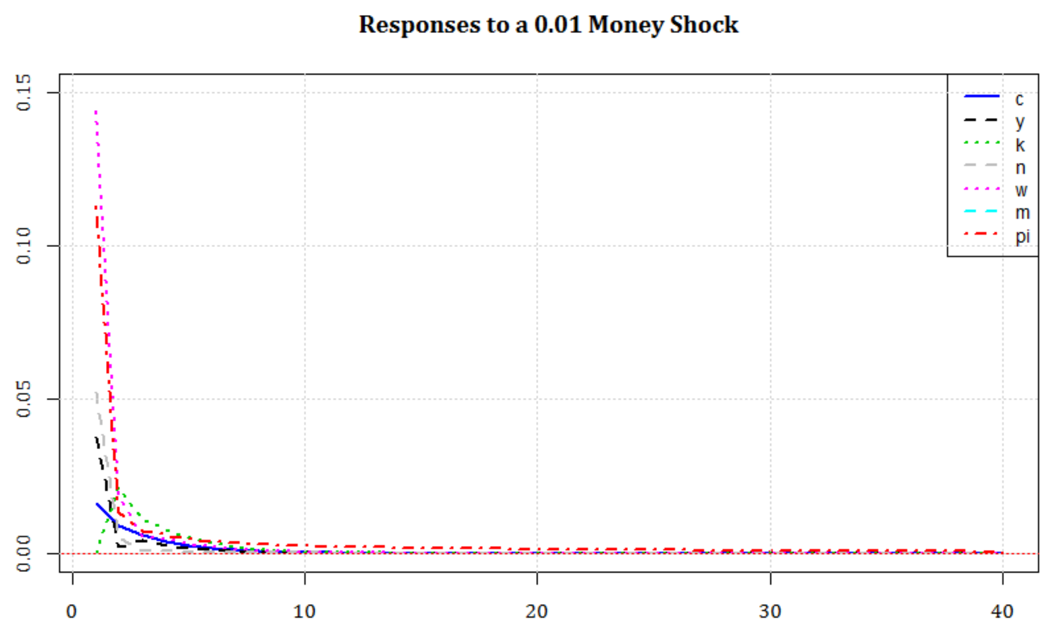

FIGURE 13-IMPULSE RESPONSE OF MAJOR VARIABLES IN A BENCHMARK MODEL TO A 0.01 MONEY SHOCK

Fig 13 shows the responses to a 0.01 money shock of major variables. Unlike the simple RBC model, we can see that all the real variables change very significantly. The response of output performs very similar as the RBC model. The growth in money supply induces an increase in output, and with the rise in marginal cost, wage also rises near to 15%, which consequently leads more people to work. Inflation also goes higher, jumps above 10% after the shock but also returns very quickly, maintaining a relatively mild inflation for a while then goes to steady state. But it still has a more sluggish movement than output, which is very typical in a staggered price model. The real money balances behave negatively due to the higher price level over a smaller increase in nominal money supply. Capital stock is what we are interested in. Some scholars (McCallum and Nelson 1999) argue that capital stock is fixed in a short term thus does not play an important role in monetary policy and business cycle model analysis. But unlike what we learned in the New Keynesian models that capital does not enter, consumption is not hump-shaped here. Because China put a large amount of resources in investment and the capital depreciation and replacement rate are relatively high, it is not wise to ignore capital accumulation in this context of analysis. Capital stock has a hump-shaped path that increases 3 percentage points after shock.

We can adopt impulse response functions to make some clearer comparisons about how the economy respond to technology and money growth shocks as θ and γ change.

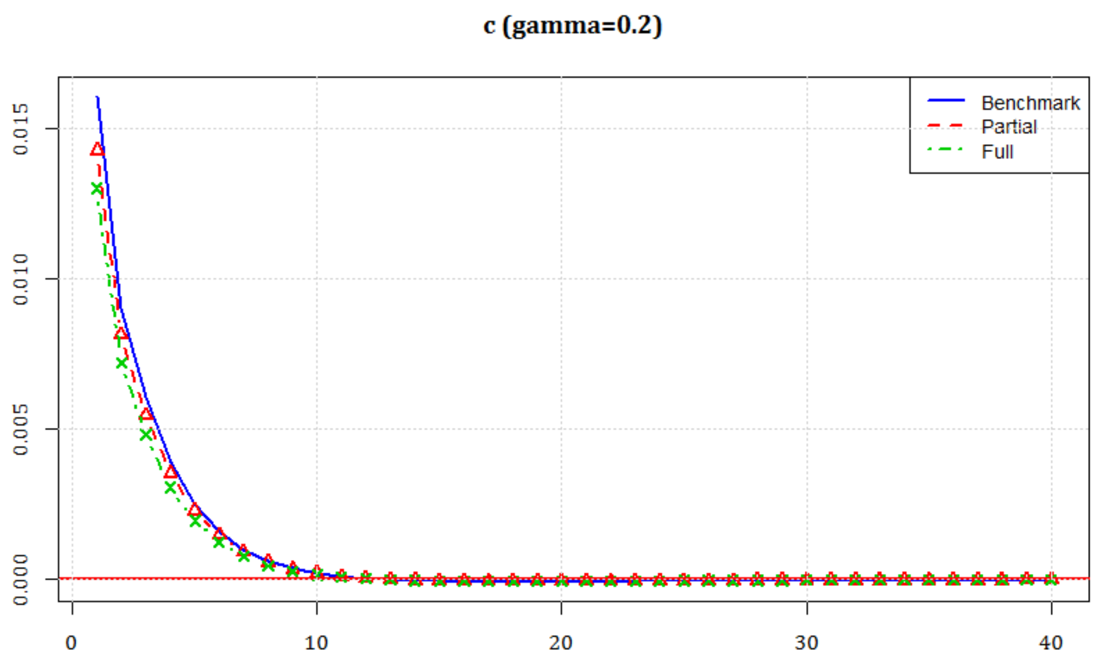

FIGURE 14-IMPULSE RESPONSE OF CONSUMPTION IN A POE-FAVOURED ECONOMY TO A 0.01 MONEY SHOCK

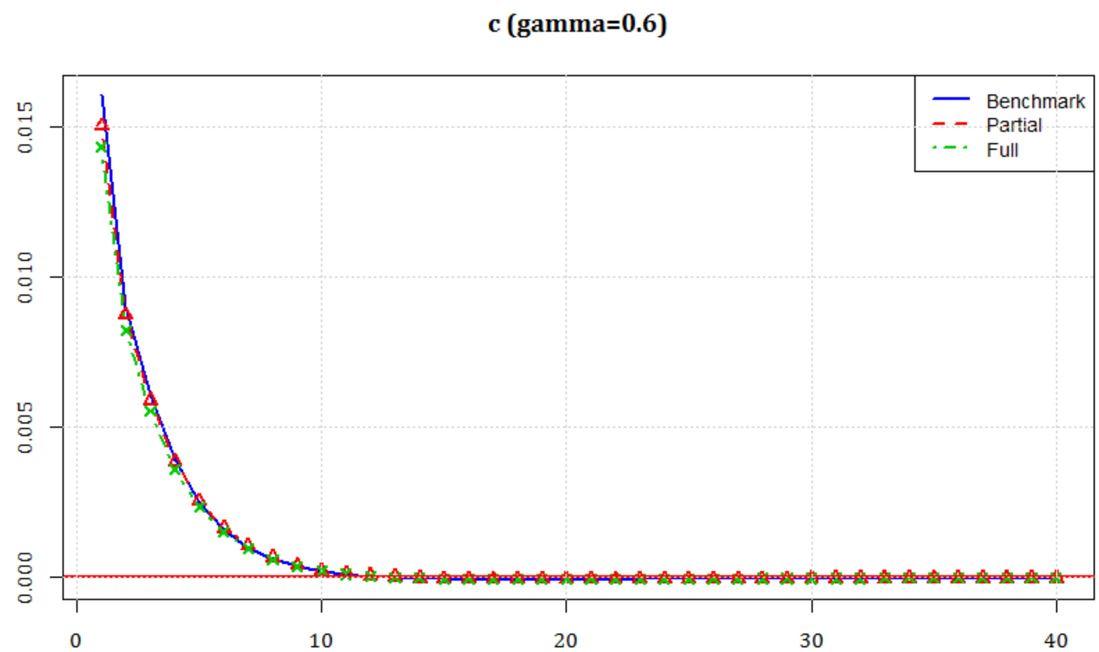

FIGURE 15-IMPULSE RESPONSE OF CONSUMPTION IN A SOE-FAVOURED ECONOMY TO A 0.01 MONEY SHOCK

Consumption in the benchmark model has the biggest deviation from steady state, above 1.5 percentage point after the shock. Case (3) are (5) very close to the benchmark model, while case (2) in the partial distortion case has the similar shape but less deviation (Fig 15 red line), and case (4) has the lowest increase in consumption (Fig 14 green line). In the partial distortion case, an increase in money supply has little effect on the difference between the benchmark model, SOEs-favored model and POEs-favored model. But the discrepancy is magnified in the full distortion case of first three periods.

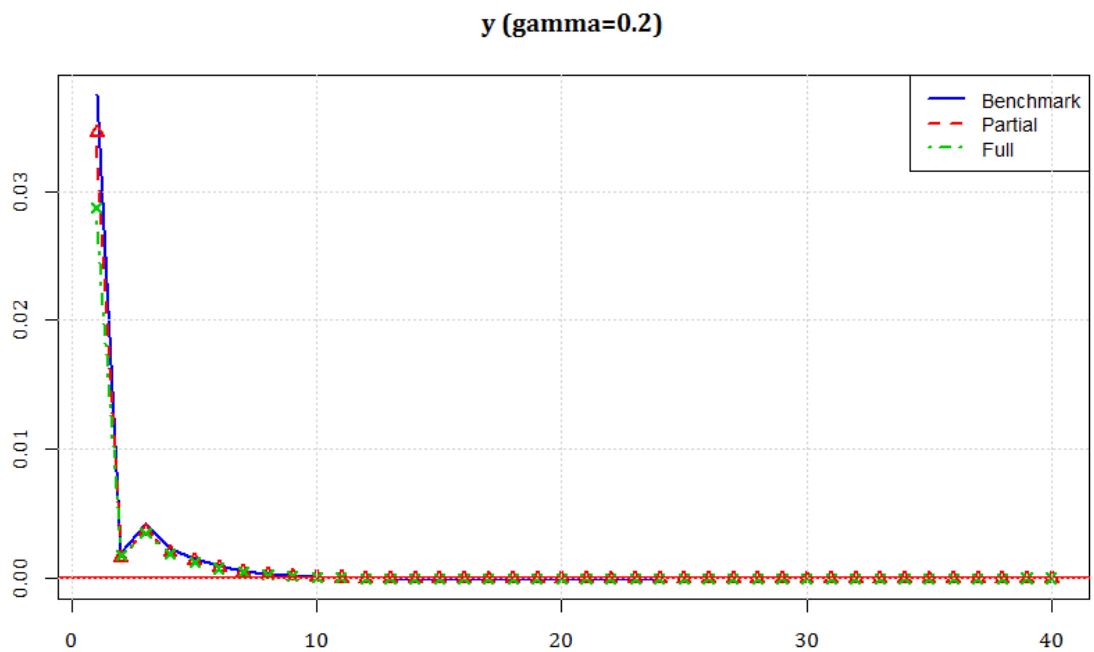

FIGURE 16- IMPULSE RESPONSE OF OUTPUT IN A POE-FAVOURED ECONOMY TO A 0.01 MONEY SHOCK

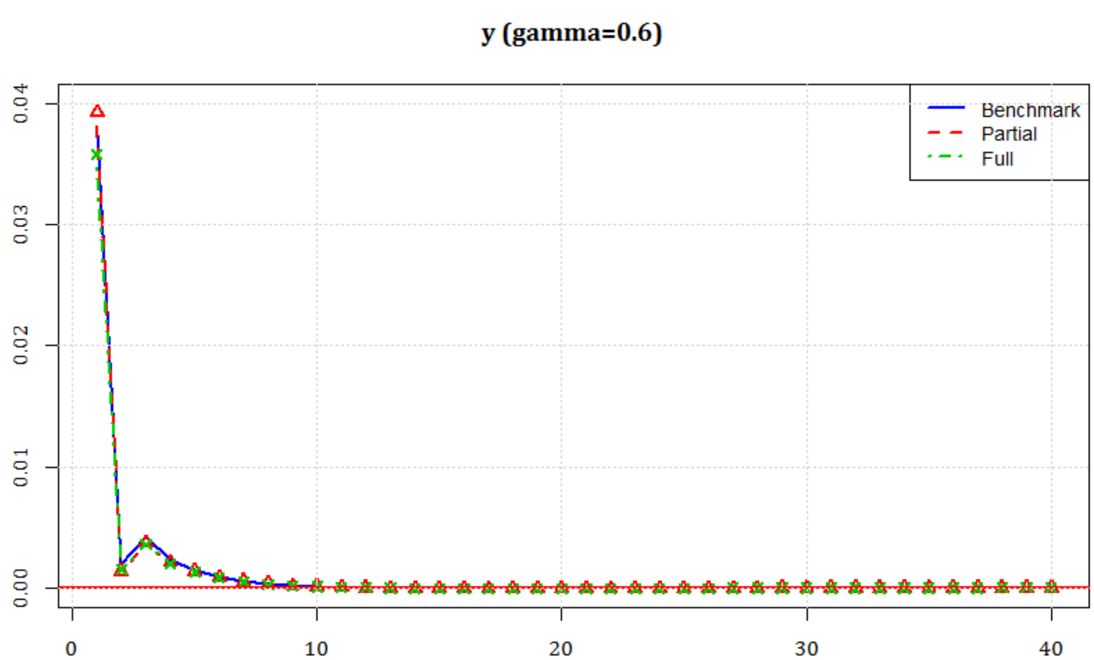

FIGURE 17- IMPULSE RESPONSE OF OUTPUT IN A SOE-FAVOURED ECONOMY TO A 0.01 MONEY SHOCK

What we can see in the response of output is after a money shock, output performs significantly different after first period in POEs-favored case and converges from second period in all cases. When $\gamma = 0.2$, under the full distortion, output only increases less than 3 percentage point where partial distortion case compensate to around 4. But in the SOEs-favored situation, the shock doesn't create so many differences between the two cases, approximately 0.3%. This result raises an important implication that we couldn't get in the simple RBC model: if we were to tax POEs, as long as the subsidy goes to SOEs and SOEs play a major role in this economy, it does not hurt the economy as much as we expect.

Due to the price stickiness, the output takes around 5 to 6 periods to return to steady state very quickly, comparing to a more sluggish return of consumption. The high inflation after the first period adjust people's expectation and hence affect the output. But the consumption preference does not change so dramatically. The same pattern also exists in capital.

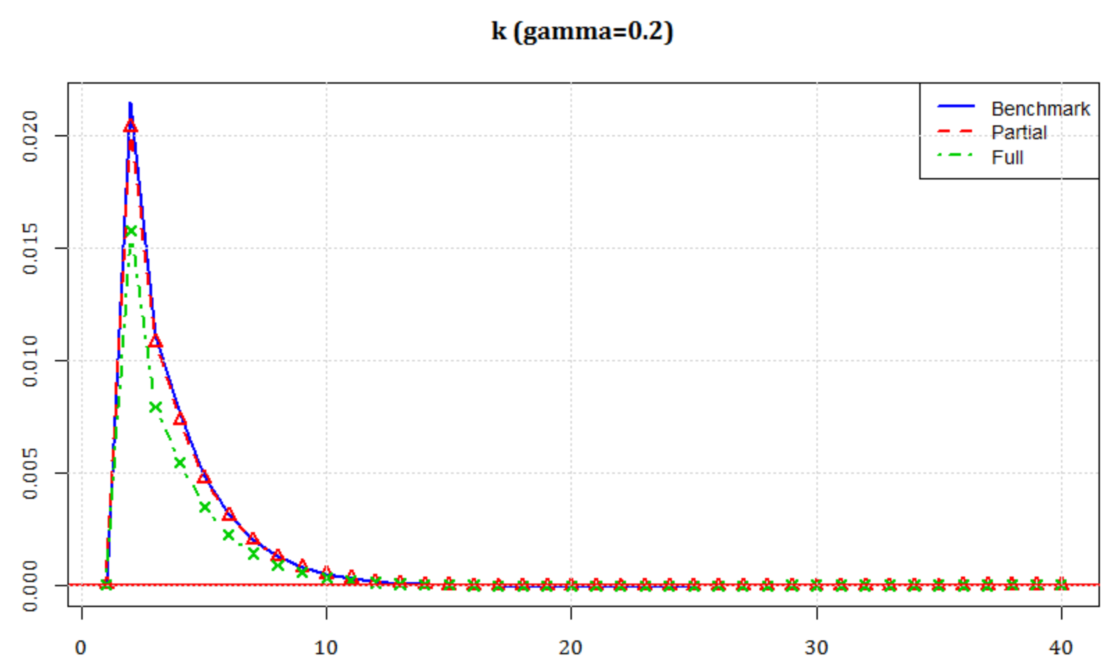

FIGURE 18- IMPULSE RESPONSE OF CAPITAL IN A POE-FAVOURED ECONOMY TO A 0.01 MONEY SHOCK

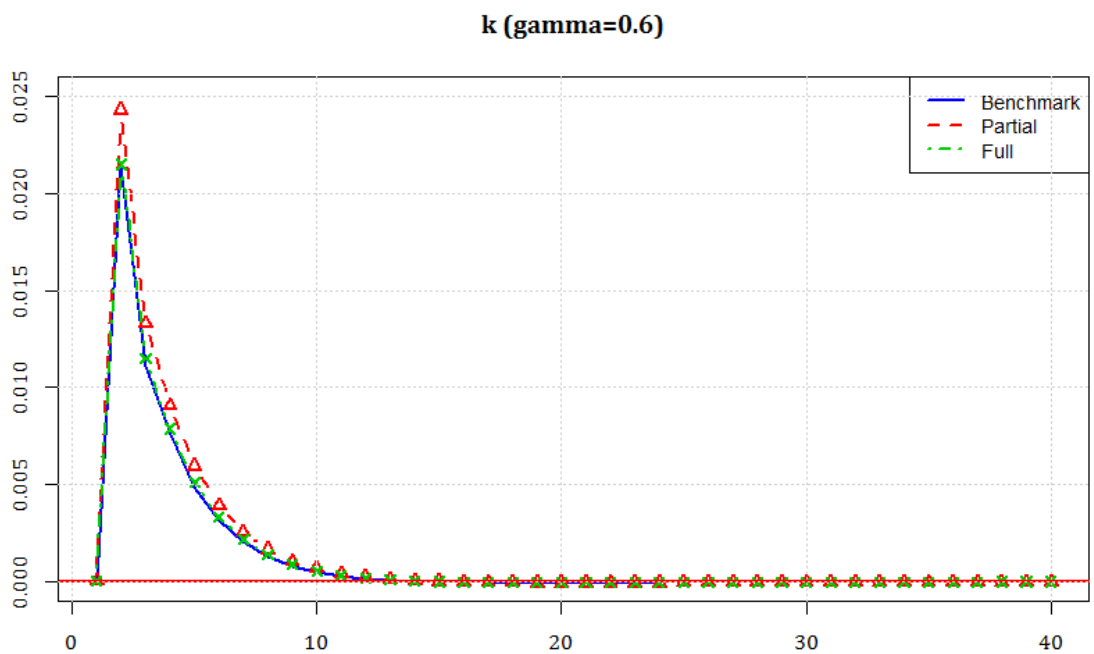

FIGURE 19- IMPULSE RESPONSE OF CAPITAL IN A SOE-FAVOURED ECONOMY TO A 0.01 MONEY SHOCK

Our result of capital stock is interesting. When γ takes 0.2 (Fig 18), the partial distortion case (red line) has a very close path to the benchmark. While under the situation where γ is 0.6, the full distortion case (green line) has the similar path to the benchmark model and partial distortion behaves better than the other two. Where the SOEs-favored situation almost does not alter from the benchmark one, where we see a huge decline in capital stock where in the POEs-favored situation. It we adopt only partial distortion, then capital stock does not change drastically and increases approximately 0.3 percentage point after the shock.

What catches our eyes is that given the share of SOEs constant, full distortion does not twist the money shock. In another word, we can achieve our stimulus target without taking that POEs are discriminated by getting financed into consideration. Output will still go up as long as we increase the size of our shock (Fig 20).

If we give a 0.03 shock to money growth and we will see that the change stands out from the previous cases. Output increases near 11 percentage point and performs better than the partial distortion SOEs-favored case with obvious distinction for 5 periods. The same results also can be found in capital and consumption.

This is the important feature we should discuss in the context of China's characteristics. During the recovery from 2008 financial crisis, we saw a very sharp spike in the money growth as well as the huge increase in inflation. SOEs benefit most from the stimulus plan while POEs are hard to get loans from commercial banks. But this did not slow down the Chinese economy and it stepped out the crisis very quickly due to the constant injection of money and consistent expansionary monetary policy.

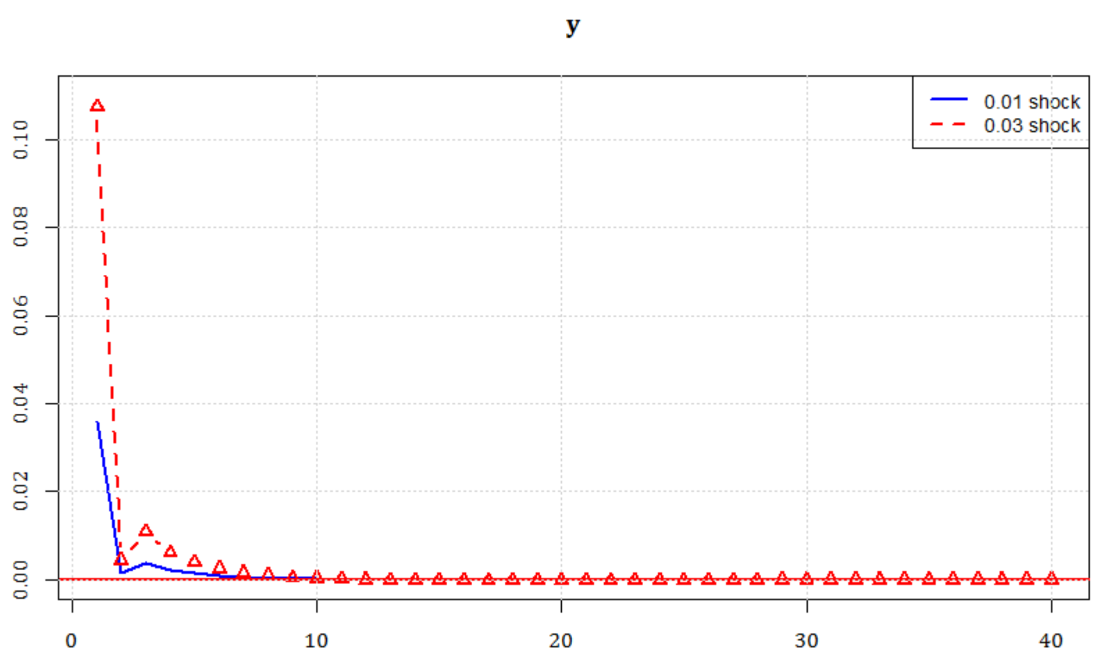

FIGURE 20- IMPULSE RESPONSE OF OUTPUT IN A FULL DISTORTION SOE-FAVOURED ECONOMY TO A 0.01 AND 0.03 MONEY SHOCK

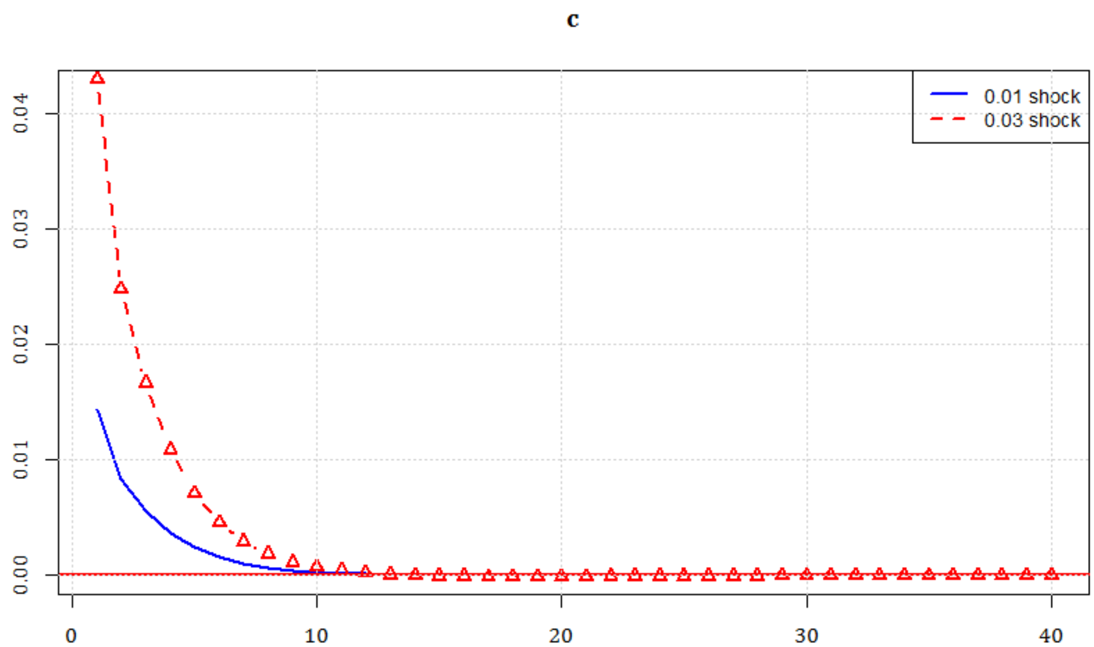

FIGURE 21- IMPULSE RESPONSE OF CONSUMPTION IN A FULL DISTORTION SOE-FAVOURED ECONOMY TO A 0.01 AND 0.03 MONEY SHOCK

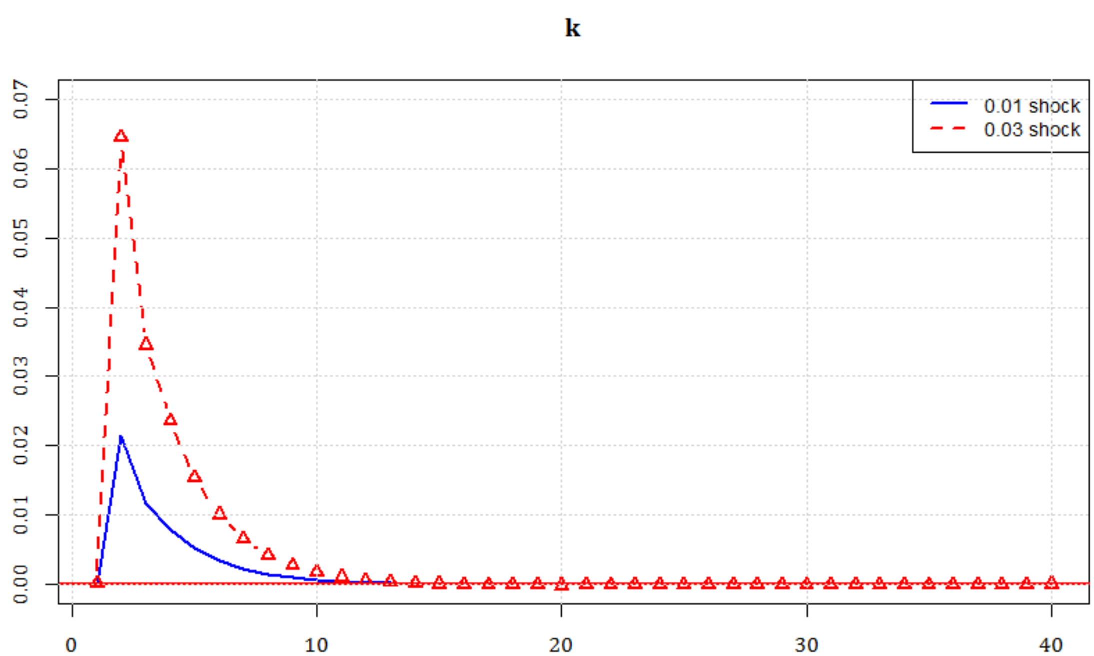

FIGURE 22- IMPULSE RESPONSE OF CAPITAL IN A FULL DISTORTION SOE-FAVOURED ECONOMY TO A 0.01 AND 0.03 MONEY SHOCK

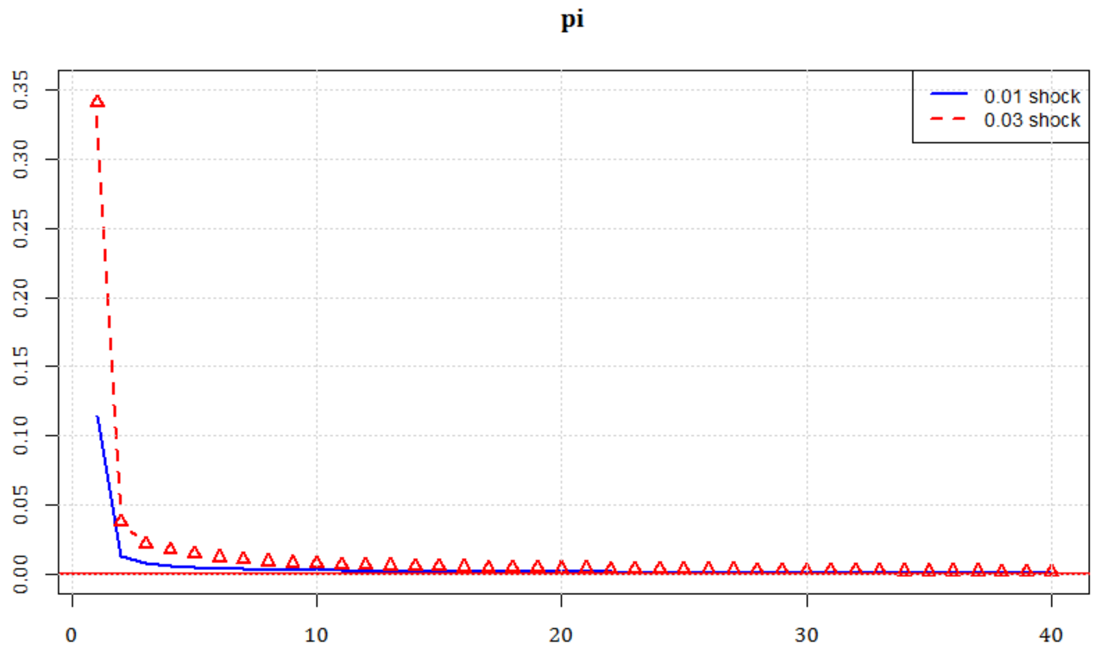

FIGURE 23- IMPULSE RESPONSE OF INFLATION IN A FULL DISTORTION SOE-FAVOURED ECONOMY TO A 0.01 AND 0.03 MONEY SHOCK

Since the higher inflation rate means the lower real interest rate with the nominal interest rate fixed, and given the subsidy to SOEs, the real interest rate is even lower. With sticky price, not all firms could be able to adjust immediately; the aggregate price level will adjust slowly. And hence it stimulus the expenditure and results in an increase in output with the same movement of consumption.

Additionally, in terms of a new Keynesian Philips Curve, we can plot the output with inflation in a same graph and see an approximate relationship between the output and inflation. We know in a traditional setting for new Keynesian Philips Curve, there is a dynamic relation between inflation, expected inflation (forward-looking inflation), and output gap. The coefficient of output gap is related with the marginal cost, which in our case, twisted by the rental cost distortion. Fig 24 and Fig 25 demonstrate that the inflation dynamics do not change so much in both $\gamma = 0.2$ and $\gamma = 0.6$ cases, the ratio of inflation and output gap keeps approximately 3 to 4.

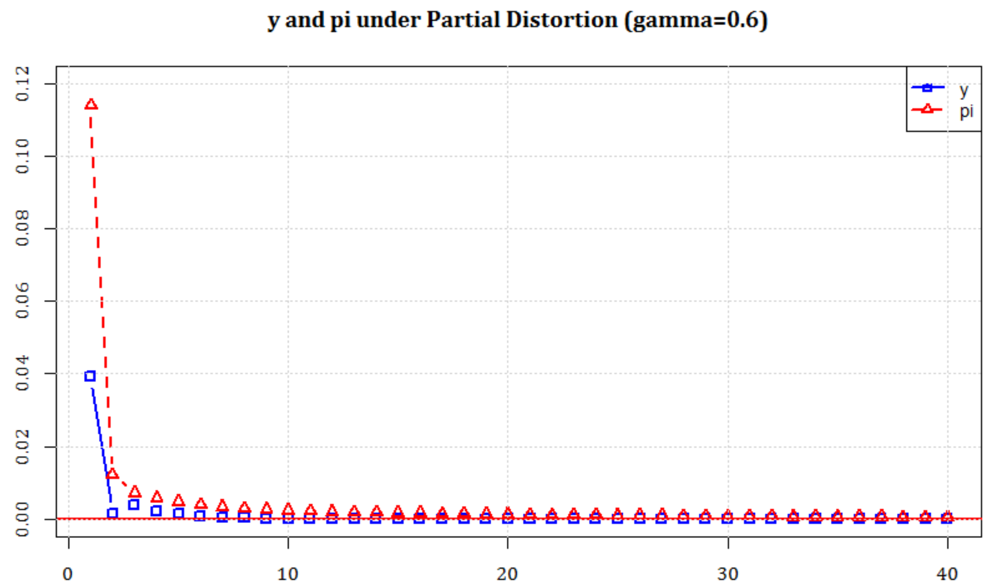

FIGURE 24- IMPULSE RESPONSE OF OUTPUT AND INFLATION IN A FULL DISTORTION SOE-FAVOURED ECONOMY TO A 0.01 AND 0.03 MONEY SHOCK

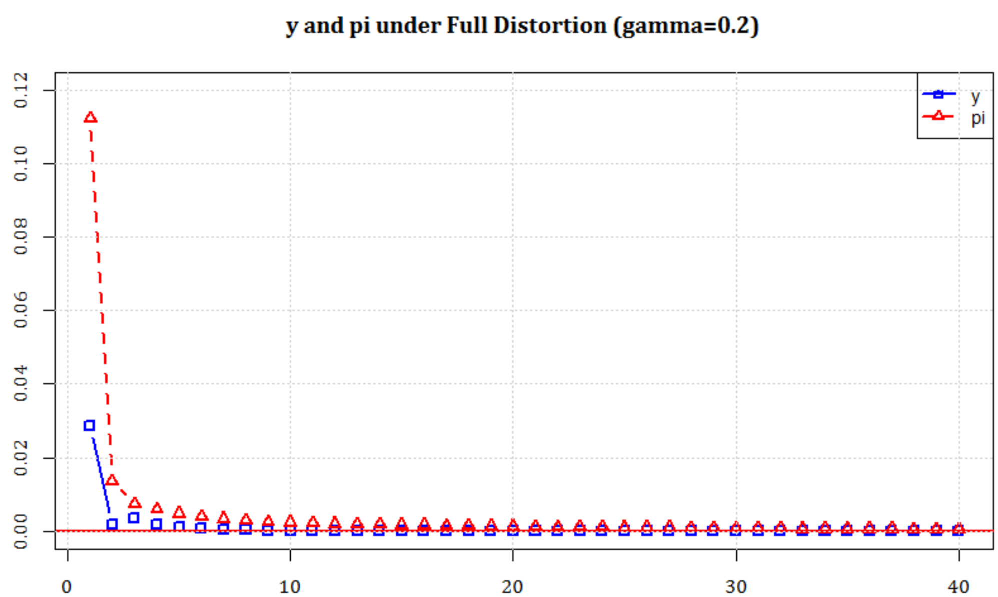

FIGURE 25- IMPULSE RESPONSE OF OUTPUT AND INFLATION IN A FULL DISTORTION POE-FAVOURED ECONOMY TO A 0.01 AND 0.03 MONEY SHOCK

5 Conclusions

Findings from this thesis can be summarized into four parts. (1) With a classical fully flexible price model, monetary policy is neutral both in short term and long term and does not affect real variables and we verified this conclusion in our first section. But a subsidy identical to all firms does reduce the cost and hence boosts the investment and capital accumulation. (2) Money shock could be able to generate an expansion in consumption, output, and capital. Full distortion performs worst among all cases, which is SOEs get real interest rate subsidy and POEs get capital rental tax. But the discrepancy between full distortion, partial distortion and benchmark case will be narrowed down if SOEs take a larger share over the economy. (3) Although monetary policy could lead to a temporary output expansion, in the medium and long term GDP does not respond so well. This result echoes the finding that quantity instrument is not as powerful as price instrument (Zhang 2009). After a 4% increase in output after first period, a 0.01 money growth shock will only lead to 0.5% increase in output and inflation. Even though we expand our shock to 0.03, the growth will still return to a relatively smaller level after the second period. (4) In terms of the lending distortion, we see some different behaviors within capital stock. As what we find in the steady states level of major variables, the capital has the biggest change comparing to other variables. The discrimination between SOEs and POEs does leave the distinctive performance under different values of γ. In a SOEs-favoured economy, even bigger distortion will not create worst outcome given that SOEs get subsidies. (5) The policy implication is if an economy is faced with distortion in different sectors and trying to recover from a recession, then giving the subsidy to the major sector will not jeopardize the economy from recovering. This somehow verifies what Chinese government did during financial crisis. State-owned sector obtained a lot of loans from state-owned commercial banks and the demand for capital and credit is created by supplying money mandatorily. But this is not sustainable because it also creates the inflation problem. As what we also witnessed in China that from 2010 to 2011, the inflation rate reached to 6%-7% monthly which is highly beyond 4%, the target announced by PBC. China also needs to reform its economic structure, since reducing the sectoral frictions is vital for achieving next level of development (Nabar and Yan 2013) and the investment-based growth contributed by large share of state-owned sector should be transformed into consumption-based growth and hence rebalance the domestic demand in China.

However, the author admits that this thesis does not address many of the issues of DSGE models and the fit of data in DSGE estimation. Future work should attempt to include more economic participants such as banking system, and distortions and frictions in financial sector and quantify the accuracy of the real data.

6 Appendix A-Mathematical Derivations

6.1 Simple RBC Model Derivation

6.1.1 Households

$$\max \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{{c_t}^{1-\sigma} - 1}{1-\sigma} + \chi \frac{(1-n_t)^{1-\xi} - 1}{1-\xi} + \nu \frac{{m_t}^{1-\zeta}}{1-\zeta} \right)$$

$$s.t.\ c_t + b_t + m_t = w_t n_t + \Pi_t + (1 + i_{t-1}) \frac{b_{t-1}}{1+\pi_t} + \frac{m_{t-1}}{1+\pi_t}$$

The Lagrangian is:

$$\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \chi \frac{(1-n_t)^{1-\xi} - 1}{1-\xi} + \nu \frac{m_t^{1-\zeta} - 1}{1-\zeta} \right.$$

$$\left. - \lambda_t \left(c_t + b_t + m_t - w_t n_t - \Pi_t - (1 + i_{t-1}) \frac{b_{t-1}}{1+\pi_t} - \frac{m_t}{1+\pi_t} \right) \right\}$$

The FOCs are:

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \iff c_t^{-\sigma} = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = 0 \iff \chi (1-n_t)^{-\xi} = \lambda_t w_t$$

$$\frac{\partial \mathcal{L}}{\partial b_t} = 0 \iff \lambda_t = \beta \lambda_{t+1} \frac{(1+i_t)}{1+\pi_{t+1}}$$

$$\frac{\partial \mathcal{L}}{\partial m_t} = 0 \iff \beta^t \nu m_t^{-\zeta} - \beta^t \lambda_t + \beta^{t+1} \lambda_{t+1} \frac{1}{1+\pi_{t+1}} = 0$$

$$\Rightarrow \nu m_t^{-\zeta} = \lambda_t - \beta \lambda_{t+1} \frac{1}{\pi_{t+1}}$$

We can simplify the first order condition by substituting the first order condition with respect to bond, which is $\beta \lambda_{t+1} \frac{1+i_t}{1+\pi_{t+1}} = \lambda_t$. Thus we get:

$$\nu m_t^{-\zeta} = \lambda_t - \frac{\lambda_t}{1+i_t}$$

$$= \lambda_t \left(1 - \frac{1}{1 + i_t}\right)$$

$$= \lambda_t \left(\frac{i_t}{1 + i_t}\right)$$

$$m_t = \lambda_t^{-\frac{1}{\zeta}} \left(\frac{1}{\nu}\right)^{-\frac{1}{\zeta}} \left(\frac{i_t}{1 + i_t}\right)^{-\frac{1}{\zeta}}$$

Since $c_t^{-\sigma} = \lambda_t$, plugging it into the equation above yields:

$$m_t = c_t^{\frac{\sigma}{\zeta}} \nu^{\frac{1}{\zeta}} \left(\frac{1 + i_t}{i_t}\right)^{\frac{1}{\zeta}}$$

6.1.2 Firms

$$\mathcal{L} = e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha} - w_t n_t - I_t$$

$$+ \mathbb{E}_t \left\{ \sum_{j=1}^{\infty} \prod_{k=1}^{j} (1 + (1 + \theta) r_{t-1+k})^{-1} \left(e^{z_t} k_{t+j-1}^{\alpha} n_{t+j}^{1-\alpha} - w_{t+j} n_{t+j} - I_{t+j}\right) \right.$$

$$\left. - \phi_t (k_t - (1 - \delta) k_{t-1} - I_t) \right\}$$

The FOCs are:

$$\frac{\partial \mathcal{L}}{\partial n_t} = 0 \Leftrightarrow w_t = (1 - \alpha) e^{z_t} k_{t-1}^{\alpha} n_t^{1-\alpha}$$

$$\frac{\partial \mathcal{L}}{\partial I_t} = 0 \Leftrightarrow \phi_t = 1$$

$$\frac{\partial \mathcal{L}}{\partial k_t} = 0 \Leftrightarrow \frac{1}{1 + (1 + \theta\gamma) r_t} \left(e^{z_{t+1}} k_t^{\alpha-1} n_{t+1}^{1-\alpha} + \phi_{t+1}(1 - \delta)\right) = -\phi_t$$

$$\Leftrightarrow (1 + \theta) r_t = e^{z_{t+1}} k_t^{\alpha-1} n_{t+1}^{1+\alpha} - \delta$$

6.1.3 Government

$$\ln \frac{M_t}{M_{t-1}} = (1 - \rho_m) \pi^* + \rho_m \ln \frac{M_{t-1}}{M_{t-2}} + \epsilon_{m,t}$$

Notice this is in nominal term, to get the real term we need to make several changes:

$$(\ln M_t - \ln M_{t-1} - \ln p_t + \ln p_{t-1} + \ln p_t - \ln p_{t-1}$$
$$= (1-\rho_m)\pi^* + \rho_m(\ln M_{t-1} - \ln M_{t-2} - \ln p_{t-1} + \ln p_{t-2} + \ln p_{t-1} - \ln p_{t-2} + \varepsilon_{m,t}$$

Since $\ln p_t - \ln p_{t-1} = \pi_t$, we have

$$m_t - m_{t-1} + \pi_t = (1-\rho_m)\pi^* + \rho_m(m_{t-1} - m_{t-2}) + \rho_m \pi_{t-1} + \varepsilon_{m,t}$$

6.1.4 Dynare Codes

```
%RBC Model

var c n w y k r inv z m i pi dm;

varexo epsz epsm;

parameters chi xi beta alpha delta rho s pis phi rhom  theta ;
beta=0.98;
alpha=0.33;
delta=0.05;
chi=2;
xi=1;
rho = 0.95;
s   = 0.007;
pis =0.02;
phi = 0.5;
rhom = 0.95;
theta=0.8;

model;
c*chi*((1-n)^(-xi)) = w;
1/beta=(c/c(+1))*(1+r);
w=(1-alpha)*(k(-1)^alpha)*(n^(-alpha))*exp(z(-1));
1+theta*r=exp(z(-1))*alpha*(k(-1)^(alpha-1))*(n^(1-alpha))+(1-
delta);
k=inv+(1-delta)*k(-1);
y=exp(z(-1))*(k(-1)^alpha)*(n^(1-alpha));
y=c+inv;
z=rho*z(-1)+s*epsz;

%Monetary side
m=c*phi*((1+i)/i);
(1+i)=(1+pi(+1))*(1+r);
dm+pi=((1-rhom)*pis)+(rhom*pi(-1))+(rhom*dm(-1))+epsm;
dm=ln(m)-ln(m(-1));
end;

initval;
epsm=0;
c =              0.921834;
n =              0.254668;
w =              1.69041;
y =              1.14713;
k =              7.50973;
r =              0.0204082;
inv =            0.225292;
z =              0;
epsm=0;
dm=0;
pi=pis;
i=0.03;
end;
```

```
shocks;
var epsz; stderr 0.01;
var epsm; stderr 0.01;
end;

steady;
stoch_simul(order=1,irf=100)
```

6.2 NK-DSGE Model Derivation

6.2.1 The Final Goods Producers

The profit maximization problem is:

$$\max P_t\left[\int_0^1 Y_t(i)^{\frac{\psi-1}{\psi}}di\right]^{\frac{\psi}{\psi-1}} - \int_0^1 P_t(i)Y_t(i)di$$

Differentiating with respect to $Y_t(i)$:

$$P_t\frac{\psi}{\psi-1}\left[\int_0^1 Y_t(i)^{\frac{\psi-1}{\psi}}di\right]^{\frac{1}{\psi-1}}\frac{(\psi-1)}{\psi}Y_t(i)^{-\frac{1}{\psi}} - P_t(i) = 0$$

$$Y_t(i) = Y_t\left[\frac{P_t(i)}{P_t}\right]^{-\psi}$$

Then we can substitute the demand back to the bundle good and get the production function as:

$$Y_t = \left\{\int_0^1\left[Y_t\left(\frac{P_t(i)}{P_t}\right)^{-\psi}\right]^{\frac{\psi-1}{\psi}}di\right\}^{\frac{\psi}{\psi-1}} = Y_t\left[\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{1-\psi}di\right]^{\frac{\psi}{\psi-1}}$$

simplifying the above equation gives

$$1 = P_t^{\psi}\left[\int_0^1\left(\frac{1}{P_t(i)}\right)^{\psi-1}di\right]^{\frac{\psi}{\psi-1}}$$

which can be written as

$$\frac{1}{P_t} = \left[\int_0^1\left(\frac{1}{P_t(i)}\right)^{\psi-1}di\right]^{\frac{1}{\psi-1}}$$

thus we get the final goods pricing formula:

$$P_t = \left[\int_0^1 P_t(i)^{1-\psi} di \right]^{\frac{1}{1-\psi}}$$

6.2.2 The Intermediate Goods Producers

$$\mathcal{L} = w_t n_t(i) + \theta r_t k_t(i) + \phi_t(i) \left\{ Y_t \left[\frac{P_t(i)}{P_t} \right]^{-\psi} - A_t k_t(i)^{\alpha} n_t(i)^{1-\alpha} \right\}$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial k_t(i)} = 0 \Leftrightarrow \theta r_t = \phi_t(i) \alpha A_t k_t(i)^{\alpha-1} n_t(i)^{1-\alpha}, \theta = \{\theta_1, \theta_2\}$$

$$\frac{\partial \mathcal{L}}{\partial n_t(i)} = 0 \Leftrightarrow w_t = \phi_t(i)(1-\alpha) A_t k_t(i)^{\alpha} n_t(i)^{-\alpha}$$

Comparing the two equations and dividing w_t by r_t eliminates $\phi_t(i)$, hence we have:

$$\frac{w_t}{r_t} = \theta \frac{1-\alpha}{\alpha} \frac{k_t(i)}{n_t(i)}$$

Rewrite the above equation yields capital-labour ratio:

$$\frac{k_t(i)}{n_t(i)} = \frac{1}{\theta} \frac{\alpha}{1-\alpha} \frac{w_t}{r_t}$$

Combining these two FOCs with production function we can solve for the optimal demand for labour and capital:

$$Y_t(i) = A_t k_t(i)^{\alpha} n_t(i)^{1-\alpha}$$

$$n_t(i)^{1-\alpha} = \frac{Y_t(i)}{A_t k_t(i)^{\alpha}}$$

From capital-labour ratio we have

$$k_t(i) = \frac{1}{\theta} \frac{\alpha}{1-\alpha} \frac{w_t}{r_t} \, n_t(i)$$

$$= \frac{1}{\theta} \frac{\alpha}{1-\alpha} \frac{w_t}{r_t} \left[\frac{Y_t(i)}{A_t k_t(i)^{\alpha}} \right]^{\frac{1}{1-\alpha}}$$

$$k_t(i)^{1-\alpha} = \left(\frac{1}{\theta}\frac{\alpha}{1-\alpha}\frac{w_t}{r_t}\right)^{1-\alpha} \frac{Y_t(i)}{A_t k_t(i)^{\alpha}}$$

$$\Rightarrow k_t(i)^* = \left(\frac{1}{\theta}\frac{\alpha}{1-\alpha}\frac{w_t}{r_t}\right)^{1-\alpha} \frac{Y_t(i)}{A_t}$$

$$n_t(i)^* = \left(\theta\frac{1-\alpha}{\alpha}\frac{r_t}{w_t}\right)^{\alpha} \frac{Y_t(i)}{A_t}$$

Substituting $Y_t(i)$ into above two equations gives us the total nominal cost:

$$TotalCost = w_t n_t(i) + \theta r_t k_t(i)$$

$$= \theta^{\alpha} \frac{w_t^{1-\alpha} r_t}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \frac{Y_t(i)}{A_t}$$

Differentiating total cost with respect to $Y_t(i)$ delivers the marginal cost:

$$mc_{1,t} = A_t^{-1}\theta_1^{\alpha} \frac{w_t^{1-\alpha} r_t^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} = \theta_1^{\alpha} mc_t$$

$$mc_{2,t} = A_t^{-1}\theta_2^{\alpha} \frac{w_t^{1-\alpha} r_t^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} = \theta_2^{\alpha} mc_t$$

Where mc_t represents the marginal cost for the non-friction case.

6.2.3 Calvo Pricing

$$\max_{P_t(i)} \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta_{t+j} \left[\frac{P_t(i)}{P_t} Y_{t+j}(i) - \theta_1^{\alpha} mc_{t+j} Y_{t+j}(i)\right]$$

$$s.t. Y_{t+j}(i) = A_{t+j} k_{t+j}(i)^{\alpha} n_{t+j}(i)^{1-\alpha}$$

Plug the budget constraint into objective function. The problem is equivalent to:

$$\max_{P_t^s(i)} \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta_{t+j} \left\{\left[\frac{P_t(i)}{P_{t+j}}\left(\frac{P_t(i)}{P_{t+j}}\right)^{-\psi} - \theta_1^{\alpha} mc_{t+j}\left(\frac{P_t(i)}{P_{t+j}}\right)^{-\psi}\right] Y_{t+j}\right\}$$

$$= \max_{P_t^s(i)} \mathbb{E}_t \sum_{j=0}^{\infty} \rho^j \Delta_{t+j} \left\{\left[\left(\frac{P_t(i)}{P_{t+j}}\right)^{1-\psi} - \theta_1^{\alpha} mc_{t+j}\left(\frac{P_t(i)}{P_{t+j}}\right)^{-\psi}\right] Y_{t+j}\right\}$$

The first order condition with respect to $P_t(i)$ is:

$$0=\mathbb{E}_t\sum_{j=0}^{\infty}\rho^j\Delta_{t+j}\left\{\left[(1-\psi)P_t(i)^{-\psi}P_{t+j}^{\psi-1}-(-\psi)\theta_1^{\alpha}mc_{t+j}P_t(i)^{-\psi-1}P_{t+j}^{\psi}\right]Y_{t+j}\right\}$$

$$\Leftrightarrow \mathbb{E}_t\sum_{j=0}^{\infty}\rho^j\Delta_{t+j}(\psi-1)P_t(i)^{-\psi}P_{t+j}^{\psi-1}Y_{t+j}=\ \mathbb{E}_t\sum_{j=0}^{\infty}\rho^j\Delta_{t+j}\psi\theta_1^{\alpha}mc_{t+j}P_t(i)^{-\psi-1}P_{t+j}^{\psi}Y_{t+j}$$

$$\mathbb{E}_t\sum_{j=0}^{\infty}\rho^j\Delta_{t+j}(\psi-1)P_t(i)P_{t+j}^{\psi-1}Y_{t+j}=\ \mathbb{E}_t\sum_{j=0}^{\infty}\rho^j\Delta_{t+j}\psi\theta_1^{\alpha}mc_{t+j}P_{t+j}^{\psi}Y_{t+j}$$

$$P_t(i)^*=\theta_1^{\alpha}\frac{\psi}{\psi-1}\frac{\mathbb{E}_t\sum_{j=0}^{\infty}\rho^j\Delta_{t+j}mc_{t+j}P_{t+j}^{\psi}Y_{t+j}}{\mathbb{E}_t\sum_{j=0}^{\infty}\rho^j\Delta_{t+j}P_{t+j}^{\psi-1}Y_{t+j}}$$

For SOEs, $P_t(i)^*=\theta_1^{\alpha}\frac{\psi}{\psi-1}\frac{\mathbb{E}_t\sum_{j=0}^{\infty}\rho^j\Delta_{t+j}mc_{t+j}P_{t+j}^{\psi}Y_{t+j}}{\mathbb{E}_t\sum_{j=0}^{\infty}\rho^j\Delta_{t+j}P_{t+j}^{\psi-1}Y_{t+j}}$.

For POEs, $P_t(i)^{\#}=\theta_2^{\alpha}\frac{\psi}{\psi-1}\frac{\mathbb{E}_t\sum_{j=0}^{\infty}\rho^j\Delta_{t+j}mc_{t+j}P_{t+j}^{\psi}Y_{t+j}}{\mathbb{E}_t\sum_{j=0}^{\infty}\rho^j\Delta_{t+j}P_{t+j}^{\psi-1}Y_{t+j}}=\frac{\theta_2^{\alpha}}{\theta_1^{\alpha}}P_t(i)^*$

6.2.4 Households

$$\max\mathbb{E}_t\sum_{t=0}^{\infty}\beta^t\left\{\frac{c_t^{1-\sigma}}{1-\sigma}-\chi\frac{n_t^{1+\eta}}{1+\eta}+\frac{\upsilon}{1-\zeta}\left(\frac{M_t}{P_t}\right)^{1-\zeta}\right\}$$

$$c_t+\frac{B_{t+1}}{P_t}+\frac{M_t}{P_t}+I_t=\frac{W_t}{P_t}n_t+r_tk_t+\Pi_t+(1+i_t)\frac{B_t}{P_t}+\frac{M_{t-1}}{P_t}+\Pi_t+T_t$$

$$k_{t+1}=I_t+(1-\delta)k_t$$

The Lagrangian is:

$$\mathcal{L}=\mathbb{E}_t\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{c}\frac{c_t^{1-\sigma}}{1-\sigma}-\chi\frac{n_t^{1+\eta}}{1+\eta}+\frac{\upsilon}{1-\zeta}\left(\frac{M_t}{p_t}\right)^{1-\zeta}\\+\lambda_t\left(\begin{array}{c}w_tn_t+r_tk_t+(1+i_t)\frac{B_t}{P_t}+\frac{M_{t-1}}{P_t}+\Pi_t+T_t\\-c_t-\frac{B_{t+1}}{P_t}-\frac{M_t}{P_t}-k_{t+1}+(1-\delta)k_t\end{array}\right)\end{array}\right\}$$

The FOCs are:

$$\frac{\partial L}{\partial c_t}=0\Leftrightarrow c_t^{-\sigma}=\lambda_t$$

$$\frac{\partial L}{\partial n_t} = 0 \Leftrightarrow -\beta^t \chi n_t^\eta + \beta^t \lambda_t w_t = 0 \Leftrightarrow \chi n_t^\eta = \lambda_t w_t$$

$$\Leftrightarrow \chi n_t^\eta = c_t^{-\sigma} w_t$$

$$\frac{\partial L}{\partial B_{t+1}} = 0 \Leftrightarrow -\beta^t \lambda_t \frac{1}{P_t} + \beta^{t+1} \mathbb{E}_t \lambda_{t+1} \frac{(1 + i_{t+1})}{P_{t+1}} = 0$$

$$\Leftrightarrow \frac{\lambda_t}{P_t} = \beta \mathbb{E}_t \lambda_{t+1} \frac{(1 + i_{t+1})}{P_{t+1}}$$

$$\Leftrightarrow c_t^{-\sigma} = \beta \mathbb{E}_t c_{t+1}^{-\sigma} (1 + i_{t+1}) \frac{P_t}{P_{t+1}}$$

$$\frac{\partial L}{\partial M_t} = 0 \Leftrightarrow \beta^t \upsilon \left(\frac{M_t}{P_t}\right)^{-\zeta} \frac{1}{P_t} + \beta^t \lambda_t \left(-\frac{1}{P_t}\right) + \mathbb{E}_t \beta^{t+1} \lambda_{t+1} \frac{1}{P_{t+1}} = 0$$

$$\Leftrightarrow \upsilon \left(\frac{M_t}{P_t}\right)^{-\zeta} \frac{1}{P_t} = \lambda_t \frac{1}{P_t} - \beta \mathbb{E}_t \lambda_{t+1} \frac{1}{P_{t+1}}$$

$$\Leftrightarrow \upsilon \left(\frac{M_t}{P_t}\right)^{-\zeta} \frac{1}{P_t} = \frac{c_t^{-\sigma}}{P_t} - \beta \mathbb{E}_t \frac{c_{t+1}^{-\sigma}}{P_{t+1}}$$

$$\Leftrightarrow \upsilon \left(\frac{M_t}{P_t}\right)^{-\zeta} = c_t^{-\sigma} - \beta \mathbb{E}_t c_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}}$$

$$\frac{\partial L}{\partial k_{t+1}} = -\beta^t \lambda_t + \mathbb{E}_t \beta^{t+1} \lambda_{t+1} (r_{t+1} + 1 - \delta) = 0$$

$$\Leftrightarrow \lambda_t = \beta \mathbb{E}_t \lambda_{t+1} (r_{t+1} + 1 - \delta)$$

$$\Leftrightarrow c_t^{-\sigma} = \beta \mathbb{E}_t c_{t+1}^{-\sigma} (r_{t+1} + 1 - \delta)$$

In summary, we have the set of households first order conditions:

$$\chi n_t^\eta = c_t^{-\sigma} w_t$$

$$c_t^{-\sigma} = \beta \mathbb{E}_t c_{t+1}^{-\sigma} (1 + i_{t+1}) \frac{1}{\pi_{t+1}}$$

$$\upsilon \left(\frac{M_t}{P_t}\right)^{-\zeta} = c_t^{-\sigma} - \beta \mathbb{E}_t c_{t+1}^{-\sigma} \frac{1}{\pi_{t+1}}$$

$$c_t^{-\sigma} = \beta \mathbb{E}_t c_{t+1}^{-\sigma} (r_{t+1} + 1 - \delta)$$

6.2.5 Government

$$\varDelta \ln m_t \equiv \ln m_t - \ln m_{t-1}$$

$$\ln m_t \equiv \ln m_t - \ln m_{t-1}$$

$$= \ln M_t - \ln P_t - \ln M_{t-1} + \ln P_{t-1}$$

$$= \ln M_t - \ln M_{t-1} - \pi_t$$

which gives $\Delta \ln M_t = \Delta \ln m_t + \pi_t$.

Hence we have

$$\varDelta \ln m_t = (1 - \rho_m)\pi - \pi_t + \rho_m \varDelta \ln m_{t-1} + \rho_m \pi_{t-1} + \varepsilon_{m,t}$$

The lump-sum transfer is defined as following:

$$T_t = M_t - M_{t-1} + B_{t+1} - (1 + i_t)B_t$$

6.2.6 Equilibrium

6.2.6.1 Aggregate Output

On the demand side,

$$Y_t(i) = A_t k_t(i)^\alpha n_t(i)^{1-\alpha}$$

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\psi} Y_t$$

$$\Rightarrow A_t k_t(i)^\alpha n_t(i)^{1-\alpha} = \left[\frac{P_t(i)}{P_t}\right]^{-\psi} Y_t$$

Integrate over i we have:

$$\int_0^1 A_t k_t(i)^\alpha n_t(i)^{1-\alpha} di = \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\psi} Y_t di$$

$$A_t \int_0^1 k_t(i)^\alpha n_t(i)^{1-\alpha} di = Y_t \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\psi} di$$

$$A_t k_t^\alpha n_t^{1-\alpha} = Y_t \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\psi} di$$

Define s_t as price dispersion:

$$s_t = \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\psi} di$$

Plugging it back to give the adjusted output:

$$Y_t = \frac{A_t k_t^{\alpha} n_t^{1-\alpha}}{s_t}$$

6.2.6.2 Capital and Labour Market

$$\int_0^1 k_t(i)di = \int_0^{\gamma} \left(\frac{1}{\theta_1}\frac{\alpha}{1-\alpha}\frac{w_t}{r_t}\right)^{1-\alpha} \frac{Y_t(i)}{A_t} di + \int_{\gamma}^1 \left(\frac{1}{\theta_2}\frac{\alpha}{1-\alpha}\frac{w_t}{r_t}\right)^{1-\alpha} \frac{Y_t(i)}{A_t} di$$

$$k_t = \gamma \left[\frac{1}{\theta_1}\left(\frac{\alpha}{1-\alpha}\right)\frac{w_t}{r_t}\right]^{1-\alpha} \frac{Y_t}{A_t} + (1-\gamma)\left[\frac{1}{\theta_2}\left(\frac{\alpha}{1-\alpha}\right)\frac{w_t}{r_t}\right]^{1-\alpha} \frac{Y_t}{A_t}$$

Similarly,

$$n_t(i)^* = \left(\theta \frac{1-\alpha}{\alpha}\frac{r_t}{w_t}\right)^{\alpha} \frac{Y_t(i)}{A_t}$$

$$n_t = \gamma \left[\theta_1 \frac{1-\alpha}{\alpha}\frac{r_t}{w_t}\right]^{\alpha} \frac{Y_t}{A_t} + (1-\gamma)\left[\theta_2 \frac{1-\alpha}{\alpha}\frac{r_t}{w_t}\right]^{\alpha} \frac{Y_t}{A_t}$$

By Walras' Law, we only need one equation of the above two to calculate the steady state.

6.2.6.3 Price Level and Inflation

$$s_t = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\psi} di$$

$$= \int_0^{1-\rho} \left(\frac{P_t^{adjust}}{P_t}\right)^{-\psi} di + \int_{1-\rho}^1 \left(\frac{P_{t-1}(i)}{P_t}\right)^{-\psi} di$$

$$= \int_0^{1-\rho} \left(\frac{\gamma P_t^* + (1-\gamma)P_t^{\#}}{P_t}\right)^{-\psi} di + \int_{1-\rho}^1 \left(\frac{P_{t-1}(i)}{P_t}\right)^{-\psi} di$$

$$= \int_0^{1-\rho} \left(\frac{\gamma P_t^* + (1-\gamma)\left(\frac{\theta_2}{\theta_1}\right)^{\alpha} P_t^*}{P_t}\right)^{-\psi} di + \int_{1-\rho}^1 \left(\frac{P_{t-1}(i)}{P_t}\right)^{-\psi} di$$

$$= \int_0^{1-\rho} \left(\frac{\left[\gamma + (1-\gamma) \left(\frac{\theta_2}{\theta_1}\right)^{\alpha} \right] P_t^*}{P_t} \right)^{-\psi} di + \int_{1-\rho}^{1} \left(\frac{P_{t-1}(i)}{P_t} \right)^{-\psi} di$$

$$= \left[\gamma + (1-\gamma) \left(\frac{\theta_2}{\theta_1}\right)^{\alpha} \right] \int_0^{1-\rho} \left(\frac{P_t^*}{P_t} \right)^{-\psi} di + \int_{1-\rho}^{1} \left(\frac{P_{t-1}(i)}{P_t} \right)^{-\psi} di$$

$$= \left[\gamma + (1-\gamma) \left(\frac{\theta_2}{\theta_1}\right)^{\alpha} \right] \int_0^{1-\rho} \left(\frac{P_t^*}{P_{t-1}} \frac{P_{t-1}}{P_t} \right)^{-\psi} di + \int_{1-\rho}^{1} \left(\frac{P_{t-1}(i)}{P_{t-1}} \frac{P_{t-1}}{P_t} \right)^{-\psi} di$$

$$= (1-\rho) \left[\gamma + (1-\gamma) \left(\frac{\theta_2}{\theta_1}\right)^{\alpha} \right] \pi_t^{*-\psi} \pi_t^{\psi} + \pi_t^{\psi} \int_{1-\rho}^{1} \left(\frac{P_{t-1}(i)}{P_{t-1}} \frac{P_{t-1}}{P_t} \right)^{-\psi} di$$

$$= (1-\rho) \left[\gamma + (1-\gamma) \left(\frac{\theta_2}{\theta_1}\right)^{\alpha} \right] \pi_t^{*-\psi} \pi_t^{\psi} + \pi_t^{\psi} \rho \int_0^{1} \left(\frac{P_{t-1}(i)}{P_t} \right)^{-\psi} di$$

$$= (1-\rho) \left[\gamma + (1-\gamma) \left(\frac{\theta_2}{\theta_1}\right)^{\alpha} \right] \pi_t^{*-\psi} \pi_t^{\psi} + \rho \pi_t^{\psi} s_{t-1}$$

Recall the reset optimal price can be written as $P_t^* = \theta_1^{\alpha} \frac{\psi}{\psi-1} \frac{X_{1,t}}{X_{2,t}}$. Define two new auxiliary variables in real terms, we have:

$$x_{1,t} \equiv \frac{X_{1,t}}{P_t^{\psi}}$$

$$x_{2,t} \equiv \frac{X_{2,t}}{P_t^{\psi}}$$

which can be written in this way:

$$x_{1,t} = u'(c_t) mc_t Y_t + \rho\beta \mathbb{E}_t \frac{X_{1,t+1}}{P_t^{\psi}}$$

$$x_{2,t} = u'(c_t) Y_t + \rho\beta \mathbb{E}_t \frac{X_{2,t+1}}{P_t^{\psi-1}}$$

In order to get inflation enter the reset price, we need to add P_{t+1} into above equations.

$$x_{1,t} = u'(c_t) mc_t Y_t + \rho\beta \mathbb{E}_t \frac{X_{1,t+1}}{P_t^{\psi}} \frac{P_{t+1}^{\psi}}{P_{t+1}^{\psi}}$$

$$= u'(c_t)mc_t Y_t + \rho\beta\mathbb{E}_t \frac{X_{1,t+1}}{P_{t+1}^{\psi}} \frac{P_{t+1}^{\psi}}{P_t^{\psi}}$$

$$= u'(c_t)mc_t Y_t + \rho\beta\mathbb{E}_t x_{1,t+1}\pi_{t+1}^{\psi}$$

$$x_{2,t} = u'(c_t)Y_t + \rho\beta\mathbb{E}_t \frac{X_{2,t+1}}{P_t^{\psi-1}} \frac{P_{t+1}^{\psi-1}}{P_{t+1}^{\psi-1}}$$

$$= u'(c_t)Y_t + \rho\beta\mathbb{E}_t \frac{X_{2,t+1}}{P_{t+1}^{\psi-1}} \frac{P_{t+1}^{\psi-1}}{P_t^{\psi-1}}$$

$$= u'(c_t)Y_t + \rho\beta\mathbb{E}_t x_{2,t+1}\pi_{t+1}^{\psi-1}$$

Plugging $x_{1,t}$ and $x_{2,t}$ into P_t^* gives:

$$P_t^* = \theta_1^{\alpha} \frac{\psi}{\psi - 1} \frac{x_{1,t}}{x_{2,t}} \frac{P_t^{\psi}}{P_t^{\psi-1}}$$

$$= \theta_1^{\alpha} \frac{\psi}{\psi - 1} \frac{x_{1,t}}{x_{2,t}} P_t$$

Dividing both sides by P_{t-1}, we can write the above equation all in inflation terms:

$$\frac{P_t^*}{P_{t-1}} = \theta_1^{\alpha} \frac{\psi}{\psi - 1} \frac{x_{1,t}}{x_{2,t}} \frac{P_t}{P_{t-1}}$$

$$\pi_t^* = \theta_1^{\alpha} \frac{\psi}{\psi - 1} \frac{x_{1,t}}{x_{2,t}} \pi_t$$

6.2.7 Summary

$$\chi n_t^{\eta} = c_t^{-\sigma} w_t$$

$$c_t^{-\sigma} = \beta\mathbb{E}_t c_{t+1}^{-\sigma}(1 + i_{t+1})\frac{1}{\pi_{t+1}}$$

$$\upsilon\left(\frac{M_t}{P_t}\right)^{-\zeta} = c_t^{-\sigma} - \beta\mathbb{E}_t c_{t+1}^{-\sigma}\frac{1}{\pi_{t+1}}$$

$$r_{t+1} + 1 - \delta = \frac{1 + i_{t+1}}{\pi_{t+1}}$$

$$Y_t = c_t + I_t$$

$$k_{t+1} = I_t + (1 - \delta)k_t$$

$$Y_t = \frac{A_t k_t^{\alpha} n_t^{1-\alpha}}{s_t}$$

$$mc_t = A_t^{-1} \frac{w_t^{1-\alpha} r_t^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}$$

$$s_t = (1-\rho)\left[\gamma + (1-\gamma)\left(\frac{\theta_2}{\theta_1}\right)^{\alpha}\right] \pi_t^{*-\psi} \pi_t^{\psi} + \rho \pi_t^{\psi} s_{t-1}$$

$$\pi_t^{1-\psi} = (1-\rho)\left[\gamma + (1-\gamma)\left(\frac{\theta_2}{\theta_1}\right)^{\alpha}\right]^{1-\psi} \pi_t^{*\,1-\psi} + \rho$$

$$\pi_t^* = \theta_1^{\alpha} \frac{\psi}{\psi - 1} \frac{x_{1,t}}{x_{2,t}} \pi_t$$

$$x_{1,t} = c_t^{-\sigma} mc_t Y_t + \rho\beta \mathbb{E}_t x_{1,t+1} \pi_{t+1}^{\psi}$$

$$x_{2,t} = c_t^{-\sigma} Y_t + \rho\beta \mathbb{E}_t x_{2,t+1} \pi_{t+1}^{\psi-1}$$

$$\ln m_t = (1-\rho_m)\pi - \pi_t + \rho_m \Delta \ln m_{t-1} + \rho_m \pi_{t-1} + \varepsilon_{m,t}$$

$$lnA_t = \rho_a \ln A_{t-1} + \epsilon_{a,t}$$

6.2.8 Dynare Codes

```
var c //consumption
    i //interest rate
    r //gross interest rate
    pi //inflation
    n  //labour
    w  //wage
    m //money
    mc //maginal cost
    z //technology
    y //output
    k //capital
    inv //investment
    s  //price dispersion
    x1 //auxillary variable 1
    x2 //auxillary variable 2
    dm //money growth
    pis; //adjusted inflation

varexo epsz //technology shock
       epsm; //money shock

parameters sigma //consumption elasticity
           chi   //labour parameter
           beta  //discount factor
           eta   //labour elasticity
           nu    //money elasticity
           delta //depreciation rate
           alpha //capital-labour ratio
           rho   //probability of keeping price
```

```
                psi   //elasticity of substitution in production
                pia   //inflation targeting
                theta1 //SOEs real interest rate parameter
                theta2 //POEs real interest rate parameter
                gamma  //ratio of SOEs towards the overall enterprises
                rhom  //money parameter
                rhoz;  //technology paramater

      sigma=0.35;
      chi=5;
      beta=0.995;
      eta=1.5;
      nu=0.02;
      delta=0.15;
      alpha=0.2;
      rho=0.75;
      psi=3;
      pia=1.02;
      rhoz=0.95;
      rhom=0.95;
      theta1=0.9;
      theta2=1.1;
      gamma=0;


      model;
      //Household FOC
      chi*n^eta=c^(-sigma)*w;
      c^(-sigma)=beta*c(+1)^(-sigma)*(1+i(+1))/(pi(+1));
      nu*m^(-1)=c^(-sigma)-beta*c(+1)^(-sigma)/pi(+1);
      //Firms FOC
      y=c+inv;
      k=inv(-1)+(1-delta)*k(-1);
      y=exp(z)*k(-1)^alpha*n^(1-alpha)/s;
      s=(1-rho)*(gamma+(1-gamma)*((theta1/theta2)^alpha))*pis^(-
psi)*pi^psi+pi^psi*rho*s(-1);
      mc=alpha^(-alpha)*(1-alpha)^(alpha-1)*exp(z)^(-1)*r^(alpha)*w^(1-
alpha);
      k=gamma*(1/theta1*alpha/(1-alpha)*w/r)^(1-alpha)*y/exp(z)+(1-
gamma)*(1/theta2*alpha/(1-alpha)*w/r)^(1-alpha)*y/exp(z);
      1+r-delta=(1+i)/pi;
      //auxillary
      pi^(1-psi)=(1-rho)*(gamma+(1-gamma)*((theta2/theta1)^alpha))^(1-
psi)*pis^(1-psi)+rho;
      pis=theta1^alpha*psi/(psi-1)*x1/x2*pi;
      x1=c^(-sigma)*mc*y+rho*beta*x1(+1)*(pi(+1)^psi);
      x2=c^(-sigma)*y+rho*beta*x2(+1)*(pi(+1))^(psi-1);
      dm=ln(m)-ln(m(-1));
      dm=(1-rhom)*pia-pi+rhom*dm(-1)+rhom*pi(-1)+epsm;
      z=rhoz*z(-1)+s*epsz;
      end;

      initval;
      c   =       0.233923;
      i   =       0.0251256;
      r   =       0.155025;
      pi  =       1.02;
      n   =       0.304125;
      w   =       0.504342;
      m   =       0.490756;
      mc  =       0.657026;
      y   =       0.267025;
      k   =       0.22068;
      inv =       0.0331021;
```

```
s   =       1.06818;
x1  =       1.40198;
x2  =       1.98565;
pis =       1.05774;
end;

steady;
shocks;
var epsz; stderr 0.01;
var epsm; stderr 0.01;
end;

stoch_simul(order=1, periods=300);
```

7 Appendix B-Tables and Figures

TABLE 1-CALIBRATION OF SIMPLE RBC MODEL

Parameter	Calibration
β	0.98
α	0.33
δ	0.05
χ	1.5
ξ	1
ζ	1
σ	1
s	0.007
ν	0.5
π^*	0.02
ρ	0.95
ρ_m	0.95

TABLE 2-STEADY STATE COMPARISON OF SIMPLE RBC MODEL

Variable	Subsidy $\gamma = -0.2$	Tax $\gamma = 0.2$
Consumption c	2.38%	-2.26%
Labour n	1.33%	-1.15%
Wage w	2.98%	-2.74%
Output y	4.35%	-3.86%
Capital Stock k	10.78%	-9.13%
Real Effective r	-20%	20%
Investment I	10.78%	-9.12%
Money Balance m	2.39%	-2.25%
Nominal Effective i	-20%	20%

TABLE 3-CALIBRATION OF NK MODEL

Parameters	Values
σ	0.35
χ	5
β	0.995
ζ	1
η	1.5
ν	0.02
δ	0.15
α	0.2
ρ	0.55
ψ	3
π	1.02
ρ_z	0.95
ρ_m	0.95

TABLE 4-STEADY STATE COMPARISON OF NK MODEL

Variable	Benchmark Model	$\theta_1 = 0.7, \theta_2 = 1$ Partial Distortion		$\theta_1 = 0.7, \theta_2 = 1.3$ Full Distortion	
	$\theta_1 = \theta_2 = 1$	$\gamma = 0.2$	$\gamma = 0.6$	$\gamma = 0.2$	$\gamma = 0.6$
	(1)	(2)	(3)	(4)	(5)
Consumption c	0.2527	0.2282	0.2484	0.1996	0.2322
Labour n	0.3020	0.3128	0.3140	0.3115	0.3132
Wage w	0.5127	0.5216	0.5404	0.4945	0.5256
Real money balance m	0.5042	0.4865	0.5012	0.4642	0.4895
Marginal cost	0.6657	0.6750	0.6944	0.6467	0.6791
Output y	0.2900	0.2651	0.2961	0.2254	0.2724
Capital k	0.2491	0.2461	0.3178	0.1720	0.2679
Price dispersion s	1.0018	1.1247	1.0632	1.2272	1.1143

TABLE 5-STEADY STATE CHANGE COMPARISON OF NK MODEL

Variable	Benchmark Model	$\theta_1 = 0.7, \theta_2 = 1$ Partial Distortion		$\theta_1 = 0.7, \theta_2 = 1.3$ Full Distortion	
	$\theta_1 = \theta_2 = 1$	$\gamma = 0.2$	$\gamma = 0.6$	$\gamma = 0.2$	$\gamma = 0.6$
	(1)	(2)	(3)	(4)	(5)
Consumption c	0.2527	-9.69%	-1.70%	-21.02%	-8.10%
Labour n	0.3020	3.60%	3.99%	3.14%	3.70%
Wage w	0.5127	1.75%	5.41%	-3.55%	2.53%
Real money balance m	0.5042	-3.51%	-0.60%	-7.93%	-2.91%
Marginal cost	0.6657	1.40%	4.31%	-2.85%	2.02%
Output y	0.2900	-8.60%	2.07%	-22.30%	-6.09%
Capital k	0.2491	-1.20%	27.56%	-30.96%	7.53%
Price dispersion s	1.0018	12.27%	6.13%	22.50%	11.23%

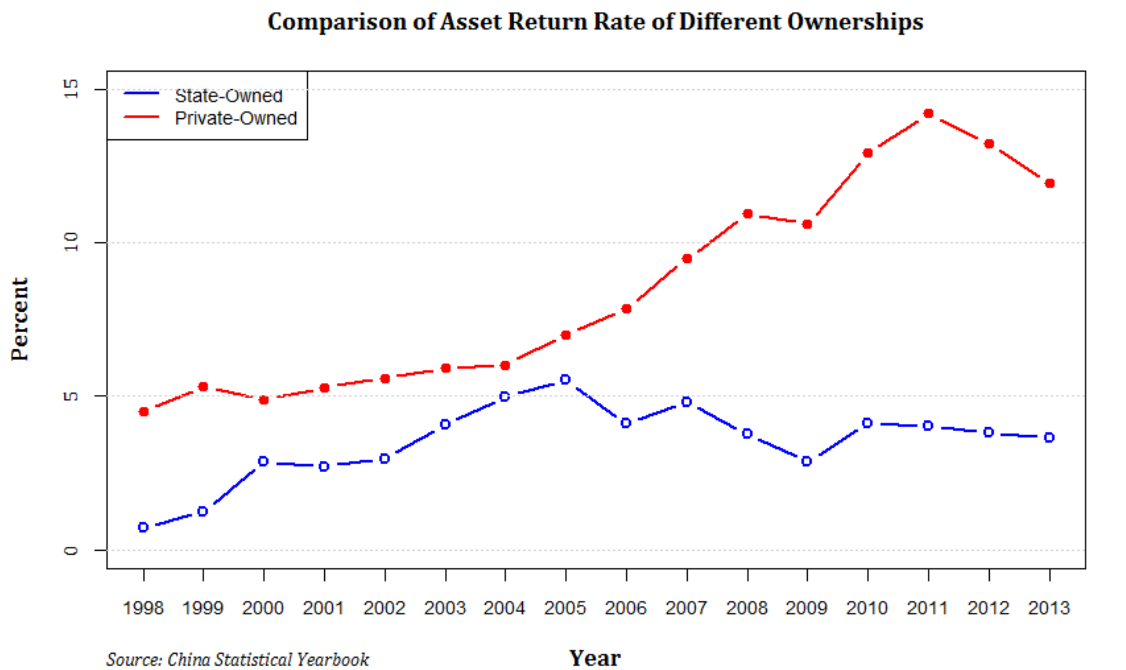

FIGURE 1-COMPARISON OF ASSET RETURN RATE OF DIFFERENT OWNERSHIPS

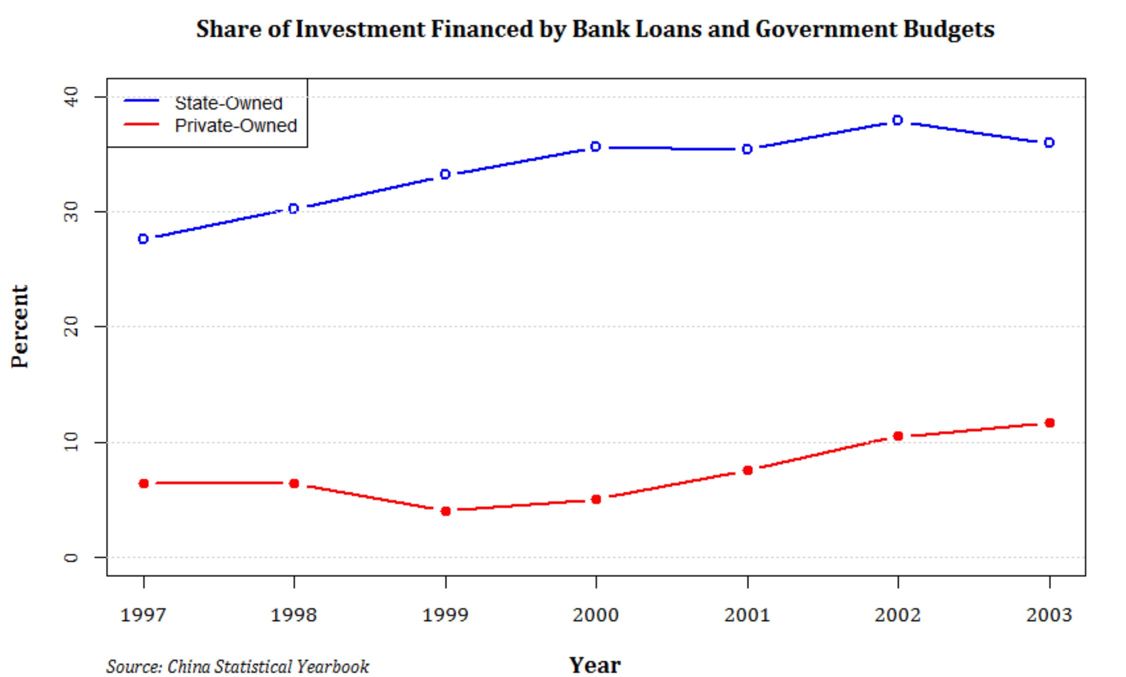

FIGURE 2-COMPARISON OF GOVERNMENT FINANCE SUPPORT OF DIFFERENT OWNERSHIPS

FIGURE 3-CHINA GDP GROWTH

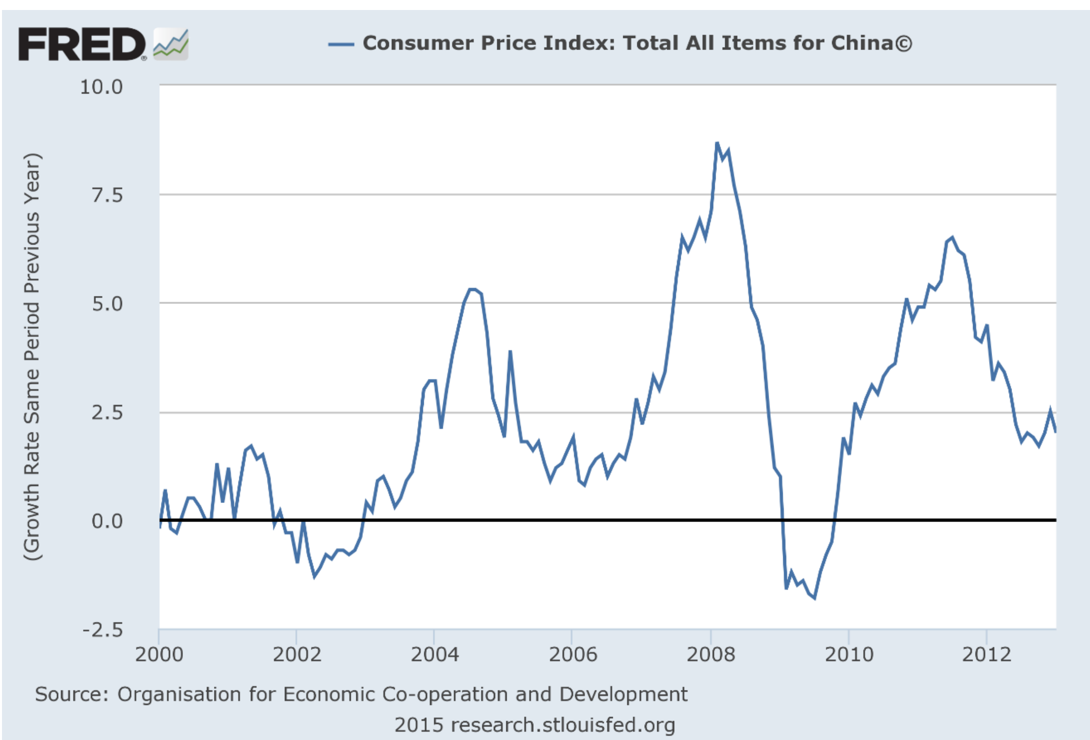

FIGURE 4-CHINA CONSUMER PRICE INDEX

FIGURE 5-CHINA MONEY GROWTH

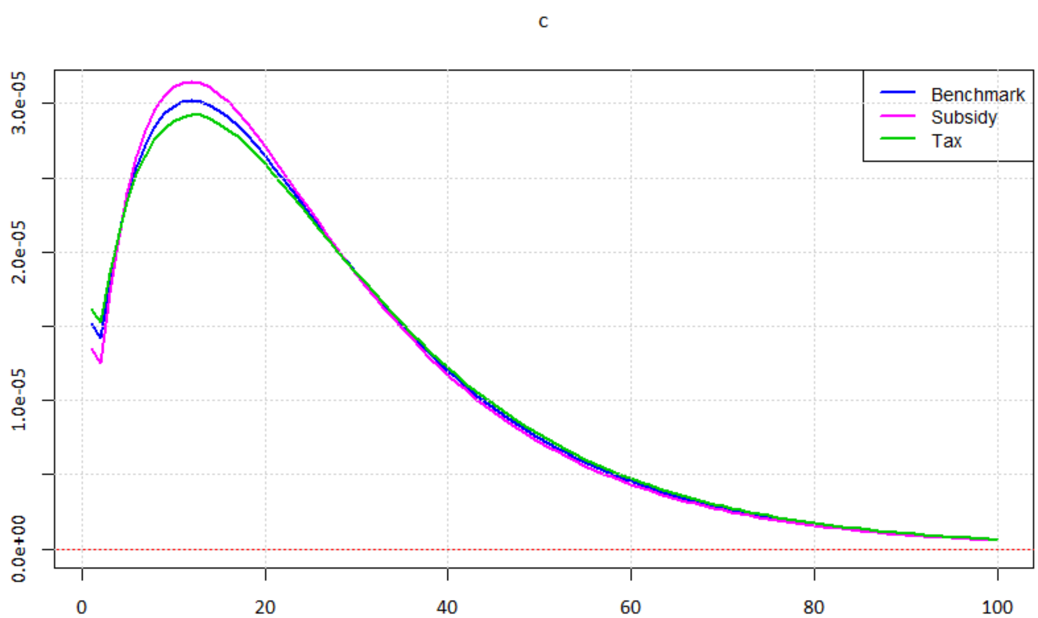

FIGURE 6-IMPULSE RESPONSE OF CONSUMPTION TO A 0.01 TECHNOLOGY SHOCK IN RBC MODEL

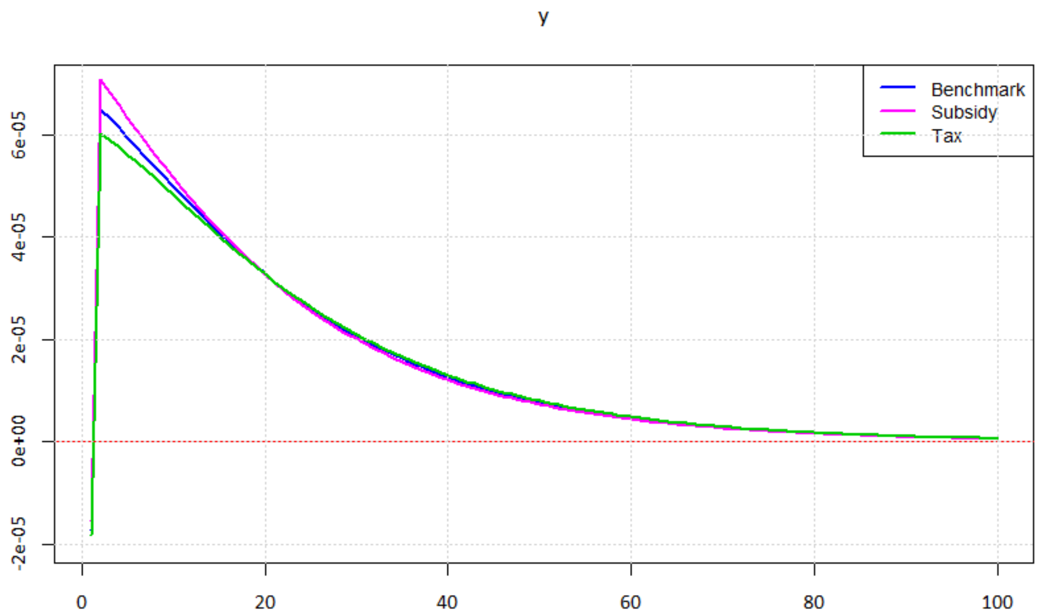

FIGURE 7- IMPULSE RESPONSE OF OUTPUT TO A 0.01 TECHNOLOGY SHOCK IN RBC MODEL

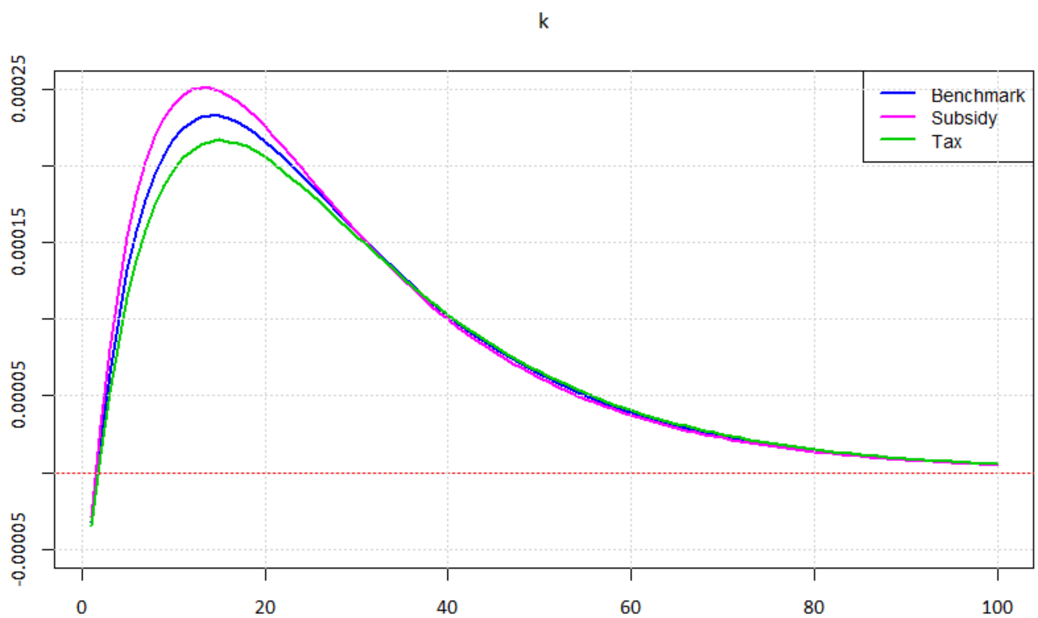

FIGURE 8-IMPULSE RESPONSE OF CAPITAL TO A 0.01 TECHNOLOGY SHOCK IN RBC MODEL

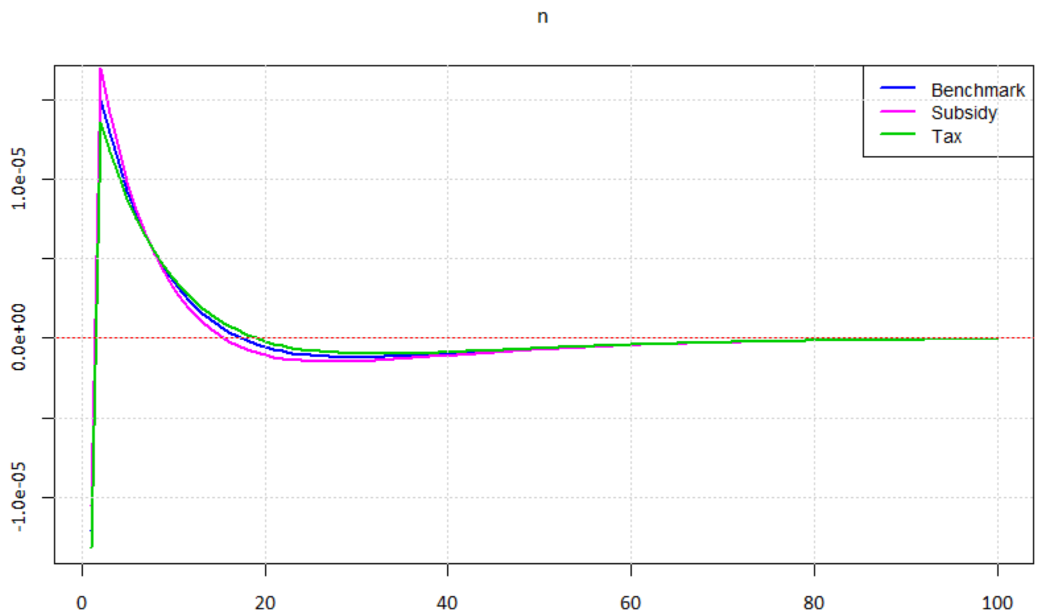

FIGURE 9-IMPULSE RESPONSE OF LABOUR TO A 0.01 TECHNOLOGY SHOCK IN RBC MODEL

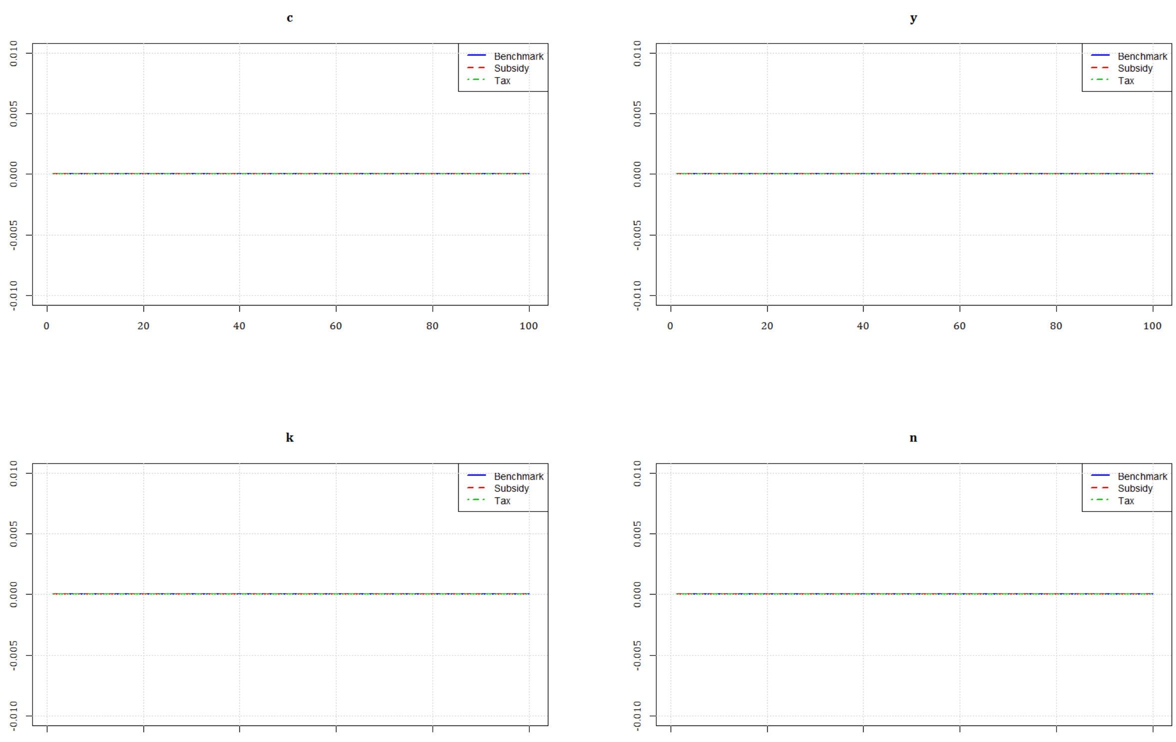

FIGURE 10-IMPULSE RESPONSE OF VARIOUS REAL VARIABLES TO A 0.01 MONEY SHOCK IN RBC MODEL

FIGURE 11-CHINA MONEY STOCK VELOCITY

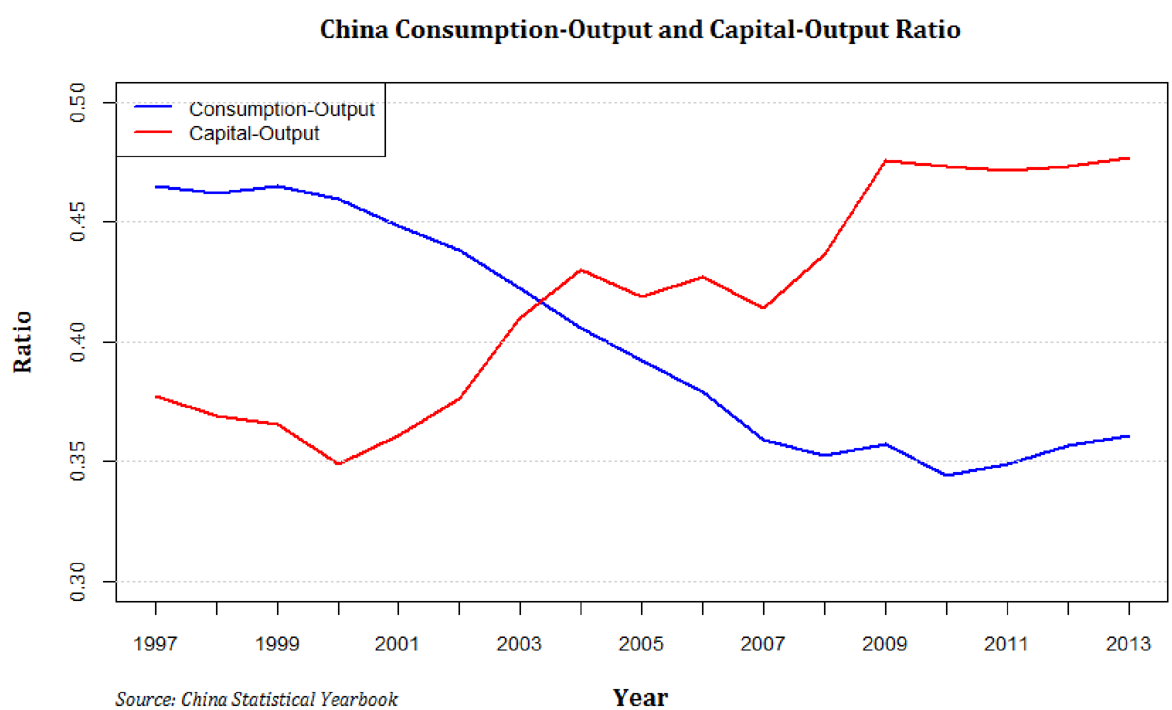

FIGURE 12-CHINA CONSUMPTION-OUTPUT AND CAPITAL-OUTPUT RATIO

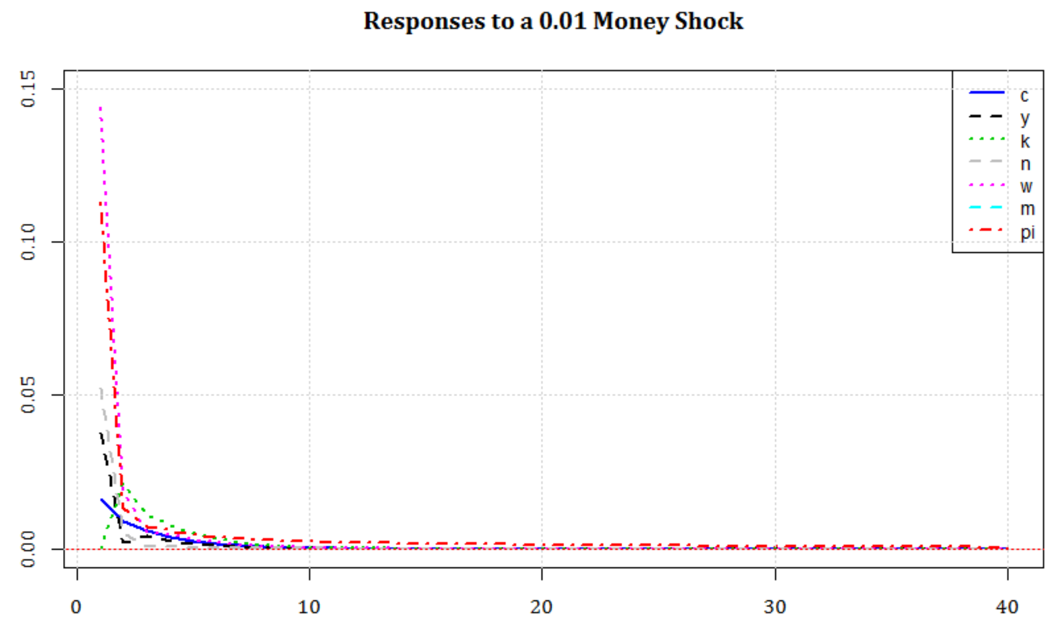

FIGURE 13-IMPULSE RESPONSE OF MAJOR VARIABLES IN A BENCHMARK MODEL TO A 0.01 MONEY SHOCK

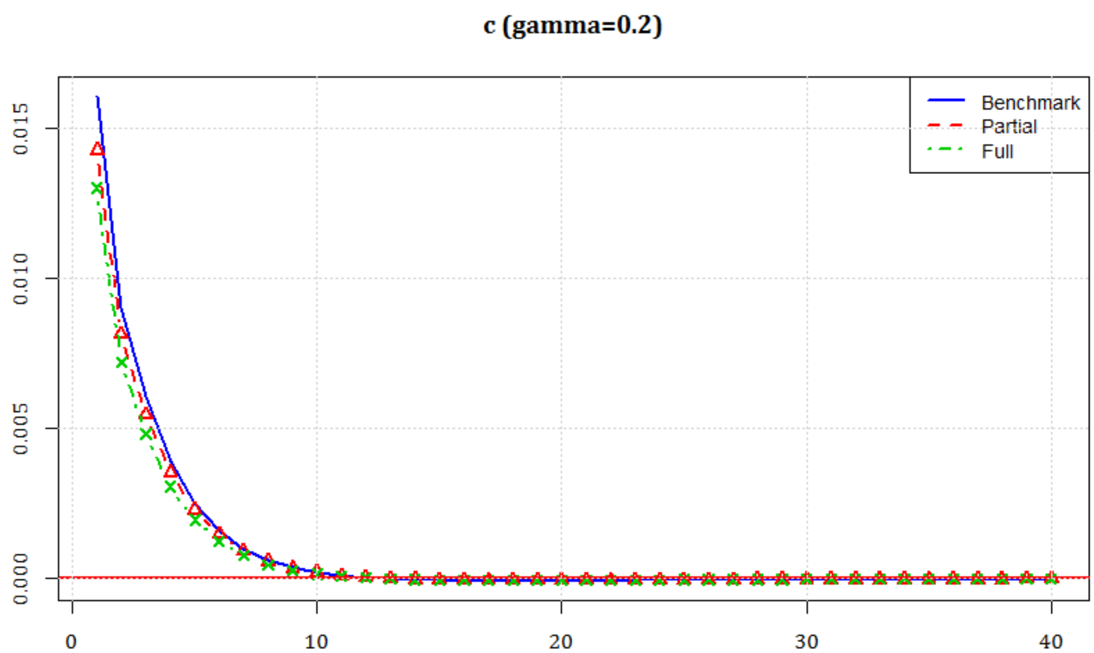

FIGURE 14-IMPULSE RESPONSE OF CONSUMPTION IN A POE-FAVOURED ECONOMY TO A 0.01 MONEY SHOCK

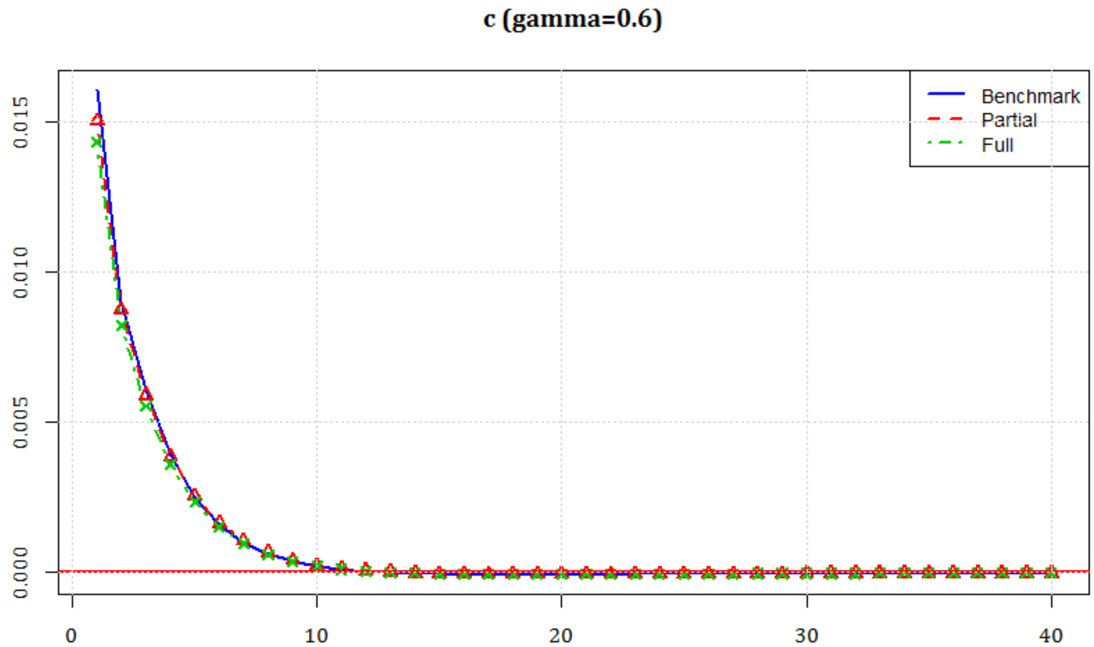

FIGURE 15-IMPULSE RESPONSE OF CONSUMPTION IN A SOE-FAVOURED ECONOMY TO A 0.01 MONEY SHOCK

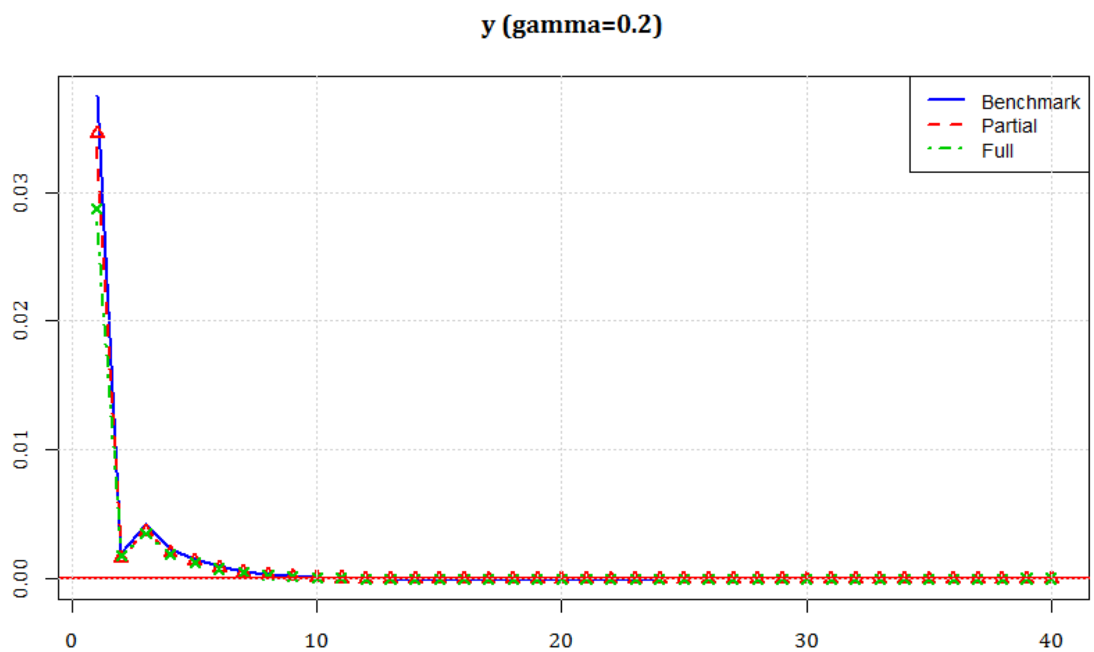

FIGURE 16- IMPULSE RESPONSE OF OUTPUT IN A POE-FAVOURED ECONOMY TO A 0.01 MONEY SHOCK

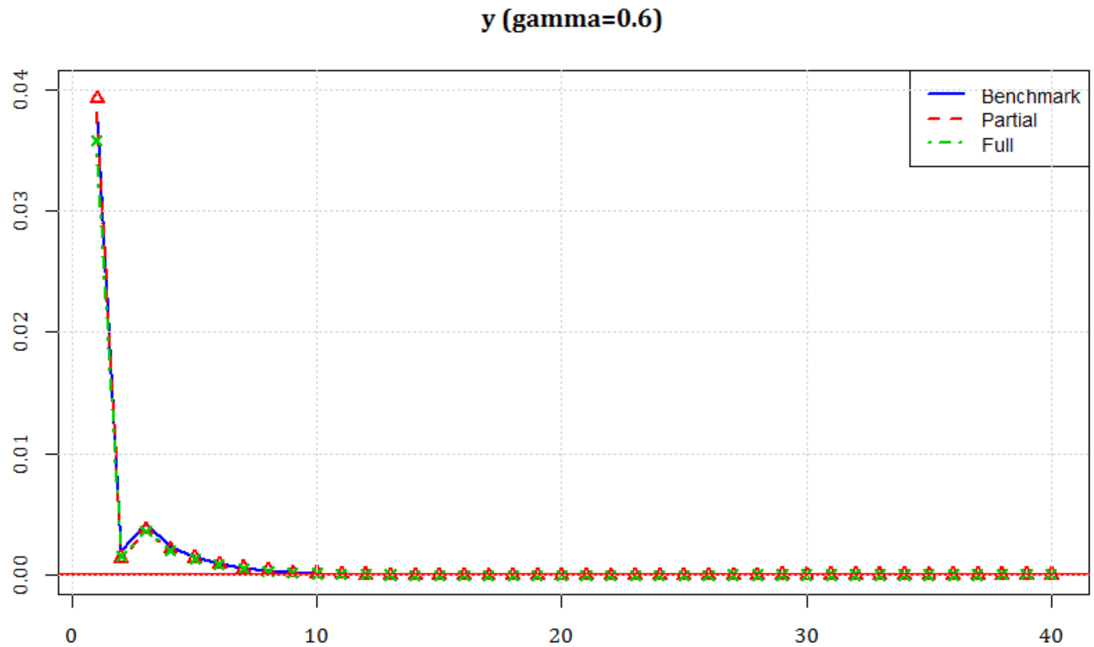

FIGURE 17- IMPULSE RESPONSE OF OUTPUT IN A SOE-FAVOURED ECONOMY TO A 0.01 MONEY

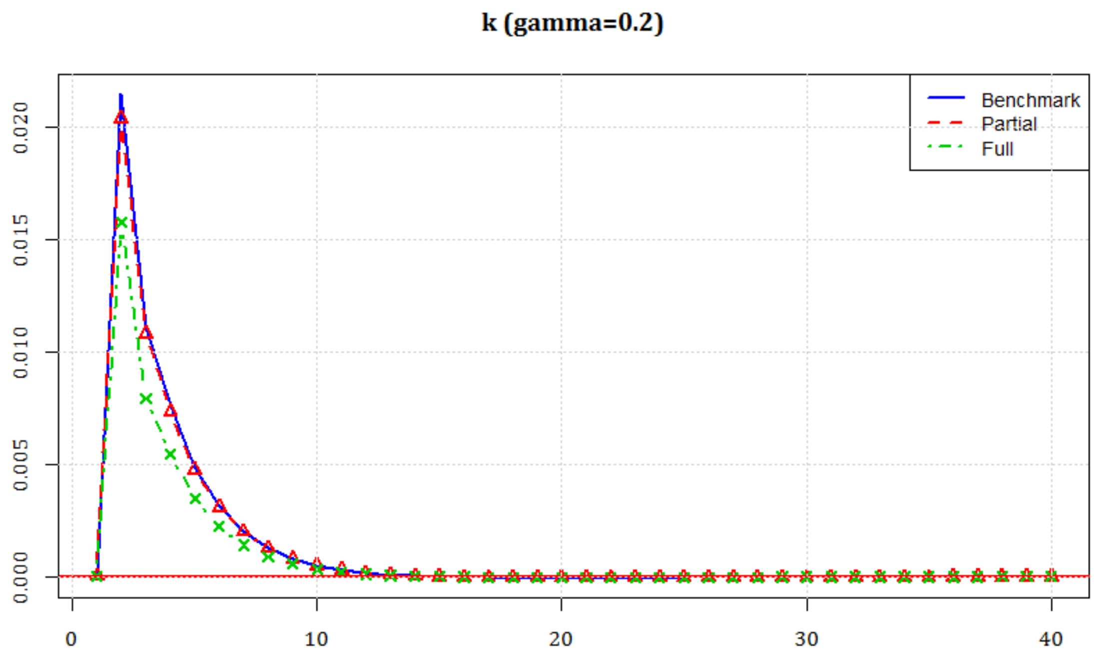

FIGURE 18- IMPULSE RESPONSE OF CAPITAL IN A POE-FAVOURED ECONOMY TO A 0.01 MONEY SHOCK

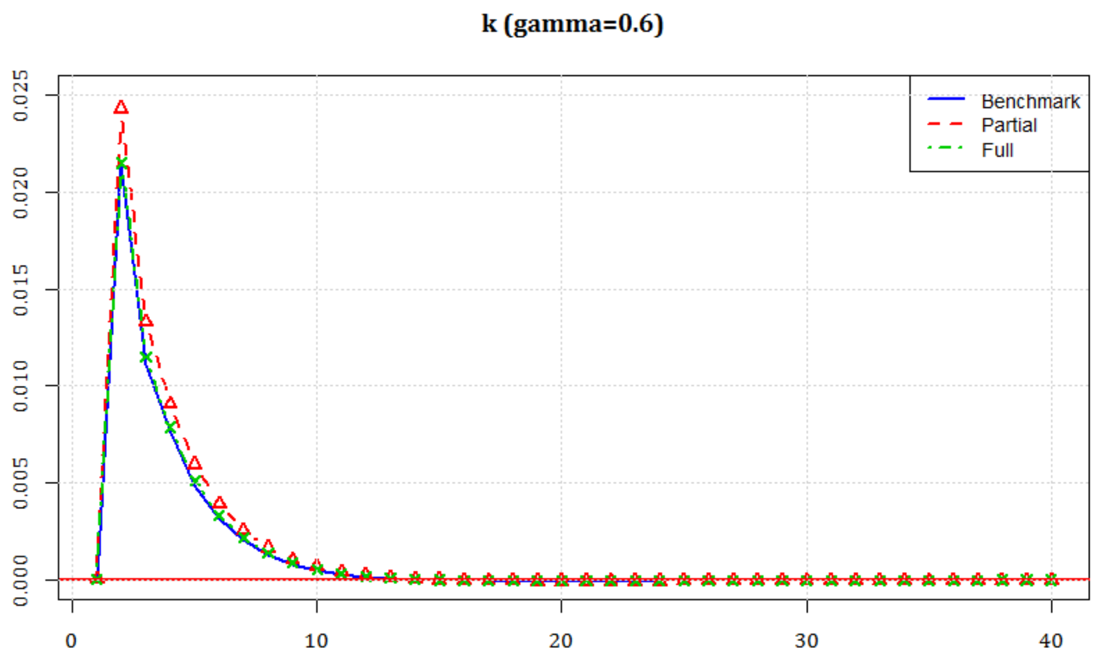

FIGURE 19- IMPULSE RESPONSE OF CAPITAL IN A SOE-FAVOURED ECONOMY TO A 0.01 MONEY SHOCK

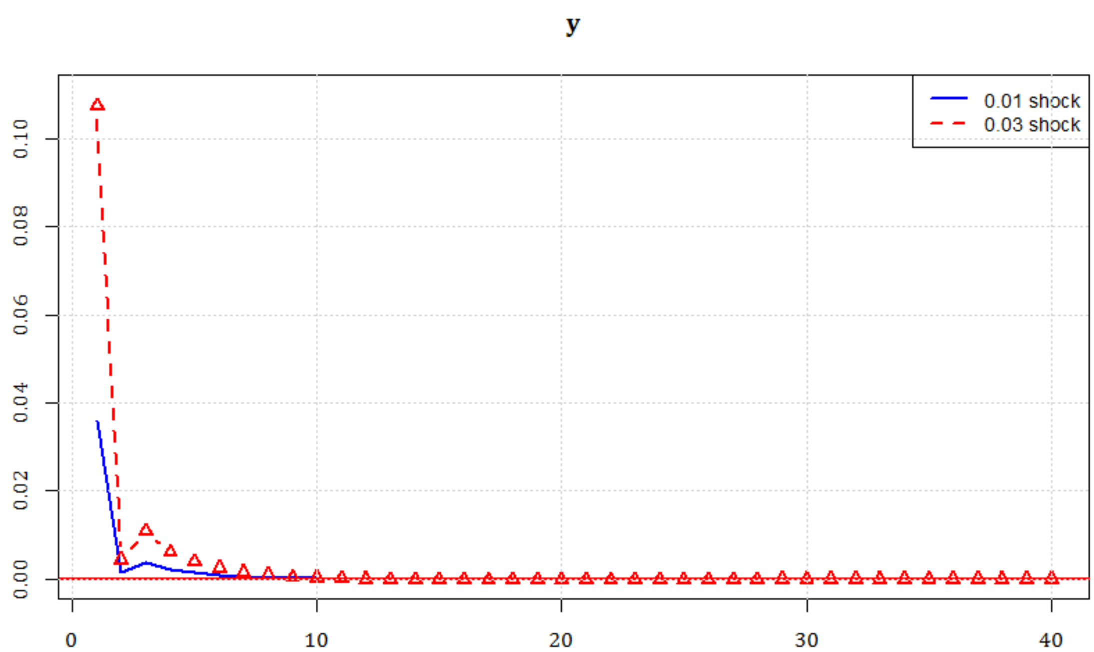

FIGURE 20- IMPULSE RESPONSE OF OUTPUT IN A FULL DISTORTION SOE-FAVOURED ECONOMY TO A 0.01 AND 0.03 MONEY SHOCK

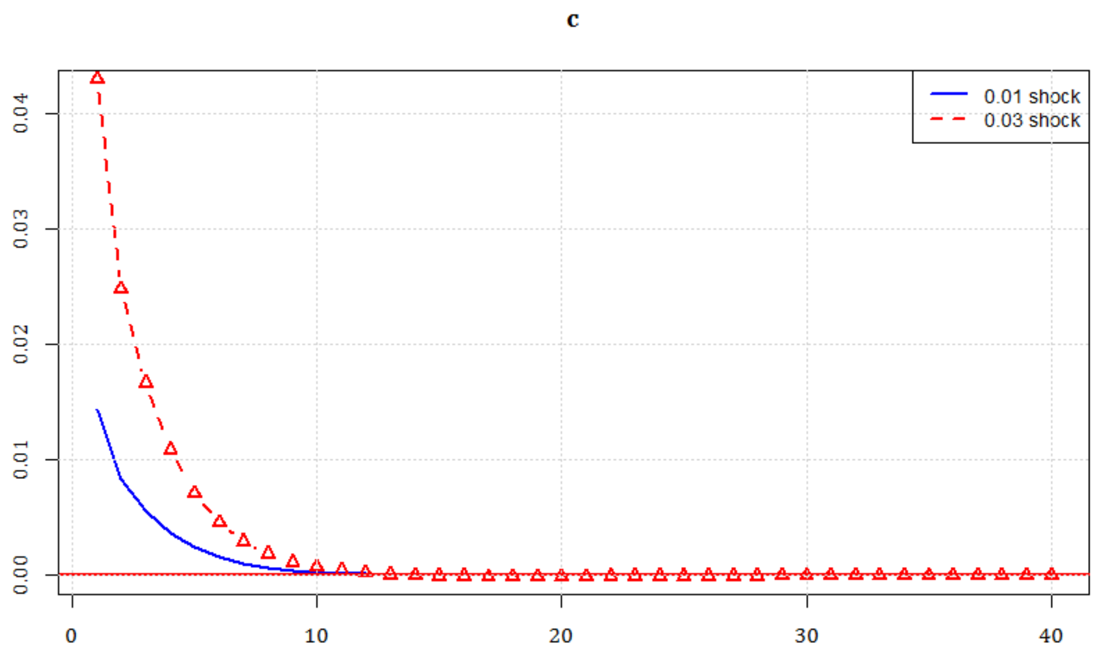

FIGURE 21- IMPULSE RESPONSE OF CONSUMPTION IN A FULL DISTORTION SOE-FAVOURED ECONOMY TO A 0.01 AND 0.03 MONEY SHOCK

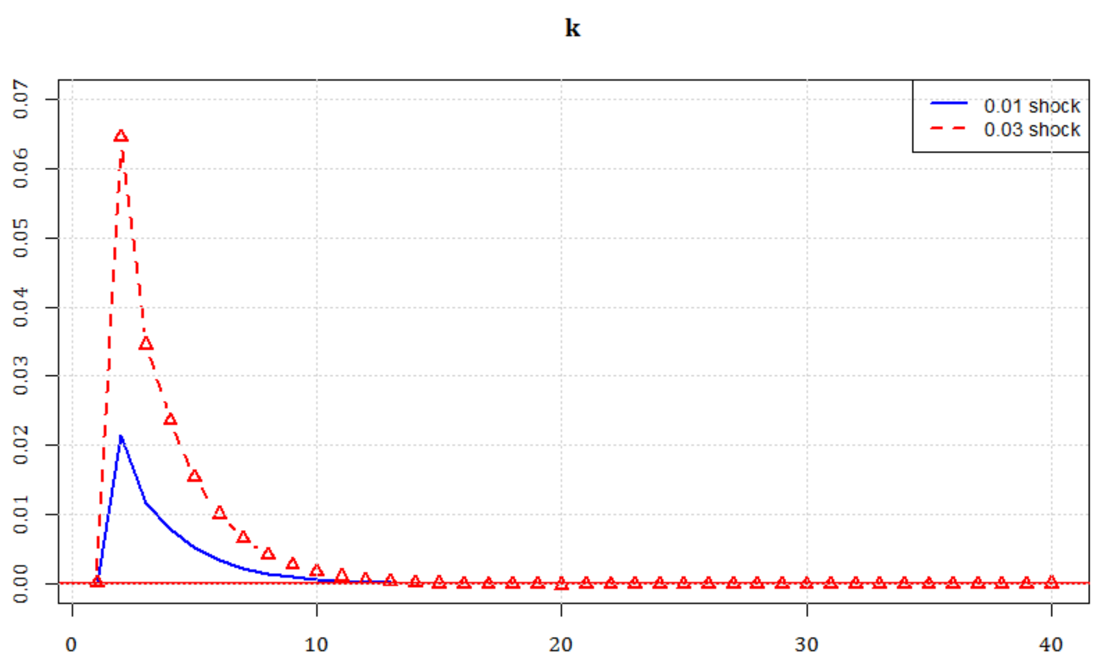

FIGURE 22- IMPULSE RESPONSE OF CAPITAL IN A FULL DISTORTION SOE-FAVOURED ECONOMY TO A 0.01 AND 0.03 MONEY SHOCK

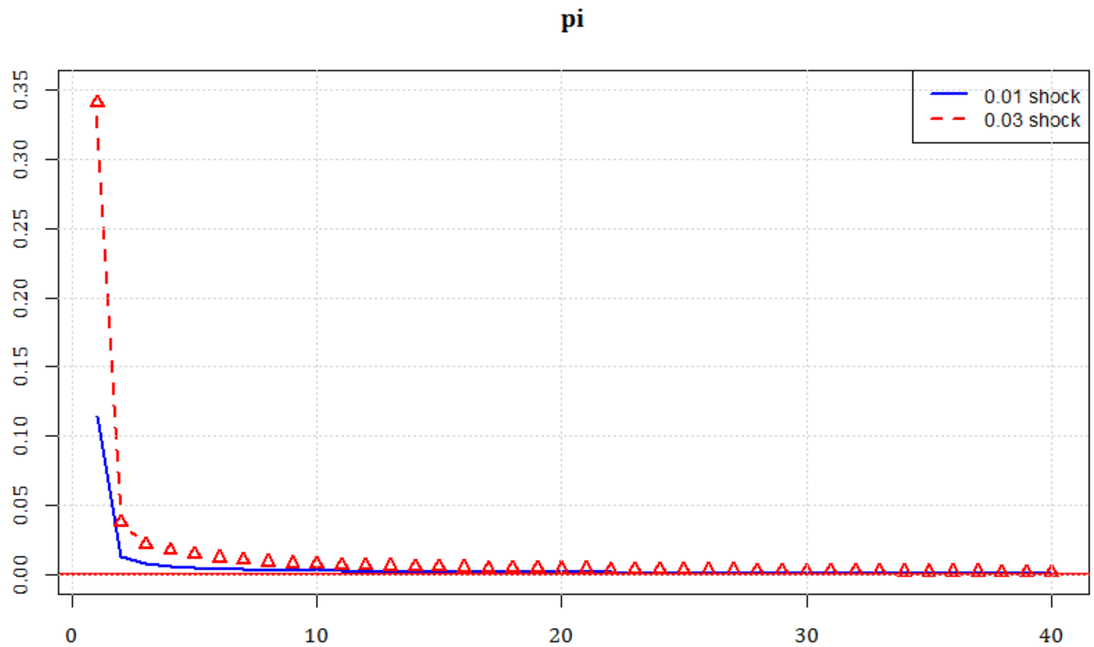

FIGURE 23- IMPULSE RESPONSE OF INFLATION IN A FULL DISTORTION SOE-FAVOURED ECONOMY TO A 0.01 AND 0.03 MONEY SHOCK

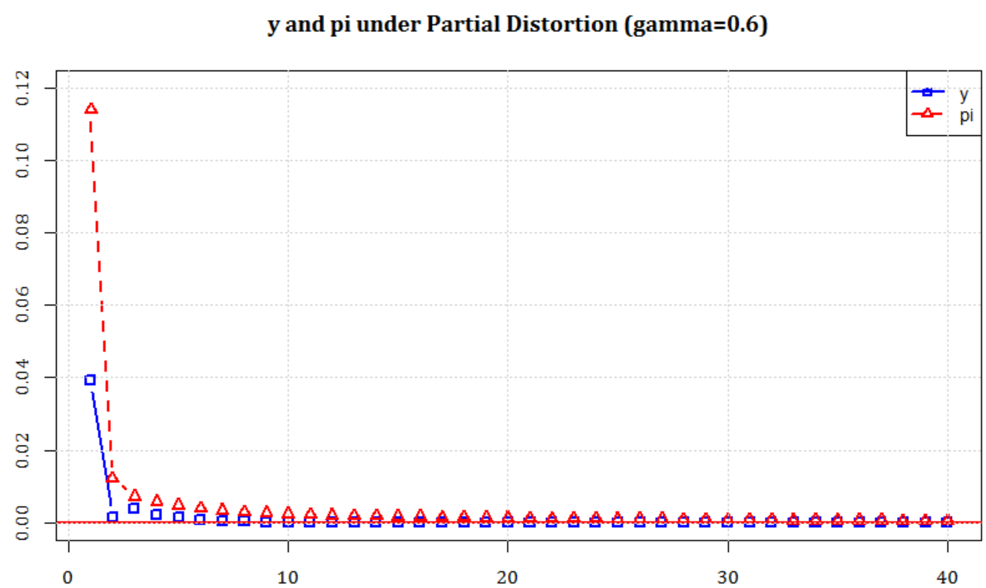

FIGURE 24- IMPULSE RESPONSE OF OUTPUT AND INFLATION IN A FULL DISTORTION SOE-FAVOURED ECONOMY TO A 0.01 AND 0.03 MONEY SHOCK

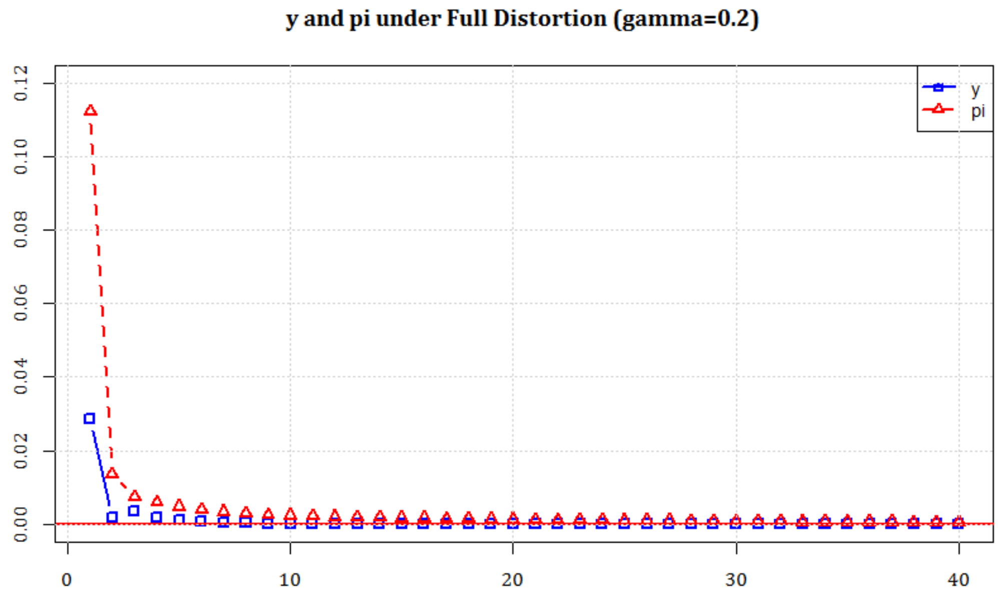

FIGURE 25- IMPULSE RESPONSE OF OUTPUT AND INFLATION IN A FULL DISTORTION POE-FAVOURED ECONOMY TO A 0.01 AND 0.03 MONEY SHOCK

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