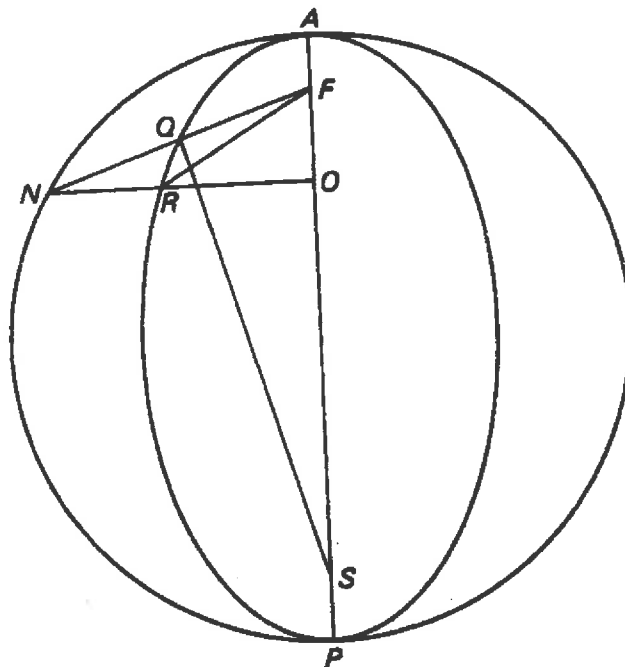


Boulliau's Cone-1645



Boulliau's 1657 Method for Obtaining
True Anomaly from Mean Anomaly

ISMAELIS
BVLLI ALDI
ASTRONOMIÆ

PHILOLAICÆ FVNDAMENTA
clarius explicata, & asserta.

ADVERSVS

*Clarissimi Viri SETHI WARDI Oxoniensis Professoris
impugnationem.*



PARISIIS,
Apud SEBASTIANVM CRAMOISY, Regis & Reginae
Architypographi,

ET

GABRIELEM CRAMOISY, viâ Iacobæâ, sub Ciconiis.

M. DC. LVII.
CVM PRIVILEGIO REGIS.

HARMONICON COELESTE:

OR,
The Cœlestial Harmony of the
VISIBLE WORLD:
CONTEINING,
An absolute and entire Piece of
ASTRONOMIE.

WHEREIN
Is succinctly handled the *Trigonometricall* Part,
generally Propounded, and particularly Applied in all Questions
tending to the *DIURNAL MOTION*.

Especially respecting, and truly subservient to the
main Doctrine of the Second Motions of the *Luminaries* and the
other *Planets*: Together with their Affections
as *ECLIPSES*, &c.

Grounded upon the most *Rationall Hypothesis* yet
Constituted, and compared with the best Observations that are
Extant, especially those of *TYCHO BRAHE*, and other
more Modern *OBSERVATORS*.

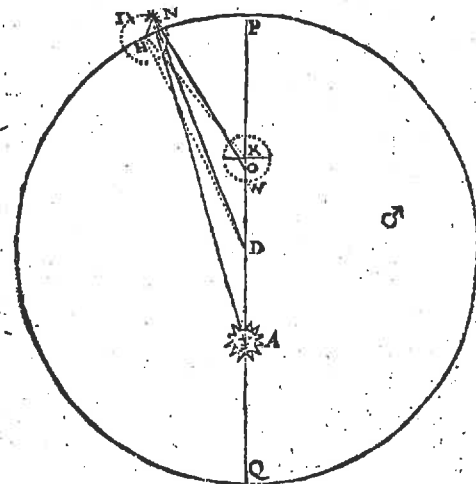
Fitted to the *Meridian* of the most Famous and Ancient *Metropolis*
LONDON, and principally intended for our English Nation, and
commended as useful to all *Scholars, Astronomers, Astrologers,*
Divines, Physicians, Historiographers, Politicians,
and *Poets*, &c.

By *VINCENT WING, Philomathemat.*

Quam respicio Caelos tuos, opus digitorum tuorum; Lunam & Stellam quam stansisti: quid est mortalis, quod memor sis ejus? aut filius hominis, quod visites eum? Psal 8.4,5.

LONDON:
Printed by *ROBERT LEYBOURN*, for the
Company of *STATIONERS*, 1651.

First Figure of Mars.



In this Diagram I number the Anomalie of δ from P to H $64^{\circ} 7' 40''$, whose Complement $173^{\circ} 52' 20''$, is the angle D X H, which given with the Side D H 152040, and the Side D X 14105, the angle D H X will be $34^{\circ} 37''$.

Side D H 152040,	5,181958
Sine of the angle D X H $173^{\circ} 52' 20''$;	9,028352
Side D X 14105,	4,149373
Sine of the angle D H X $34^{\circ} 37''$.	7,995767
Simple Anomalie P X H,	$64^{\circ} 7' 40''$
Angle D H X Substr.	34 37
Anomalie æquated P D H.	5 33 37
Motion of the Epicycle I H N.	11 7 14

In the Triangle D H N.

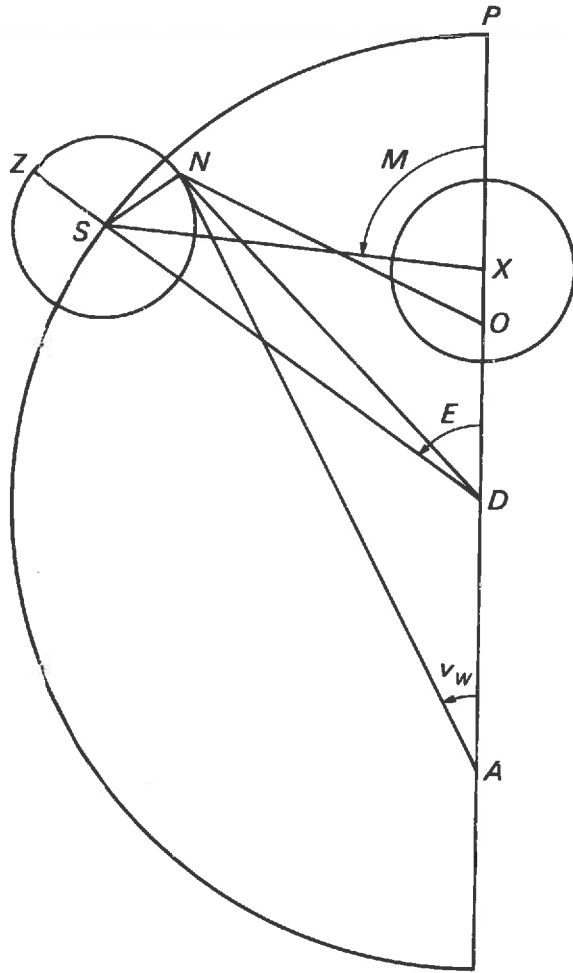
Summe of D H and D N 152500,	5,183270
Difference 151580;	5,180642
Tangent of $5^{\circ} 33' 37''$,	8,988343
Tangent of $5^{\circ} 31' 37''$.	8,985715

Aggregate 11 5 14. viz. Angle H N D,
Difference 2 0. viz. Angle H D N,

Side of the angle H N D $11^{\circ} 5' 14''$,	5,283985
Side D H 152040;	5,181958
Sine of the angle D H N $168^{\circ} 52' 46''$,	9,285274
Side D N 152492:	5,183247

Anomalie æquated P D H.	5 33 37
Angle H D N Substr.	2 0
Anomalie Co-æquated P D N.	5 31 37

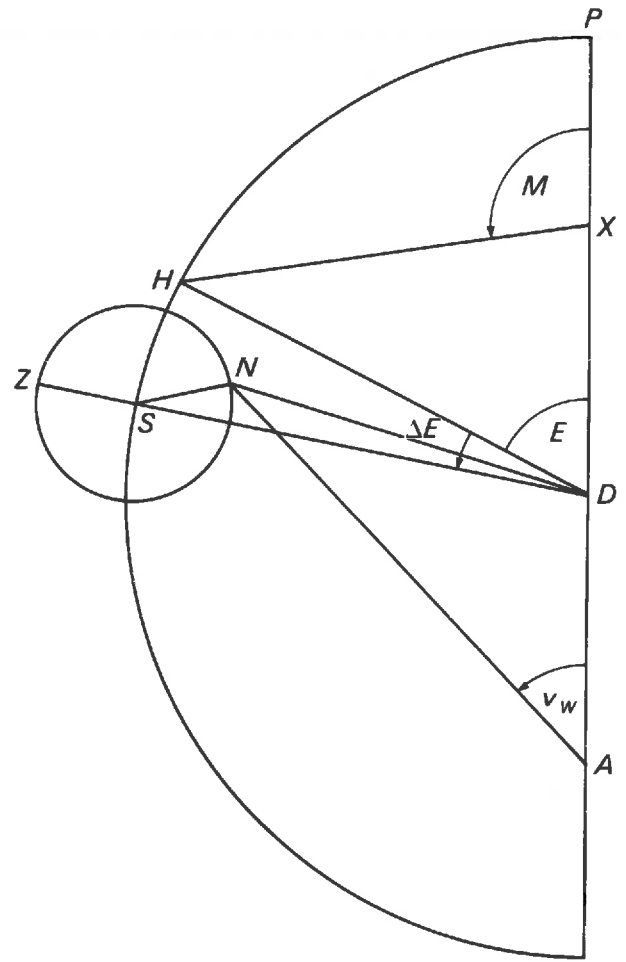
In the former Diagram δ is *Supra-Diacentron*, therefore I number the motion of the Epicycle $11^{\circ} 7' 14''$ in the nether part of the Equant from X to O, then I say,



10.6. Vincent Wing's procedure, in his *Harmonicon coeleste* of 1651, for deriving true anomaly (v_W) from mean anomaly (M).

newly devised by the Authour, wherein is plainly and succinctly delivered . . . how to calculate the Motions of all the Planets Trigonometrically, wherein I much dissent from all other Authours that have treated hereof in other Languages, and have delivered the same more methodically for practice, than any hath done before me

Wing's new procedure is in fact a modification of Boulliau's. In Figure 10.6 the ellipse is produced by an epicycle of radius $\frac{1}{4}e^2$ moving on a circle of radius $1 - \frac{1}{4}e^2$; M is the mean anomaly, and E the "equated anomaly", determined by the relation $\sin(M - E) = e \sin M / (1 - \frac{1}{4}e^2)$. The angle ZSN of epicyclic motion is $2E$. The eccentricity $DX = AD = e$ is varied by subtracting a sinusoidal term $XO = \frac{1}{4}e^2 \sin 2E$, and the total equation of centre is given by $\angle OND + \angle DNA$. The resulting true anomaly v_W can be shown to differ from the Keplerian value by



10.7. Wing's improved procedure, in his *Astronomia instauratione* of 1656 and his *Astronomia Britannica* of 1669, for deriving true anomaly (v_W) from mean anomaly (M).

$$v_K - v_W = \frac{1}{4}e^2 \sin 2M - \frac{1}{4}e^2 \sin M \sin 2M - 2e^3 \sin M + \frac{5}{3}e^3 \sin^3 M - \frac{1}{2}e^3 \sin^4 M.$$

In the case of Mars, this error rises to 5' in the second quadrant of anomaly.

By the time Wing published his *Astronomia instauratione* in 1656, he had detected the error in this theory by comparing it with acronychal observations of Mars. Moreover, he had found a way of eliminating most of this error; it consisted in adding to the angle E a correction term equal to $k \sin 2E$, where k was to be determined empirically. The value of k should be about $\frac{1}{2}e^2$; in the case of Mars, Wing in his calculation takes it to be $14' 55'' \approx \frac{1}{2}e^2 + \frac{2}{3}e^4$. The new theory, which is also that of the *Astronomia Britannica* of 1669, is represented in Figure 10.7. Once again the radius DS of the deferent is $\frac{1}{2}(1 + \sqrt{1 - e^2}) \approx 1 - \frac{1}{4}e^2 - \frac{1}{16}e^4$, so that the radius SN of the epicycle is $\frac{1}{4}e^2 + \frac{1}{16}e^4$, while, with $\angle PDH = E$, $\angle HDS = (\frac{1}{2}e^2 + \frac{2}{3}e^4) \sin 2E$; and the

ASTRONOMIA BRITANNICA:

IN QUA

Per **Novam, Concinnioremq;** Methodum, hi
quinq; Tractatus traduntur.

I. LOGISTICA ASTRONOMICA, quæ continet Doctrinam Fractionum Astronomicarum integram, tum in Numeris Naturalibus, tum Artificialibus.

II. TRIGONOMETRIA, seu Doctrina Triangulorum, (Analytica & Practica) quæ comprehendit Dimensionem omnium Trigonorum, tam Planorum, quam Sphæricorum, cujus ope, Dimensiones Cœli, Terræ, univèrsiq; Mundi Orbis (modo mirabili) dignoscantur.

III. DOCTRINA SPHÆRICA, quæ exhibet Longitudines, Latitudines, Declinationes, Ascensiones, Ortus, Occasus, Intercapedines, Parallaxefq; singulorum Planetarum ad cujuslibet Sphære positum, & quo pacto Figuræ Cœlestes erigi possint.

IV. THEORIA PLANETARUM, quæ Novâ, accuratâq; Methodo super Hypothesi *Copernicanâ*, veros Motus & Configurationes omnium Planetarum computare docet.

V. TABULÆ NOVÆ ASTRONOMICÆ, ex quibus Singulorum Planetarum Motus, & Luminarium Eclipses, mirâ promptitudine colligantur.

Congruentes cum Observationibus accuratissimis Nobilis
TYCHONIS BRAHÆI.

Cui accessit Observationum Astronomicarum *Synopsis Compendiaria*, ex quâ *Astronomiæ Britannicæ* certitudo affatim elucescit.

Opus exoptatum, non modò Astronomis, Astrologis, sed & Theologis, Historiographis, Nautis, Medicis & Poetis, perutile & jucundum.

Cui additur Postscriptum de *Refractiōe*.

Authore **VINCENTIO WING**, Mathem.

LONDINI,

Typis *Johannis Macock*, Impensis *Georgii Sawbridge*, prostantq; venales apud locum vulgò *Clerkenwel-Green* dictum. 1669.